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THE JOURNAL OF THE
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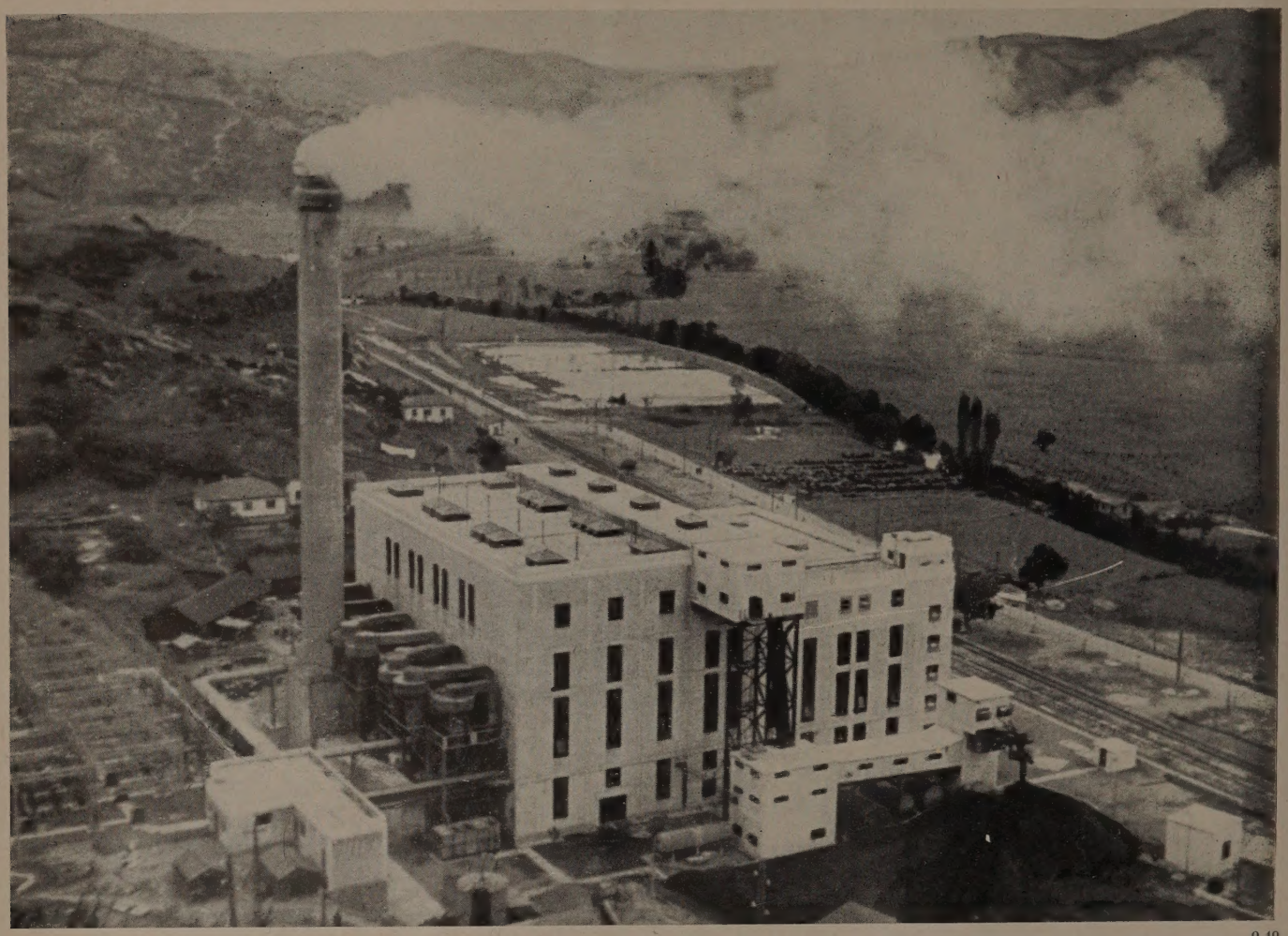

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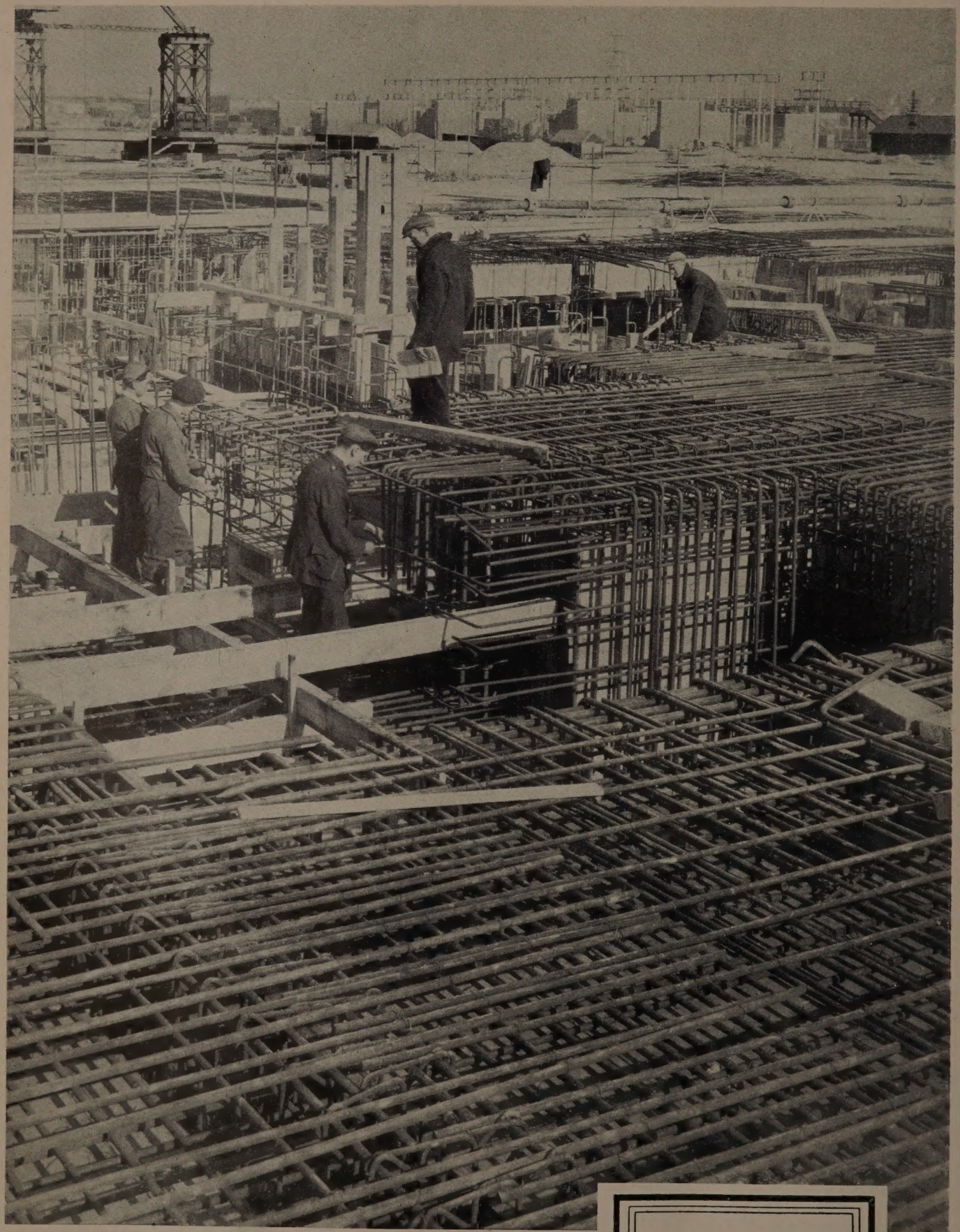

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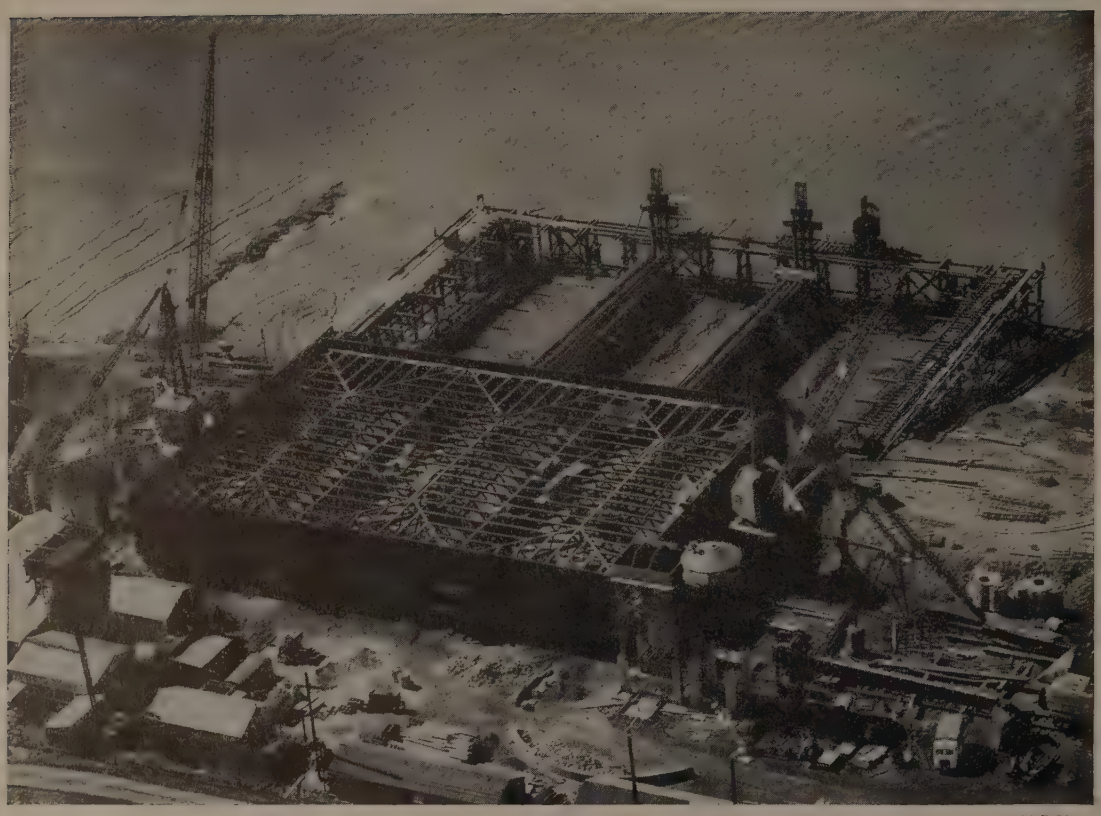

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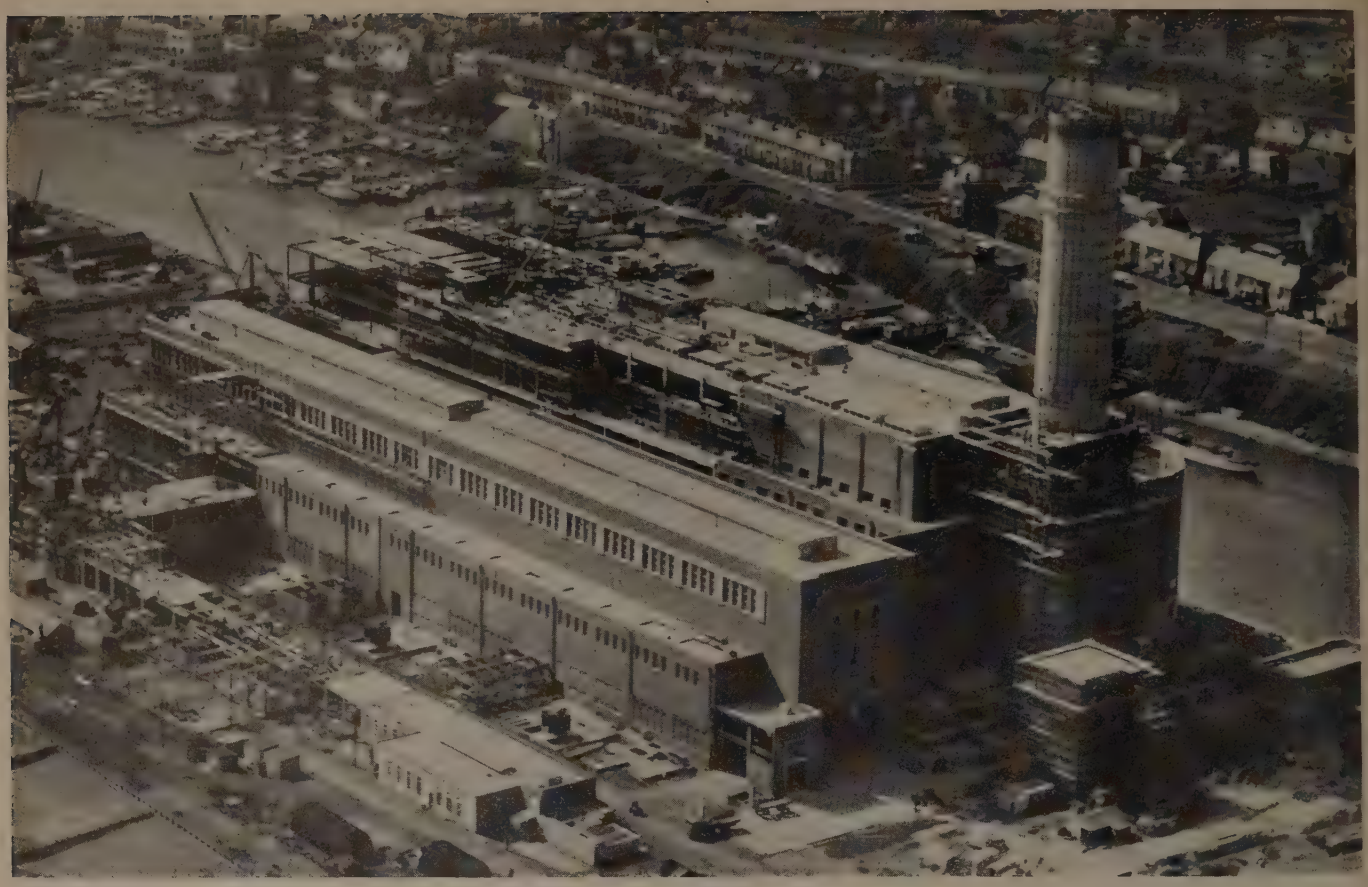

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A Comparison of the Bearing Power of Footings on Dry and Inundated Sand*

By W. Eastwood, B.Eng., Ph.D., A.M.I.Struct.E.

Synopsis

This investigation has shown that for narrow footings on inundated sand the ultimate bearing pressures are between 80 and 90 per cent. of those for footings on dry sand. According to the usually accepted theories, bearing pressures on inundated sand should only be about 50 per cent. of those on dry sand[1] due to the reduction in the apparent density of the sand and also (possibly) to a slight reduction in the angle of internal friction. The reason for the discrepancy between the theoretical and experimental bearing values is that the actual mechanism of failure differs from that assumed in any of the several theories, and further is slightly modified when the footing becomes inundated. The locus of the sliding surface was determined experimentally for the dry sand and computed bearing pressures based on the inferred failure mechanism agree with the experimental loads.

Introduction

The principal object of this investigation was to compare experimentally the bearing power of narrow footings on dry and inundated sand.

It is usually accepted that the ultimate bearing capacity of footings on a non-cohesive soil is directly proportional, amongst other things, to the density of the sand. Consequently a footing founded on an inundated sand, the apparent density of which is reduced by the buoyancy due to the water, would be expected to fail at a lower load than a similar footing on dry sand. Indeed, Terzaghi and Peck[1] have stated, "If the water table rises . . . to the surface of the sand, the effective unit weight of the sand γ is reduced to its submerged unit weight γ^1. The submerged unit weight is roughly equal to one half of γ. Hence the rise of the water table to the surface of the ground reduces the bearing capacity by about 50 per cent." Since inundation will also reduce the angle of internal friction $\varnothing$ for most sands, and according to all the accepted failure theories a small reduction in $\varnothing$ gives a large reduction in bearing power, it appeared possible that inundation may in certain cases reduce the failing load by more than half.

There was, however, no experimental confirmation of Terzaghi's statement, and since it is probable that a large proportion of footings become inundated either by a rising ground water level or by flooding at some stage of their life, it was obviously important to investigate its truth.

The results of the investigation will not of course be important in designing footings of large breadth since in general the factor limiting the load on such footings is the permissible settlement rather than the ultimate failing load, but they will be applicable to many medium-sized footings and the majority of small footings.

In addition they will be useful in designing road and runway pavements on frictional soils by the Golder[2] method. In this method the thickness of the runway is chosen so that its load spreading effect and its surcharge weight are just sufficient to prevent ultimate failure of the soil immediately beneath a wheel load. The wheel load is assumed to be spread over a rectangular area instead of the more usual circle or ellipse, and the ultimate failing load is calculated by either Prandtl or Terzaghi's formulæ for the ultimate bearing power of frictional soils, both of which are discussed later. Golder states that where necessary due account should be taken of the possibility of the failing load being *halved* by inundation.

It was originally intended to carry out only a few tests on dry sand in order to correlate the results with those obtained by Golder[2], and then to carry out a comprehensive series of experiments on inundated sand for comparison. The tests on dry sand indicated, however, that Golder's results are possibly unreliable, and so a comprehensive series of tests had to be made on the dry sand as well.

In order to compare the mechanism of failure with the mechanisms suggested by Prandtl, Terzaghi, Krey and Ritter, the location of the surface of sliding was also determined opposite the centre of each footing. This proved very useful when analysing the results of the investigation, since it facilitated the explanation of the unexpectedly high bearing power of the inundated sand.

Previous Investigations and Theories

The more important experimental investigations concerning the bearing power of sands which had been carried out prior to 1940 were summarised by Golder[3] when describing tests made at the Building Research Station, the purpose of which was to clear up some uncertainties and contradiction which still existed. The three most important investigations summarised by Golder were those by Fellenius[4], Kogler[5], and Meischeider[6].

Fellenius used rectangular and elliptical footings ranging up to 30 cms. in width and 59 cms. long. His principal conclusion was that "on a purely frictional soil the maximum possible load per unit area of a rectangular or elliptical area is determined by the smallest dimension of the area and is proportional to this dimension."

Kogler's tests were made on circular footings up to 16 cms. diameter and on a series of rectangular footings of different shapes but all of the same area. He concluded that for circular footings the ultimate pressure is proportional to the diameter of the footing, whilst for footings of equal area the ultimate pressure increases as the footings become more compact. Some of Kogler's experimental results are shown in Fig. 1, from which it will be seen that the graph of ultimate pressure against the area/perimeter ratio for rectangular footings is a curve.

Meischeider obtained exactly similar curves using three similar series of rectangular footings, each series of which was of constant area but of different area/perimeter ratios. Now, if the bearing pressure were directly proportional to the breadth of the footing, but independent of the length, the full line curve in Fig. 1 should be exactly similar in shape to the dotted line

* *Paper to be read before the Institution of Structural Engineers at 11 Upper Belgrave Street, London, S.W.1, on Thursday, January 11th, 1951, at 6 p.m.*

curve of Fig. 1 obtained by plotting the footing breadth against the area/perimeter ratio. The typical experimental curve is not, however, exactly similar to the dotted line, and the difference in shape may be due to one or possibly both the following factors. Either the

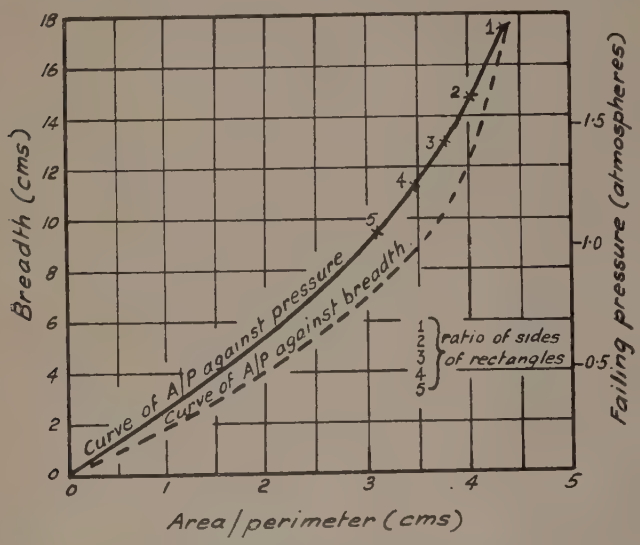


Fig. 1.—Comparison of ultimate load—A/P curve obtained by Kögler with A/P-breadth curve

rate of increase of bearing power decreases with increasing footing width, or for a given footing width the bearing power may increase with increasing length. Plotting Meischeider's results in another way as in Fig. 2 indicated that the latter might be the case, since there does appear to be a tendency for the squares to give pressures below the average for rectangular footings of the same width.

To clear up this point Golder[3] carried out tests on 6 in. wide footings of different lengths up to 30 in., and came to the conclusion that the bearing pressure was independent of length. Having established this fact it appeared that the difference between the two curves shown in Fig. 1 was entirely due to the rate of increase of pressure decreasing with increasing width.

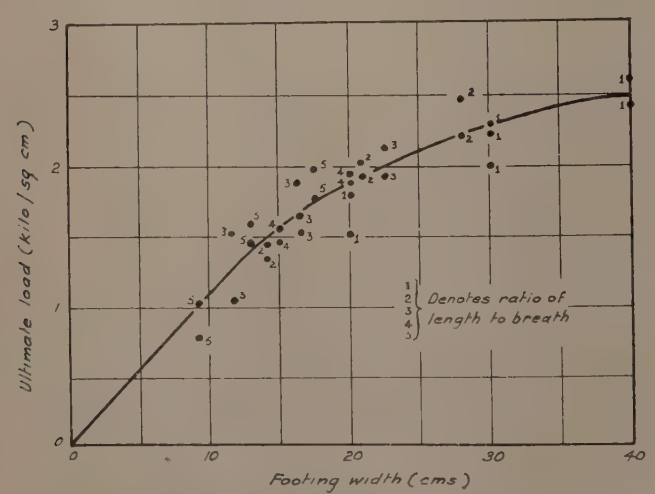


Fig. 2.—Meischeider's results and effect of width of footing

Golder suggested that this decline in the rate of increase of bearing pressure may be connected with the decrease in the angle of internal friction of a dense sand with increase in pressure. If this suggestion is correct no decrease would be expected with loose sand, whose internal friction does not vary very much with pressure. This might explain why Fellenius had obtained a straight line relation between "p" and "b" since he does not definitely state that his sand was compacted before testing. As will be shown later, however, the straight line relation might have been caused by Fellenius carrying out his tests in a box which was too small. Further, although the present tests were made on a sand whose angle of internal friction did not vary much for pressures up to 1.5 ton per ft.^{2} a distinct decrease in the slope of the p-b graph was obtained for pressures in this range. In view of this, another possible explanation of the fact that the ultimate pressure does not increase uniformly with footing width is put forward later.

At the time of Golder's experiments the best-known methods of calculating the bearing pressure of footings were those due to Prandtl[7], and Krey[8], and he compared the values given by these methods in his experimental results. A further formula due to Ritter was also examined but it gave bearing values which were obviously inaccurate and will not be mentioned further.

The original Prandtl formula was derived in connection with researches into the plastic flow of metal under the action of the cutter of a machine tool. Fig. 3 shows the assumed mechanism of failure, the wedges GAF, ABC and BED being assumed to behave as rigid wedges, whilst the sectors AFC and BDC deform plastically.

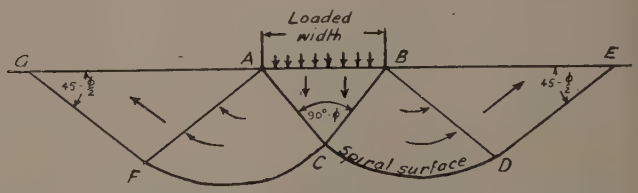


Fig. 3.—Diagrammatic representation of Prandtl's condition of failure of loaded ground

The curves CF and CD are logarithmic spirals. Since the weight of the small mass of metal being deformed was negligible compared with the applied load, Prandtl took no account of it in his calculations. His expression for the ultimate bearing capacity q is

$$q = c \cot \phi \left[\frac{1 + \sin \phi}{1 - \sin \phi} e^{\pi \tan \phi} - 1 \right]$$

where c is the cohesive shear strength and ϕ the angle of internal friction of the material. It will be seen that for $c = 0$ the formula gives $q = 0$, and if the formula were applied to soils it would mean that a cohesionless sand would have no bearing capacity. This, of course, is not true and the assumption chiefly responsible for the discrepancy is that the soil is weightless. Taylor has, however, applied a correction to Prandtl's formula by adding a "weight factor" $b \gamma \cot \left(\frac{\pi}{4} - \frac{\phi}{2} \right)$ to the term $c \cot \phi$, where 2b is the footing width and γ is the bulk density of the soil. The formula then becomes

$$q = \left[c \cot \phi + b \gamma \cot \left(\frac{\pi}{4} + \frac{\phi}{2} \right) \right] \left[\frac{1 + \sin \phi}{1 - \sin \phi} e^{\pi \tan \phi} - 1 \right]$$

Krey suggested a graphical method, the surface of sliding being assumed to consist of a circular arc under the footing, terminating in a tangent at $(45 - \phi/2)$

degrees to the horizontal (see Fig. 4). Krey stated that the centre of the most dangerous circle would lie on the same level as the underside of the footing and various trial centres are taken at this level. Golder, however, calculated that the bearing pressures would be lower if centres of rotation above the level of the bottom of the footing were used in the calculations.

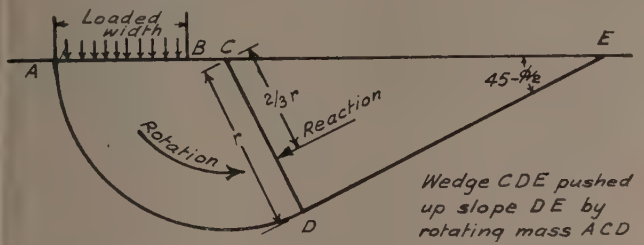


Fig. 4.—Failure mechanism, according to Krey

Fig. 5 compares the bearing pressure for a 6-in. wide footing on cohesionless soil according to Prandtl and Krey with the experimental value obtained by Golder. It will be seen that Prandtl's formula gives the correct value if $\phi = 42\frac{1}{2}°$ and Krey is correct if $\phi = 44°$, but because of the steepness of the curves the two methods give bearing pressures which differ by as much as 30 per cent. for any one value of ϕ.

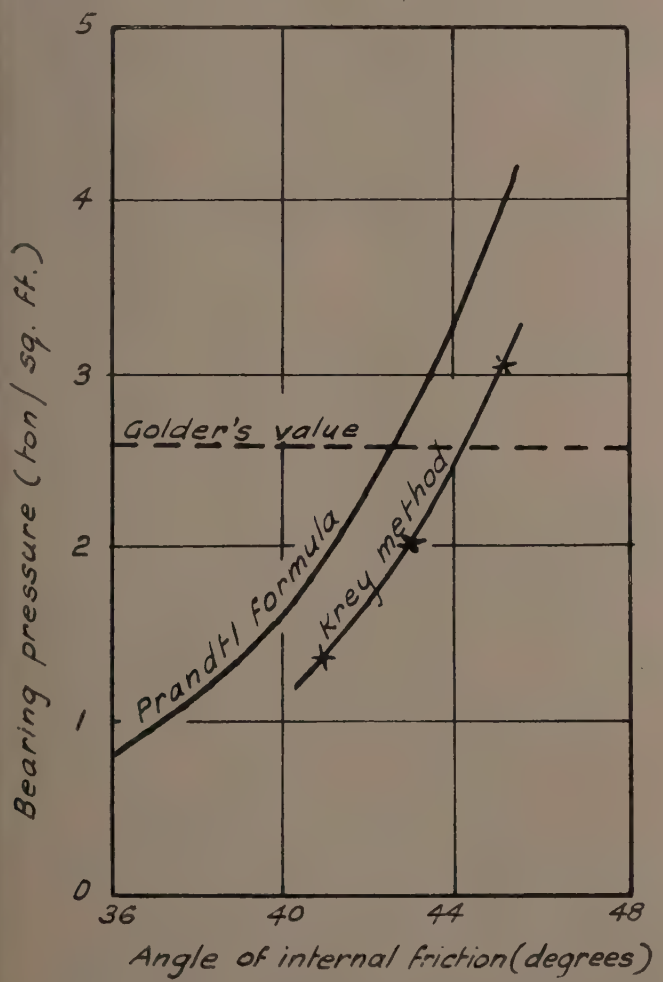


Fig. 5.—Golder's observed values compared with various theories

The measured value of ϕ varied between 48° and 39° as the normal load increased from zero to 4 ton per sq. ft., and since it is difficult to estimate the mean pressure along the surface of sliding it can only be said that either $42\frac{1}{2}°$ or 44° was a possible value for ϕ in the experiments. Thus it was impossible to say whether the Krey or Prandtl method was more correct.

Unfortunately, Golder carried out his experiments in a box which was far too small for the footings used. As he himself points out, the theoretical position at which the surface of sliding cuts the sand surface is well outside his box according to both Krey and Prandtl. Although Golder did not think this important the present investigation has shown, as will be seen later, that it probably caused an increase in the failure loads.

Since the publication of Golder's paper Terzaghi[7] has also published a general theory applicable to footings on any type of soil. Fig. 6 shows the supposed mechanism of failure and the failing load is given by the expression

$$Q = b\,(cN_c + \gamma\, D_f\, N_q + \tfrac{1}{2}\,\gamma\, bN_\gamma)$$

The symbols c, q, b and γ have the same significance as before, D_f is the depth of the footing below the soil surface and N_c, N_q and N_γ are called bearing capacity factors. These bearing factors are dimensionless and depend only on the value of ϕ. Terzaghi has therefore computed them once for all and has plotted their values on a chart.

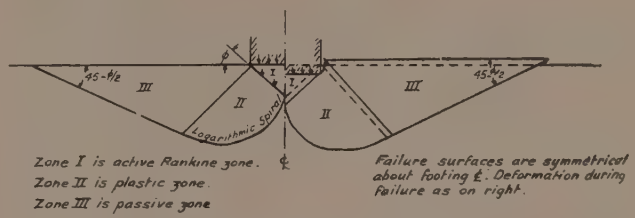


Fig. 6.—Mechanism of failure according to Terzaghi

Since in the present case c and D_f are both zero the value of ultimate load per unit length according to the Terzaghi formula becomes $\frac{1}{2}\gamma b^2 N_\gamma$, and the variation in pressure with ϕ for a 6 in. wide footing is as in Fig. 7. Comparison with Fig. 5 shows that the Terzaghi and Prandtl formulæ give very similar results.

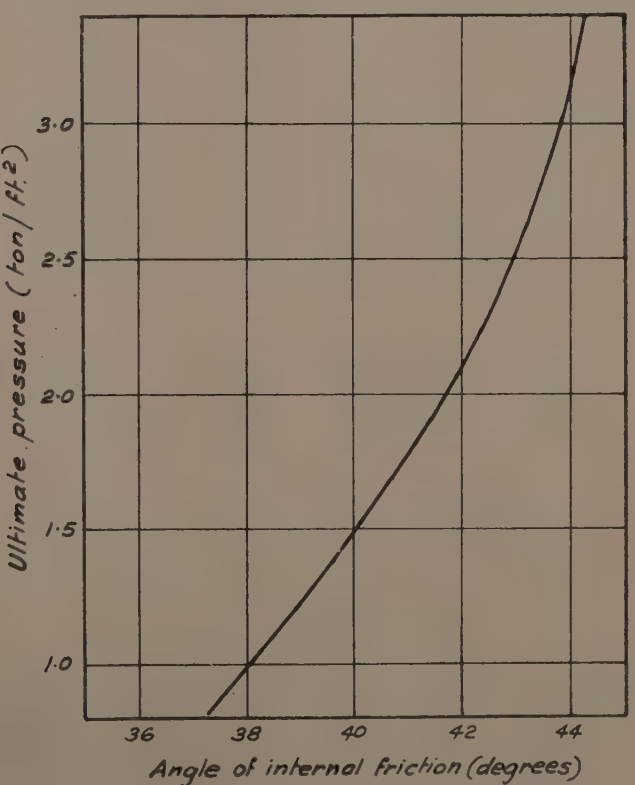


Fig. 7.—Bearing pressures by Terzaghi formula for 6 in. wide footing on sand

Although the ultimate pressures according to Prandtl, Terzaghi and Krey give fair agreement with Golder's results on 6 in. wide footings, it is the author's opinion that this agreement is entirely fortuitous, since in all three cases the assumed mechanics of failure appears to be incorrect in the light of the present investigation.

Description of Apparatus Used in Present Investigation

The apparatus used in the tests was very similar to that used by Golder. The sand was contained in a box whose inside dimensions were 4 ft. long by 3 ft. 6 in. wide by 16 in. deep. The upper edges of the box sides were carefully planed so that, by running a straight-edge over them, the surface of the sand could be levelled. To keep the box watertight a lining of bituminous sheeting was inserted. For the tests on inundated sand, some of the sand was removed so that the sand surface was one inch below the top of the box and the standing water level just below the top of the box. Levelling of the sand surface was effected by running a straight-edge over the box sides as before but with a 1 in. deep recess cut in each end.

Loads were applied to the footings by means of a screw-jack bearing against the horizontal beam of a frame built round the box as indicated in Fig. 8, their

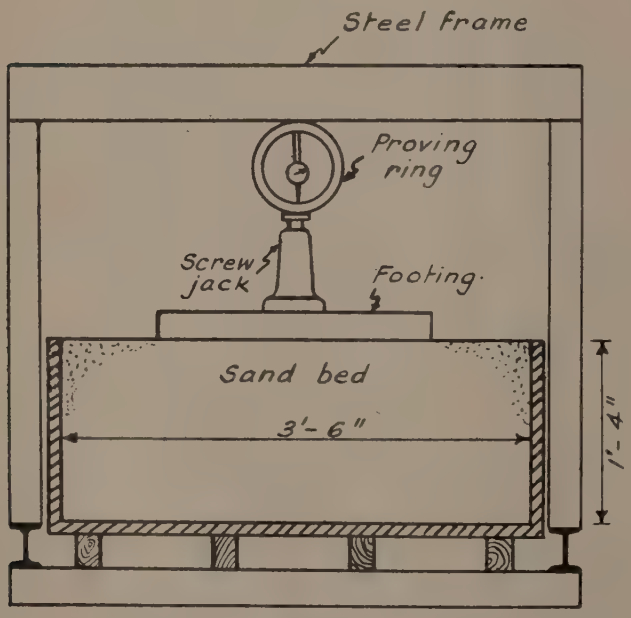


Fig. 8.—Arrangement for testing

magnitude being measured by a calibrated "proving ring." The footings themselves consisted of various lengths of 3, 4 and 6 in. wide steel channel, and also squares and circles cut from mild steel plate. The strip footings varied between 6 in. and 30 in. in length by 3 in. increments, but since the 30 in. ones sometimes gave unsatisfactory results due to one end sinking first their results are not quoted.

The grading curve of the sand used in the investigation was as in Fig. 9. The angle of internal friction of the sand when dry and at the dry density used in the loading experiments, i.e., 110 lb. per cu. ft., was $42\frac{3}{4}°$ and was virtually constant for all pressures up to 1.5 ton per sq. ft. The shear box test was also carried out under water but it was difficult to obtain the same dry density as in the loading tests (again 110 lb. per cu. ft.), and so the angle of internal friction which was obtained (39°) was almost certainly a little low. The mean specific gravity of the sand particles was 2.63, so that the dry density of 110 lb. per cu. ft. corresponds to a voids ratio of 0.49.

Description of Tests

The tests on dry sand were carried out first. After each test the sand was dug out to the full depth of the box and then recompacted after being replaced. The main compaction was carried out using a hand rammer weighing 16 lb., the final compaction being made with a lighter, 10 lb., rammer of larger area which caused less disturbance of the sand surface. The weight of sand in the box throughout the dry sand experiments was maintained at about 2,010 lb., and compaction was continued until not more than about 1 lb. of sand was

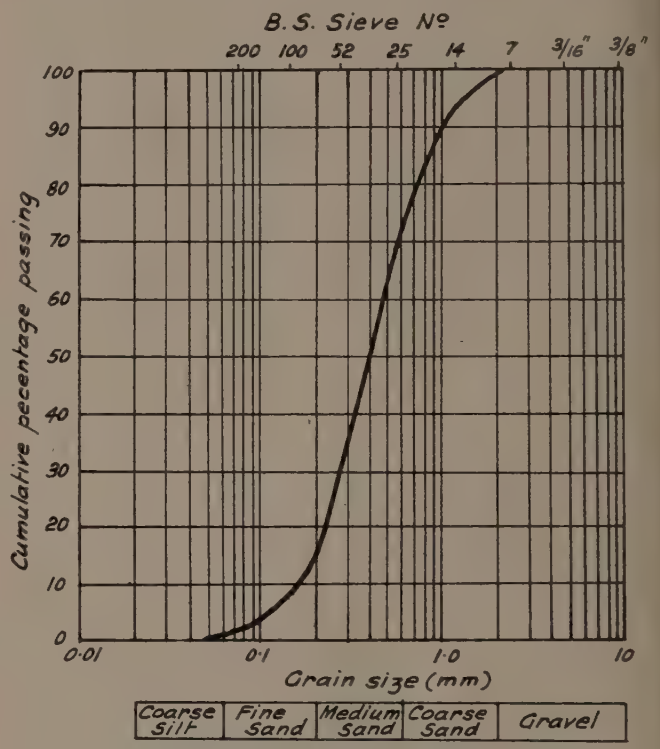


Fig. 9.—Grading curve for sand used in investigation

scraped off by the straight edge. These scrapings were put back in the box before the start of compaction for the next test, and hence the mean density of the sand was, practically speaking, the same in all the tests. The variation in density from one part of the box to another was measured by inserting an open-topped can of known volume, and digging it out after the test and weighing the sand it contained. On the average the density obtained by this method 108 lb. per cu. ft., was rather lower than the known value of 110, but the variation in measured density was relatively small, the limits being from 106 to 110 lb. per cu. ft. The variation in measured density was largely independent of the location of the sample and seemed to be due chiefly to experimental error.

Load was applied to the footings by turning the screw of the jack at an approximately constant rate. The load steadily increased to a maximum and then quite suddenly dropped to between 50 and 80 per cent. of the maximum value. This sudden drop was accompanied by a distinctly audible sudden slip in the sand, and very little further penetration by the footing was sufficient to show up the outcrop of the slip planes on the sand surface.

In some of the tests, measurements were made of the heave of the sand at various locations in the box for different penetrations of the footing. In addition the

ertical and lateral movements of the footing were ecorded.

The procedure for locating the slip planes was as ollows. After compacting the sand, strips of thin issue paper about $\frac{1}{4}$ in. wide and 10 in. long were placed etween two strips of steel whose ends were bevelled o that the two together gave a sharp point. The ointed end was pushed down into the sand, after which he steel strips were pulled apart, thus allowing sand to low between them and to surround the tissue paper. The steel was then pulled slowly out of the sand, leaving the paper behind. This was repeated at various locations, after which the top end of all the paper strips was trimmed level with the sand surface. After carrying out the bearing test, the paper strips were carefully dug out. At the sliding surface, a kink was found in the

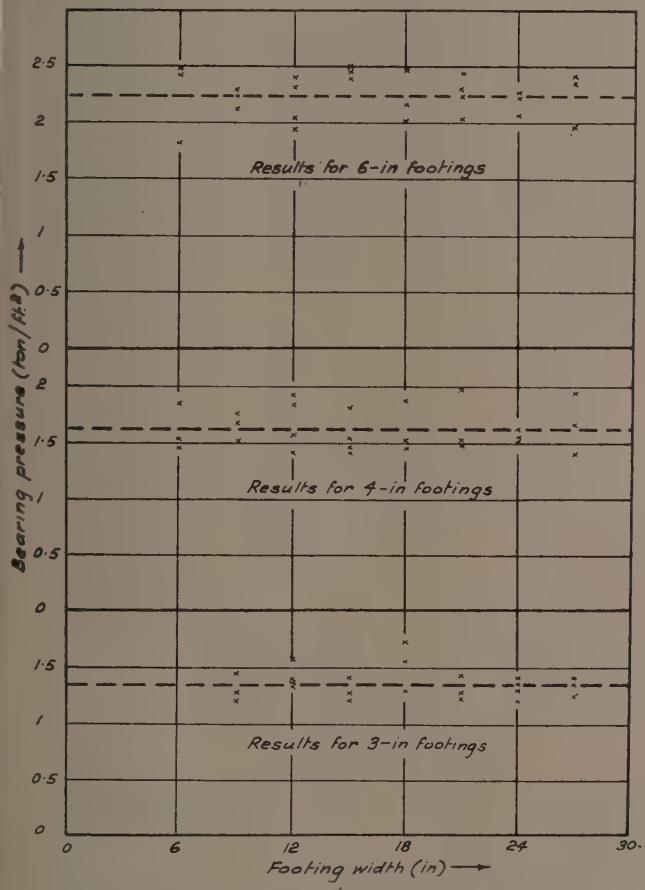


Fig. 10.—Values of bearing power for rectangular footings of various widths on dry sand

paper, or occasionally a tear, so that the depth of the slip plane was easily determinable at each point. Considerable care and practice was needed before the technique became sufficiently reliable, but eventually the failure surface could be traced fairly accurately except under the footing itself. In this region the sand movements appeared to be too confused, and it is probable that there was no distinct surface of sliding.

For the inundated tests 1/16th of the sand was removed from the box, so that the voids ratio would be exactly the same as in the dry tests provided that the sand surface was just one inch below the top of the box. Sufficient water was then added to inundate the sand completely.

After each test the sand was spaded over to its full depth and then thoroughly puddled and vibrated with a spade. The puddling was done from the centre of the box outwards, finishing in the corners, since it was found that the sand density at any point was increased most during the puddling of adjacent areas, and it was therefore advisable to have the last areas to be worked in the corners where the lower density did not affect the footing failure. Great care had to be taken to remove all air from the sand since it is probable that an excess of small air-filled voids might give the soil some cohesive strength due to surface tension effects. In addition, they would add to the buoyancy of the sand and therefore decrease its apparent density.

Quite fortuitously, thorough puddling of the inundated sand produced exactly the same voids ratio that had been obtained by thorough compaction of the dry sand. This was very fortunate since it made the results of the two series of test directly comparable.

As in the dry sand tests, lateral movement of the footings and the heave of the surrounding soil were measured for various penetrations. The determination, as in the previous tests, of the location of the slip planes in the sand mass was not of course possible, but the general mechanism of failure was shown to be very similar by the shape of the slip plane outcrops and the movement of the footings.

Test Results

(a) *Bearing values for dry sand.* The ultimate bearing pressures for 6 in., 4 in. and 3 in. wide footings of various lengths are shown in Fig. 10. For each of the footing widths the spread of the results is about plus or minus 20 per cent. of the mean. In all cases the ultimate pressure appears to be independent of the length of the footing.

The ultimate pressure is plotted against the least dimension of the footing in Fig. 11. For the rectangular

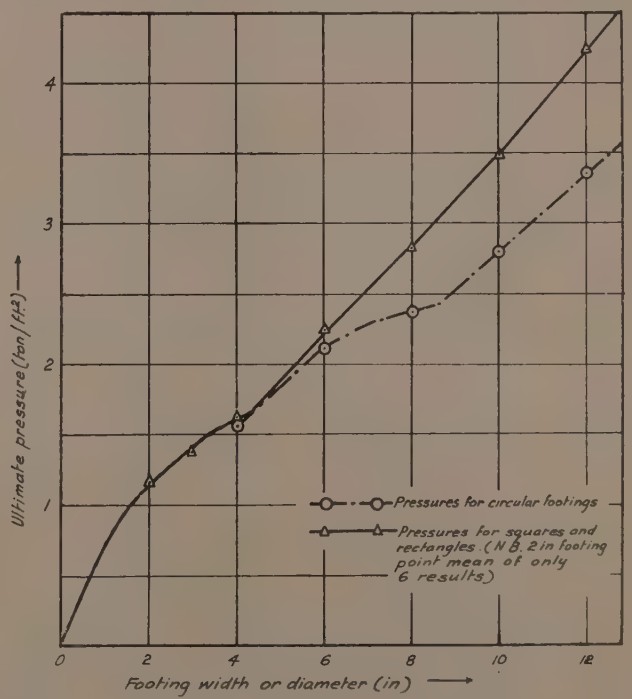


Fig. 11.—Ultimate pressure against least dimension (dry sand)

and square footings the rate of increase of ultimate pressure with increasing breadth falls off fairly rapidly for footings up to 4 in. diameter, after which it appears to increase approximately uniformly with increasing footing width. The reason for this discontinuity will be discussed later. For 4 in. and 6 in. diameter circular footings the bearing pressure was only a little less than the pressure for a rectangle of the same width. With

greater diameters, however, the bearing capacity fell off rapidly compared with the square footings. Indeed, the bearing pressure for a 10 in. diameter circle was only approximately 0.78 (i.e., $\pi/4$ approx.) of that for a 10 in. square, but the author does not think that this indicates that the bearing pressure is proportional to the *mean* width for reasons which will be given later.

The ultimate load was not nearly so distinct for the square and circular footings as in the tests on rectangles. Instead of a distinct drop in the load after it had reached the ultimate value as was obtained for rectangles, the load-penetration curve merely flattened out for a time, after which the load began to increase again.

(b) *Bearing values on inundated sands.* Fig. 12 shows the ultimate bearing pressures obtained with 3 in., 4 in. and 6 in. wide footings on inundated sand. As for the dry sand, the bearing pressure appears to be independent of length.

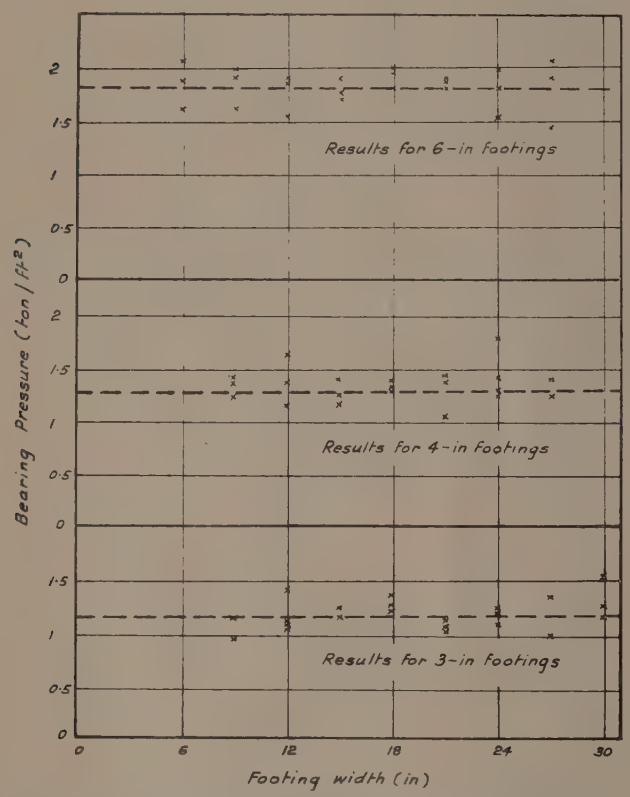


Fig. 12.—Values of bearing power for rectangular footings of various widths on inundated sand

The variation of bearing pressure with least dimension is shown in Fig. 13, but the results for 10 in. and 12 in. plates are not given since there was no distinct ultimate pressure, the load increasing continuously though not steadily with increasing sinkage of the footing.

(c) *Mechanism of failure and location of slip surfaces.* (1) Circular and square footings not less than 6 in. across. On both dry and inundated sand the larger square and circular footings sank vertically into the sand without sideways movement. The sand surface rose evenly all round the footings and heave was apparent over almost the whole box area, even in the corners. No distinct failure planes were obtained, and it appeared that the sand was in a state of plastic flow to a considerable depth with general relative movement between the sand particles rather than just localised relative movement along definite slip surfaces. (2) Rectangular footings. The 3 in. and 4 in. wide rectangular footings failed as follows: Initially they sank vertically into the sand, causing an approximately equal heave on each side. Immediately after the ultimate load had been reached, having ceased at one side and the footing began to move sideways a little, but this sideways movement was not pronounced until the sinkage was equal to nearly half the breadth of the footing. The outcrop of the slip surface appeared on *one side* of the footing only, shortly after the ultimate load had been passed, becoming more pronounced as the footing was driven further in. For both sets of footings on inundated sand, the outcrop was about $\frac{1}{2}$ in. further from the footing on the average than it was in the tests on dry sand. Although there was quite considerable spread in the individual measurements, the distances were taken as the mean of a great number of measurements and it is probable that the difference is a real one.

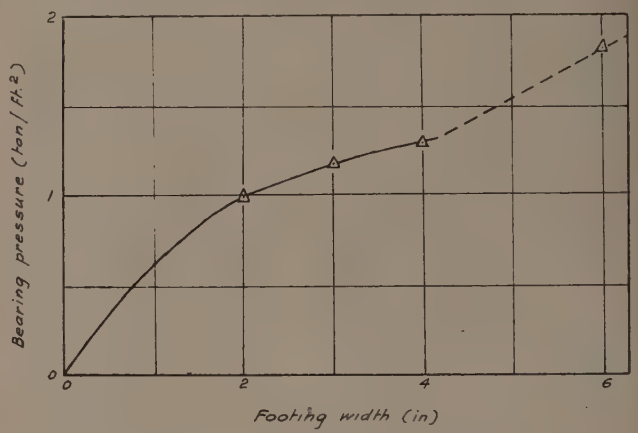


Fig. 13.—Variation of bearing pressure with footing width for footings on inundated sand

The average distance from the edge of the footing to the failure surface outcrop, opposite the footing centre line, was, for 3 in. footings, about $15\frac{1}{2}$ in. on the dry sand, the range being from 11 in. to 20 in. In general the distance was slightly smaller for the shorter footings than for the long ones. The corresponding range for 4 in. footings was 16 in. to 22 in. (i.e., the edge of the box) the mean position being about 20 in.

The photographs in Figs. 14 and 15 show typical failures for 3 in. and 4 in. footings. In these tests the depth of water standing on the surface of the sand initially was purposely limited to about $\frac{1}{8}$ in., so that as the voids ratio increased due to the sand being disturbed during failure the water level fell to a little below the sand surface. Thus the outcrop line was very marked since the cohesion of the damp sand on the

Fig. 14.—Failure of 4 in. wide footing

surface was sufficient to cause a sharp edge to form. This facilitated the taking of the photographs. The footings have also been driven much further into the sand than in an ordinary test to make the mode of failure show more clearly.

It was significant that the footing did not tilt more than one or two degrees in any of the tests despite the fact that the loading arrangement did not impose any restraint on rotation.

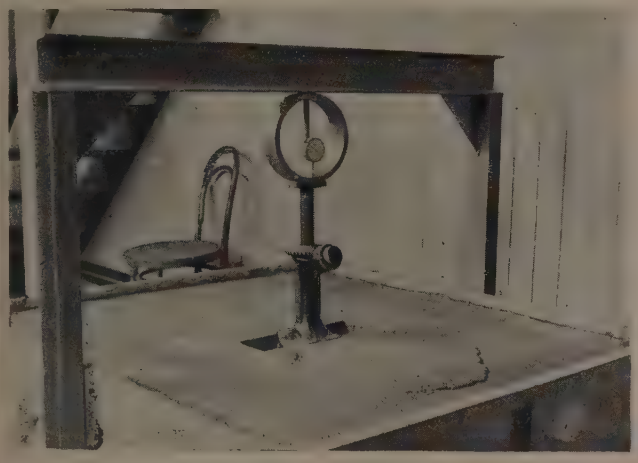
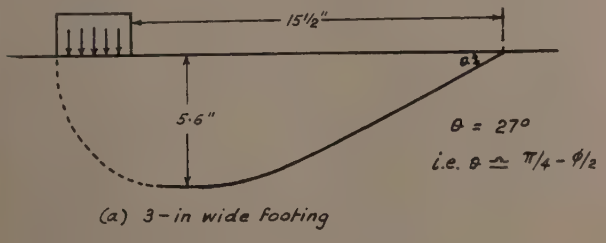


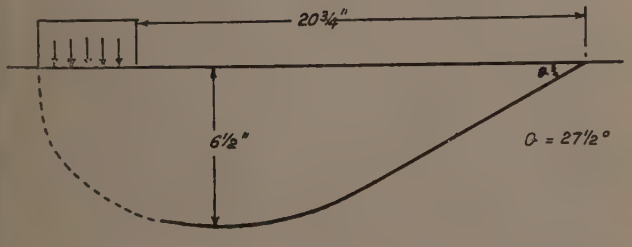
Fig. 15.—Failure of 3 in. wide footing

Typical slip plane locations for 3 in. and 4 in. footings on dry sand are given in Fig. 16. It will be seen that they agree fairly closely with the shape assumed by Krey and are very different from those suggested by Terzaghi and Prandtl. But since tilting of the footings did not occur the *mechanics* of failure must be different from those suggested by Krey.

The 6 in. footings usually failed in a similar way to the 3 in. and 4 in. footings except that the box was so small relatively that the failure plane outcrop was right up



(a) 3-in wide footing



(b) 4-in. wide footing.

Fig. 16.—Typical locations of failure surfaces opposite footing centreline. Full lines show measured locus. Dotted lines show inferred locus

against the end of the box (see Fig. 17). The failure planes were obviously being distorted but unfortunately the slip surfaces could not be located accurately by the paper strip method since it was not possible to insert the strips sufficiently deeply.

Very occasionally a failure surface was formed on both sides of the 6 in. footings, the two outcrops being between 16 in. and 21 in. from the footing. Golder reported that this usually occurred in his tests on 6 in. footings in a box 42 in. long. Since the least dimension of the box used in these tests was 42 in. some of the footings were turned through 90° and re-tested. Dual failure sur-

Fig. 17.—Failure of 6 in. wide footing

faces were now obtained more often although failure to one side was still more frequent. An important feature of these tests was that the mean ultimate load was 6 per cent. higher on dry sand and 5 per cent. higher on inundated sand than with the footings facing the other way. Thus, cutting down the box dimensions in the direction of failure by 6 in. had increased the ultimate load significantly.

Analysis and Discussion of Test Results

It is apparent that the mechanism of failure was affected by the box being too small for all the footings wider than 4 in. The results of the tests on 3 in. and 4 in. wide footings will therefore be discussed first.

The values of the ultimate bearing pressure for 3 in. and 4 in. wide footings, according to Terzaghi, Prandtl and Krey (assuming that $\varnothing = 42\frac{3}{4}°$), are compared with the experimental values in Table 1. The theoretical values of bearing pressure on inundated sand were obtained by multiplying those for dry sand by 1.63/2.63 i.e., in the ratio of the apparent densities.

TABLE 1.—Comparison of Theoretical and Actual Bearing Pressures for 3 in. and 4 in. wide Footings.

Method	Footing Width	Failing pressure (ton/ft.²)	
		Dry Sand	Inundated Sand
Experiment ...	3 in.	1.35	1.18
	4 in.	1.62	1.30
Terzaghi	3 in.	1.25	0.77
	4 in.	1.67	1.03
Prandtl	3 in.	1.35	0.84
	4 in.	1.80	1.11
Krey	3 in.	1.02	0.64
	4 in.	1.36	0.85

It will be seen that the experimental values of the bearing pressure on inundated sand are 87½ per cent. and 80½ per cent. on those on dry sand for the 3 in. and 4 in. wide footings. The value, according to Terzaghi, Prandtl and Krey should be not more than 62 per cent. for each, even if ∅ were not decreased by the inundation. It is obvious therefore that the bearing value has not decreased proportionately to the apparent density of the bearing medium.

The Terzaghi and Prandtl bearing value agree very well with experimental values on dry sand, whilst the Krey formula gives values which are too low. Since, however, all three formulæ give values which are very low when the apparent density is decreased, it is probable that the agreement shown by Terzaghi and Prandtl on dry sand is quite fortuitous.

It is, of course, easy, as with most analytical treatments of soil mechanics problems, to find flaws in the theories which will explain why they are not universally applicable. For example, both Terzaghi and Prandtl assume that part of the surface of sliding is a logarithmic spiral (see Figs. 3 and 6). This is very convenient mathematically since the resultant force on the spiral surface during sliding passes through the spiral centre and therefore has no moment about it. There is, however, no justification other than mathematical convenience for the spiral assumption. With regard to the Krey theory, very few footings would be free to rotate sufficiently far about the point C in Fig. 4 for failure to occur. In addition, it seems likely that the line of action of the reaction perpendicular to CD would be very near D once CDE started to move, instead on only 2/3 of the way along CD. The method also completely ignores the stresses along CD due to the footing load and their effect on the force polygon for the wedge CDE.

Since the accepted theories did not fit the experimental results for both dry and inundated sand, an attempt was made to find a mode of failure which would fit them. Referring to Fig. 16 it will be seen that the failure surface follows a path similar to that predicted by Krey, but there was the important difference that the footings did not tilt ; instead they sank with their base more or less horizontal. This suggested that the wedge ABC in Fig. 16 was being forced up the slope BC whilst the sand underneath the footing was merely extruded plastically towards AB. The lines of flow would be as in Fig. 18. The measured rise in the sand surface at various points tended to confirm this theory since the surface AB in Fig. 18 rose fairly uniformly once the ultimate load was passed, i.e., a slip surface had been formed.

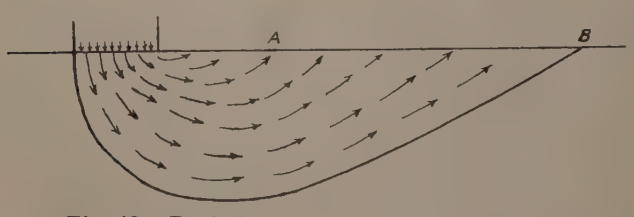


Fig. 18.—Probable lines of flow under footing

Further confirmation of this supposed mechanism of failure has been found in a recent paper by Peynircioglu[9]. He tested footings on dry sand in a glass-sided box with one end of the footing right up against the glass. By taking a time exposure photograph as the footing was driven into the sand the movement of the sand particles was recorded. Although the sand movements at the end of the footing are not absolutely typical since the distance from the footing to the outcrop of the slip surface is not so great at the end of the footing as in the centre, and there will also be friction effects against the glass face, the locus of the sand particles in Figs 11c of his paper (reproduced here in Fig. 19) will be seen to be very similar to that suggested in Fig. 18. Unfortunately Peynircioglu did not measure the failing loads so they cannot be compared with those obtained in this investigation. It is also unfortunate

Fig. 19.—Locus of sand particles during failure as photographed by Penirciogla

that, having discovered that failure occurred by slipping to one side only, he tried to reconcile this with the Terzaghi theory by saying that Terzaghi's failure plane was the right shape but that a plane was formed on one side only. If that were so the failing load would always be much less than that predicted by Terzaghi, which this investigation has shown to be untrue.

An analysis was made of the forces acting on the wedge ABC of Fig. 20, to determine whether the wedge would be on the point of moving with the ultimate load applied to the footing. It was unnecessary to consider the equilibrium of the mass ADB since if ABC moved the soil under the footing would flow out allowing it to settle.

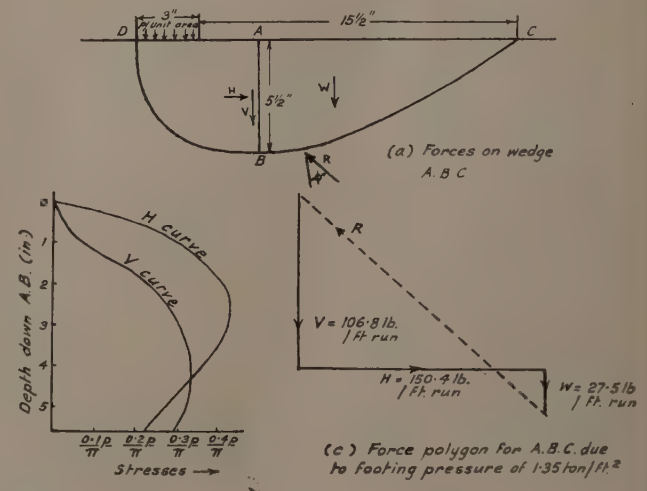


Fig. 20.—Equilibrium calculations for wedge ABC for 3 in. footing

If the soil can be considered to behave as a homogeneous isotropic elastic semi-infinite mass, and the load on the underside of the footing is uniformly distributed, the stress distribution along AB is easily calculable.

Boussinesq[10] derived expressions for the stress intensity at any point inside such a mass due to a point load on the surface, and the stresses due to a point load can be integrated to give those due to square, circular or long strip loaded areas. For a loaded area of infinite length the two principal stresses in a plane at right angles to the footing are given by

$$\sigma_1 = p/\pi\ (\beta + \sin\beta)$$
$$\sigma_{111} = p/\pi\ (\beta - \sin\beta)$$

where p is the pressure applied to the loaded area and β is the " sight angle " as shown in Fig. 21. The major

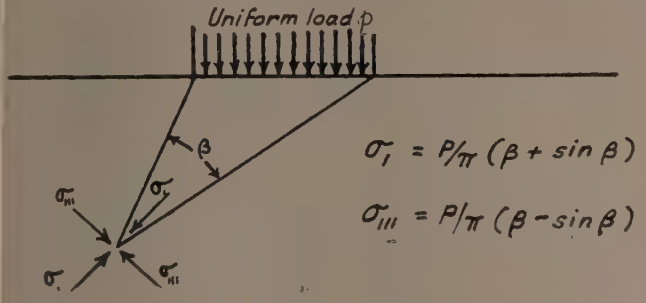


Fig. 21.—Principal stresses beneath infinitely long strip subject to load p per unit area

principal stress bisects the angle β and the minor principal stress is, of course, at right angles to σ_1. Since the soil is not perfectly elastic, however, the stress distribution due to a strip load will be somewhat modified by plastic flow. Also the stresses will be further modified by the fact that the contact pressure across the base of a rigid footing is not uniform. For example, a perfectly rigid strip footing on a medium of infinite elastic range would have a contact pressure distribution as shown in Fig. 22a, the pressure at the edge being infinite. There would also be a concentration of load under the ends of the footing (Fig. 22b), in the case of footings not of infinite length.

For a footing on the surface of a sandy soil, however, the pressure distribution on the underside of the footing is probably as shown in Fig. 22c, since plastic flow of the sand will occur near the edges, thus reducing the pressure in that region. The stress distribution in the soil mass will, therefore, be a modification of the distribution obtained by integration of the Boussinesq equations, the major principal stress directly under the footing centre line being somewhat increased at all depths, and the stresses at points away from the footing being very slightly reduced. Except in the immediate vicinity of the underside of the footing, however, the general stress distribution will probably be very similar to that for a uniform base load of of the same total magnitude. Hence it has been assumed in working out the stresses on AB of Fig. 20, that the load on the underside of the footing was uniformly distributed and the soil has been treated as if it were perfectly elastic.

The principal stresses have thus been worked out at several points on AB for the case of a 3 in. wide footing carrying the ultimate load found in the experiments. By means of the Mohr's circle construction the direct stress on, and tangential stress along, AB were determined at each of the points. These stresses are shown in Fig. 20b. The forces acting on unit length of the wedge ABC would therefore be (i) its own weight vertically downward, (ii) the shear force on AB also vertically downwards, (iii) the horizontal force on AB, and (iv) the reaction from the underlying soil on BC. The polygon of forces for the wedge ABC for the case of dry sand would therefore be as in Fig. 20c. To close the polygon the reaction on BC would have to be as shown by the dotted line. If BC is just on the point of moving, this line should be at an angle $\varnothing$ to the normal to BC at the centre of pressure for BC. Since most of the pressure on BC will be near B because of the stresses set up by the footing load, the centre of pressure should be close to B on the nearly horizontal part of the curve. Thus the location obtained by choosing a normal to BC which is at $\varnothing$ to the force-polygon closing line is quite feasible (see Fig. 20a). Accordingly, it would appear that the wedge ABC could reasonably be expected to be on the point of moving under the experimental footing load, thus allowing sand to flow from underneath the footing and permitting it to sink.

It will be noticed from Fig. 20b that near B the shear stress on AB is becoming increasingly greater than the direct stress so that there would be no tendency for larger wedges than ABC to slide. Smaller wedges near the surface will of course tend to slide earlier than ABC but the instant these wedges yield, the horizontal force causing sliding will decrease, resulting, possibly, in an increase in the forces lower down AB. Yield is probably progressive, therefore, until eventually the whole wedge ABC moves. This would explain why heaving starts fairly near the footing initially and then slowly spreads outwards as the load on the footing is increased.

The mechanism of failure in Fig. 18 having been found to be compatible with the experimental results on dry sand, similar calculations were made for the inundated footings. It was assumed in these calculations that the shape of the sliding wedge ABC was as in Fig. 23, the angle ABC being 2° less than in Fig. 20 because of the lower $\varnothing$ value. Owing to this lower $\varnothing$ value and to the point C being $\frac{1}{2}$ in. further from the

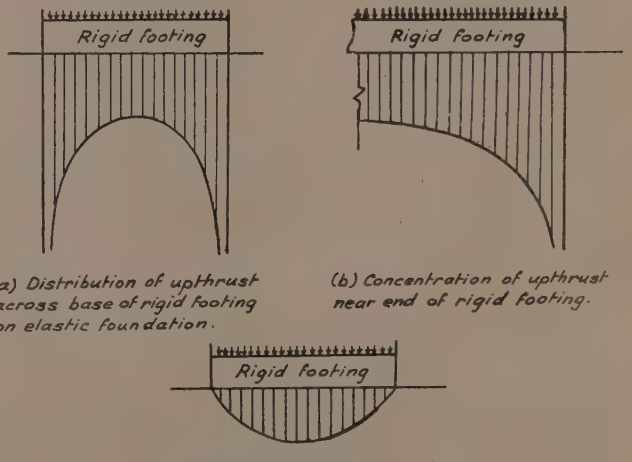

Rigid footing
Rigid footing
(a) Distribution of upthrust across base of rigid footing on elastic foundation.
(b) Concentration of upthrust near end of rigid footing.
Rigid footing
(c) Probable distribution of upthrust on base of rigid footing on sand.

Fig. 22

footing than for dry sand, B was about $\frac{1}{2}$ in. deeper than in Fig. 20. Fig. 23b shows the force polygon for ABC, the forces on AB being calculated from the known experimental failing load. It will be seen that the direction of the reaction on BC given by the force polygon is again reasonably possible. Thus, if the stresses in the sand are in accordance with the elastic theory the wedge ABC should be on the point of sliding just as in the case of the footing on dry sand.

Although the validity of the assumed mechanism of failure could only be justified by calculations based on the false assumption that the stresses in the sand are as given by the elastic theory, there appears to be no reason why it should not be equally valid when the

stresses are somewhat modified by some areas of the sand behaving plastically. Indeed, in view of the strong experimental evidence there appears to be little doubt that the lines of flow for the sand shown in Figs. 18 and 19 are the correct ones.

It would not be justifiable to assume that the failure surface would be geometrically similar for all widths of footing and sand densities, so no attempt was made to produce a general expression for the failing load in terms of these two variables. Calculations were made,

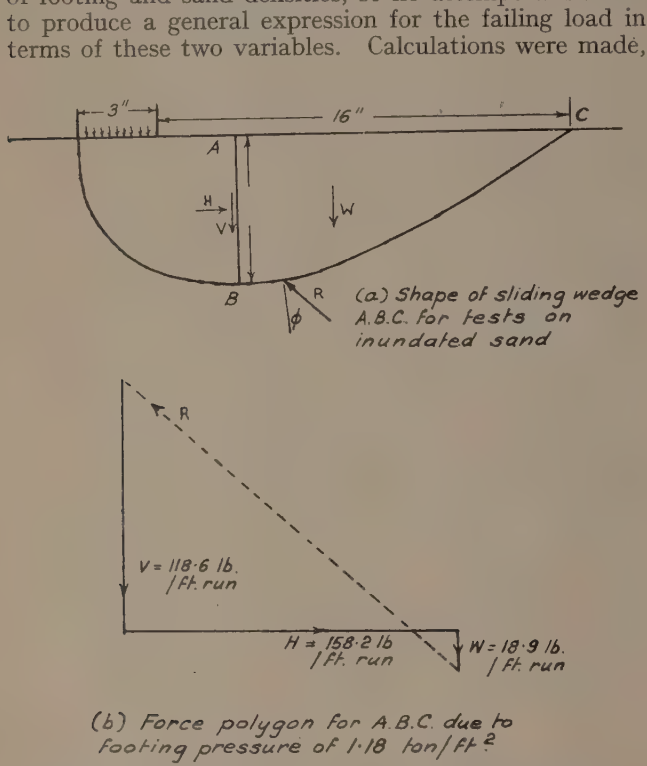


Fig. 23.—Forces on wedge ABC for inundated sand

however, to determine the failing load for 3 in. wide footings on sands with various angles of internal friction. The density of the sand was assumed to be 110 lb. per cu. ft. In these calculations the failure surface was assumed to be exactly as in Fig. 20 except that the angle ACB varied as ∅ varied, the value taken being $(45 - \varnothing/2)$. The direction of the reaction of BC was also turned through an appropriate angle to take account of the rotation of BC and the change in ∅. The magnitude of the shear and direct forces on AB was unknown since the footing load was unknown, but the ratio of one to the other and hence the direction of their resultant is constant provided that the elastic theory applies. It was therefore possible to draw the triangle of forces for the wedge ABC since the magnitude and direction of one of the forces acting on it was known, and also the direction of the other two. The magnitude of the forces on AB which would cause sliding with various values of ∅ having been determined, the load on the footing causing these forces was easily calculated.

The calculated ultimate bearing pressures for various values of ∅ are compared with those obtained by the Terzaghi, Prandtl and Krey methods in Fig. 24. For values of ∅ greater than 40° the agreement is reasonably good, but there is wide disagreement for ∅ values between 35° and 40°. This is probably due to unjustifiably extrapolating the results of the present experiments, but it is also possible that the various formulæ give values which are too low. Another investigation using a material of lower internal friction would be needed to elucidate this point.

Even if it had been possible to obtain a general expression for the failing load in terms of the sand density and internal friction, it would have been difficult to take account of the decreasing rate of increase of bearing power with footing width. For example, the ratio of the bearing power of the 4 in. footings to that of the 3 in. footings was found to be 1.20. If the bearing power had been proportional to footing width this ratio would have been 1.33. A *possible* explanation of this is that the distribution of the pressure on the underside of the two footings is not geometrically similar. For example, it is conceivable that the limiting

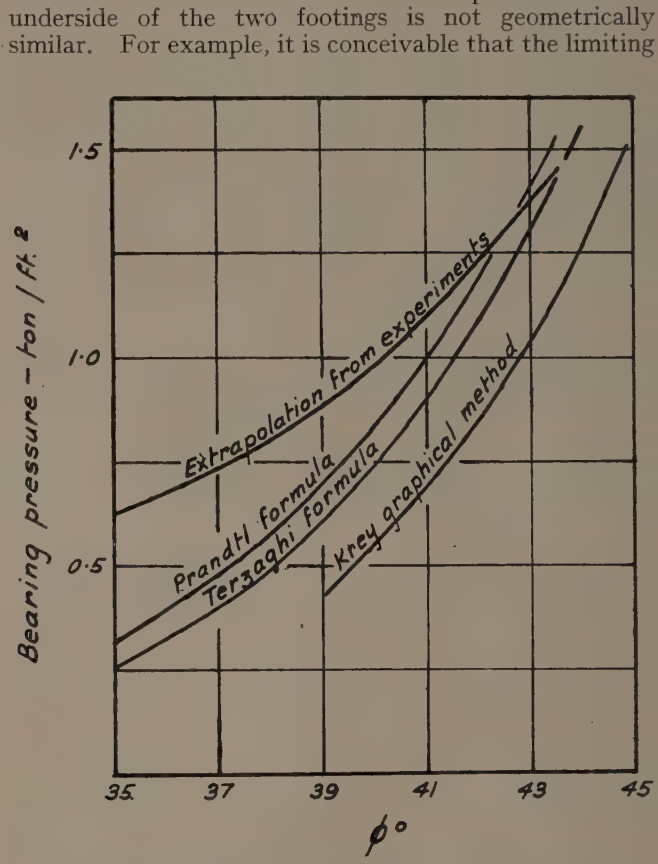


Fig. 24.—Comparison of bearing pressures for 3-in. wide footings obtained by various theories and by extrapolation from experimental results

load near the footing edge which causes yield in the region of the edge is independent of the footing width. Thus the pressure near the edges of a wide footing might be the same as for a narrow footing and only in the centre would the pressure be proportionally greater (see Fig. 25). Thus the *mean* ultimate stress would not be proportional to the footing width. While this

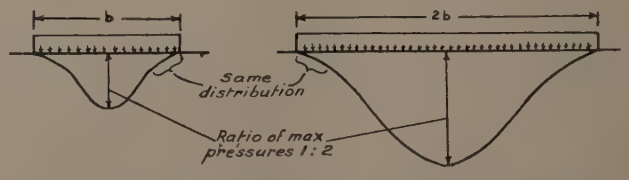


Fig. 25.—Possible upthrust distribution which would not give mean pressures proportional to breadth

is mere conjecture, it does seem that the alternative explanation put forward by Golder (that the falling off was due to a decrease in ∅ at higher pressures) did not hold in this case since ∅ only varied slightly with pressure.

Referring now to the test results for the footings of greater width than 6 in. (see Figs. 11 and 13) it will be seen that the decreasing rate of increase of ultimate footing pressure is no longer evident. This is almost certainly due to the box being too small for failure to occur without interference from its sides. Even the

6 in. footing failure surface was obviously very distorted (see Fig. 17) and with wider footings the effect was more pronounced. It seems likely that if the box had been large enough the slope of the ultimate pressure—least dimension curve would have continued to decrease, but the increasing degree of cramping prevented this. It is interesting to see that the slope of the circular footing curve continued to decrease up to the 8 in. footing size, after which it became nearly constant. It seems that partial inhibition of failure did not occur in the case of the circular footings except for diameters of about 10 in. and greater.

The ratio of the ultimate pressure on inundated sand to that on dry sand for the 6 in. footings was 82 per cent., which is similar to the ratios obtained on 3 in. and 4 in. wide footings. This figure also shows that although the ratio was higher for the 3 in. than for the 4 in. footings, there is not a general tendency for the ratio to decrease with increasing footing width.

Since Golder tested 6 in. wide footings in a box smaller than the one used in the present investigations it is doubtful whether his results have great quantitative value. His conclusion that the failing pressure for rectangular footings is independent of width is, however, borne out by the present investigation.

Conclusions

The following are the principal conclusions to be drawn from the investigation.

(1) For narrow, shallow footings on sand the ultimate bearing pressure is reduced by less than 20 per cent. if the footing is inundated. This is very different from the usually assumed value of 50 per cent.

(2) The mechanism of failure assumed by Krey, Prandtl and Terzaghi does not agree with that observed in the experiments, but for dry sand the Prandtl and Terzaghi formulæ give bearing values within 7 per cent. (lower for 3 in. footings and higher for 4 in. footings) of the experimental loads. Krey's method gives bearing values which are 25 per cent. too low. All three theories give values which are much too low for inundated sand.

(3) The movement of the sand is probably as indicated in Figs. 18 and 19 for both dry and inundated sands, and approximate calculations show that the observed bearing values were not incompatible with this mechanism.

(4) The rate of increase of ultimate bearing pressure seems to decrease with increasing footing width, but the extent of this decrease could not be gauged for the 6 in. and wider footings because the bearing pressures were affected by the sand-box being too small.

(5) Golder's conclusion that the ultimate pressure of rectangular strip footings is independent of their length is confirmed.

In the opinion of the author, the safest way to design footings on sand is to carry out a bearing test and to apply a suitable factor of safety to the result. Instead of halving the bearing pressure so obtained if the footing is likely to be inundated, it is only necessary to multiply by, say, 0.75. A similar factor should be used when designing aerodrome runways by the Golder method.

If a bearing pressure is determined by calculation using the Terzaghi, Krey or Prandtl methods a higher factor of safety is necessary than where design is based on actual bearing tests since these methods are likely to give excessively high bearing values for wide footings.

Acknowledgements

The experimental work described in this paper was carried out in the Engineering Laboratories of the University of Aberdeen. The photograph in Fig. 19 is published by kind permission of Dr. Peynircioglu, of the Istanbul Technical University, and the author is greatly indebted to Prof. J. Allen for his constant advice and encouragement, and to Dr. G. A. Taylor, who was responsible for the photographs in Figs. 15, 16 and 17. Fig. 19 is reproduced by kind permission of Prof. H. Peynircioglu.

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Book Reviews

Architects', Builders' and Civil Engineers' Reference Book. General Editor, E. Molloy. (London, Newnes, 1950.) 868 pages, 11 in. by 8½ in. 63s.

In the third edition of this valuable encyclopædia and book of reference, a new arrangement has been made and there are now twenty-two sections, which include chapters on basic building materials, structural steelwork and piling, reinforced concrete and brickwork, contractors' plant, formwork and scaffolding and lighting, water supply and sewage disposal.

All the technical articles have been brought up-to-date, and a number of entirely new articles have been added.

The book has been well indexed and contains many excellent illustrations, and is strongly bound.

Applied Mechanics : Definitions and Formulae for Students, 2nd Edition, by E. H. Lewitt (London, Pitman, 1950.) 44 pages, 5¾ in. by 4¼ in. 1s. (paper cover).

This booklet contains definitions and formulæ for motion, force and energy, strength of materials, structures, machines, and mechanics of fluids. New work on beams has been added to the section on strength of materials, and a strain-energy method of solution for redundant frames has been added to the structures section. The former section on Hydraulics has been renamed Mechanics of Fluids and a considerable amount of new matter has been included.

There have been twelve impressions of the first edition since publication in 1929, showing how useful the booklet has been found.

A Fundamental Approach to Prestressed Concrete Design*

By J. W. H. King, M.Sc., A.M.I.C.E., A.M.I.Struct.E.

Introduction

It has appeared to the author for some considerable time that a reluctance on the part of designers to depart from the well implanted formula $\frac{M}{I} = \frac{f}{y}$, and the modular ratio concept of reinforced concrete design was leading them into considerable difficulties and perplexities when tackling prestressed concrete work. As a result one frequently hears the questions, " How do you find 'I' for a prestressed beam? "—" What is the modular ratio for a concrete stressed so highly as to have considerable plastic deformation? "—" How can one deal with composite construction where part of the work is prestressed and part cast in situ, the concretes being of different qualities? "—etc. More and more elaborate formulæ appear daily, which try to weigh the imponderables involved, and " guestimation " plays a large part in much design work as a consequence, the errors being safely masked by that ever to be cherished factor of " ignorance."

The work which follows is an attempt to lead the subject back to fundamentals, when it will be seen that most of the perplexities about how to find the answer will disappear, provided the properties of the materials used are known, or can be closely estimated. All that is required is a stress strain curve for each grade of concrete used, and for the steel, the concrete curves being required for instantaneous deformation only for most problems, since they give conservative designs, but being advisable on a long-term basis also (i.e., including full plastic deformation or creep) for consideration of some aspects of the work. Given this information (without which design by any method must be purely guesswork) the only laws to be followed are the fundamentals of all bending theories, viz. :—

1. That total compression on a cross-section—total tension on the same section = externally applied thrust (= zero in pure bending problems).

2. That the moment of these two about any point = the externally applied moment at the same point.

3. That plane sections remain plane after bending—a fact abundantly confirmed by experiment except in such cases as have freak re-distribution of strain due to local failure. It is not implied that stress variation across the section is therefore linear. This is the case, of course, for materials being worked within the range where their stress-strain curve is linear. In the main this is not so in prestressed concrete work, where both concrete and steel are highly stressed.

No attempt will be made in what follows to deal with the practical details of prestressed concrete construction, since there is an abundant literature dealing with this already, and only a simple approach to the basic design principles will be attempted, by a method which it is shown can easily deal with such problems as creep, non-linear stress-strain relationships, composite construction, hysteresis, and the possibility or otherwise of developing tensile stress (as distinct from strain) in concrete. The significance of errors in one or two cases are also demonstrated, such as the case of too low a concrete strain, or of faulty tensioning, and the principles of a logical approach to " non-bonded " construction are also indicated.

The following terms are used in the text, and their significance should be noted. " Pre-stressed concrete " is any concrete in which the steel has been placed in tension relative to the concrete adjacent to it. Where the steel has been tensioned against permanent anchorages before the concrete has been placed in intimate contact round it, and has subsequently been released from those anchorages after developing full bond with the hardened concrete, it is said to be " pre-tensioned." Where the tensioning is done against the already hardened concrete (for instance when placed freely in a longitudinal cavity in a beam, then tensioned by jacks against the ends of the beam and subsequently wedged) it is said to be " post-tensioned." Such " post-tensioned " work may be left with the steel free except at the wedged or anchored ends, when it is " non-bonded," but it is usually " bonded " by introducing a liquid mortar into the cavity under pressure, which when set, ensures that steel and concrete strain together at any particular section. The stage at which concrete first carries the load due to steel tension is spoken of as " transfer." It is, of course, the same time as tensioning in the case of post-tensioned work, and the time of release of steel from permanent anchorages in the case of pre-tensioned work.

Fundamental Principles

Suppose that a concrete has stress strain characteristics as Fig. 1. At some stage of loading and steel stressing, a beam having a cross-section as Fig. 2 (i) might have a strain distribution as Fig. 2 (iii). Its stress distribution would then be a replica of the appropriate part of Fig. 1, between stresses c_1 and c_2 corresponding to strains e_1 and e_2, and would be non-linear in the instance given. The strain at steel level, e_3, is easily found from the fact that the strain distribution *is* linear.

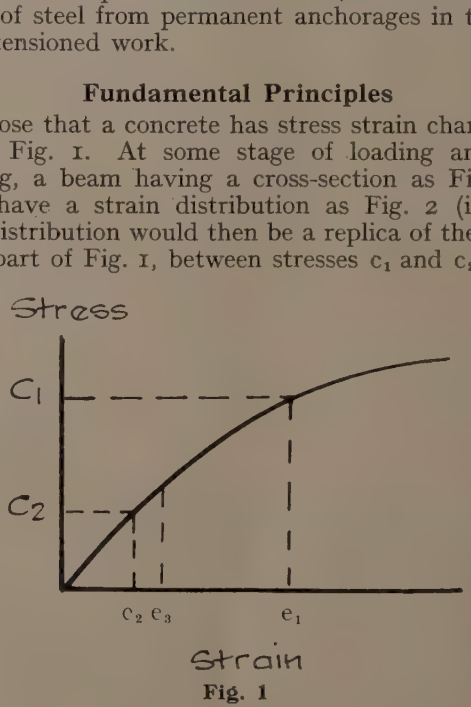

Fig. 1

At such a stage, the total compression being developed " C " is clearly equal to b x the area of diagram 2 (ii),

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nd for pure bending is equal to the total steel tension T." The bending moment being developed is thus C or T) × (a) where " a " is the distance from steel to he centre of area of Fig. 2 (ii), or the " compression " entre. At another stage, say at " transfer," the stress

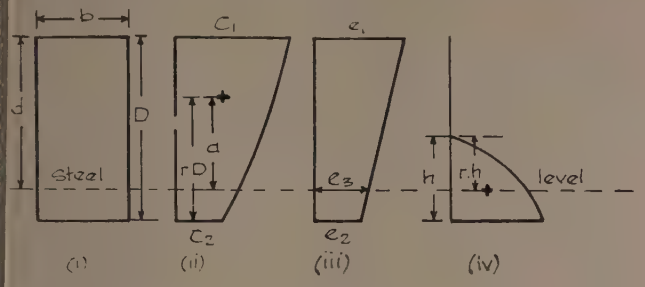


Fig. 2

istribution might be as in Fig. 2 (iv). Here it has been ssumed that concrete, although developing tensile strain, does so by developing numerous hair cracks, and herefore does not develop tensile stress. At this stage, herefore, since there is assumed to be no external moment, the centre of compression must be on the steel centre line, whilst the total values of C and T must again be equal, although not, in general, the same as for Fig. 2 (ii).

The connection between the two stages is provided by the fact that for bonded work or for non-bonded work where the section and external moment are both constant throughout the beam, the steel at any point and the locally adjacent concrete must strain by the same amount between any two stages. (The general non-bonded case is dealt with in Example 12.) Provided these fundamentals are adhered to, no further principles are required, and pre-stressed beam design becomes a simple case of watching that C = T, aC = M and steel and concrete strain together, where bonded together. It is of course necessary to see that the stresses involved are not beyond the strength of the material at all stages, but within this limitation there is no restriction placed on beam size, neutral axis position at any loading stage first considered, steel area, steel stress, etc. In general it would be usual to arrange that c_1 was equal to the ultimate stress of the concrete in the beam, and " t " the steel stress as high as practicable at the bending moment given by a " factored " load. This stage is referred to below either as " at failure " or at " ultimate load," or " ultimate moment." It will be seen (Examples 1 and 2) that it is not always practicable to stress the steel to its ultimate, this being possible only where the neutral axis at failure of the beam is very high, owing to the great extensibility compared with concrete, of even the work-hardened high tensile steels usually used in pre-stressed work. This naturally means under-reinforced beams, and heavy designs. It will usually be found better to work on a lower maximum stress in steel, say the .2 per cent. proof stress, the yield stress (where present), or, say, 75 per cent. of the ultimate strength.

Various examples of application of these principles are given below. In cases where it is necessary to find the area of a stress diagram which is not linear, it has been found from the well-known approximation that if the ordinates of a continuous curve at x = 0, x = $\frac{L}{2}$ and x = L are respectively a, m and b, the area between x = O and x = L is given by

$$\text{Area} = \frac{L}{6}(a + 4m + b)$$

A further approximation is also used to find the proportional distance " r " of the centre of area from the x = O point,

$$\text{viz.: } r = (b + 2m)/(a + b + 4m).$$

More rigid methods are not usually warranted, but can of course be employed if necessary.

For all examples, the stress strain curves for steel and concrete are taken to be those of Figs. 3 and 4. The

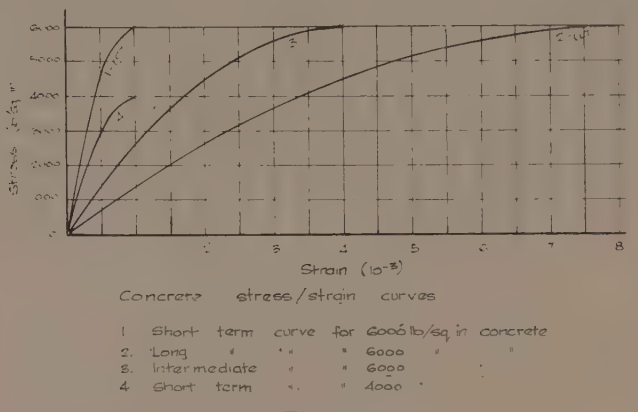

Strain (10⁻³)
Concrete stress/strain curves
1. Short term curve for 6000 lb/sq in concrete
2. Long " " " 6000 " "
3. Intermediate " " 6000 "
4. Short term " " 4000 "

Fig. 3

steel curve is that of general strain, not the localised value present at the necking close to the fracture, the two curves being the same up to a stress in the region of 200,000 p.s.i. for the steel used in this instance. The

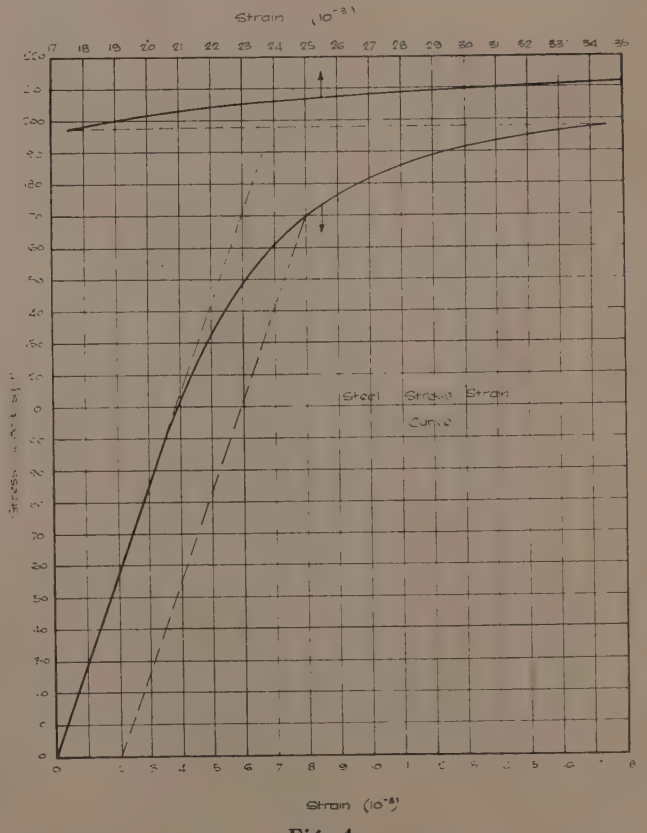


Fig. 4

concrete curve marked (ST) is that assumed for sudden loading, where no plastic flow can occur. It is difficult to obtain in practice, since creep is immediately present, and has had to be estimated bearing in mind that lower strains were the ones leading to safer designs, and that concrete of a strength better than that specified, or of a greater age, would have lower strains than the true values for 6,000 p.s.i. concrete at 28 days. The (LT)

curve is a reasonable one for such a concrete under long-sustained loading, being nearly approached by increasing normal laboratory test strain after about two hours, by some 25 per cent. The significance of errors in these values will appear in the examples below.

For the first example, certain simplifying assumptions are made which are either dealt with particularly in subsequent examples, or whose effect is at once appreciated after dealing with the particular cases below. These initial assumptions are :—

1. Steel is bonded to concrete between transfer and ultimate load, the beam being post-tensioned, after all concrete shrinkage has taken place, and being of constant cross-section.

2. The moment caused by self weight of the beam on tensioning is ignored.

3. Tensioning of the steel, and application of a load such as would cause failure are both assumed to be rapid, so that, for safety, the ST curve is used.

4. Concrete maximum stress at failure to be 6,000 p.s.i. whilst that in steel at the same time = 75 per cent. of the ultimate, i.e., 160,000 p.s.i.

5. Concrete tensile stresses are assumed non-existent, although strains may be present (as in normal RC design).

Example 1.

A beam is required 6 in. wide, to take a working moment of 537,600 lb., with a safety factor of 2.5. In order to lighten construction, it is desired to have the neutral axis at failure at .8 of the effective depth (i.e., n = .8d), and a rather high concrete prestress of 3,750 p.s.i. will be permitted.

At failure, concrete stress distribution will be as Fig. 5 (i), therefore total compression (referring to Fig. 3 for values = $6n. \frac{1}{6}(0 + 4.4500 + 6000) = 24000n = 19200d$.

also $r = \frac{6000 + 2.4500}{24000} = \frac{5}{8}$

Hence " a " = .2d. + 5/8 (.8d) = .7d.

Total moment = .7d. × 19200d = 2.5 × 537600 = 1344000

Hence d = 10 in.

Area of steel needed = $\frac{19200 \times 10}{160000}$ = 1.2 sq. in.

Concrete maximum strain at top of beam (ST curve) = .001.

therefore strain at steel level = $\frac{2}{8} \times .001$ = .00025

At transfer (Fig. 5) (iii)—Referring to Fig. 3 for values.

Total compression = $\frac{6h}{6}(0 + 3750 + 4.2100)$ = 12150 h

and $r = \frac{3750 + 4200}{12150} = .65$

Concrete strain at bottom of beam (ST) = .0004

therefore at steel level = .65 × .0004 = .00026

Hence change in steel strain between transfer and failure of beam = .00025 + .00026 = .00051

Steel strain at beam failure (Fig. 4) for 160,000 p.s.i. = .00685

Hence strain after transfer = .00685 — .00051 = .00634.

corresponding to a stress of 154,500 p.s.i. and a load of 185,400 lb.

Hence we have 12150 h = 185400, or h = 15.25 in

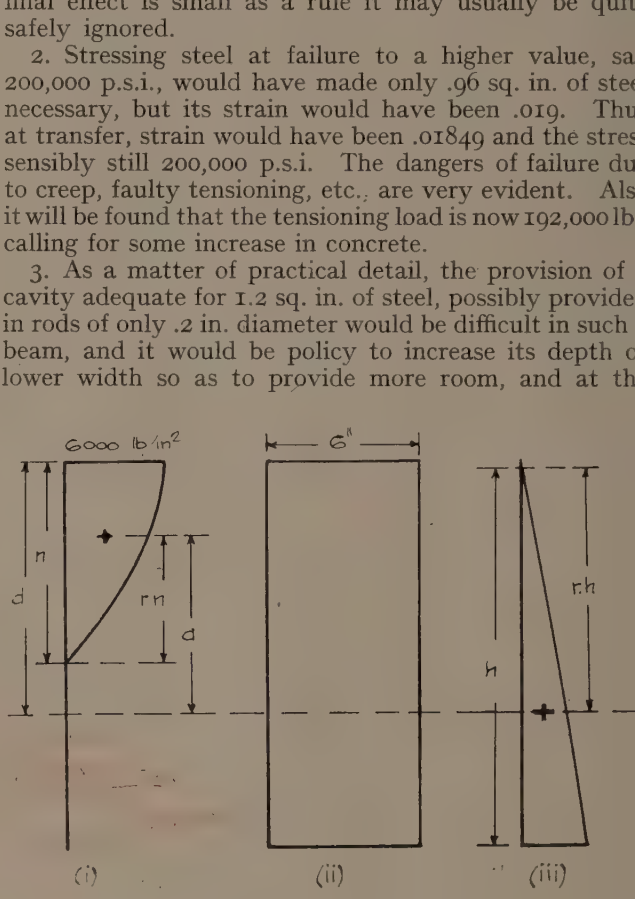
Of this, .65 h is above the steel (= 9.9 in., which i < 10 in. and possible) and 5.35 in. below the steel.

Hence the beam should be 6 in. × 15.35 in. wit 1.2 sq. in. of steel 10 in. down, tensioned to 154,50 p.s.i.

N.B. 1. Strictly an allowance should have been mad for the cavity left for the steel at tensioning. As th final effect is small as a rule it may usually be quit safely ignored.

2. Stressing steel at failure to a higher value, sa 200,000 p.s.i., would have made only .96 sq. in. of ste necessary, but its strain would have been .019. Thu at transfer, strain would have been .01849 and the stres sensibly still 200,000 p.s.i. The dangers of failure du to creep, faulty tensioning, etc., are very evident. Als it will be found that the tensioning load is now 192,000 lb calling for some increase in concrete.

3. As a matter of practical detail, the provision of cavity adequate for 1.2 sq. in. of steel, possibly provide in rods of only .2 in. diameter would be difficult in such beam, and it would be policy to increase its depth o lower width so as to provide more room, and at th



Beam section and stresses for Example 1.

Fig. 5

same time cut down the pre-compression in the concrete which is higher than many designers will allow. Les steel at the expense of more concrete is also possible b decreasing the ratio of n to d. This is done in Example 2 where concrete stress at tensioning is also made lowe than that used above.

Example 2.

Modify the design of Example 1, so that the neutra axis is relatively higher at failure (n = .4d), and concret stress at transfer is not greater than 2200 p.s.i.

At failure. Total compression = 24000 n = 9600 d and lever arm = .6d + 5/8 (.4d) = .85d

Hence 9600 × .85d² = 1,344,000 or d = 12.8 in.

Steel required = $\frac{9600 \times 12.8}{160000}$ = .768 sq. in.

Concrete strain at steel level = $\frac{6}{4} \times .001$ = .001

t transfer. Compression = h (0 + 2200 + 4.1125) = 6700 h
and r = (2200 + 2250)/6700 = .662
whilst maximum concrete ST strain = .00021.
therefore Strain at steel level = .00021 × .662 = .00014
therefore Change in steel strain = .0015 + .00014 = .00164.
But Failure strain = .00685, therefore Tensioning strain = .00521 corresponding to a stress of 138,000 p.s.i. and a total tension of 106,000 lb.

$$\text{Hence } h = \frac{106000}{6700} = 15.8 \text{ in.}$$

Depth below steel = 15.8 × (1 — .662) = 5.35 in.
Hence total depth = 12.8 + 5.35 = 18.15 in.
Hence beam is 6 in. × 18.15 in. with .768 sq. in. of steel at 12.8 in. depth, tensioned to 138,000 p.s.i.

N.B. Trial of a steel stress of 200,000 p.s.i. at failure in this case would still show a need for a prestress of 98,500 p.s.i.; too high a value for practice. It is possible to increase somewhat above 160,000 p.s.i. if desired, with consequent steel economy. This is left as an exercise for the reader.

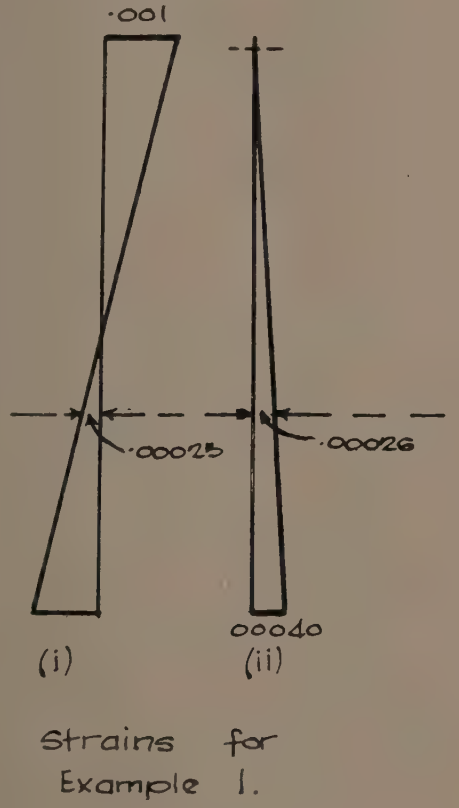


Fig. 6

Example 3. *Effect on design of slow tensioning.*

In the case of post-tensioned beams by the Magnel system for instance, the tensioning operation is not sudden, but takes several hours. For such a case, the ST curve is clearly not correct on tensioning, an intermediate curve having strains equal to half the LT values being about right. The effect of using this curve for tensioning the beam of Example 2 is examined herewith. (NB Strains are 4 times those of the ST curve.)

At failure. Conditions are as Example 2, and those figures apply.

At transfer. Strain at steel level now = .00056.
Hence total change = .00150 + .00056 = .00206.
therefore Steel pre-tension strain = .00685 —.00206 = .00479 corresponding to 130,000 p.s.i. or a load of 100,000 lb.

$$\text{Hence } h = \frac{100000}{6700} = 15 \text{ in., and depth below steel} = 5.09 \text{ in.}$$

therefore Total depth of beam = 17.89 in. as against 18.15 in., and Prestress in steel = 130,000 p.s.i. as against 138,000 p.s.i., not a very great difference.

N.B. A similar calculation applied to Example 1 would show an economy in depth to 14.94 in. instead of 15.35 in., and a reduction of steel prestress from 154,500 p.s.i. to 142,500 p.s.i. — a rather larger economy but not a very serious matter.

Example 4. *Effect of a lower concrete "modulus" throughout the design.*

The trend will clearly be towards economy as in Example 3. For instance if the ST strains were twice as large in Example 2, the total steel strain change becomes .00328, giving a tensioning strain of .00357, a stress of 101,000 p.s.i. and a load of 78,000 lb. The required value of "h" is then 11.6 in. with depth below steel 3.93 in., and total depth 16.73 in., i.e., beam is 16.73 in. deep and steel prestress is 101,000 p.s.i., compared with 18.15 in. deep and steel prestress of 138,000 p.s.i.—a saving of nearly 8 per cent. of concrete.

Example 5. *Modification for pre-tensioned work and concrete shrinkage.*

In pre-tensioned work, during the stage before transfer, the concrete, bonded to the steel, endeavours to shrink on maturing. This it cannot do because of its bond to the steel, and it is therefore strained in tension immediately before transfer. Since moreover transfer is normally at an early age, this shrinkage is not completed at that stage, and further shrinkage takes place between transfer and failure. Typical figures for these two "strains" are .0003 and .0002 respectively.

Using these values, Example 2 would require modification as follows :—

Steel strain change between transfer and ultimate must be decreased by .0002 to .00144, requiring a strain in steel after transfer of .00541. As a result the stress in steel becomes 142,000 p.s.i. instead of 138,000 p.s.i., and we find "h" must now be 16.3 in., and D = 18.3 in.

Initial tension required must also take into account the earlier shrinkage of .0003, and hence the strain must be .00571, giving a stress before transfer of 146,000 p.s.i.

Example 6. *Allowance for tensile stress in concrete.*

In the author's opinion it is a debatable point whether concrete in practice can ever be relied on to take tensile stresses when subjected to the normal hazards of everyday use. As will be seen below, the effect of allowing for them in design is likely to be small, whilst the calculations are made somewhat more difficult, although not unduly so, were reliability and economy ensured.

The changes required are (a) that the concrete stress strain curve should be continued into the tensile zone, and (b) that tensile stresses are allowed for in the same way as compressive stresses. The effect is to reduce the total concrete compression, and to move the centre of compression further from the neutral axis, as compared with the case of compression only, with the same maximum stress and neutral axis. It is generally

agreed that failure or ultimate conditions should be worked out on the basis of a "cracked" section, and it is therefore only at transfer that any amendment of the calculations arises.

Assume that the tensile stress strain curve is similar to the ST compression curve, but with stresses and strains both reduced to $\frac{1}{10}$, i.e., that failure occurs at 600 p.s.i., with a strain of .0001, and consider the effect on Example 2.

Assume for the moment that the concrete tensile zone falls entirely within the beam depth, and stops abruptly at a stress of 600 p.s.i. Then we have on the compression side, strains vary from .00021 to zero, and on the tension side up to .0001 where stressed.

Hence length of tensile zone $= \frac{10}{21} \times h = .475\ h.$

Centre of tension will be at 5/8 of this from the Neutral Axis $= .2975\ h$

and total tension $= .475 \times \frac{2}{3} \times 600 \times 6\ h = 1140\ h.$

Hence resultant compression $= 6700\ h - 1140\ h = 5560\ h.$

And position of centre of compression for all concrete stresses will be distant from the NA; $h\ (1140 \times .2975 + .662 \times 6{,}700)/5{,}360. = .86h.$

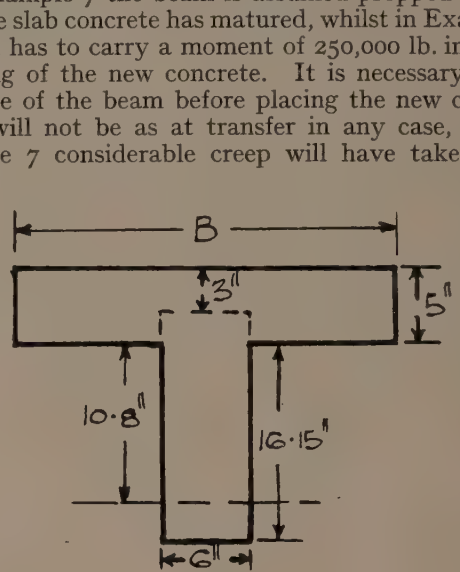
It appears at first sight as if the effect of this will be great. However, two facts emerge. Firstly concrete strain at steel level $= .86 \times .00021 = .00018$, which compares with .00014 originally. The effect on the steel stress is to change it to 136,000 p.s.i. instead of the original 138,000 p.s.i.—a small matter. It will furthermore be found that an attempt to line up the steel and the centre of compression with a total depth above steel of $.475\ h + .86\ h = 1.335\ h$, and h this time $= \frac{104{,}000}{5{,}560} = 18.7$ in., is not possible. Hence concrete tension must be much less than 600 p.s.i. As a rough guide, assume the NA stays at 2.35 in. from the top as at tensioning example 2. Concrete tensile strain is then $\frac{2.35}{15.8} \times .00021 = .0000313$, corresponding to a stress of about 315 p.s.i. and a total load of about 2,220 lb., which has but a small effect on the original total compression of 106,000 lb. As actually the NA will move upwards, the effect is clearly even smaller, its precise value needing to be found by trial and error, assuming a NA position as a starting point as in the manner of Examples 7 and 8 below. It is simpler and safer to rely only on the method of Example 2, and to ignore tensile stresses in concrete.

Examples 7, 8 and 9. Composite Construction.

In this type of work, prestressed beams may be embodied in cast in situ floors, thus forming T beams in which steel, some of the slab, and all of the rib are stressed in some manner either at no load where beams are propped, or at a low loading where they are not propped, whilst at the same time the main slab, owing to shrinkage on hardening, develops tensile stresses, or rather tensile strains, as we shall again ignore stresses of this type. Furthermore, the slab concrete is often of lower strength than the original beam concrete, and has different stress strain characteristics. The following three examples deal with cases where the beam is, and is not propped, and also where the original beam does, and does not develop stress at ultimate load. The cast in situ concrete is assumed to have characteristics obtainable from the curves of Fig. 3, by taking 2/3 of the stress value at any given strain, i.e., its short-term curve shows a strain of .001 at 4,000 p.s.i. The beam to be used is that obtained in Example 2, whilst the shrinkage figure again totals .0005.

The composite beam will be taken as Fig. 7, B having a value of 36 in. in Examples 7 and 8, and 12 in. in Example 9.

In Example 7 the beam is assumed propped securely until the slab concrete has matured, whilst in Examples 8 and 9 it has to carry a moment of 250,000 lb. in. during maturing of the new concrete. It is necessary to find the state of the beam before placing the new concrete which will not be as at transfer in any case, since in Example 7 considerable creep will have taken place



Composite Beam

Fig. 7

whilst in 8 and 9 it is loaded. The method of doing this by trial and error is shown. In general it is sufficient to form a rough estimate of a neutral axis position in the beam, and to work out values of compression and tension. If these do not balance, they are used as a guide to a new NA position, which is nearer the truth. In general a plot of T and C against NA position then gives the correct values for all these points by the intersection of the T and C curves, usually assumed straight.

Example 7. Propped case with slab 36 in. wide. Initial state of PSC beam.

Having presumably been stressed for some time before being embodied in the floor, the concrete will have "crept" and stresses in steel and concrete will have decreased, whilst strains in concrete will have increased the LT curve being now the correct one.

It will be seen from Example 2 that the steel strain at zero local concrete strain $= .00521 + .00014 = .00535$. This will of course continue to be the case. As a trial assume the maximum concrete strain has reached about 7/8 of the LT value at the original stress say .0015, corresponding to a stress of 2,000 p.s.i. As the concrete stress is low, we can for practical purposes assume a straight line diagram at this stage, so that we have $h = 3 \times \frac{5.35}{6} = 16.05$ in., and total compression $= 2{,}000 \times \frac{6}{2} \times 16.05 = 96{,}300$ lb.

Concrete strain at steel level $= 2/3 \times .0015 = .001$

therefore Steel strain $= .00435$; stress $= 122{,}000$ p.s.i. and load $= 93{,}500$ lb.

Clearly the strain in concrete is lower.

Trying .00146 we get maximum concrete stress = 1,950 p.s.i. and total load = 94,000 lb. Steel strain =

00438 this time, giving a stress of 122,500 p.s.i. and a oad of 94,000 lb. Hence initial beam conditions are : no noment, top strain

$= \frac{-2.1}{16.05} \times .00146 = -.00019$, bottom strain = .00146, steel level strain = .00097 and steel strain = .00438. At the same time slab strain, everywhere = —.0005.

Possibilities of failure are : (1) Top of slab reaches 4,000 p.s.i. ; (2) Top of original beam reaches 6,000 p.s.i., and (3) Steel reaches 212,000 p.s.i. In general a very wide slab will fail by (3), a moderately wide one by (1), and a narrow one by (2), but all should be kept in view.

Assume in this case that failure will be by (1) or (3). The latter involves a total tension of .768 × 212,000 lb. = 163,000 lb.

Hence try NA depth in slab so as to develop this amount, i.e., make $36 \times \frac{n}{6} (0 + 4.3000 + 4000) =$

$36 \times \frac{2}{3} \times 4000\ n = 163000$, i.e., assume depth =

$(163000) \div (\frac{2}{3} \times 36 \times 4000) = 1.7$ in.

We then have, working now on the ST curves :—

Strain at slab top = .001 ∴ Strain change from initial = .0015

Strain at slab NA = 0 ∴ Strain change from initial = .0005

Hence depth of plane of no change in strain

$= \frac{15}{15-5} \times 1.7 = 2.55$ in.

∴ Strain change at top of PSC beam

$= \frac{3-2.55}{2.55} \times .0015 = .000265$ tensile.

∴ Strain in beam top = (.000265 + .00019) tensile

Strain change at beam bottom $= -\frac{18.6}{2.55} \times .0015 =$ — 0.0109

∴ Strain at beam bottom = .00146 —.0109—again tensile.

Hence original beam concrete can be neglected.

Strain change at steel level $= \frac{13.25}{2.55} \times .0015 =$ .0078 tensile.

∴ Steel strain = .0078 + .00438 = .01218.

giving a stress of 189,000 p.s.i. and a load of 145,000 lb. compared to compression of 163,000 lb.

Hence NA in slab must be higher.

Try 1.5 in. from slab top.

Total compression $= 36 \times 1.5 \times \frac{2}{3} \times 4000 =$ 144,000 lb.

Concrete strain change at slab top is again = +.0015.

Concrete strain change at 1.5 in. down (NA position) is .0005.

Hence depth of plane of no strain change = 2.25 in.

Strain change in beam top $= \frac{.75}{2.25} \times .0015 =$ .0005 tensile.

∴ Strain in beam top = —.00069, i.e., tensile.

Strain in beam bottom is also again tensile.

∴ Beam ignored.

Strain change at steel level $= \frac{13.55}{2.25} \times .0015 =$ .00903 tensile.

∴ Total steel strain = .00438 + .00903 = .01341 tensile, corresponding to a stress of 192,000 p.s.i. and a load of 147,000 lb.

Plotting as straight lines the values of compression and tension against NA position gives an intersection at 146,600 lb. and 1.53 in.

Hence ultimate moment = $146600 (15.8 - \frac{3}{8} \times 1.53) = 146600 \times 15.23 = 2{,}245{,}000$ lb. in., and failure is clearly by mode (1).

Example 8. As Example 7 but with beam not propped, and subjected to a moment of 250,000 lb. in.

The PSC beam will be under this moment for several hours before the new concrete has set and hardened considerably, hence we can assume the intermediate curve used in Example 3 to get the beam condition corresponding to the shrinkage strain of .0005 in the slab.

As a first estimate, assume steel stress in the beam at this stage = 150,000 p.s.i., corresponding to a total load of 115,000 lb.

To develop a moment of 250,000 lb. in., the lever arm must be 2.18 in., giving the centre of compression 7.53 in. from the beam bottom.

Clearly the concrete stress will be fairly low, and moderately uniform so that a straight line distribution may be assumed with considerable accuracy. If we let stresses in concrete at beam top and bottom be 'a' and 'b' respectively, we thus have,

Total compression $= \frac{a+b}{2} \times 6 \times 18.15$

$= 115000 \quad \therefore a + b = 2110$

And $r = \frac{2a+b}{3a+3b} = \frac{7.53}{18.15} \therefore \frac{a}{b} = \frac{1.48}{4.57}$

Trial values of a and b are thus 515 and 1591 p.s.i.

Stress at steel level is then $\frac{1}{18.15} (5.35 \times 515 + 12.8 \times 1595) = 1280$ p.s.i.

∴ Concrete strain at steel level (intermediate curve) = .0005.

But steel strain at no concrete strain = .00535 (as in Example 7).

∴ Total steel strain = .00485, corresponding to stress of 132,000 p.s.i., which is lower than the initially assumed 150,000 p.s.i.

Hence the lever arm must be increased, and total compression and tension decreased.

Assume steel stress = 133,000 p.s.i., giving total load 102,000 lb.

Lever arm is then 2.45 in., i.e., 7.8 in. up from the bottom.

Hence $\frac{2a+b}{3(a+b)} = \frac{7.8}{18.15}$ giving $\frac{a}{b} = \frac{1.75}{4.30}$

and a + b = (102000)/(3 × 18.15) = 1880 ∴ a = 540 and b = 1,340 p.s.i.

Concrete stress at steel level = (5.35 × 540 + 12.8 × 1340)/18.15 = 1,100 p.s.i.

∴ Concrete strain = .0004, steel strain = .00495, steel stress = 133,000 p.s.i. which is as assumed.

Hence position when slab concrete has hardened is :—

Beam concrete strain at top = .00020 — (stress 540 on intermediate curve)

Beam concrete strain at bottom = .00052 — (stress 1340 on intermediate curve)

Steel strain = .00495, and strain in slab = —.0005 throughout.

N.B. In actual fact this state of affairs would induce a larger BM in the beam, but as this has no effect on the calculations for ultimate load, there is no point in dealing with it here.

At failure. Concrete strain change in slab top = .0015 as Example 7. Assuming slab NA at 1.6 in. down, strain change there = .0005,

Hence plane of no change is 2.4 in. down.

$$\text{Strain change at beam top} = \frac{.6}{2.4} \times .0015 = .000375 \text{ tensile.}$$

∴ Total strain there is .0002 — .000375 = .000175 tensile. Similarly bottom strain is also tensile, therefore beam concrete can be neglected.

$$\text{Strain change at steel level} = \frac{13.4}{2.4} \times .0015 = .00844.$$

∴ Steel strain = .00844 + .00495 = .01339.

∴ Stress = 192,000 p.s.i. and load = 147,000 lb.

$$\text{Concrete compression} = 1.6 \times 36 \times \frac{2}{3} \times 4000 = 154{,}000 \text{ lb.}$$

Hence NA assumed too low.

Try at 1.5 in. down.

Referring to Example 7 it will be seen that steel strain change = .00903 for this case. Hence total strain = .01398, stress = 193,000 p.s.i., and load = 148,000 lb., whilst concrete compression = 144,000 lb. as in Example 7.

Plotting these two trial cases gives the correct solution as NA at 1.5375 in. down with load of 147,600 lb.

∴ Ultimate moment = 147,600 $(15.8 - \frac{3}{8} \times 1.5375)$

= 147,600 (15.225)

= 2,250,000 lb. in.

virtually the same value as with the propped beam in Example 7. This is due to the fact that large strain changes in steel at the high stresses involved cause little change in the stresses, so that the effect of propping is completely masked, by the time ultimate load is reached. This would usually be the case with wide slabs.

Example 9. The case of a lower slab neutral axis, owing to the use of a smaller slab width.

For this example, B will be taken as 12 in., and the unpropped case will be considered, carrying the same moment of 250,000 lb. in. during construction of the slab.

As a trial assume the slab NA is 4.0 in. down.

Then, as in Examples 7 and 8, we shall find the plane of no strain change is at 6 in. down, whilst the top strains .0015.

Hence at beam top, strain change = .00075 (compressive)

at beam bottom strain change

$$= \frac{15.15}{6} \times .0015 = .00379 \text{ tensile}$$

and at steel level strain change

$$= \frac{9.8}{6} \times .0015 = .00245 \text{ tensile}$$

∴ Strain in beam at top = .00075 + .0002 = .00095

,, ,, bottom = .00052 — .00379 = —.00327

,, ,, steel = .00495 + .00245 = .00740

Hence steel maximum stress is 165,500 p.s.i., and maximum load 128,000 lb., whilst the beam has a maximum stress of practically 6,000 p.s.i. (ST), and has a neutral axis of its own at a depth of $\frac{95}{95 + 327} \times$ 18.15 in. = 4.08 in. in the beam.

$$\text{Total compression} = 12 \times 4 \times \frac{2}{3} \times 4000 + 4.08 \times \frac{6}{6} (6000 + 4.4300)$$

$$- 6 \times \frac{1}{6} (1800 + 4.900) \text{ lb.,}$$

(i.e., complete slab 12 × 4, + beam 6 × 4.08 — 6 × 1 common to both, and stressed at 1,800 p.s.i. at its top and 900 p.s.i. at its mid depth)

= 128,000 + 95,000 — 5400 = 217,600 lb.

which is far too high.

N.B. It should be noted that this case is approaching the case of failure by beam concrete before slab concrete, i.e., method (2).

Taking the slab at N.A. 3 in. down we get

Plane of no strain change is at 4½ in. down, whilst strain change at slab top = .0015 as above

Hence total strain in beam top = .0002 + .0005 = .0007 (corresponding to ST stress of 5,450 p.s.i.)

and total strain in beam bottom

$$= .00052 - \frac{16.65}{4.5} \times .0015$$

$$= .00052 - .00555$$

$$= - .00503$$

Hence beam has its own NA at a depth of

$$\frac{70}{573} \times 18.15 \text{ in.} = 2.21 \text{ in.}$$

$$\text{Total compression} = 12 \times 3 \times \frac{2}{3} \times 4000 + 2.21 \times \frac{6}{6} (5450 + 4.3450) = 96{,}000 + 42{,}500 = 138{,}500 \text{ lb.}$$

$$\text{Strain change in steel} = \frac{11.3}{4.5} \times .0015 = .00377$$

∴ Steel strain = .00495 + .00377 = .00872 corresponding to a stress of 175,000 p.s.i. and load of 134,000 lb.

Plotting will give the correct values as

slab NA depth = 2.95 in. Beam NA depth 2.12 in.

and beam load (compressive) = 40,000 lb.

whilst total load = 134,500 lb.

Hence very nearly, maximum moment

= 94500 $(15.5 - \frac{3}{8} . 2.95)$ + 40000 $(12.8 - .35 \times 2.12)$

= 94500 × 14.69 + 40000 × 12.06

= 1,390,000 + 482,000 = 1,872,000 lb. in.

Example 10. Effect of tensioning errors

In a practical case, steel tension is measured as a rule by noting the pressure recorded on the dial of a hydraulic jack, with a check provided by recording the extension of the steel, relative to the concrete. A little thought will show that errors of the order of 5 per cent. or more can easily arise, and some idea of their effect on the final construction is useful. In order to save a certain amount of new calculation, we may consider the case of Example 1, were the steel only tensioned to such a stage

s would make the concrete maximum stress 2,200 p.s.i. s in Example 2 instead of the intended 3,750 p.s.i.

For Example 2 stresses, r = .662, hence $h = \frac{5.35}{.338} =$ 5.75 in., and so the total load is 106,000 lb., and steel ension 88,300 p.s.i. on 1.2 sq. in., with strain of .00305 nstead of 154,000 p.s.i. and .00634 as intended. Con- rete strain at steel level will still be .00014 as in Example 2.

The actual beam state at ultimate load must usually e found by trial, but certain direct calculations will pply if steel remains within its linear range. As this will nearly be the case, here, it will be used as a first attempt at a solution.

At ultimate load, concrete compression = 24000 n (as Examples 1 and 2), and maximum strain = .001 (ST). Steel is 10 in. down.

Hence change in strain at steel level between transfer and failure $= .00014 + \frac{(10-n)}{n} \times .001$

With steel in its linear range .00035 strain = 10,000 p.s.i. stress, or 12,000 lb. load.

Hence $24000\,n = \frac{12000}{.00035}\left(.00014 + \frac{.01}{n}.001\right) + 106000$

leading to $n^2 - 3.19\,n - 14.29 = 0$ or $n = 5.7$ in.

For this value compression = $24000 \times 5.7 = 136{,}800$ lb.

whilst steel strain $= .00305 + .00014 + \frac{4.3}{5.7} \times .001 = .00394$ corresponding to a stress of 112,000 p.s.i. (just outside the linear range) and a consequently low load of 134,400 lb.

A close estimate of the correct values would be NA depth 5.6 in. and total load 134,800 lb , giving a moment of 134,800 (10 — 3/8 × 5.6) = 134,800 × 7.9 = 1,060,000 lb. in. compared with the intended 1,344,000 lb. in. of Example 1, i.e., a reduction in prestresses in steel and concrete of nearly 45 per cent. have affected the final moment by less than 20 per cent. The effect of any normal errors will thus be of no practical significance.

Broken wires would be dealt with similarly, but the steel would be working in a higher stress range at failure and the cases are not exactly parallel. It is clear, however, that a small loss in area from this cause would be in part compensated for by the higher stresses so that the percentage moment change would not be as great as the percentage loss in area of steel.

Example 11. *Beams not rectangular in section.*

Where such beams can be divided into rectangles, the obvious method is to do this, and add together the various portions of compression, and moment, derived from the separate parts, in a manner similar to that used in Examples 7, 8 and 9.*

If such division is not possible, the best thing is to plot the total stress at any level, i.e., stress × width, and to use the resulting curve for calculations, using an approximate method of calculation appropriate to the case. It will be realised that the picture will vary as the position of the Neutral Axis varies, and the process may be tedious in some cases, although not intrinsically difficult.

Example 12. *Non-bonded beams.*

On tensioning beams of uniform section, the moment is everywhere sensibly constant (except for the self weight moment of very long and heavy beams). Consequently steel strain and concrete strain at steel level are constant throughout the beam length. This has been assumed in all the post-tensioned examples worked out above. Subsequent bonding has then ensured that the same condition of equal variation of steel and concrete strain at any section persists when the moment is not uniform at the later stages.

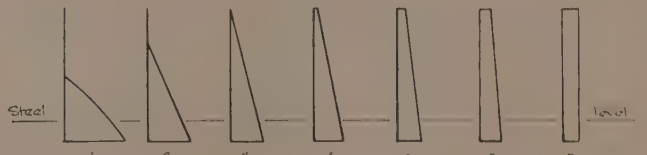
If non-bonded beams are subjected to a varying moment along their length, or are not uniform in cross section, or have a varying steel position in the cross section, the controlling conditions for cases of pure bending are :—

(1) Total tension in steel is constant throughout the beam length, and equal to total compression.

(2) Total tension or compression × lever arm = external moment, at all points.

Hence the compression diagram changes shape, but not area, at different parts of the beam, and concrete strain at steel level varies from point to point whilst steel strain is constant at all points. Change in steel strain between any two stages of loading is thus equal to change in mean concrete strain at steel level, and this has to be found. It is not, as might be thought at first, the concrete strain at the section of mean moment. It must be remembered also that the total compression can vary between stages of loading, although it cannot vary along the beam. Hence a particular moment may be arrived at in any case by a high compression and a small lever arm, or vice versa, and only a very tedious trial and error process will provide a solution in the general case.

In order to indicate the general lines on which it must proceed, Fig. 8 indicates possible stress distributions across a section 12 × 6 with steel 2 in. from the bottom, tensioned to a total load of 72,000 lb., together with the strains at top, bottom and steel level, and the resulting moment. Fig. 9 shows the plot of strains against moment. Given a particular moment diagram, it is thus possible to find the strain in concrete at steel level at all points of the beam and so the average value for this one total tension of 72,000 lb. A series of such curves is required in practice to cover a range of tensions.



Stress Diagram	1.	2.	3.	4.	5.	6.	7.
Min^{m} Stress (lb/sq.in.)	0	0	0	200	500	700	1000
Max^{m} Stress (lb/sq.in.)	3700	2800	2000	1800	1500	1300	1000
Hr. of Comp^{n} centre (in.)	2·07	3·03	4·00	4·40	5·00	5·40	6·00
Lever Arm (in)	·07	1·03	2·00	2·40	3·00	3·40	4·00
Moment (1000 lb. in.)	5·04	74·2	144·0	172·5	216·0	245·0	288·0
Concrete Strain at steel level (10^{-3})	2·06	1·56	1·25	1·12	1·00	0·90	0·75
For Diagrams inverted							
Lever Arm (in)	7·93	6·97	6·00	5·60	5·00	4·60	4·00
Moment (1000 lb. in.)	570	500	432	403	360	330	288
Concrete Strain at steel level (10^{-3})	-2·06	-0·22	0·75	0·38	0·50	0·60	0·75

Possible stress diagrams for a total compression of 72,000 lb. on a 12′ × 6′ beam, with resulting moments and strains at steel level. (Steel 2′ from nearest face)

Fig. 8

* *For an actual example see "Design of Prestressed Concrete Beams from Fundamental Principles"—J. W. H. King—Concrete and Constructional Engineering* 45, *Sep.* 1950 *p.* 312.

Subsequent correlation must ensure that the mean strain changes in concrete correspond to the tension changes in steel between stages, and it is clearly a very long process in the general case. Some simplification does arise where the beam is uniform, and one stage is that of transfer, as conditions are constant along the beam, and may be dealt with as heretofore at that stage. Also, for the most efficient ultimate design, clearly the concrete should be on the point of failure both at the

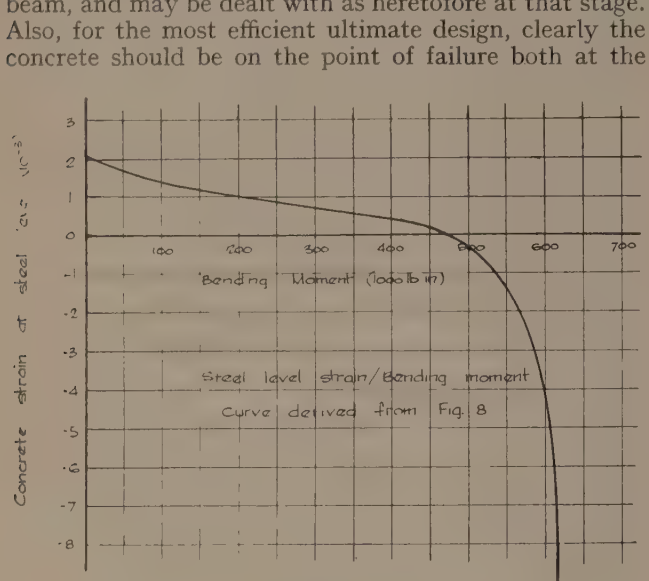


Fig. 9

ends where the moment is zero, and at the point of maximum moment. This leads at once to the necessity for the lever arm to be (2d — D), since the compression diagrams must be similar but inverted. These facts will help simplify the work, but it is nevertheless long, and since in general, the resulting beam will develop a single large crack if overloaded, and may fail suddenly, such designs are not normally advised. It is hoped that the above information will be of service if such designs must be contemplated.

Example 13. *Hysteresis.*

The extent to which account need be taken of hysteresis is problematical. It is well known that steel stressed beyond its elastic limit, and concrete stressed at all for any length of time will become plastically strained and will not recover their original state on removal of the load.

With the examples given, the concrete placed in compression at transfer is mainly either in tension and therefore ignored, or more compressed at ultimate load, so that hysteresis effects would be small. In the case of post-tensioned beams, the steel is always higher stressed at failure than at transfer, so that again hysteresis does not arise.

With pre-tensioned beams, however, the initial tension may be higher than say the tension present when embodying in a composite beam. The method to be used is as follows. From the point of maximum stress, draw a straight line parallel to the initial portion of the stress strain curve, and use this for all cases of lower stress subsequently. For example, suppose the prestress to be 170,000 p.s.i. For subsequent lower stresses the curve to be followed would be one joining this point on Fig. 4 to .002 strain at zero stress. Thus the strain at a subsequent stress of 85,000 p.s.i. would be .005, not .003 as on the original curve. Changes of this nature are all that is required, the original curve serving for all stresses above the original maximum.*

* *Strictly hysteresis should have been allowed for in Examples* 7, 8 *and* 9, *as the steel stress during bonding to the new concrete was in each case below* 138,000 *p.s.i. The effect is small.*

Example 14. *Deflection, etc., depending on EI.*

Such properties of a beam as its deflection under a given load system, and its stiffness and carry-over factor in moment distribution calculations depend on E I, and M, the last two being found from the application of a particular bending moment diagram which gives unit rotation at one end when the other is fixed. The uncertainties about the values of both E and I are overcome if the methods used above have been employed, since we have a readily available method of finding the strain change at the beam top and bottom for any given moment change. This gives at once the value of $\frac{M}{E\,I}\left(=\frac{1}{R}\right)$ which can then be used in the normal way. As an example we have already obtained sundry data for the beam of Example 2, as follows:—

At zero moment, immediately after transfer

Bottom strain = .00021 Neutral axis ht = 15.8 in.

$$\therefore \frac{1}{R_1} = \frac{.00021}{15.8} \text{ (convex upwards)} = .0000132$$

At 250,000 lb. in. moment —(medium term case in Example 8)

Top strain = .00020 Bottom strain = .00052

Depth = 18.15 in.

$$\therefore \frac{1}{R_2} = \frac{.00032}{18.15} \text{ (convex upwards)} = .0000176$$

Hence change of $\frac{M}{E\,I}$ value at point of maximum BM between transfer and part loaded stage $= \frac{1}{R_2} - \frac{1}{R_1}$

= .0000044 (hogging)

(N.B. Here the effect of creep is greater than the effect of loading, and the beam bends upwards.)

Similarly, at ultimate load in Example 2, the concrete strain at the top = .001 and the neutral axis depth is at 5.12 in. down.

Hence $\frac{1}{R_2} = \frac{.001}{5.12}$ (concave upwards) = .0001955

Hence $\frac{1}{R_2} - \frac{1}{R_1}$ = .0002087 (sagging).

It is of course necessary to find these values for a sufficient number of points along the beam to enable a diagram of $\frac{1}{R_2} - \frac{1}{R_1}$ to be plotted. It can then be used in the same way as the normal $\frac{M}{E\,I}$ diagram in order to find slopes, deflections, stiffness, etc.

Conclusion

It is hoped that the foregoing will prove useful in leading to clearer thought on not only prestressed concrete work, but also normal concrete work where subjected to high stresses and consequent plastic flow, since the cases are identical. The uncertainty which is present at the moment about the actual stress strain characteristics of a particular concrete has been shown to have some measurable effect on design, and a method of obtaining this information quickly and accurately is very desirable. In the meantime, assessment on the "stiff" side will give safe design and should be used whilst assessment on the "plastic" side will give larger deflections.

Steel Economy*

Discussion of Mr. L. R. Creasy's Paper

The AUTHOR introduced his paper by saying that his view economies in material ought not to be hieved by a disproportionate expenditure of human ıergy or loss of amenities, and he hoped that if the ıper had been studied in this light there would be und much of interest which was not necessarily nfined to a period of steel shortage.

He asked that he might be allowed to make his own ntribution to the detail of the subject by referring at me length to that Cinderella of steel frame construc-on—the semi-rigid method of design and this theme as developed at some length with the aid of the slides f Figs. 1 to 11, which are reproduced here with the ommentary.

The PRESIDENT proposed from the chair a hearty ote of thanks to the author and declared the meeting pen for discussion.

Mr. JOHN MASON (Hon. Secretary) wanted first to dd his congratulations to those already expressed by he President to the author for the paper they had just eard. After seeing the slides and hearing the theoreti-al exposition illustrated by those slides he felt that he paper did less than justice to the work the author nust have put into this subject. He personally hoped hat they would have a written article on the theoretical basis behind the method of design which they had had expounded to them and which gave rise to the design ules shown in Table 2 on page 512 of the Journal. As he author had presented it, it had been easy enough o follow, but it was a little hard on the audience to be expected to absorb such theoretical developments at short notice or to be able to form a critical assessment of them.

Steel economy was a subject with which he had had a great deal to do recently. Two economies had been mentioned by the author. One might be described as piecemeal economy or economy applied to the individual member and was achieved by paring that member down, and the other was economy of method. The author, however, had not mentioned a third economy, which he himself designated as economy of disposition. It could be illustrated in this way. Architects explained when they brought in a scheme which had a number of internal spans and two rather longer end spans that the traditional architectural method is to have a uniform spacing of the inner bays and for the whole mass to be supported by rather bigger bays at the ends. When pointed out that this is not the most economical disposition for continuous beams because the end spans require a moment of resistance of WL/10, which becomes the critical section for design, whilst the internal spans only require a moment of resistance of WL/12, whereas if the external spans were smaller their WL/10 moment would be no greater than WL/12 of the internal spans, the architect replies that if only he had known in time he could have designed in the modern manner so as to fit in with the structural desideratum. By linking up the constructional requirements with the architectural treatment there was, he suggested, a true economy obtained by proper disposition. He thought it was becoming recognised that more early co-operation between the architect and the engineer would give bigger dividends by way of economy.

It was sometimes said to be an economy to increase the stress from 8 tons to 10 tons, but he thought that such a proposition should be looked at very carefully. He doubted himself whether it could be called true economy.

Economy was a word of Greek origin which means "good housekeeping." If one's wife had been in the habit of cooking three sausages for breakfast and one morning she produced only two that was called economy, Mr. Mason observed, amid laughter, but was it true economy? What had happened was that one simply got a poorer breakfast.

The author had devoted much of his introduction to the semi-rigid design and it was a method of design on which he had spent a great deal of time some years ago when Steel Code 113 was under consideration. He wanted to draw attention to Table 2 because he thought it was important. Table 2 was largely arbitrary. An arbitrary reduction or relief moment in the bending moment of the beam was allowed. He had no quarrel with that in general but he would like to ask the author why in the column restraint moment Z was taken and not the same figure as that allowed for the relief moment. For example, suppose it so happened that for some reason a rather heavier beam than the minimum necessary to carry the load at full stress was employed, then only M/10 was allowed for the relief moment on the beam whereas Z, which was greater than the relief moment, was to be taken on the column. He would like to know whether he was correctly interpreting the table. Again, did one take both the restraint moments Z and balance one against the other or were they taken separately, on one hand at the upper end of the beam and on the other hand at the lower end, because, if so, it would give rise to single curvature in the column which led to a very heavy column design.

He had to say that he was very attracted by this idea of taking an arbitrary beam relief moment and applying an equivalent moment to the column, and in fact it passed through his mind at the time the Code was being prepared. He was not sure that it was not a good rough rule, but unfortunately it was not suitable for applying in a Code that could be put into practice. The rules put forward in the paper purported to be more precise and whilst he thought the idea in general was an excellent one, he did not think it was sufficiently complete. There was one other point. He did agree with the author that the composite construction of combined steel and concrete had been neglected. He thought perhaps one of the reasons was that at one time the steel people were told their job was steel, and he imagined the reinforced concrete people had been told their job was reinforced concrete, and perhaps that was why composite construction got left out in the cold.

Mr. DONOVAN LEE (Member) said he felt all present agreed Mr. Creasy's paper had been excellent and he would like to add his personal thanks to those tendered on their behalf by the President.

Read before the Institution of Structural Engineers at 11, Upper Belgrave Street, London, S.W.1, on Thursday, January 12th, 1950, at 6 p.m. Mr. Leslie Turner, B.Sc., M.I.C.E., M.I.Struct.E. (President), in the Chair. Published in THE STRUCTURAL ENGINEER, *Vol. XXVII, No. 12, pp. 503-527. (December, 1949.)*

Now that the world had an adequate steel output and shortages in this country were mainly confined to material little used by structural engineers, he assumed Mr. Creasy did not suggest more expensive or less satisfactory alternatives should still be used merely to save steel.

After all, the use of steel, per capita of population, closely followed the degree of industrial development and the standard of living. For example, in the U.S.A. the steel consumption was about 4/7 ton per annum, in Great Britain about 1/3, and in India (allowing for imports), say 1/50 ton.

With a steel production now over double the average of the ten years before the war he hoped steel economy was to be understood as the most efficient use of steel and labour, taken together, the measure of this being cost and quality and *not* a question of the weight of steel.

With regard to the approximate method of design proposed, he felt members had not had enough time to study it, but it seemed likely to be a most valuable contribution. It was needed because the method of the Final Report of the Steel Structures Research Committee was much too tedious and even unworkable in really large structures. Also the roughly approximate method of taking the beam moment as WL/9 instead of WL/8 and treating the difference as the eccentricity on the stanchions, was much too erratic in accuracy to be used except within narrow limits.

It was to be noted the American Institute of Steel Construction had recently also proposed an approximate method.

He himself had found that with the usual semi-rigid connections the eccentricity on stanchions could not be related in any simple terms of the beam span, beam depth, stanchion stiffness, etc., but was surprisingly easily forecast within narrow limits by a graphical method and the eccentricity only varied within a fairly narrow range for all ordinary beams and stanchions.

As regards cold-formed steel it seemed there might be great scope for quantity production using the latest methods of ridge and multi-spot welding and corrosion protection but little had been done in this direction yet. It must not be overlooked, however, that cold-formed and pressed sections were generally inferior, not superior, to hot rolled sections owing to the inability to avoid webs and flanges of the same thickness.

For this, and other reasons, such as cost of the material the scope for cold-formed structures was generally for light frameworks, where timber would normally have been used, and not in competition with frameworks of hot-rolled steel sections.

The Boyd beam, he thought, involved too much cutting and welding; so although it was very useful in making deep sections for large spans, it seemed otherwise too much trouble to save some steel in the web.

He was quite impartial between steel, reinforced concrete and prestressed concrete, and he felt that cost, time and quality should be the sole basis of choice between them. The paper that evening had reminded all present of the wide scope for different types of construction.

Mr. G. M. Boyd (Associate Member) wanted to correct a few points raised by Mr. Lee. The Boyd Beam was not introduced first in America. As regards the economy angle, he thought the proof of the pudding was in the eating. A third was that the construction was covered by British patents, and fourthly, he was no longer financially interested in it.

The paper was extremely interesting and contained some weighty implications. Economy was one of the most vital and difficult subjects, and therefore he congratulated the author on his temerity in attacking it so efficiently. There was another means by which economy if not of steel, at least of time, could be achieved and that was by simplification of orders. He was a little out of touch with the commercial aspect at the moment and he was not sure whether steel manufacturers made allowances to their customers for large quantities, but he believed they ought to. It would give some encouragement to simplifying orders if they did, and that applied not only to sections but to plates where a multiplicity of thicknesses occurred.

Apropos the reference to Professor Baker's method,* he felt that the author had fallen into the same error as he had and treated it as an indirect method of design. In other words the author had stressed the distinction between finding the load that a given structure would carry, and designing a structure to carry a given load. He thought if the author would study Professor Baker's paper again he would realise that actually Baker had introduced a big simplification and had given us a method by which we could design directly. (See discussion on Baker's paper.)

On the question of the fixing moment it had always been a puzzle to him why beams were so frequently framed into the stanchions so that any restraining moment which was put into the beams must be allowed for in designing the stanchions. In his experience of this kind of design he had always endeavoured to obtain the fixing moment for one beam out of the adjacent beam in so far as it was possible, by carrying the beams either past or through the stanchions, and thus avoiding the necessity for putting undue moment on the columns. The only justification he imagined for putting such moments on to the columns would be if they were required for stability of building.

He had one other very simple point to raise and that was to draw attention to the missing line on page 508. The author had made a very important observation about the use of high tensile steel for sections, that is that one could not get the fullest advantage of them due to the fact that whereas in high tensile steel the yield and ultimate were increased the modulus of elasticity was not. It had occurred to him that there might be a long-term advantage to be gained by designing special deep sections to be rolled from high tensile steel, so that the stiffness would be increased in proportion to the working stresses.

Mr. Geoffrey Palmer (Member) was glad to have the opportunity of congratulating Mr. Creasy on a most interesting and provocative paper. The part that interested him most was the question of semi-rigid design, because he had recently been working on the design of a steel frame building in this way and he thought members would be interested to know how the method appealed to the designer in practice.

First of all he had one small criticism. The method as shown in Table 2 on page 512 was to deduct from the free bending moment in a beam a percentage (usually 10 per cent.) and to apply to the stanchion a restraint moment which was stated to be numerically equal to the section modulus of the beam. He pointed out that the restraint moment obtained in this way was not quite sufficient ; it should be at least equal to the relief moment applied to the beam and that meant that it should be at least 11 per cent. more than the beam modulus.

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is point was that Table 2 assumed that $\frac{M}{10}$ and Z ere the same ; but they were not. The beam modulus, was only $\frac{9}{10}$ of $\frac{M}{10}$.

However it did not make very much difference in fact, ecause in practice the restraint moment on the stanchn was determined by the strength of the connection, id as connections were not infinitely variable it was und that the restraint moment applied to the stanchion generally from 11 per cent. to 100 per cent. over the eam modulus (averaging about 50 per cent.) but, as e had already pointed out, with a definite minimum of t per cent. where the full relief moment was taken for ie beam.

As to beam design, the method did not appreciably ngthen the calculations because one put down an xpression for the free bending moment in ton feet in ie usual way, and instead of multiplying by 1.2 one ultiplied by 1.2×0.9, equals 1.08, and this was all one in one operation on the slide rule.

Stanchion calculations however were a different natter, and took a good deal more time because one equently had five separate stresses, namely, the direct tress, the eccentricity both ways, and the stress due to he restraint moments both ways. It followed that it as much more difficult to select the right stanchion ection at the first shot.

He reminded members that with beam calculations ne could work out the modulus which nine times out f ten decided the size of the beam, and then select a eam to suit, but with stanchion calculations the section ad to be selected first, before the stresses developed ould be calculated.

It had been customary years ago, at least in his office, o deal with stanchion design in a much more simple way by working out equivalent concentric loads based n web and flange factors, and then merely selecting a section from a table of loads and heights. This method certainly took less than a quarter of the time that the more modern methods entailed, and he personally was not entirely satisfied that when used with discretion, the old method was appreciably less accurate in the final result.

There was also a point about eccentricity calculations for stanchions designed under the semi-rigid method. He was instructed that the method was to calculate the eccentricity stresses, and to add the stresses due to the restraint moments. But it seemed to him that if the restraint moments were properly applied it should not be necessary to take eccentricity stresses as well. The effect of applying a restraint moment to the stanchion was to put an imaginary hinge in the beam at some distance from the stanchion at the point where the base line of the bending moment diagram (raised 10 per cent.) cut the free bending moment curve. The beam reaction was then applied through the imaginary hinge and it appeared that this covered the question of eccentricity.

His own feeling and that of his partner and staff, about the semi-rigid method described by Mr. Creasy, was that it was ingenious and that there was a decided economy in the beams, which was offset to some extent by increased weight in the stanchions. He felt that the overall economy was scarcely worth the extra time involved. In other words, he felt that the method was a little timid, and that to make semi-rigid design worthwhile the method should be extended to something nearer the recommendations of the Steel Structures Research Committee of 1936. It would be necessary however to use more modern stresses and avoid some of the complications involved in the Steel Structures Report.

Finally, he entirely agreed with Mr. Creasy's statement that " in order to permit the development in the drawing office of semi-rigid designs they must be based on the results obtained by ' simple ' methods manipulated in a straightforward and uncomplicated manner."

Mr. J. S. TERRINGTON (Associate Member) wanted to join in the thanks to Mr. Creasy for his paper, which he felt sure had been read with considerable interest. It was important that engineers should continually clear their minds about what form of construction was really best in any given circumstances. Mr. Creasy had not only dealt with the subject of steel economy but he had also drawn attention to the various amenities which could be expected by the client, because after all the client and architect expect certain qualities from the finished building. He was not going to join in the heated discussion on end restraint in beams but perhaps he could refer to one or two other points. The author had rather suggested that the acceptable degree of serviceability in plastic design was only reached in the plastic range, whereas surely this degree of unserviceability was really reached in the elastic stage and also that the stresses under service conditions were still within the elastic range even in plastic design.

A further point was, Mr. Creasy seemed to suggest that deflection in a single-storey building was less important than in a brick multistorey building, but where very heavy cranes are used trouble had been experienced due to deflection and distortion of the crane girders so that, in that type of single-storey building, deflection and distortion was really important. He noticed that he had referred to the term " guaranteed yield." He believed that the steel makers rather avoided the term guaranteed now, and referred to minimum yield stress, as yield is not guaranteed. He was sure they all agreed with Mr. Creasy in achieving the maximum economy in steel design by the use of high tensile steel in the welded frame and he would be grateful to learn of any good examples.

Mr. Creasy had asked for a column formula ; surely, if a space frame was used one only needed to employ Clause 22 of B.S. 449, where one simply estimated direct without any relation to the length of the member, as in rigid arch construction.

Referring to Sketch 8, Mr. Terrington asked how the concrete was put into the inverted channel in that position, and secondly, in that sketch he noticed that the 11 in. wall was on the inside supported at floor level (at 10 ft. heights), and on the outside it ran right the way down to the ground, so that, as there was a different method of support, one might expect that there would be differential settlement in the wall.

Mr. R. P. MEARS (Member of Council) said that on page 505 were shown beams of 20 in. depth, apparently designed as simple spans and at the top, as shown on the detail, they are attached to the stanchions by a cleat having one rivet each side both to the beam top and to the stanchion. Then on page 509, with 15 in. beam, where he understood that the restraint moment was taken into the calculation, the cleat contained four rivets in shear in the beam and again only two pulling on their heads in the stanchion, and he believed that they were not rivets but only bolts, and, Mr. Mears added, amid laughter, considering how important it was that they should be very tight, he would like to know whether the Consulting Engineer himself was expected to go round and tighten up these bolts himself. On page 511, where the full rigid frame was dealt with, beams were

reduced from their original 20 in. depth to 12 in. Further, on page 521, they were told that in reinforced concrete design depth is always 50 per cent. greater than in steel design, but if one looked at the picture on page 517, the depth of this reinforced concrete beam was shown as 1 ft. 0½ in., so it seemed to him to compare very favourably with complicated welded design.

Then they were told that the deflection was expected to be up to 1/325th of a span, or in other words on the 20 ft. span it would be ¾ in. It therefore seemed to him that it would be advisable to deflect the beam by artificial means in advance to perhaps half this amount so that the final deflection should not adversely affect architectural treatment in the form of plastering, and so on.

Mr. S. M. Reisser (Member) drew attention to a statement made by the author which had surprised him very considerably. In showing a slide of what had been described as a " semi-space frame " of welded construction, it was stated that this type of construction had many advantages but was some 20 per cent. heavier—a contention which appeared to contradict practical experience. A number of such structures have been built in recent years both in this country and in Belgium, and all compared more than favourably with the weight of alternative designs. Perhaps he had misunderstood the author and would be glad if the matter could be cleared up.

Mr. A. J. Mitchell (Member) had two small points on which he wanted clarification. One he thought might be a misprint, on page 521, where comparison was made between the amount of steel used in ordinary structures, reinforced and prestressed structures. The ratio given was 16 : 8 : 1 ; he took it that possibly 16 : 8 : 4 was intended, because the amount of steel saved by prestressing was approximately half under normal conditions.

In Table 2 the Column Restraint Moment was given as Z. Now Z was purely a numerical value, while $\frac{M}{10}$ involved the stress variable. Perhaps this would be better shown as Z multiplied by something.

He concluded by saying that he thought this problem had lost some of its urgency now that more steel was available, owing to the gratifying achievements of this industry in regard to output.

Mr. Geoffrey Palmer rose again to point out that a mistake had been made on page 515 where the proportion of depth to span was given. He said that the denominator should be 192 divided by the stress, that is, $\frac{\text{span}}{19.2}$ for 10 tons stress and other stresses proportionately.

Written Discussion

Mr. C. V. Blumfield (Member), expressing appreciation of the paper, writes :—I would, however, like to query Mr. Creasy's statement regarding the weight of steel in a barrel vault roof, which I do not agree approximates to that required for a sheeted steel roof. In my experience it is generally appreciably lower and particularly so if the columns are added in both cases. The difference is more marked on the smaller spans and widths and in some examples it is only 50 per cent. of the weight of structural steel. I would also say that the weight of 6½ lb. per sq. ft. for a barrel vault with a bay 80 ft. × 40 ft., appears to me to be very heavy, and I cannot feel that the barrel vault roof designed in this case was done with due regard to steel economy.

On the question of foundations, I would like to suggest that in a very large number of cases it is possible to use mass concrete bases unreinforced for any but quite heavy columns. In fact I would go so far as to suggest that reinforcement should only be used in foundations where load has to be transmitted over appreciable distances to equalise ground pressure. Mass concrete base has a further advantage of allowing the general contractor to proceed unhindered, and this naturally shortens the time where frequently work is being done under difficult conditions.

Before the war I was responsible for the design of a number of buildings in London using a composite construction consisting of a structural steel skeleton clothed in reinforced concrete. Generally speaking, I found that the weight of steel was mid-way between structural steel and reinforced concrete, and the same applied to the cost. These buildings have all now been up to 10-16 years, and have proved quite successful in use, and particularly resistant to the effects of bombing. I have not made a post-war comparison of this method of construction with structural steel now that the allowed stresses are rather different, but I feel it probable that under present-day conditions it would be worth doing.

Mr. H. A. Cadwell (Member) writes :—There are two matters of detail in Mr. Creasy's paper on which I would like to comment, although they have little significant bearing on the main theme of steel economy.

First, Mr. Creasy says, on page 506, that the " omission of some relief (in working stress) in the design of encased beams, would appear . . . etc." This is not strictly correct, since B.S. 449-1948 permits an increase in working stress for properly cased beams of up to 50 per cent. more than that for uncased beams, on the same principles as for cased stanchions. It is true that no advantage can be derived where a beam is built solidly into a floor since the working stress would be limited to the maximum value of 10 tons per sq. inch. But in wall beams which are not laterally restrained by floor or other construction, the increase obtained from the casing allowance is appreciable.

Secondly, on page 514, Mr. Creasy says, in connection with rigid frame joints, that " there is considerable need here for further experience in the development of a satisfactory detail." I am sure the author will forgive me if I remark that the welded detail shown in sketch No. 4 seems to me far from satisfactory. In fact, I am at a loss to understand the precise function of the various reinforcing and cover flats, and the manner in which the welds would be made.

This detail makes no attempt to overcome the difficulty that the cover flats and beam flanges would connect to the outside faces of the flange plates of the compound stanchions (shown in the sketch as a plain R.S.J.) and not to the main shafts, under which conditions the flange plates would be incapable of transferring the tensile forces across the stanchion section. Further, the detail does not take into account the erection clearances which would probably be required at the ends of the beams.

I should be grateful for any additional information Mr. Creasy can give on the connections shown.

Mr. D. H. May (Member) writes :—Mr. L. R. Creasy drew some rather surprising conclusions concerning " the value of prestressed concrete work as applied to general building construction. . . ." These conclusions must not be allowed to pass unchallenged. Surely, be

annot be serious in his comparisons with a timber :ructure.

Few examples appear to exist of the application of pre-:ressing to building frames. The majority of Con-inental examples are flat roofs supported on reinforced oncrete columns. It is fair to say that Sighthill, vhich is apparently the building referred to in sketch Jo. 7, is probably the first major building structure mploying extensive prestressing for carrying heavily oaded floors. Mr. Creasy is, therefore, drawing con-lusions between two comparatively well-developed nethods of construction and one that, in terms of time, s in its infancy.

I hasten to add that my remarks must be construed is not applying to other examples of prestressed concrete :onstruction, such as bridges, where there is no doubt ibout its supremacy over reinforced concrete construc-:ion, as has been proved on numerous contracts.

The examples of prestressed concrete building construc-:ion given by Mr. Creasy represents a step into a new tech-nique. Those who have been associated with the work know that the members are not heavy bearing in mind :he superimposed loads, the column spacing and other restrictions placed upon the designer by his terms of references, e.g., nearness to airport restricting height, and architectural requirements. One hesitates to draw a comparison with a similar construction in " English Oak," as Mr. Creasy suggests, as there is no doubt that such a structure would be " vastly heavy and ugly." One can imagine Mr. Creasy's difficulties in attempting to accommodate the very heavy shears that are present.

There is no doubt that the prestressed frame of the not too distant future will present a very different picture and only when such frames are being constructed commercially will he be justified in making such conclusive comparisons between the various construction methods.

Dr. P. W. Abeles (Member) writes :—The alleged ugliness and approximation (of prestressed concrete work) to oak construction are in the writer's view by no means established. In his view oak structures designed in accordance with the strength of the oak need not be ugly ; but in any case there is no general common basis between oak and prestressed concrete. In fact, one of the main features of this new material is to allow the use of beams of reduced construction depth without excessive deflection by the provision of an initial counter-deflection.

The appearance of prestressed concrete constructions may vary to a great extent according to the assumptions upon which the design is based and on the suitability of the cross sectional shape selected. Thus, it is not impossible that beside slender and shallow constructions some structures may be found which are heavier in appearance than would be expected by normal standards; but this would not give justification for a generalisation as expressed in Mr. Creasy's conclusions.

It may be added that only where bending moments may occur in two opposite directions, very slender and shallow constructions are generally not obtained, since a uniform prestress must be applied and consequently there is no initial counter-deflection. However, this loading which may occur with masts is unusual in ordinary construction.

It is quite true that the successful application of prestressed concrete to normal building construction is relatively restricted at present, as Mr. Creasy says, but prestressed concrete is still in a state of development and it may be expected that in the near future standard units in prestressed concrete will be available for use.

As against Mr. Creasy's conclusions, the writer would like to mention the following main advantages of prestressed concrete as compared with some other materials.

1. Serviceability after loading up to nearly the failure load in view of complete flexibility and resilience ;
2. Possibility of freedom from cracks ;
3. No maintenance ;
4. Reduced construction depth, if designed with temporary upward camber ;
5. Saving of steel ; and
6. Increased economy when designed on economical principles.

The Chairman, winding up the discussion, said that the cost of design had been mentioned. As the amount of design work increased, so did design costs. The aim was to reduce overall costs. Thus closer analysis and higher stresses resulted in a higher proportion of total cost going to design charges. This change in incidence of cost, proportionately more to design and less to shop or works, was a gradual but continuous process and it was essential that this question should be squarely faced and accepted if progress is to be supported and if real overall economy is desired. This aspect, again, can only be adequately dealt with and explained to clients by engineers, and it must be undertaken as one of the engineer's many duties to the public and the profession.

The conception that reinforced concrete takes longer than steelwork appears to be general, but it is not so well founded on fact and it should be qualified. Given the materials, proper organisation and the engineer in charge of the job, the reverse has often been the case, particularly when drawing office time is included.

The Author's Reply

Mr. Creasy began by saying how grateful he was to members for making in the discussion so vigorous a contribution to the questions which had been developed in the paper. A good deal of the discussion had centred on the question of semi-rigid construction, and he proposed to leave that to the last. Some questions he thought had overlapped and some had followed one another so quickly that he had not perhaps been able to make a note of them all. He hoped that he would be forgiven if he appeared at this discussion to ignore some of the points and promised to repair the omission, if such did occur, when the report of the meeting came to be printed.

Clearing up the incomplete sentence on page 508, this should have read as follows :—" Those who decry this method of design well shown by mathematical analysis that plastic yield occurs in the end restraint cleat at an early stage in the loading of the beam and hence that quite small variations in super load may relieve the beam of all restraint and even introduce a moment of opposite sign into the adjacent beam. This theoretical analysis cannot be substantiated by tests," etc.

Concerning the question of steel economy and its relation to the cost of sausages ; he was not sure whether this was meant to be taken seriously, but he thought that it did represent an economy if one who was used to living on three sausages subsequently managed to reduce this number to two, although, he added, amid laughter, that the discussion rather reminded him of the man who was teaching his donkey to live on less and less, and just when the animal had learnt to live on nothing—it died.

He was fully in agreement with Mr. Mason on the necessity for early co-operation between the Engineer and Architect, and he had tried to bring this out in the section on ancillary economies though not, perhaps,

with the same forceful emphasis. That relatively unnoticed part of his paper was intended to deal with the problems of that early stage in the development of a scheme when so many of the fundamental issues which affect economy were decided, almost haphazardly, and often without consultation between all concerned.

Referring to Mr. Donovan Lee's important contribution to the discussion which had ranged over a wide field, the author hoped that in the introduction to the paper that evening his interpretation of the term "economy" as applied to structural design had reassured Mr. Lee on this point. On the question of cold formed steel: he supposed it was fair to say that in this country this type of structure was only used for relatively small spans of up to 30 feet or so and that even there the use of this method did impose a severe rigidity on the layout.

In the author's experience the cold formed type of steel truss was extremely suitable as mass-produced frames for the temporary office building type of construction, but it would not lend itself to a building requiring only a few trusses as one of non-regular layout.

Mr. Boyd had wondered whether there was a reduction in the cost of steel for a large tonnage of the same joist section, and he thought he was right in saying that there was no such reduction but the views of the steel manufacturers would be welcome on this point.

On the relative merits of framing beams into or past the column: it would be seen that, in the former case, the stability of the column was improved with a consequent increase in the permissible working stress and with a reduction in the column section.

Regarding the saving of material which was to be achieved by using high tensile steels, the author recalled one actual example of a riveted multi-storey high tensile steel frame recently constructed in which a saving had been achieved of 20 per cent. in material over a corresponding frame in mild steel.

A query had been raised concerning the detail of a semi-rigid top cleat and the difference between the numbers of rivets in the vertical and horizontal legs of the bracket: the semi-rigid form of cleat had been developed empirically and did not lend itself to theoretical analysis. The vertical leg of the cleat, however, was shop riveted to the stanchion and the rivets in the horizontal table were driven at site.

Regarding the depth of the reinforced concrete beams shown in one of the sketches; these could not be compared with the steel beams since as indicated in the typescript they were designed for quite different super loads.

Concerning the "plastic" method of design; the author agreed that at working loads the maximum stress induced in a beam so designed was quite possibly within the elastic range but, nevertheless, that stress was probably higher than that at present permitted for beams designed according to the "simple" method and on that account the deflection was correspondingly greater. This led to the need for closer research into the limits of deflection which were acceptable when considering the possibility of damage to the structure on the one hand and, on the other hand, of inconvenience to the user.

The relevant clause in B.S. 449 on "deflection" was unsatisfactory in this respect and a more specific direction was required, for example, on permitted limits of deflection for separate dead and live load conditions. In many cases the former could be easily masked by the casing, or by an initial camber, whereas the latter was more important in its relation to the damage which might be caused to the architectural treatment. The query which had been raised in respect of the quoted beam-depth-to-span ratios arose from a difference in the value used for Young's Modulus which, in this case had been taken at 13,000 lb. per sq. in., to allow for the additional deflection from the induced shear stresses.

Mr. Reisser had raised a question on the welded semi space frame in which the latticed girder in the third and horizontal plane was omitted and replaced by the rigidity of the inter-connection between the other two planes, at their junction. The author had intended to say that a welded frame of this type would be lighter than a similar frame which was riveted, but would be heavier than a full space frame of normal type for exactly comparable conditions and form of construction.

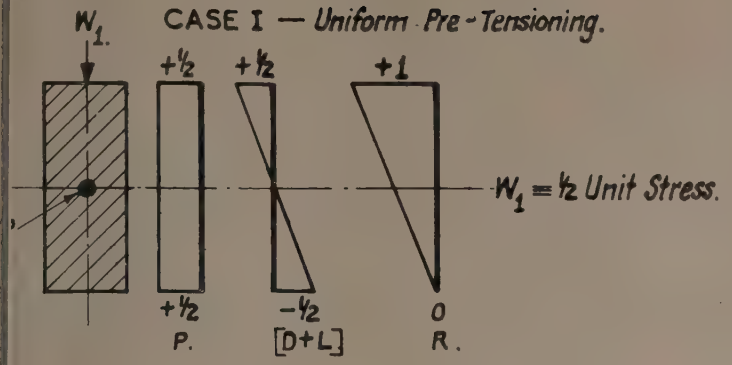
The figures quoted in the paper on the relative weight of steel required for a sheeted latticed steel frame and barrel-vault roof had been taken from actual example but the author appreciated a contribution from Mr Blumfield's wide experience in this field. Mr. Blumfield had also referred to his previous experience with composite steel and reinforced concrete construction. The author had already expressed a disappointment that the latest edition of the B.S.S. 449 should ignore the possibilities in this field. The contribution of a reinforced casing to the strength of a member was recognised for the unrestrained condition only and contrary to the impression given by Mr. Cadwell it would appear that small advance had been made in this direction. Mr Cadwell's reference to the unsatisfactory rigid joint a sketched was well merited but was to some extent answered by his own quotation from the paper—that there was "a very considerable need for further experience in this field." The actual joint indicated was based on certain aspects of American practice in which the joists were notched to facilitate site welding.

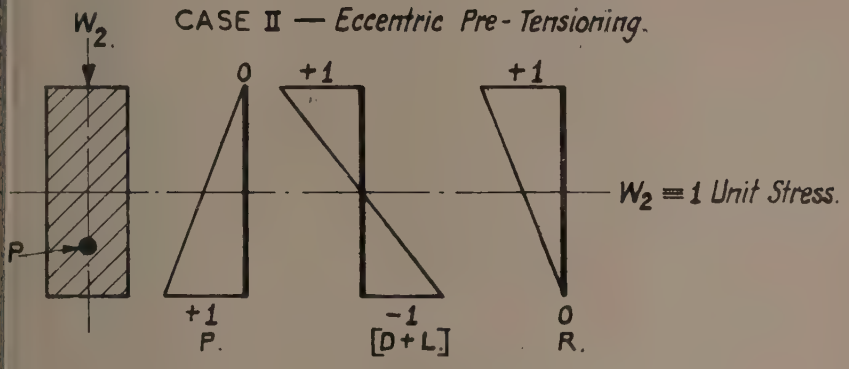
Turning to the subject of Prestressed Concrete: the ratios of 16 : 8 : 1 which had been quoted in the paper for the weight of steel reinforcement in a steel beam, as compared with one of normal reinforced concrete and one of prestressed concrete assumed, as was stated that the beams were cast and prestressed by the long-bed method. In cases where the beams were cast and tensioned individually with their own permanent end anchorages the weight of these must be included and, in consequence, the weight of steel required then approached the ratios of 16 : 8 : 4.

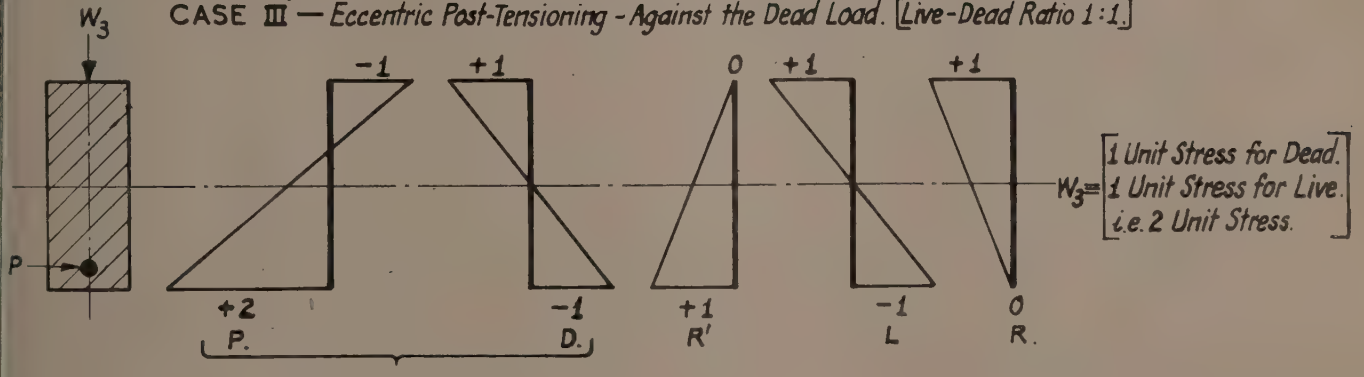
The author was grateful to Dr. Abeles for reminding Mr. May that the references in the paper referred to the use of prestressed concrete as applied "at present in the field of "normal building construction." The diagrams of Cases I to IV indicated the basis for the comparison which had been drawn between a prestressed concrete beam and one of a material, such as high quality oak in which the same compressive stress in flexure was permitted of, say, 2,000 lb. per sq. in.

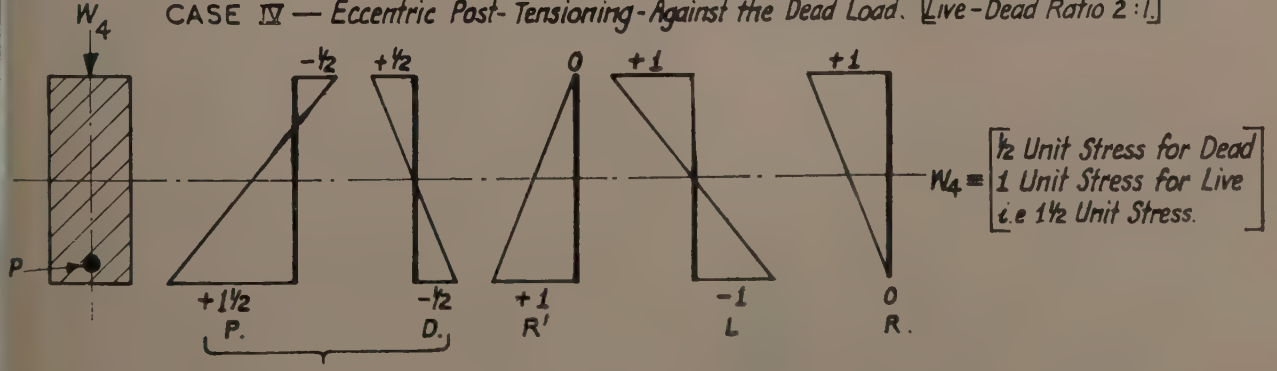
It would be seen from a study of the diagrams that in the simple case of pre-tensioning or of post-tensioning against the relatively small dead weight of beam only the strength in flexure of the two beams would be similar. In favourable circumstances, where the beam was post-tensioned against the dead weight of beam and slab construction and when the ratio of live to dead wood approached to unity the strength in flexure of the prestressed concrete beam might be twice that of the oak beam. These were, however, special loading conditions and the process of tensioning the beams after the construction of the slab was one of some complexity when introduced into normal building operations.

The author thought that he could now turn to the very vexed question of semi-rigid construction. He proposed in the first place to explain how Table 1 was intended to be used. The inter-restraint moment









PRESTRESSED CONCRETE BEAMS

Conditions of Stressing :

Maximum permitted compressive stress in concrete = +1.

Permissible limits of stress in concrete at all stages of stressing and loading = $> +1$ and $< \pm 0$.

Nomenclature :

P = Stresses induced by Pre or Post Tensioning.

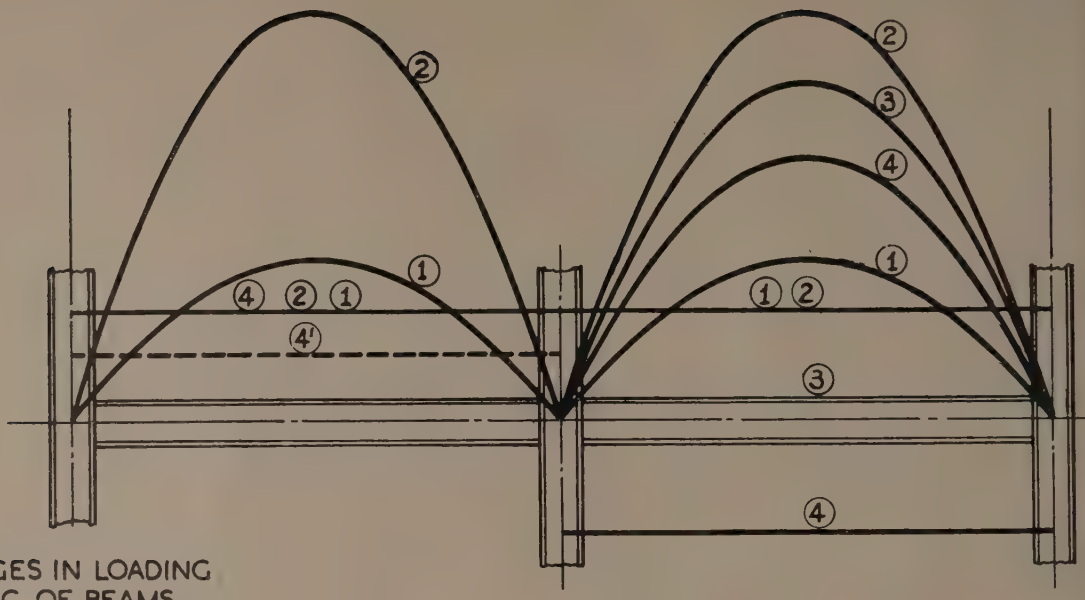
D = Stress induced by Dead Loads.

L = Stress induced by Live Loads.

R^1 = Resultant Stress Condition before application of Live Load.

W = Total Load which can be carried by the Beam in Flexure.

R = Final Stress Condition under Full Load.



SUCCESSIVE STAGES IN LOADING AND UNLOADING OF BEAMS.

1. Beam loaded up to elastic limit of top cleat.
2. Full load on beam - top cleat under plastic yield.
3. Beam load relieved - top cleat freed of stress.
4. Beam load fully relieved - reverse restraint moment at elastic limit in top cleat.

Figure 1.

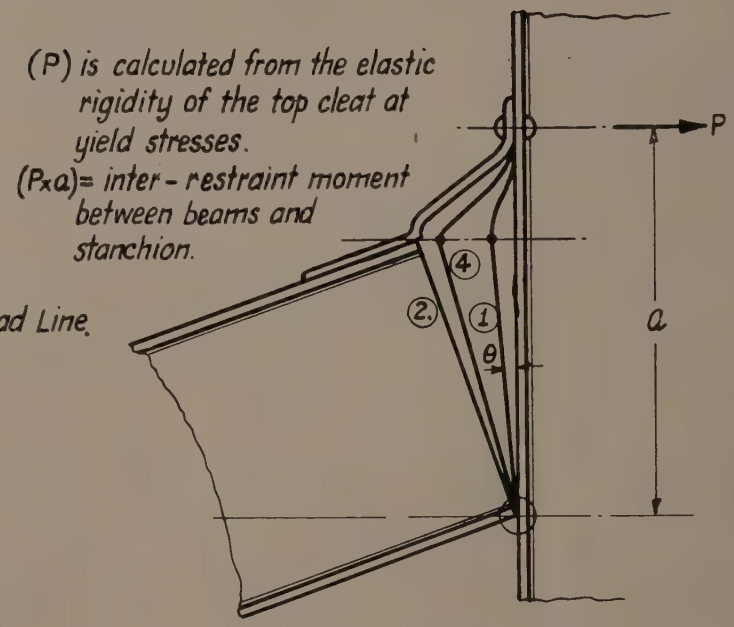


Figure 2.

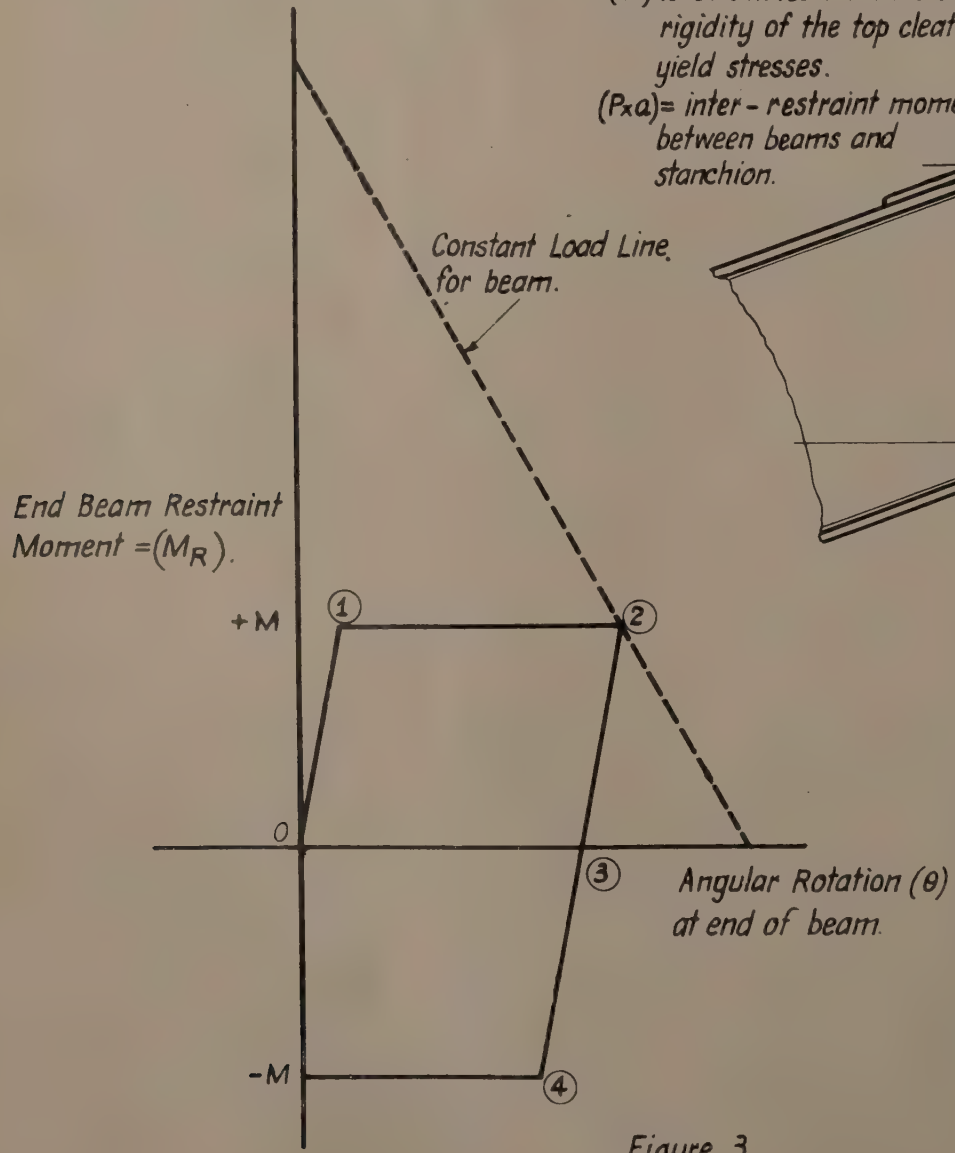


Figure 3.

THEORETICAL ARGUMENT

Fig. 1.—Portion of Twin Span bent showing the assumed theoretically induced moments under progressive loading of the beams.

Fig. 2.—Successive stages in the deflection of the end cleat connection.

Values of (P) are determined from the elastic stiffness of the root of the top cleat at yield stresses.

Values of (M_R) are calculated from the product of the force (P) acting about the fulcrum of the toe of the beam.

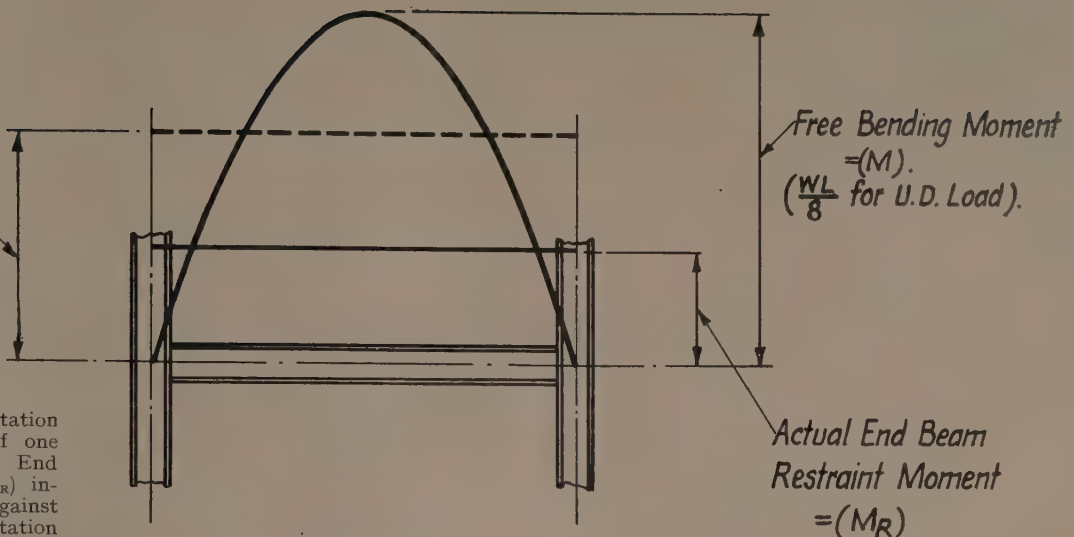

Theoretical End Restraint for full rigidity =(M_F) (WL/12 for U.D. Load).
Free Bending Moment =(M). (WL/8 for U.D. Load).
Actual End Beam Restraint Moment =(M_R)

Figure 4.

Fig. 3.—Graphical representation the progressive loading of one am plotting values of the End am Restraint Moment (M_R) in- ced by the semi-rigid cleat against lues of the end Angular Rotation the Beam (θ).

" Constant Load Line " shows values of) for all possible values of (M_R) for the same ad on the beam. Thus :—

Full end fixity—zero rotation.

Zero end fixity—full rotation.

Beam-Load (1). The application of the am load of a magnitude to induce yield resses in the top cleat :

Full Restraint Moment from cleat Fig. 1.

Partial rotation of the end of the beam—Fig. 2.

Limit of straight line deformation—Fig. 3. From theoretical considerations this might cur at say 1/20 of beam load at working resses.

Beam-Load (2). Beam carrying full load working stresses.

The Restraint Moment (M_R) in beam remains unchanged—Fig. 1.

Max. end rotation—Fig. 2.

Constant (M_R) with max. end rotation—Fig. 3.

Beam-Load (3) and (4). Cleat will perate elastically through the full range of eformation from positive yield stress to egative yield stress whilst values of (M_R) ill pass successively to zero and finally to a ioment of equal but opposite sign—thus elieving the restraint moment in adjacent eam with consequent overstress—Fig. 1.

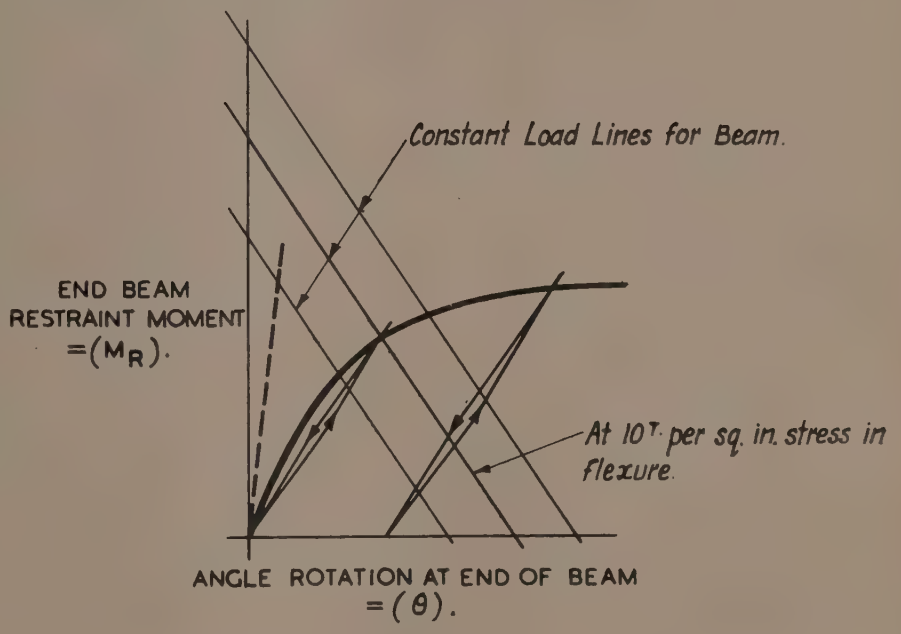

Constant Load Lines for Beam.
END BEAM RESTRAINT MOMENT =(M_R).
At 10 T. per sq. in. stress in flexure.
ANGLE ROTATION AT END OF BEAM =(θ).

Figure 5.

SEMI-RIGID TESTS

The " theoretical " argument is not con- rmed by tests.

Fig. 4 is a reminder of the nomenclature in use.

Fig. 5 shows graphically by full lines the actual elationship between (M_R) and (θ) as obtained from ests on beam end connections subject to pro- ressive conditions of loading and unloading. The lotted line indicates the relation between (M_R) and θ) to be anticipated from theoretical considera- ions. Within normal working stresses in the beam he cleat behaves in an elastic manner for all con- litions of loading and unloading of the beam vithout evidence of marked residual rotation.

Fig. 6 shows broadly the relative rigidity $\left(\frac{M_R}{M_F}\right)$ or different cleats when applied to various beam groups.

The graphs shown have been plotted from tests n which the columns were completely rigid. These graphs suggest that in a frame where the columns are completely rigid with end cleats of $\frac{3}{8}$ in., $\frac{1}{2}$ in. and $\frac{5}{8}$ in. thick applied to light, medium and heavy beams respectively then a Restraining Moment (M_R) of about 1/3 of the Fixed Moment or 1/5 of the Free Moment on the beam will be induced.

Further investigations suggest that this value of 1/5 is reduced to nearer 1/10 when allowance is made for relative beam and column stiffness of the normal range.

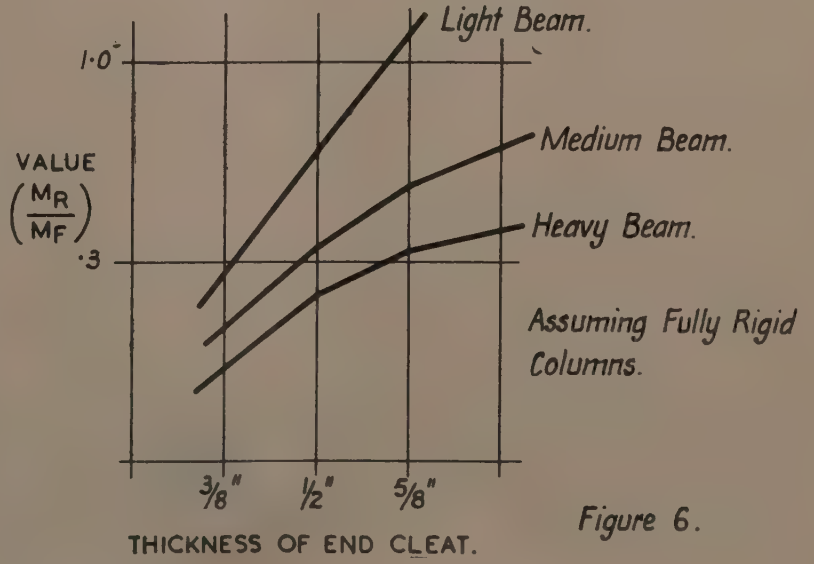


Figure 6.

between beam and stanchion is assumed to relieve the free bending moment in the beam and to induce a corresponding moment in the stanchion of equal magnitude. The value of this relief moment was chosen empirically to equal one-tenth of the free moment in the beam which for convenience of computation was assumed to be numerically equal to the modulus of the beam section. The stanchion was then designed for this latter value, in addition to the moments assumed to be induced, by the eccentricity of the beam load as for " simple " design and the algebraic sum of all these moments on each side of the column at each floor must be suitably proportioned between the upper and lower column lengths ; again as for " simple " design.

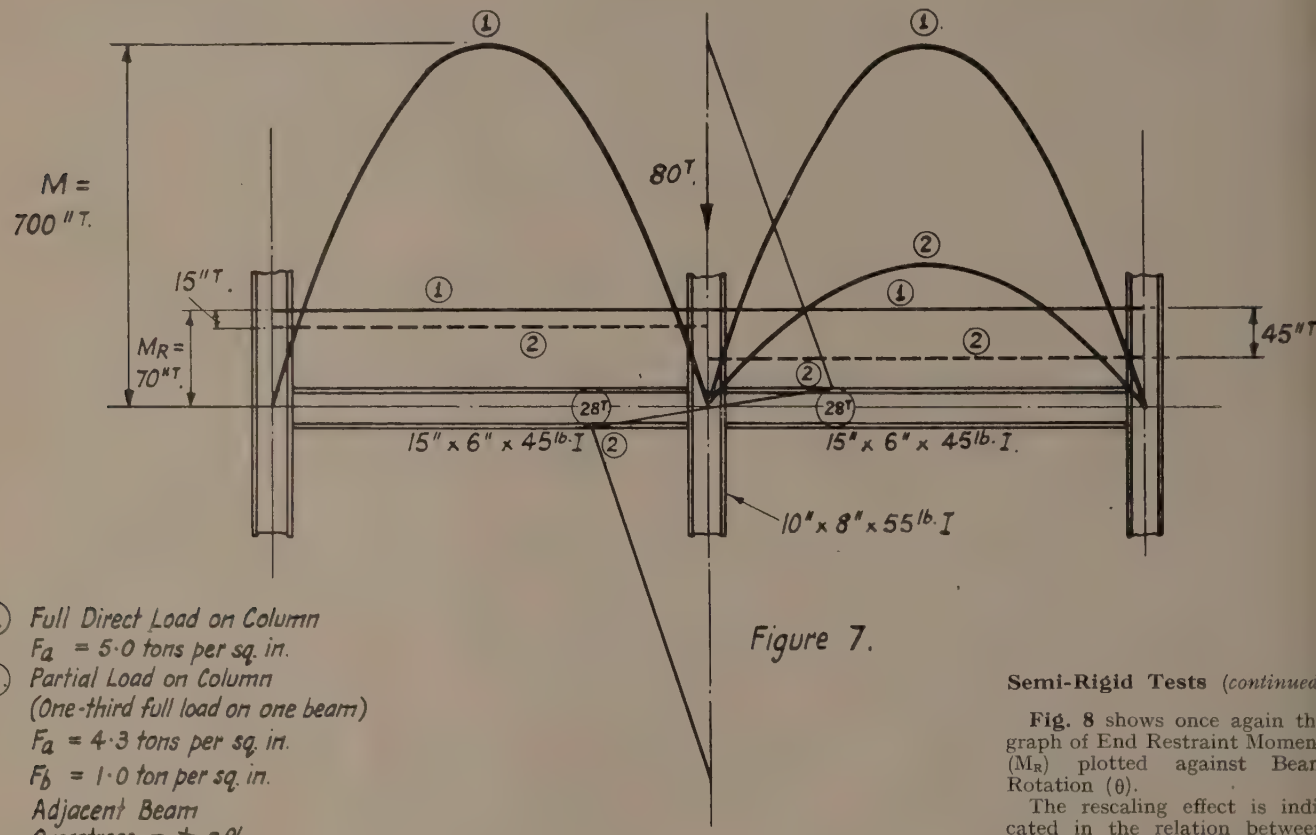

M = 700 "T.
15"T.
MR = 70"T.
80T.
45"T
28T
28T
15" x 6" x 45lb. I
15" x 6" x 45lb. I
10" x 8" x 55lb. I
1. Full Direct Load on Column
Fa = 5·0 tons per sq. in.
2. Partial Load on Column
(One-third full load on one beam)
Fa = 4·3 tons per sq. in.
Fb = 1·0 ton per sq. in.
Adjacent Beam
Overstress = 7 2%.

Figure 7.

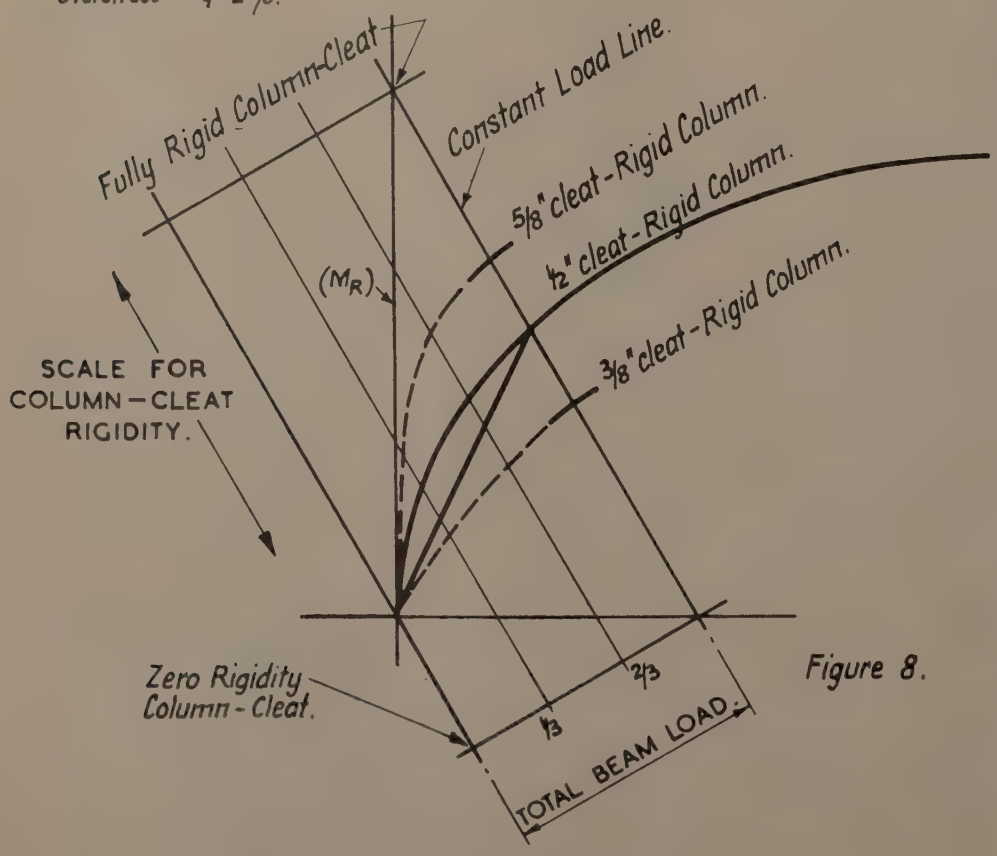


Figure 8.

Semi-Rigid Tests *(continued)*

Fig. 8 shows once again the graph of End Restraint Moment (M_R) plotted against Beam Rotation (θ).

The rescaling effect is indicated in the relation between (M_R) and (θ) when allowance is made for relative beam and column rigidity.

For the cleat system proposed tests suggest that the End Restraint Moment (M_R) approaches to 1/10 of the Free Moment on the beam for a wide range of relative beam to column rigidity.

Fig. 7 shows a working example of the stress induced in a portion of a semi-rigid frame subject to variations in applied load.

The Restraint Moment (M_R) is proportional to the load on the beam for any fraction of the full beam load.

Full allowance is made in the calculations for the beam and column for an Inter-Restraint Moment of 1/10 of the Free Moment in the beam.

Under the full beam load the column is stressed to 5T. per sq. in. and the beam to 10T. per sq. in.

The load on one beam may be reduced by 2/3 without increasing the stress in the column and with a small overstress in the adjacent beam.

The difference between the value of one-tenth of the e bending moment and the numerical value of the am modulus amounted to approx. 1 per cent. of the e moment in the beam at maximum permissible esses. The magnitude of the free moment could not, of course, be assessed to this degree of accuracy and the device was recommended for its simplicity. In cases where the beam was not fully stressed and, in fact, in all cases, if so desired, the same numerical value for the inter-restraint moment would be used in the design of

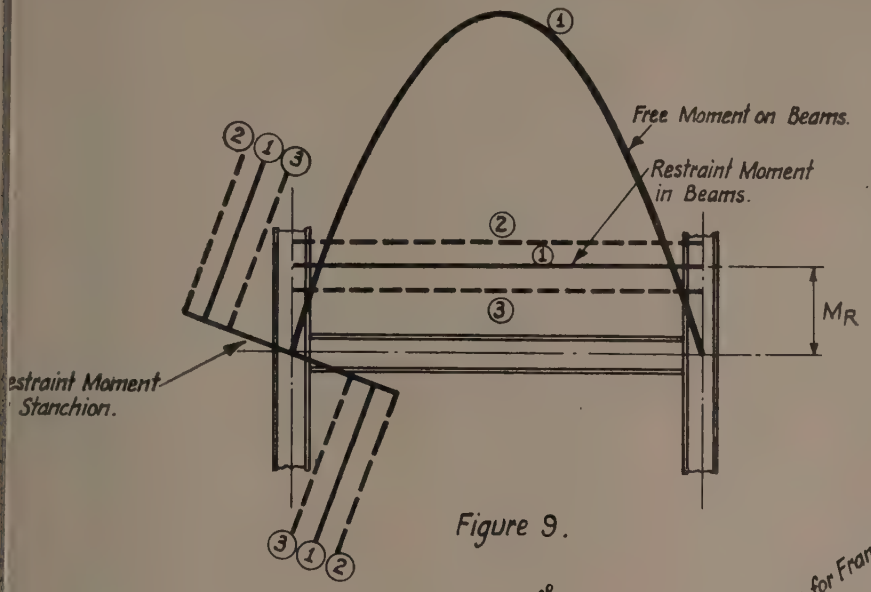


Figure 9.

Righting Tendencies where (M_R) tends to vary from the Desired Value

Consider the stability of the frame if the actual induced Inter-Restraint Moment between beam and column is greater or less than the arbitrary chosen fraction of the Free Moment in the beam. **Fig. 9** illustrates these alternative conditions of Restraint Moment.

Figs. 10 and 11 show by alternative illustrations the "righting" tendencies which conspire to restore the frame into the selected condition of equilibrium for which it was designed.

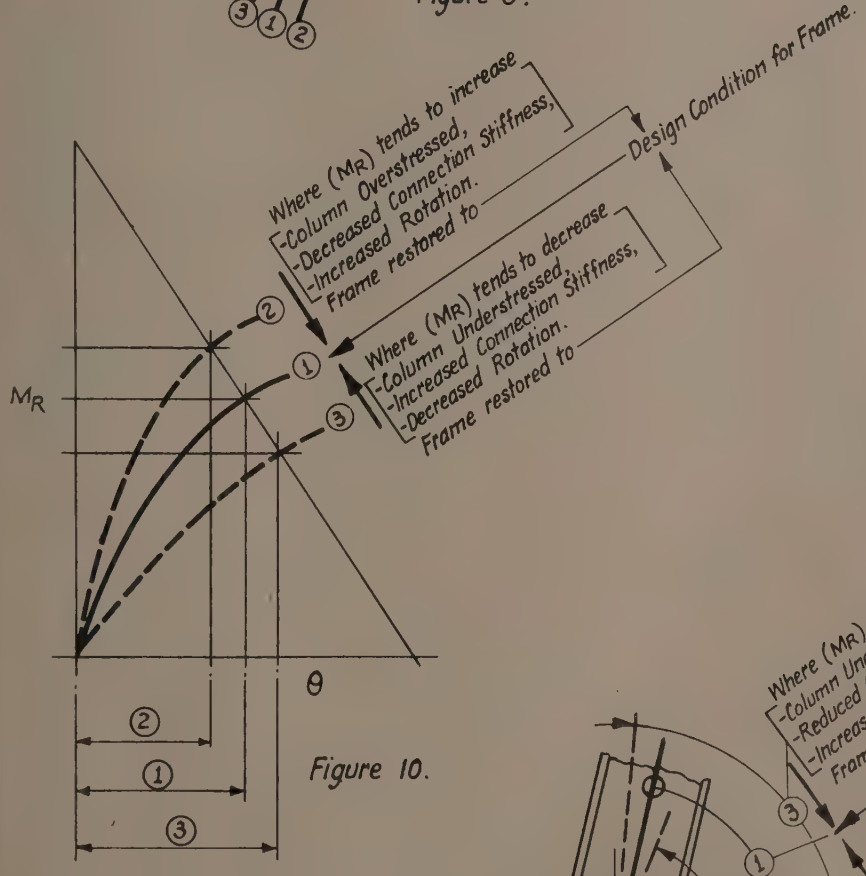


Figure 10.

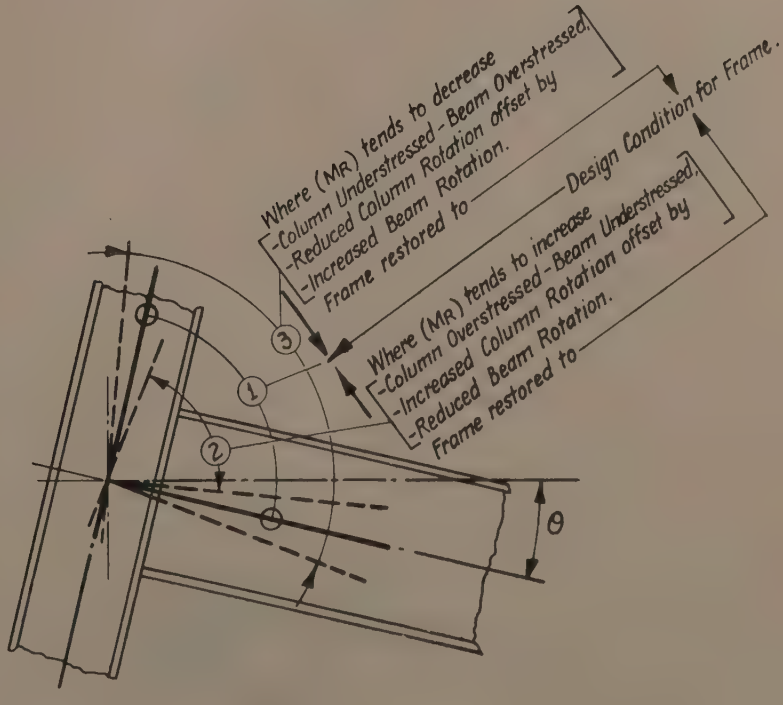


Figure 11.

beam and stanchion. It was, of course, essential that these calculations were made in the same units throughout.

One question to be answered referred to the reason why the proposed method of semi-rigid design tended to use less steel than the simple method. There was of course an obvious saving in the beams which was offset, in part, by an increase in the outside stanchions. The section of the internal columns which, in general, were subject to a uniform loading would not be increased, however, and there was, in consequence, a net overall saving of material.

The diagrams shown with commentary in Figs. Nos. 1 to 11, outlined the basis upon which the semi-rigid design procedure indicated in Table 2 had been developed. This rested largely on the results of tests performed in this country and in America which had led to certain broad conclusions and which could be summarised as follows :—

That the actual behaviour of semi-rigid frames could not be deduced from a theoretical analysis :

That an inter-restraint moment must be induced between beam and stanchion, however slender the proportion of the top cleat detail.

That an allowance ought to be made for this effect in the design of the frame by the super position of an arbitrary relief moment into the beam and of a restraint moment of equal numerical value into the stanchion :

That the actual magnitude of these moments was not critical provided they were relatively small in proportion to the load to be carried and were statically consistent.

Concluding with a general review of this difficult problem : it was the ever-pressing concern of the engineer to balance a too rigid adherence to tradition with the claims of each new scientific " fashion." The semi-rigid method of frame design here proposed was intended to be a middle course between the conventional " simple and the complex " rigid " or " plastic " method design. It could be claimed to reflect more nearly tha the one the actual behaviour of a frame and to be les intricate to use than the other. One could not have both ways : it was irrational to object to the sma difference between the numerical value of the bea modulus and one-tenth of the free moment and then advocate a return to the approximate method of stanch ion design using " equivalent " loads to object to th additional operations involved in this method of sem rigid design as compared with " simple " design an then to advocate the use of the far more complex Sem Rigid Code of the Department of Scientific and Industri Research. The wide interest which had been show in the proposed method of semi-rigid was extremel gratifying. It was thought to be the first time that suc a relatively simplified approach to the semi-rigi problem had appeared in print and the close attentio of engineers was welcomed to the method, for only b such means could new ideas be developed and improve

Finally, the author said that he wished to than members for their interest and patience in comin along that evening to listen to what he had to say an in giving their views so freely and in such a helpf manner.

The PRESIDENT, terminating the meeting, thanke Mr. Creasy for his unperturbed and clear response the questions which had been launched at him fro right, left, and centre. He hoped he would be publish ing the diagrams shown on his final slides, with th necessary explanations of his particular theory an method, and he also thanked all members who ha taken part in the discussion that evening.

Book Reviews

The Rigid Frame Bridge, by Arthur G. Hayden and Maurice Barron. (London, Chapman & Hall; New York, Wiley.) Third Edition, 1950. 240 pages, 9 in. by 6 in. 40s.

This book is probably the only work in English on the Rigid or Portal Frame, and this third edition, though generally similar to the first two editions, contains much revision. The first chapters are the same as in the two first editions. They might well be revised using strain energy. This makes the derivation of the equations much simpler and more brief, and also reduces the possibility of confusion over signs.

A new departure is the use of elastic weights and conjugate structures to lighten the arithmetical work. The principles underlying the use of these are not clearly explained. Presumably knowledge of other work is implied, though no references are given, and they might well be included.

The method of design of skewed frames has been enormously simplified. The general conclusion being that they can be designed for the square span, and a " turning " process applied to the design of the reinforcement, temperature stresses being based on the square span stresses multiplied by the secant of the skew angle. This is very welcome, since the Rathbun method was very laborious. It is claimed that the new method and the exact method agree very well, and with model analysis. One proviso must be made, however, that the present reviewer has never felt really confident of the Rathbun method on the grounds of doubt of the soundness of the method of arriving at the " free " moment, and he has never felt really happy as to the validity of the temperature theory.

Minor criticisms are that the method of calculatin earth pressure effects is needlessly complicated, and tha the symbol M has been used for both " free " an " total " moment, and in cases in which it was n entirely clear from the context which effect was i tended. On the whole, however, the book is a mode and the general layout and printing are admirable.

J. L. L.

Highway Engineers' Reference Book, 1950-5 General Editor, E. Molloy. (London, Newnes, 1950 464 pages, 11 in. by 8½ in. 45s.

A new edition of this useful book has been publishe in which there has been a complete revision and r arrangement. The book is divided into eleven sectio which cover road design and construction from th initial project to lighting, traffic control and mai tenance.

Special features of this edition are new articles The Special Roads Act, 1949, Table of Principal Statut governing Highways and Streets, and a completel new section on Bridges, which includes articles on Brid Design both in Steel and Concrete, and Steel She Piling and Bearing Piles, with particular applicatio to Bridge Design.

The subject of Traffic Control forms a separa section which includes a resumé of the Ministry Transport Departmental Committee's Report on Traff Control.

Manufacturers' Data, covering progress and develo ment in each sphere of highway construction an equipment, follows at the end of the individual section

The book is well illustrated.

Institution Notices and Proceedings

ORDINARY GENERAL MEETING

An Ordinary General Meeting of the Institution of ructural Engineers was held at 11, Upper Belgrave reet, London, S.W.1, on Thursday, November 23rd, 950, at 5 p.m., Mr. J. E. Swindlehurst, M.A., M.I.C.E., .I.Struct.E. (President), in the Chair.

The following members were elected in accordance ith the Bye-Laws. Will members kindly note that the ections, as tabulated below, should be referred to when nsulting the Year Book for evidence of membership ?

Students

DAMS, Peter Henry, "Summerville," Derek Gardens, Southend-on-Sea.

TTWOOD, Roy Leonard, 98, Chaddesden Park Road, Derby.

ANK, Nathan, 72, Derby Road, Judith Paarl, Johannesburg, South Africa.

LACKBURN, Keith Chalkley, 4, Woodside Road, Woodford Green, Essex.

OOTH, William Harold, 5, Highroyd, Moldgreen, Huddersfield, Yorks.

LARK, Peter Jerome, 58, Brookvale Road, Olton, Birmingham, 27.

OOPER, Ivan Maurice, 37, Africa Street, Oaklands, Johannesburg, South Africa.

OXHEAD, Peter Leonard, 186, Clock House Road, Beckenham, Kent.

AVIES, Colin, 7, Senghenydd Street, Treorchy, Glam.

OWELL, Walter Roy, 145, Kingston Road, Willerby, E. Yorks.

ZIEWULSKI, Jerzy, 26, Spruce Hills Road, Walthamstow, E.17.

ARLEY, Michael George Gilbert, 1, Vine Gardens, Milehouse, Plymouth.

AWLER, Stanley William, c/o Hydro Design Office, Ministry of Works, Wellington, New Zealand.

REEN, Donald Herbert, "Arlington," Silver-Sides, Brigg, Lincs.

ALLOWS, Brian Hector, 23, Dorset Avenue, Romford, Essex.

OUGHTON, Tom Keith, 2, Buxton Street, Hazel Grove, Stockport, Cheshire.

AMES, Dennis Frederick, 29, Barriedale, New Cross, London, S.E.14.

KACZKOWSKI, Tadeusz Adam, 43, Bromley Road, Walthamstow, E.17.

KERR, David Sydney Mackay, 3, Fourth Street, Delarey, P.O. Maraisburg, Transvaal, South Africa.

KWIATEK, Zbigniew, 43, Bromley Road, Walthamstow, E.17.

MORTON, James, 11, Hillside Drive, Bishopbriggs, Glasgow.

NIGHTINGALE, Alan Francis, 20, Laings Road, Lower Hutt, New Zealand.

PULLER, Malcolm John, 63, Highams Station Avenue, South Chingford, E.4.

READ, Roy Benjamin, 827, Old Kent Road, Peckham, London, S.E.15.

ROBERTS, David John, 278, Devonport Road, Tauranga, New Zealand.

ROWLEY, John Charles, 63, Pelham Road, Wimbledon, S.W.19.

SKUPIN, Henry, 13, Westbury Road, London, E.17.

SUTTON, Derek Harold, 65, Beresford Road, Chingford, E.4.

THISTLE, Sydney Alan, 3, Richmond Terrace, Monkseaton, Whitley Bay, Northumberland.

WALKER, Brian Colin, 8, Elizabeth Avenue, Sandringham, Johannesburg, South Africa.

WALKER, James, 53, Ashdown Avenue, Scunthorpe, Lincs.

WILKINSON, Leslie Alan, 356, Ormond Road, Gisborne, New Zealand.

WOJCIECHOWSKI, Jan, 63, Rectory Road, Walthamstow, E.17.

Graduates

ANDERSON, Albert William, Marine Guest House, 1, Marine Terrace, Havre des Pas, Jersey, C.I.

BANKS, Douglas Henry, 156, High Street, Coleshill, Birmingham.

BANKS, John Bertram Wallace, 68A, Mount Road, Wallasey, Cheshire.

BOLTON, Arthur, B.Sc.(Tech.), Manchester, 78, Bluestone Road, Moston, Manchester, 10.

BOOTH, Alan Sutherland, 23, Normanton Avenue, Salford, 6, Lancs.

BREWINGTON, Charles Bernard, B.Sc.(Hons.), Birmingham, 16, Glenfield Grove, Selly Park, Birmingham, 29.

CHRAPOWICKI, Konstanty Antoni, 118, Cromwell Road, London, S.W.7.

COHEN, Stanley Ernest, B.Sc., Rand, 802, Sunninghill, Koch Street, Joubert Park, Johannesburg, South Africa.

CORP, Roy, B.Sc.(Eng.), London, 56, Hawthorn Avenue, Anlaby Road, Kingston-on-Hull.

CUSSON, Jeffrey Raymond, 1, Lynwood Gardens, Walton, Liverpool, 9.

DOUGLAS, Thomas Harrison, 14, Murrayfield Avenue, Edinburgh, 12.

FEWSTER, Howard Morgan, A.M.I.C.E., 56, Fidlas Avenue, Llanishen, Cardiff.

HOLLAND, Ronald Wilber, B.A.(Hons.), Cambridge, 9, Magda Road, Stockport, Cheshire.

JOLLY, Alwyn Grimshaw, B.Sc.(Eng.), London, A.C.G.I., 46, Egmont Road, Sutton, Surrey.

JONES, Gweurydd, 72, Raeburn Road, Blackfen, Sidcup, Kent.

KERR, James Cowie, B.Sc.(Eng.), Edinburgh, c/o Moir, 24, Little Russell Street, London, W.C.1.

KHAN, Abdut Tawwab, B.Sc.(Eng.), Aligarh, 3, Mariam Manzil, Tirathdas Road, Artillery Maidan, No. 1, Karachi, Pakistan.

LAING, Denis, 10, Irving Close, Woodsmoor, Stockport.

LEIBOWITZ, Bernard, B.Sc.(Eng.), Rand, 12, Hillway, Kingsbury, N.W.9.

MADAN, Madhukar Yeshwant, B.E.(Civil), Bombay, 225, Khetwadi Main Road, Bombay, 4.

MANDELZWEIG, Gordon, B.Sc.(Eng.), Rand, 59, 8th Avenue, Highlands North, Johannesburg, South Africa.

MARTYN, John, B.E., Ireland, 24, St. Gabriel's Road, London, N.W.2.

MIRVIS, Lionel, B.Sc.(Eng.), Rand, Box 6749, Johannesburg, South Africa.

NADEN, James Anthony, 11, Turls Hill Road, Sedgley, Nr. Dudley, Worcs.

PAUL, Peter Richard, 48, Belmore Road, Eastbourne, Sussex.

PAYTON, Alan William, B.Sc.(Eng.), London, 60, Beechwood Gardens, Ilford, Essex.

RAIJI, Prithviraj Diliprai, B.E.(Civil), Bombay, Noserwan Hall, Station Road, Santa-Cruz, Bombay, 23, India.

STEPHENS, Francis David, B.A., Oxford, 307, Burton Road, Derby.

TELLER, Otto George, 22, Park Square East, Regent's Park, N.W.1.

WALKER, Eric Stuart, B.Sc.(Eng.), Glasgow, Reinforcing Steel Co., Ltd., P.O. Box 848, Bulawayo, Southern Rhodesia.

WILLIAMSON, James Wilson, B.Sc.(Civil), Edinburgh, 3, Duggan Avenue, Toronto, Canada.

WOLSTENCROFT, Derek, B.Sc.(Eng.), (Hons.), London, 61, Battlefield Road, St. Albans, Herts.

ZUNZ, Gerd Jack, B.Sc.(Civil), Rand, c/o "Wild Woods," Crews Hill, Nr. Enfield, Middlesex.

MEMBERS

NEWFIELD, Philip, B.Sc.(Eng.), Rand, Box 7233, Johannesburg, South Africa.

PARKMAN, Claude Charles, O.B.E., M.I.C.E., A.M.I. Mech.E., A.M.I.Mun.E., 38, The Temple, Dale Street, Liverpool.

SPILKIN, Joseph Hyman, B.Sc.(Eng.), Rand, P.O. Box 7233, Johannesburg, South Africa.

STRONGMAN, Frank Spensley, A.M.I.Mun.E., P.W.D., P.O. Box 662, Head Office, Nairobi, Kenya Colony.

TRANSFERS

Students to Graduates

BRUNDRITT, Alan, St. Augustine's Vicarage, Lynton Road, London, S.E.1.

LANE, Alan Sidney, 229, Old Shoreham Road, Southwick, Sussex.

STEEL, Richard William Charles, 15, Bramley Road, Sutton, Surrey.

Associate-Members to Members

ANDERSON, Gordon Maconachie, A.M.I.C.E. (S.A.), P.O. Box 9542, Johannesburg, South Africa.

BARBER, Walter, 20, Selwyn House, Manor Fields, Putney Hill, London, S.W.15.

BENNETT, Denis John, A.M.I.C.E., Kelham, Bath Lane, Fareham, Hants.

HOROWITZ, Ludwig, LL.D., DIP.Eng., A.M.I.C.E., 6, Millstone Lane, Leicester.

KANG, Chandan Singh, A.M.I.E.(India), Executive Engineer, Anandpur Division, P.O. Rupar, Dist. Ambala, Punjab, India.

MACDONALD, Archibald Parker, B.Sc., Dalhousie, A.M.I.C.E., 115, Pound Court Drive, Orpington, Kent.

MARQUES, Charles Albert, M.B.E., Culverwood House, Little Berkhamsted, Nr. Hertford, Herts.

SARMA, Kasi Govind, B.Eng., B.Sc.(Eng.), Manchester, A.M.I.C.E., Chief Engineer's Office, Bombay Port Trust, Ballard Road, Fort, Bombay, 1, India.

RE-ADMISSION

Associate-Membership

WEYMOUTH, Robert Issell, Trinidad House, Coventry Street, Birmingham, 5.

OBITUARY

The Council regret to announce the deaths of Field-Marshal The Rt. Hon. Jan Christiaan SMUTS (Honorary Member); George Walker ATKINSON, Selwyn George BENNETT, Joseph BISSET, Donald BROOKE, Walter MACDONALD, Douglas Henry REMFREY, Robert Dudley WOOD (Members); William Ewart THOMAS, A. E. VICKERY (Retired Members); Frank Guy BURS (Associate); James Percival CORNELIUS, Ernest Edm LARKIN (Associate-Members).

EXAMINATIONS, JULY, 1950

(OVERSEAS CENTRES)

The Institution's Examinations were held in J 1950, in Auckland (N.Z.), Beirut, Bombay, Ca Calcutta, Cape Town, Christchurch (N.Z.), Colom Aligarh (Delhi), Dunedin (N.Z.), Durban, Hong K Johannesburg, Kuala Lumpur, Lahore, Luckn Madras, Mosul, Penang, Port Elizabeth, Salisb (Southern Rhodesia), Shillong, Singapore, Tel-A and Wellington (N.Z.).

Twenty-four candidates entered for the Graduate Examination and sixty-two for the Associate-Mem ship Examination, making a total of eighty-six. these twelve passed the Graduateship Examination nineteen passed the Associate-Membership Examinat

The names of the successful candidates are :—

GRADUATESHIP EXAMINATION

EDWARDS, Paul Harper Dickinson, HERBERT, Ro William, JAMES, Bryan Wilson, MAK AH CHAN, MOD Sydney, POGSON, Albert Leonard, REED, Rob William, RUDD, Arthur Claude, SHAPIRO, Myer Jac THOMAS, David Lewis, WATSON, Ian Dissington, WYL Gerald Alfred.

ASSOCIATE-MEMBERSHIP EXAMINATION

ARMSTRONG, Stanley George, CHATTERJEE, Ra Mohan, CHELVADURAI, Kanakenthiram, CLIFF, De Rydal, CROSSLEY, Bernard Trevelyan, DOBSON, Patr Arthur, DUBASH, Ruton Jamshedi, EHRLICH, Rob ERULKAR, Jonathan Eliab, GEDYE, Noel Trev GOLDIE, John Barkley, KHARE, Shrikrishna Tryamb MATTHEWS, Samuel Wauchope Armour Charles, PITR Kenneth John, RYSS, Jurie, SINGH, Kishen, V TINTEREN, Boudewijn, WONG GAI HONG, YAP LE KEE.

PRIZE LIST—JULY, 1950 EXAMINATIONS

ANDREWS PRIZE. (For the candidate who obtains highest aggregate of marks in the Associate-Members Examination, passing in all subjects).

Leslie Kenneth BOND, of London.

HUSBAND PRIZE (For the candidate who takes whole of the Associate-Membership Examination, pa in all subjects, and obtains the highest marks in paper "Structural Engineering Design and Drawing

Henry Victor WELLS, of London.

WALLACE PREMIUM (SENIOR) (For the candidate v takes the whole of the Associate-Membership Exami tion, passes in all subjects, and obtains the high marks in the paper "Theory of Structures (Advanced

Leslie Kenneth BOND, of London.

WALLACE PREMIUM (JUNIOR) (For the most succes candidate in the Graduateship Examination, pass in all subjects).

Maurice Grant BAGULEY, of Bexleyheath, Kent.

EXAMINATIONS, 1951

The Examinations of the Institution will be held centres in the United Kingdom and Overseas on Janu 9th and 10th, and July 17th and 18th (Graduatesh and January 11th and 12th, and July 19th and 2 (Associate-Membership).

REPRINTS FROM THE JOURNAL

Authors who wish to obtain reprints of their pa published in the Journal, are requested to communic

th the publishers at least one calendar month before e date of publication of the paper in question.

DRURY MEDAL AWARD

The third competition for the above award will take ace in 1951, and the subject will be the design of a nforced concrete garage with provision for erection ove it of future superstructure, the garage being bjected to unbalanced earth pressure.

Graduates and Students of the Institution who wish compete are invited to apply for full details to the cretary ; envelopes to be marked in the top left-hand rner " Drury Medal Award."

The closing date for the competition is October 1st, 51.

The general conditions of the competition are as llows :—

1. The competition shall be for Graduates and udents of the Institution of not more than 25 years age.

2 The subject of the competition shall be a design of structural character, that is to say, primarily structural sign, not planning.

3. The subject of design and conditions shall be repared and issued biennially by a group of five mem- ers appointed by the Council.

4. The Literature Committee shall appoint a jury of ot less than five to examine the works submitted and interview candidates, if found necessary.

5. In order to show that the work submitted is solely he work of the competitor, the documents submitted hall be countersigned by a corporate member of the nstitution, or, failing this, shall be accompanied by a eclaration on a prescribed form signed by the candidate i the presence of a Justice of the Peace or a Commis- ioner for Oaths.

THE MACLACHLAN LECTURE

General Conditions

Through the generosity of Mr. John MacLachlan Retired Member), the Council was able in 1948 to nstitute an Annual Lecture to be competed for by Associate-Members. The conditions of the presentation re as follows :—

1. The Institution of Structural Engineers shall nstitute a written lecture to be known as the MacLachlan ecture and to be held annually.

2. The subject of the Lecture may be on any aspect f Structural Engineering so long as in every second ear the subject shall be confined to steel structures.

3. Entrance into the competition for the Lecture shall e confined to Associate-Members of the Institution.

4. All papers entered for the competition shall be ubmitted to assessors to be appointed by the Council of he Institution and all such papers (including the prize- winning Lecture) shall be available for publication in the Journal of the Institution at the discretion of the Council.

5. The winner of the competition shall be required to present the Lecture to a meeting of the Institution at which he will be presented with the sum of £17 10s.

6. In the event of there being no winner of the competition in any one or more years, whether because no lecture is submitted or because no lecture submitted is considered to be of sufficient merit to warrant an award, or for any other reason, the Institution shall transfer this sum to the Research Fund of the Institution.

Particulars of the Competition for 1951

1. The MacLachlan Lecture will be given at a meeting of the Institution to be arranged towards the end of 1951.

2. The subject of the Lecture shall be confined to Steel Structures.

3. The work should be submitted as the script of a lecture which the author, if successful in the competition, will deliver before an audience in the course of about one hour. The development of mathematical formulæ and detailed calculations should be avoided as far as possible in the text ; if they are essential they should be embodied in appendices. Photographs, drawings, graphs, etc., which would appear as illustrations to the Lecture in published form, should accompany the script. If additional illustrations would be shown as slides, a list of these should be included.

4. Six copies of each Lecture should be submitted and should be addressed to the Secretary of the Institution.

5. The closing date for the receipt of entries by the Institution is Friday, March 30th, 1951.

Mr. F. E. DRURY (PAST-PRESIDENT)

The Council announces with deep regret that Mr. F. E. Drury, O.B.E., M.Sc., who was President of the Institution for the Session 1945-1946, died suddenly at his home in Orpington on November 19th, 1950, at the age of 72.

Francis Edson Drury was born in Yorkshire in 1878, and was trained for a teaching career. His first appointment was in Oldham, after which he moved to London, where he gave advanced lectures on structures and applied geometry at the City and Guilds College. Feeling the need for practical experience, however, he resigned his teaching appointment to enter a building contractor's business, where he spent six or seven years. In 1907 he returned to teaching as Chief Lecturer in Building at the Halifax Technical College. In 1913 he became Head of the Building and Civil Engineering Department at the Royal Technical College, Salford, and from there he went as Head of the Building Department, to the Manchester College of Technology. From 1915 to 1922 he carried out considerable research work on concrete, for which he was awarded the degree of M.Sc.(Tech.), in 1923.

From 1918 to 1927, Mr. Drury was senior partner in the firm of Drury & Gomersall, Architects, Surveyors and Consulting Engineers. He was then appointed Principal of the London County Council School of Building, an appointment which he held until December, 1943, when he retired. During this period, Mr. Drury took an active part in the work of the Institution, of which he had been a member since 1913, as an examiner, as a member of the Literature and Science Committees, and as Chairman of the Education and Examinations Committee and of the Membership Committee. He was elected to the Council in 1940, became Honorary Secretary in 1941, Vice-President in 1943, and President for the Session 1945-46.

For many years, Mr. Drury served on Committees set up in connection with building education and he represented the Institution on the Building Apprenticeship and Training Council. He was the author, or joint author, of a number of books on construction, quantities, building science and mathematics, and for his paper on " The Structural Engineer and his Vocational Training," read before the Institution in 1931, he was awarded a Silver Medal.

In 1946, the King honoured Mr. Drury for his public services by awarding him the O.B.E.

LAPSED MEMBERSHIP

The membership of the undernoted has lapsed in accordance with the provisions of Bye-Law 31 :— Cyril Murton Croft, Cyril Hawthorn Mitchell,

Sidney Thomas SMITH, William Francis TIPPET (Members); Oliver Stanley WEBB (Retired Member); William Benford MOORE (Associate); John Bryce Wesley ABELL, Manual ACKERMAN, Cecil Vance BOWLES, John BRENNAN, Ivor BROOKS, A. D. CLARK, Krishnaji Ganesh DEODHAR, Naum FISER, Edward Thomas HANRAHAN, Chung Ying NIEN, Robert TURNER, Ramjee Prasad VERMA, Noel John WALLIS (Associate-Members); Richard Purcell BLOW, Donald CARTWRIGHT, Harold Anthony EDWARDS, Stanley James FERRIS, Cyril Rowland HARMAN, Lawson Herbert Beese HATHERELL, Peter Frederick HODGES, Siang LI, Harry LUCAS, Geoffrey OSBORNE, Herbert Harry SPENCER, George William TAYLOR, Arthur Cecil TIVEY, George Albert WADLOW, Alastair Claude WICKRAMASINGHE (Graduates); Norman Harry BALL, Leonard Brokensha BURROWS, Robert Dennis CHERRY, Robert Michael HIGGIN, Robert Mortimer LLOYD, Denis George Stewart LOOSEMORE, C. K. Radha Krishna SASTRY, Victor Ayoade SOLANKE, Shri Narayan TIWARI (Students).

CORRIGENDUM

1950/51 edition of the Year Book, page 4, and Sessional Programme 1950/51, page 3.

The entries under the name of Mr. H. C. Husband should be amended to read as follows:—

H. C. Husband, B.Eng., M.I.C.E., M.I.Mech.E.

FORTHCOMING MEETINGS

The following meetings will be held at 11, Upper Belgrave Street, London, S.W.1:—

Thursday, January 11th, 1951

Ordinary Meeting at 6 p.m., when Dr. W. Eastwood, B.Eng. (Associate Member), will present a paper on "A Comparison of the Bearing Power of Footings on Dry and Inundated Sand."

Thursday, January 25th, 1951

Ordinary General Meeting for the election of members, 5.55 p.m., followed by Ordinary Meeting at 6 p.m., when Mr. J. W. H. King, M.Sc., A.M.I.C.E. (Associate Member), will present a paper on "A Fundamental Approach to Prestressed Concrete."

Thursday, February 8th, 1951

Ordinary Meeting at 6 p.m., when Mr. F. G. Etches, B.Sc.(Eng.), A.M.I.C.E. (Member), will present a paper on "The Construction of an Extension to a Factory at Silvertown."

Wednesday, February 14th, 1951

Joint Meeting with the Reinforced Concrete Association, 6 p.m., when Professor A. L. L. Baker, B.Sc., M.I.C.E. (Member), will present a paper on "The Ultimate Strength of Prestressed Concrete."

Thursday, February 22nd, 1951

Ordinary General Meeting for the election of members, 5.55 p.m., followed by Ordinary Meeting at 6 p.m., when Dr. G. G. Meyerhof, M.Sc., Ph.D., A.M.I.C.E. (Associate Member), will present a paper on "Building on Fill, with Special Reference to the Settlement of a Large Factory."

Thursday, March 8th, 1951

Ordinary Meeting at 6 p.m., when Mr. C. L. a'Court, M.I.C.E., will present a paper on "Special Treatment for Portland Cement Concrete."

Thursday, March 29th, 1951

Ordinary General Meeting for the election of members, 5.55 p.m., followed by Ordinary Meeting at 6 p.m., when Mr. E. Roland Hole (Member), and Mr. F. C. Ea (Associate Member), will present a paper on "T Construction of the Royal Festival Hall."

Members wishing to bring guests to the Ordina Meetings of the Institution announced above requested to apply to the Secretary for tickets of mission.

LONDON GRADUATES' AND STUDENTS' SECTIO

The next meeting will be held at 11, Upper Belgra Street, London, S.W.1, on Tuesday, January 23 1951, at 6 p.m., when Mr. J. E. Swindlehurst, M. M.I.C.E. (President), has kindly consented to addr the section.

Hon. Secretary: W. S. Twelvetrees, 6, Dell Ro Enfield Wash, Middlesex.

BRANCH NOTICES

LANCASHIRE AND CHESHIRE BRANCH

The following meetings have been arranged:—

Tuesday, January 16th, 1951

Third Ordinary Meeting. Open Discussion Meeti at the College of Technology, Manchester, at 7 p.m.

Thursday, February 15th, 1951

Fourth Ordinary Meeting. Joint Meeting with Institution of Civil Engineers, North-Western Assoc tion. Mr. J. Cunningham, B.Sc., A.M.I.C.E., on "T Britannia Tubular Bridge over the Menai Straits," the Engineers' Club, Albert Square, Manchester, 6 p.m.

Friday, March 16th, 1951

Fifth Ordinary Meeting. Details to be announc later.

Hon. Secretary: G. Greenlees, 16, Mayfield Aven Stretford, Lancs.

GRADUATES' AND STUDENTS' SECTION

The following meetings have been arranged:—

Wednesday, January 10th, 1951

Third Ordinary Meeting. Mr. P. N. Reed on "T History and Development of the Manchester Sh Canal."

Friday, March 30th, 1951

Annual General Meeting. Details to be announc later.

Hon. Secretary: F. H. Turner, 11, Colwell Aven Stretford, Lancs.

MIDLAND COUNTIES' BRANCH

The following meetings have been arranged:—

Friday, January 26th, 1951

Mr. A. Sutcliffe and Mr. G. E. Garrard on "So Aspects of the Construction of a Large Modern T plate Cold Reduction Plant."

Friday, February 23rd, 1951

Mr. A. J. Francis, M.Sc.(Eng.), Ph.D., A.M.I.C (Associate Member), on "Distribution Methods Structural Analysis."

Friday, March 30th, 1951

Mr. W. P. Andrews and Mr. P. G. Bowie, A.M.I.C (Member), on "The Solution of the Car Parking Probl Here and in the U.S.A."

All the above meetings will be held at the Jam Watt Memorial Institute, Birmingham, at 6 p.m.

Hon. Secretary: E. R. Deeley, Arranmoor, Adshe Road, Dudley, Worcs.

GRADUATES' AND STUDENTS' SECTION

The following meetings have been arranged :—

aturday, February 24th, 1951

Visit to the Round Oak Steel Works, at 10 a.m.

uesday, March 20th, 1951

Annual General Meeting. Films : " The Failure of he Tacoma Narrows Bridge," and " The Collapse of the Remagen Bridge," at the James Watt Memorial Institute, Birmingham, at 7 p.m.

Hon. Secretary : M. H. Evans, B.Sc., 107, Metchley Lane, Harborne, Birmingham, 17.

NORTHERN COUNTIES' BRANCH

The following meetings have been arranged :—

Wednesday, January 10th, 1951

Joint Meeting with the Institution of Civil Engineers. Mr. E. F. Bowen and Mr. T. P. Reeves (Associate Member), on " Reconstruction in Concrete of a Steel Pipe Bridge," at Middlesbrough.

Wednesday, January 17th, 1951

Joint Meeting with the Northern Architectural Association. Discussion : " The Aesthetics of Structural Engineering," at Newcastle.

Tuesday, February 6th, 1951

Mr. N. J. Ruffle (Graduate), on " Concrete Control for Small Works," at Middlesbrough.

Wednesday, February 7th, 1951

Mr. N. J. Ruffle (Graduate) will present the same paper at Newcastle.

Tuesday, March 6th, 1951

Mr. P. L. Capper, T.D., M.Sc., A.M.I.C.E. (Associate Member), on " Testing Methods in Structural Engineering," at Middlesbrough.

Wednesday, March 7th, 1951

Mr. P. L. Capper will present the same paper at Newcastle.

All meetings commence at 6.30 p.m., preceded by tea at 6 p.m., the Middlesbrough meetings being held at the Cleveland Scientific and Technical Institution, Corporation Road, and those at Newcastle in the Neville Hall, near the Central Station.

Hon. Secretary : J. A. Williams, A.M.I.C.E., 57, Reid Park Road, Jesmond, Newcastle upon Tyne, 2.

NORTHERN IRELAND BRANCH

The following meetings have been arranged :—

Tuesday, January 9th, 1951

Details of the lecture to be announced later.

Tuesday, February 6th, 1951

Mr. W. Shearer Smith, A.M.I.C.E. (Associate Member), on " Cold Formed Sections in Structural Practice with a Proposed Design Specification."

Tuesday, March 6th, 1951

Film : " Military Pile Driving."

The above meetings will be held in the College of Technology, Belfast, at 7.15 p.m.

Hon. Secretary : S. G. Duckworth, "Lisleen," 13, Finaghy Road North, Belfast.

SCOTTISH BRANCH

The following meetings have been arranged :—

Friday, January 12th, 1951

Joint Meeting with the Institution of Civil Engineers. Mr. S. Packshaw, M.I.C.E. (Member), on " Recent Developments in Steel Piled Construction," at Institution of Engineers and Shipbuilders, Glasgow, 6.15 p.m.

Friday, January 26th, 1951

Joint Meeting with the Institution of Civil Engineers. Mr. S. Vaughan, B.Sc., M.I.C.E. (Member), on " The New House of Commons—Foundations and Structure," at the Royal Technical College, Glasgow, 6 p.m.

Tuesday, February 20th, 1951

Mr. H. B. Sutherland, S.M., A.M.I.C.E. (Associate Member), will present a paper at Ca'doro Restaurant, Glasgow, 6 p.m. (Title to be announced later).

Hon. Secretary : D. G. Drummond, B.Sc., A.M.I.C.E., 11, Woodside Terrace, Glasgow, C.3.

SOUTH-WESTERN COUNTIES' BRANCH

Hon. Secretary : E. W. Howells, 10-12, Market Street, Torquay.

WALES AND MONMOUTHSHIRE BRANCH

The following meetings have been arranged :—

Friday, January 19th, 1951

Mr. D. J. Maclean, B.Sc., A.Inst.P., on " Soil Mechanics," at Colwyn Bay.

Tuesday, February 6th, 1951

Mr. W. S. Atkins, B.Sc., M.I.C.E., on " Modern Trends in Structural Design," at Cardiff.

Tuesday, February 13th, 1951

Mr. Norman Thomas, M.B.E., A.R.I.B.A., on " Planning Factories for Light Industry," at Swansea.

Friday, February 24th, 1951

Details of the lecture to be announced later, at Colwyn Bay.

Tuesday, March 6th, 1951

Mr. G. R. Brueton, A.M.I.C.E. (Associate Member), will present a paper at Swansea. (Subject to be announced later).

Wednesday, March 14th, 1951

Mr. G. R. Brueton will present the same paper at Cardiff.

Friday, March 30th, 1951

Mr. A. V. Hooker, A.M.I.C.E. (Associate Member), on " Structural Engineering at Abbey Works," at Colwyn Bay.

The meetings in Cardiff will be held at the South Wales Institute of Engineers, Park Place, at 6.30 p.m.

The meetings in Swansea will be held at the Mackworth Hotel, at 6.30 p.m.

The meetings at Colwyn Bay will be held at the County Buildings, at 6 p.m.

Hon. Secretary : E. R. Steward, Edrom, Ashleigh Road, Blackpill, Swansea.

WESTERN COUNTIES' BRANCH

The following meetings have been arranged :—

Friday, January 5th, 1951

Mr. N. G. T. Ball (Associate Member), on " The Reconstruction of the Colston Hall, Bristol," at Grand Hotel, Bristol.

Friday, February 2nd, 1951

Joint Meeting with the Institution of Civil Engineers. Professor Andrew Robertson, D.Sc., F.R.S., M.I.C.E., on " A Chapter of Engineering a Century Ago," at College of Technology, Bristol.

Friday, March 2nd, 1951

Mr. F. W. L. Heathcote and Mr. A. H. Chilver, B.Sc. (Student), on " Light Gauge Structural Steel Sections," at College of Technology, Bristol.

The above meetings will be held at 6.15 p.m., and will be preceded by tea at 5.30 p.m.

Hon. Secretary : C. E. Saunders, Dunkery, Edward Road, Walton St. Mary, Clevedon, Somerset.

YORKSHIRE BRANCH

The Second Ordinary Meeting was held on Wednesday, November 15th, 1950, at 6.30 p.m., in the Great Northern Hotel, Leeds, when Mr. H. C. Husband, B.Eng., M.I.C.E., M.I.Mech.E. (Member), was in the Chair, and 40 members and visitors were present. Mr. L. Scott White, M.I.C.E. (Member), gave a lecture on " Moving King Henry VIII's Wine Cellar." At the conclusion of the lecture a vote of thanks was proposed by Mr. James Bussey (Member), and seconded by Captain J. B. Lawson, M.B.E., A.R.I.B.A. (Member).

The following meetings have been arranged :—

Wednesday, January 17th, 1951

Mr. D. J. Davies, B.Sc.(Eng.), A.M.I.C.E. (Associate Member), on " Welded Highway Bridges."

Wednesday, February 21st, 1951

Junior Branch Prize Paper.

Friday, February 23rd, 1951

Annual Dinner and Dance, at Castle Grove Masonic Hall, Headingley, Leeds, at 7 p.m.

Wednesday, March 21st, 1951

Joint Meeting with the Yorkshire Association of the Institution of Civil Engineers. Mr. J. Guthrie Brown, M.I.C.E. (Member), on " Hydro-Electric Development, Pitlochry," at Leeds University, at 7 p.m.

All meetings, except the Joint Meeting on March 21st, 1951, will be held at the Great Northern Hotel, Leeds, at 6.30 p.m.

Hon. Secretary : E. Wrigley, City Engineer's Department, Civic Hall, Leeds.

UNION OF SOUTH AFRICA BRANCH

Branch Hon. Secretary : A. E. Tait, B.Sc., A.M.I.C.E., P.O. Box No. 3306, Johannesburg. During week-days Mr. Tait can be contacted in the City Engineer's Department, City Hall, Johannesburg, 'phone 34-1111 ext. 257.

Natal Hon. Secretary : E. G. Bennett, c/o Reinforcing Steel Co., Ltd., P.O. Box 478, Durban.

Cape Section Hon. Secretary : R. Stubbs, P.O. Box 1692, Cape Town.

ADDITIONS TO THE LIBRARY

The following publications have been added to th Library :—

AMERIKIAN, A. Analysis of Rigid Frames (An Applica tion of Slope Deflection). Washington, 1942.

BAKER, A. L. L. Reinforced Concrete. London, 194 Presented by Professor R. H. Evans.

BAUER, E. E. Plain Concrete. 3rd Edition. Ne York and London, 1949. Presented by Mr. R. Wilkins.

BLAKEY, Joseph. University Mathematics. Londo and Glasgow, 1949. Presented by Mr. F. A. Gerar

BUTTERWORTH, S. Structural Analysis by Momer Distribution. London, 1949. Presented by Mr. D. Williams.

FAIRWEATHER, G. Structural Economy for the Arch tect and Builder. London, 1949. Presented b Mr. F. S. Mumford.

GLOAG, J., and BRIDGWATER, D. A History of Cas Iron in Architecture. London, 1948. Presented b Mr. S. B. Hamilton.

HARRIS, C. O. Strength of Materials. Chicago, 194 Presented by Mr. H. A. Warren.

HARRIS, G. W. Clerks of Works Manual. London, 194 Presented by Mr. Scott H. Hume.

HERZKA, Leopold. Statik der Formanderungen vo Vollwandtragwerken. Vienna, 1948. Presented b Dr. P. W. Abeles.

Iron and Steel Institute. Corrosion of Iron and Steel b Industrial Waters, and its Prevention. Londo 1949. Presented by Mr. R. W. Schofield.

KLOUCEK, C. V. Distribution of Deformation (A Ne Method of Structural Analysis). Translated from th Czech and German editions by A. H. WADDELL ZALUD and F. H. ZALUD. Prague, 1949. Presente by Dr. Hajnal-Konyi, together with the Czech an German editions.

LEGAT, A. W., DUNN, G., and FAIRHURST, W. A. Desig and Construction of Reinforced Concrete Bridge London, 1949 Presented by Mr. Malcolm Glover.

Lincoln Electric Company. Procedure Handbook Arc Welding Design and Practice. Cleveland, Ohi 1942. Presented by Mr. J. V. Taylor.

LOW. B. B. Strength of Materials. London, 194 Presented by Mr. K. E. Walker.

MINIKIN, R. R. Structural Foundations. Londo 1948. Presented by Mr. Gower B. R. Pimm.

MORGAN, W. Elementary Reinforced Concrete Desig for Students of Architecture and Building. Londo 1949. Presented by Mr. T. J. Bray.

REECE, P. O. An Introduction to the Design of Timb Structures, London, 1949. Presented by the Autho

SMITHELS, C. J. Metals Reference Book. Londo 1949. Presented by Mr. E. P. Featherstone.

SULLY, A. H. Metallic Creep and Creep Resista Alloys. London, 1949. Presented by Mr. A. Vivian.

WOOD, R. D. Principles of Quantity Surveyin London, 1948. Presented by Mr. E. R. Knight.

The Construction of an Extension to a Factory at Silvertown*

By Frank G. Etches, B.Sc., M.I.Struct.E., A.M.I.C.E.

Introduction

The site of the factory extension to be described xtends from North Woolwich Road to the River Thames nd covers approximately eleven acres. A wharf xtends along almost the whole of the river frontage nd is provided with 2-ton travelling electric cranes. lthough it was necessary for the owners to have access o this wharf for outgoing goods during the progress of he works, the wharf was not used to receive materials r plant, and all such traffic was by road.

During 1948, the owners, Venesta Limited, decided to econstruct the south end of their site up to the river. t that time this portion of the factory was used for the roduction of plywood and metal-faced ply, but this was lanned to be moved to another site some miles further own the river.

Block E (shown on the site plan) had a five-span timber Belfast truss roof, boarded and felted, carried on steel lattice girders on steel stanchions.

It was known to be carried on substantial concrete strip foundations, but these had sunk under each stanchion, causing the concrete floor to follow in a series of undulations.

One of the essential requirements of the buildings was that they should be provided with travelling cranes, but there was not sufficient headroom in E Block, even if the foundations had been capable of carrying the additional load.

Block J was the newest of the three buildings and was of normal light steel trusses and stanchions covered with corrugated steel sheeting. The mass concrete foundations here also showed clear signs of settlement.

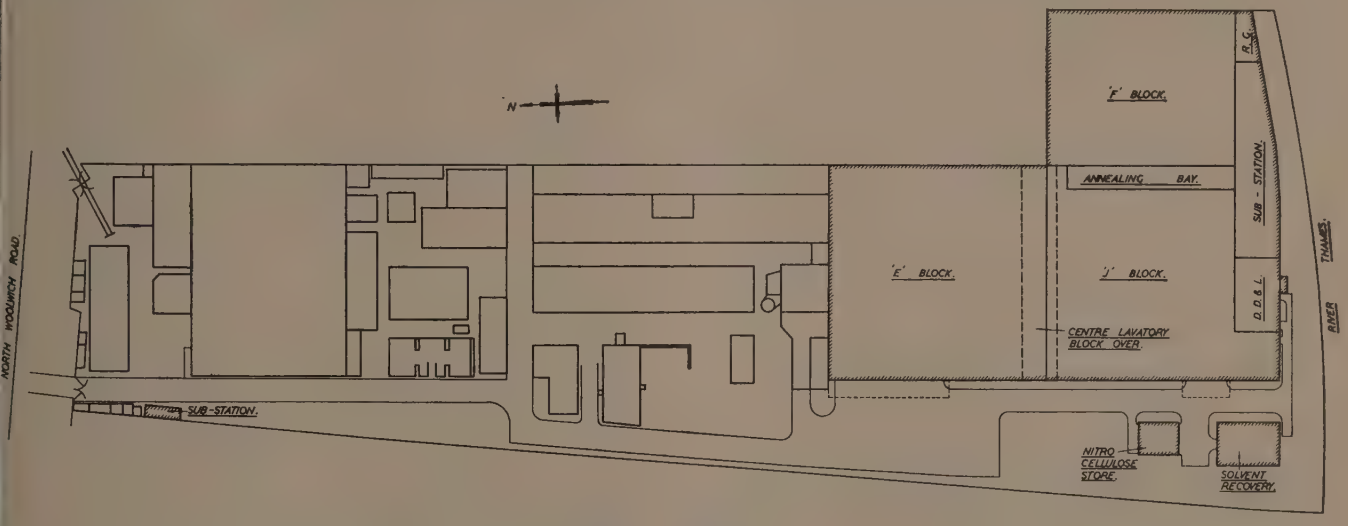


Fig. 1

The aluminium foil rolling section then in operation at the north end of the site had reached the limits of its expansion. It was to be moved to the south end of the site, to the space previously occupied by plywood production, where sufficient room was available for immediate needs and some future extension.

The change of use, however, demanded different conditions and the three main single-storey buildings were reviewed to see if any use could be made of them as they stood, or after reconstruction.

All the buildings had been built or re-built after the Silvertown explosion which occurred on the adjoining site during the 1914-18 War. These buildings were of different forms of construction and had evidently been built or extended at various times.

Block F was a reinforced concrete framed building with brick panel walls carried on piled foundations. The roof was slated and boarded with continuous glazing and ridge lanterns carried on steel trusses supported by the concrete beams. Although the concrete floor had sunk in some areas, there was no indication of any serious settlement of the piled foundations; the headroom would be sufficient for cranes.

The result of this investigation was a decision to retain F Block with some modifications, and to demolish both E and J Blocks. The roof of E Block was intended to provide all the timber which would be needed for the new work; this intention was not fully realised, as the quality was later found to be lower than was hoped. It did, however, provide all the rough shuttering for foundations and for much of the concrete casing for steelwork, where this could not be avoided.

The steel frame of Block J was taken down, cleaned, primed and marked and removed to the second site

Paper to be read before the Institution of Structural Engineers at 11, Upper Belgrave Street, London, S.W.1., on Thursday, February 8th, 1951, at 6 p.m.

referred to above. Here it was re-erected and now forms part of the manufacturing buildings there.

The finished buildings are to be used in the following way: Block F for heavy rolling machinery, Block J for printing and lacquering, and Block E for finishing and packing.

Site Investigation

Before the demolition was started, a number of exploratory pre-piling tests were made in each building and in the adjoining areas. These established that the top 40 feet of the site was made up of soft river deposits and made ground. Below this was about 10 feet of loose gravel with firm clay below. An existing record of a boring for an artesian well made some years ago a little further north on the site confirmed this.

It was clear that all heavy load concentrations would have to be carried on piles, and these would have to be up to 55 feet long. Three 14 in. × 14 in. precast concrete test piles were accordingly driven in such positions that they could be incorporated in the finished work. One of these was selected for a static loading test.

A temporary timber crib was built over the pile and this was loaded with steel kentledge. The top of the pile was brought to a level surface to receive a 200-ton hydraulic jack with pressure gauge reading tons force. A light metal frame was securely bolted near the head of the pile which carried four vertical strips of graph paper, so arranged that one occurred approximately at each corner. This can be seen in Fig. 2.

A theodolite was set up at sufficient distance from the pile to be unaffected by ground movements, and pile settlement under load could be read at each corner, so that any tendency to deflect sideways or buckle would be seen. Settlements could be estimated to .01 inch with ease. The pile actually carried 112 tons with a total settlement of two inches after 24 hours.

Piling

Owing to the possibility of bad weather, the contractor elected not to strip the roof of E Block at once, but to

Fig. 2.—Kentledge and hydraulic jack ready for pile loading test

begin casting concrete piles under cover on the existing floor screeded up level. The precast concrete piles were 14 in. × 14 in. in section having four $1\frac{1}{8}$ inch diameter main rods and cast in 1 : $1\frac{1}{2}$: 3 concrete.

Pile driving started from the river front and worked northwards. Some 250 precast reinforced concrete piles varying in length from 45 to 55 feet were driven from late December, 1949, to March, 1950, starting with one pile frame, but with a second frame in use during January. Fig. 3 shows a view of the site on completion of driving.

While these piles were being driven, 72 bored piles were sunk in F Block to carry the foundations for the rolling mills. The bored piles were 17 in. diameter reinforced with six $\frac{3}{4}$ in. diameter main rods and cast in 1 : $1\frac{1}{2}$: 3 concrete.

General Description of Buildings

The main entrance to the three new shops is on the west elevation at the junction of E and J Blocks. A

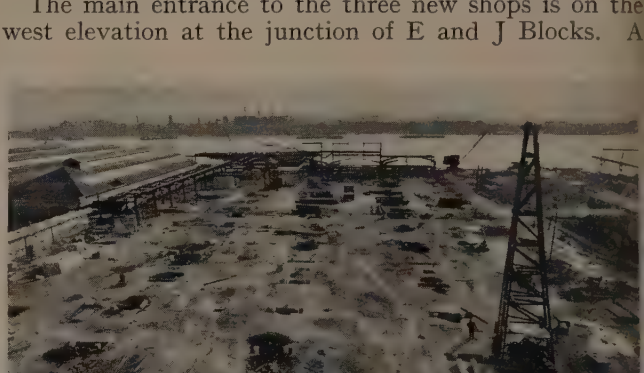


Fig. 3.—General view of site on completion of piling

first floor set immediately over the separating wall between these blocks houses the lavatories and is shown in plan on Fig. 4.

The south end of J Block comprises part of the river elevation and is of three storeys, housing solvents, inks and dyes and provides storage, mixing and weighing rooms.

Just to the east is the H.T. switchroom, transformer cubicles and L.T. switchroom. On the extreme east is the roll grindery.

It is proposed now to describe each of these sections in more detail.

Block F—Rolling Section

The foil rolling mills required very extensive pits up to 10 feet deep below floor level, on and over which the machinery weighing up to 45 tons per machine was mounted. Work started in November, 1949, and some seventy-two 17-inch diameter pressure piles were sunk to a depth of about 50 feet below existing F Block floor level, but the steel and concrete were stopped about eight feet below the surface. They were completed in two months.

The choice of a bored type in-situ pile was dictated by the necessity to avoid damage to the enclosing structure and also by the headroom restriction of the existing roof.

Light steel trench sheeting was then driven around the perimeter of each of the pits. After the sheeting had been strutted with timber, the dumpling was excavated and the pile heads exposed. Concrete 6 in. thick and reinforced with a light fabric was laid on this bottom and $4\frac{1}{2}$ in. brick walls built up to receive a tanking of three-coat asphalt, against which the reinforced concrete walls and bottom were cast. Shuttering for the rather complicated internal surface of the pits was then constructed under easy conditions.

As the pits neared completion, work started on the existing 14 in. square concrete columns which divided the building into three 51 feet bays by 40 feet centres north to south.

The columns were now required to carry 2-ton travelling electric floor operated cranes in each bay, in addition

the weight of the existing roof. In order to strengthen em and provide brackets for the crane beams and ils, their surfaces were hacked back to expose the isting rods and a new reinforced surrounding layer st on with the new brackets. The finished columns are

The method of erection is shown by the diagrams in Fig. 6; the stanchions were erected first and were followed by the Y-shaped valley section welded in place. The centre rafter was composed of 10 in. × $4\frac{1}{2}$ in. joist with $\frac{3}{8}$ in. plates intermittently welded top and bottom

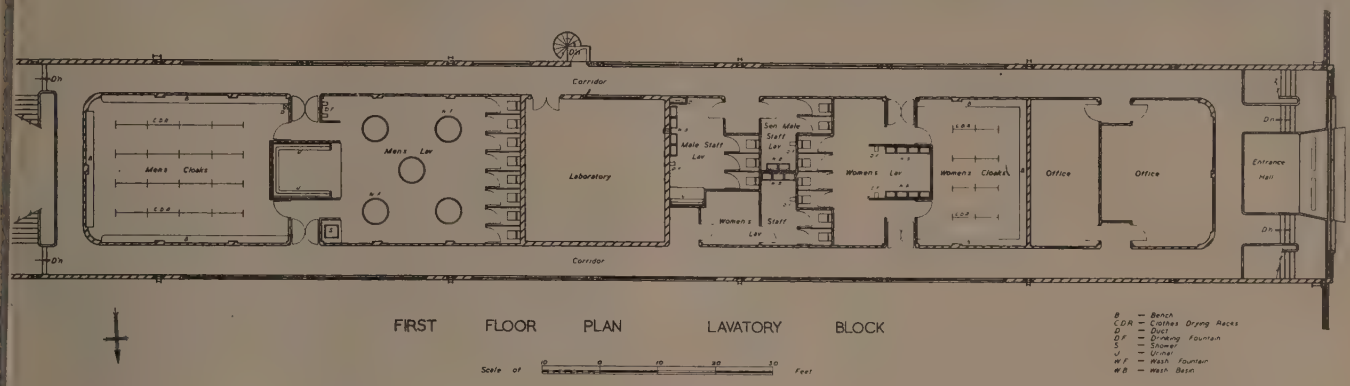


Fig. 4

1 in. square below the brackets. As the existing olumns were neither truly in line nor were the lines ruly parallel, ample pockets were left in the brackets o receive $\frac{3}{4}$ in. diameter mild steel U bolts bolted hrough the bottom flange of the crane beams. These vere grouted up when the beams had been levelled and et to line. Some of the finished columns with crane eams erected are shown in Fig. 5.

Most of the existing service pipes hanging from the oof trusses were taken down, but some of the material vas overhauled and re-used where possible, particularly nit heaters.

Steelwork for Blocks E and J

These long single-storey shops were designed as 10° pitch four 50 feet span welded steel rigid frames. A preliminary analysis was made to arrive at approximate sizes of members. This was followed by solving the three redundances at the foot of each stanchion by means of a model cut from stress-free Perspex approximately 1/10 in. thick, with the depth of the model in proportion to the moment of inertia of the frame at every point.

Fig. 5.—Existing F Block with columns strengthened for cranes

Although it was known that the moments at the foot of each stanchion in the completed structure would be small, the bases were treated as fully fixed to help during erection, particularly for the method chosen to reduce site welding to the minimum.

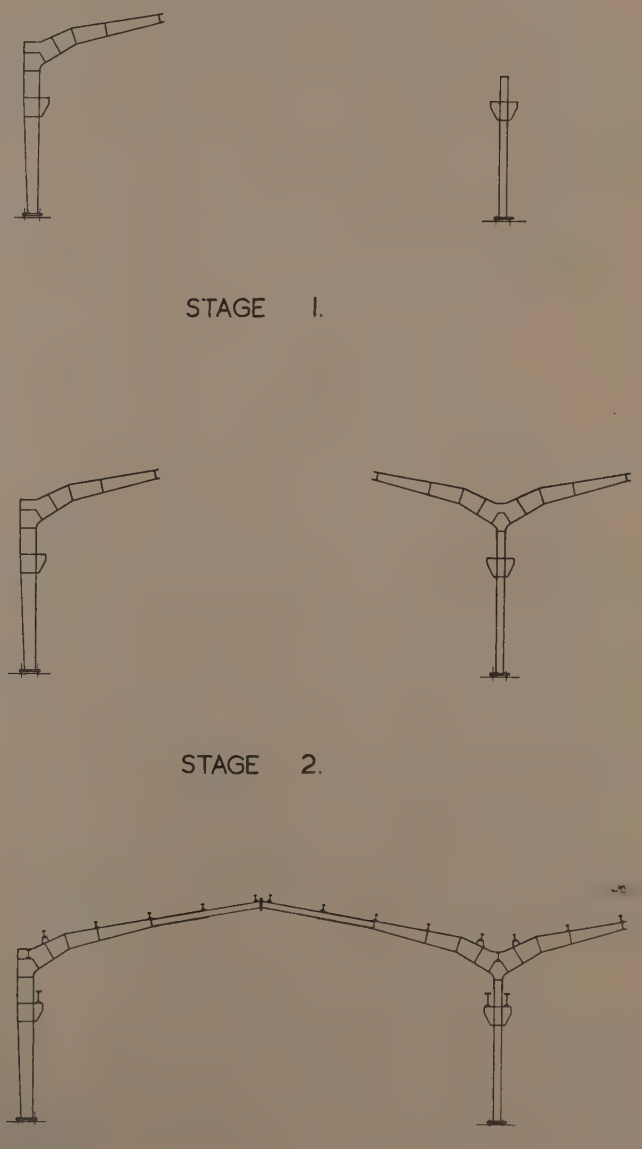


Fig. 6

for part of the length, and bolted through ½ in. meeting plates at the ridge.

The two halves of this rafter were bolted together and lowered into position and site welded to the valley sections on either side. The 8 in. × 4 in. joist purlins were then bolted on to give stability to the completed frames.

The moments in the outermost legs of the continuous frames are comparatively high, and a built-up welded member was chosen as being the most economical solution. It was made up of a ½ in. web tapering from 18 in. down to 10½ in. at the foot, with 10 in. × ¾ in. flange plates.

The moments in the internal stanchions due to bending in the frames are small, but as it was possible for two cranes to lift on either one or both sides of each internal stanchion these, under some conditions, would carry a load of 43 tons and a maximum moment of 370 tons inches due to the eccentricity of the crane reactions. A 10 in. × 10 in. broad-flanged stanchion section was found to give the best solution, and this gave an ideal width of flange to receive the site fillet welds to the 7 in. wide flanges of the upper part of the frame. Fig. 7 shows this connection and the welded crane bracket.

A flat-roof section runs along the east side of both E and J Blocks. The roof adjoining E Block was at high level, and approximately at the valley beam level of the main frame. As the self weight of this roof was considerable, its eccentricity was taken advantage of to reduce the moment in the outer frame leg on this side, requiring a section similar to that used internally.

The roof adjoining J Block, which covers an annealing furnace and cooling tunnel was unavoidably lower owing to the level of F Block roof which abuts it. The loading from it to the main frames was applied too low to give the same advantage as in E Block, and these frame legs were therefore built-up as in other external positions. Fig. 11 shows a typical cross-section through J Block.

The main frames are at 25 ft. 3 in. centres to suit the overall length of the two shops, and each purlin is continuous over one frame and connected at about the

Fig. 7.—Welded rigid steel frames in J Block

quarter point of the span in order to reduce deflection. One bay of the roof is cross-braced in the plain of the rafters with 3 in. × 3 in. × ⅜ in. angles to provide some additional stiffness against wind, although the longitudinal forces due to crane surge and braking are adequately resisted by the crane beams themselves and also by valley beams below the gutters.

The lines of stanchions in E Block occur immediately over the existing strip foundations previously mentioned. Since these were about nine feet below floor level at the north end and were a minimum of 2 ft. 6 in. thick, it was considered more economical to leave them in place, drive piles on each side and straddle them with suitable pile caps. As the bases in J Block were isolated and just below floor level, they were broken up where necessary and a normal size pile cap was adopted.

The steelwork in each shop was kept entirely free and 2 in. clear of brickwork and all other restraint on one side and one end, while beams and purlins occurring on the other side and end were provided with sliding joints to

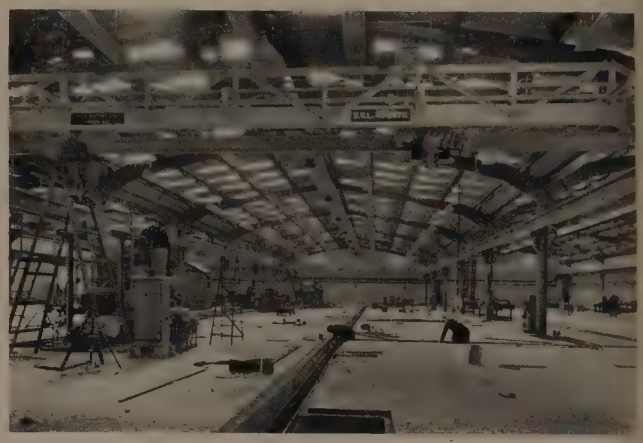


Fig. 8.—New J Block nearing completion

provide for all changes of length due to temperature variations.

Canopies of 20 feet overhang are provided over bays where goods are to be loaded or unloaded, thus affording reasonable protection from the weather. The cantilever arms are of cut and welded 18 in. × 6 in. joist sections site welded to the outer main frames. Temporary cleats were provided to position these cantilevers while being welded to the main frames. A general view of J Block nearing completion is shown in Fig. 8.

The whole of the steelwork is of mild steel to B.S.S. 15 and the design was made in accordance with C.P. 113. All the welded frames and other steelwork were delivered to the site unpainted, and after being cleaned were painted with one coat of Miraculum red lead filler applied on the site where it could be properly supervised. All this steelwork was finally painted two coats of light green to suit the general colour scheme. Encased steelwork was not painted.

Roofing

The roof sheeting is all of Robertsons' Protected Metal of vee beam section laid direct on ½ in. fibreboard lining to a slope of 10°. This is the minimum slope recommended for this section, which spans between purlins at 6 ft. 9 in. centres.

The area of glazing is only 12 per cent. of the plan area of the roofs of E and J Blocks ; it is provided in the form of fixed sheetlights glazed with wired-cast glass. This type of glass diffuses direct rays of sunlight, and the intensity of daylight illumination is good and very uniform. On an overcast day during November, at 1.30 p.m., a series of readings taken with a light-meter at a level of 3 ft. above floor level gave the intensity of daylight as lying between 24 and 27 lumens per square foot throughout E and J Blocks.

The glass area adopted was checked at the design stage by means of the D.S.I.R. daylight protractors, and this relatively small area of glass represents a correspondingly small amount of heat loss, which is an extremely

portant consideration in any large single-storey ilding.

The flat roofs generally are of hollow precast concrete its screeded to falls with a layer of ½ in. insulation ard covered by two layers of roofing felt bedded in t bitumen and finished with coarse chippings. Lantern hts are provided where required, supported on in-situ ncrete kerbs.

Roof Drainage

In order to avoid internal downpipes and also to avoid far as possible breaking up the existing concrete or for drains, large gutters were provided falling ghtly to one end of each 250 feet long shop where a ngle connection was made. Their capacity is sufficient hold the equivalent of ½ in. of rainfall over the whole the roof area should the 6 in. outlets at any time be ocked.

Each gutter is made up of galvanised ⅛ in. M.S. plate ith welded stiffening brackets and bolted connections etween sections, bedded in bitumastic compound. The oss-section is trapizoidal, with a top dimension of ft. 6 in. A walk-way rests on the stiffening plates ade up of removable sections of framed expanded etal, 1½ in. mesh No. 23, which will also serve as a ow arrestor when required. These walk-ways provide sy access for all roof maintenance and re-glazing with inimum chance of damage to the sheeting. Fig. 9 ows a view of the partly-completed roof with gutters xed ready to receive the walk-ways.

Floor Trenches

Wherever possible the various services running to ndividual machines, such as steam and condense, ompressed air, fresh water and cooling water, are carried n floor trenches of varying size. The walls of these renches were built in concrete bricks and fitted with oncrete-filled cast iron Elkington removable trench

Fig. 9.—Partly completed roofs of E and J Blocks

overs of a weight sufficient to carry the normal expected raffic and occasional heavier point loads. Figs. 5 and 8 how some of these trenches during construction.

A system of under-floor ducts constructed with brick valls and reinforced concrete base and top is also rovided below J Block to convey air charged with olvents drawn from the printing machines to the olvent recovery building to be described later.

Centre Lavatory Block

In order to provide lavatory accommodation within asy reach of all parts of this extension, a two-storey ection is introduced at the junction of E and J Blocks.

Lavatories sufficient for 250 men and 60 women, and cloakrooms for 400 men and 100 women allow for some overlap at changes of shifts. Lavatories are also provided for supervisory and office staff.

Also in this area is a test laboratory which is required to be as near as possible to the printing machines.

A double staircase is provided at each end of this section with a corridor running the length of both sides. This serves the dual purpose of giving access to the various lavatories and also directing the movements of personnel off the factory floor. The floors and walls up to five feet height of these rooms and corridors are finished in terrazzo, except for the cloakrooms and laboratory, which have wood-block floors.

Fig. 10.—South elevation of solvent recovery building

Five washing fountains 52 in. diameter are provided in the men's lavatory as being most suitable and easily kept clean, but wash-basins have been provided elsewhere.

Two drinking fountains are installed in both the men's and women's lavatories, although several others have been provided on the floor of the shops. The lay-out of these lavatories is shown in Fig. 4.

River Front

The south end of J Block ends with a three-storey building facing the river. The top floor contains tanks for oils and solvents supported on open steel bearers over a reinforced concrete sub-floor and walls which form a tank designed to catch these liquids in an emergency.

The floors below are for storage and the wet and dry mixing of inks.

To the east of this three-storey building is the H.T. room, followed by cubicles for six oil-filled 750 kVA transformers, and room for future expansion. Each transformer stands over an area laid to falls and drained away to a concrete pit, so that in the event of fire, the burning oil may be damped and quickly removed to a safe distance. Behind the transformer cubicles is the L.T. distribution and switchroom, and at the extreme south-east of the site, i.e., south of F Block, is the roll grindery, where the rolls used in the foil rolling mills are re-ground after wear or damage.

Ancillary Buildings

A lofty single-storey building to the south-west of the site houses absorbers and fractionating still for the recovery of solvents.

As previously mentioned, the air extracted by hoods over the printing machines is drawn along a system of ducts below the floor of J Block and into this recovery building, which is shown in Fig. 10 nearing completion.

After the various solvents have been recovered and separated, they are pumped back underground and up to the tanks in the three-storey block from which they gravitate for re-use.

The structure consists of frames of 15 in. × 5 in. joists welded together and pin-jointed at their base. The span is 46 feet and height to ridge 32 feet. The roof is also of 10° pitch and sheeted with Robertsons' Protected Metal, but is not lined, as no heating is provided in the building other than that given off by various items of steam-consuming plant.

One other small building, sited just north of the last, is to contain nitro-cellulose in steel drums. These are off-loaded at one end of the building and stored on inclined racks, where they are allowed to roll down an easy slope to the despatch end. The object of their being stored on inclined racks is to ensure that all drums in a line rotate when the lowest of them is withdrawn, so ensuring that the contents remain uniformly moist.

The structure of this building consists of frames of 10 in. × 5 in. joists welded together. The roof is sheeted with Robertsons' Protected Metal, and in this case is lined with ½ in. insulation board immediately below the sheeting to reduce the internal temperature during hot weather.

Drains

Before the present extension, the whole of the factory soil and surface water was emptied into a sewer running along North Woolwich Road. Since this sewer is already heavily overloaded, it was decided to separate soil and surface water into two systems in the new work, and to discharge as much as possible of the surface water into the River Thames.

To achieve this, it has been necessary to discharge at five points under the deck level of the existing jetty through tidal flap valves.

The whole of the soil drains connect to the existing combined system.

Roads and Footpaths

The entire road surface at the south end of the site has been relaid. The sub-soil is poor and variable, and a minimum of 12 in. of rolled hardcore has been used. Some of the softer spots were given 3 in. of ashes before the hardcore was laid, but reliance was placed on well rolling with a 10-ton roller until a stable surface was obtained. A semi-flexible surface of asphalt with granite chippings was then laid.

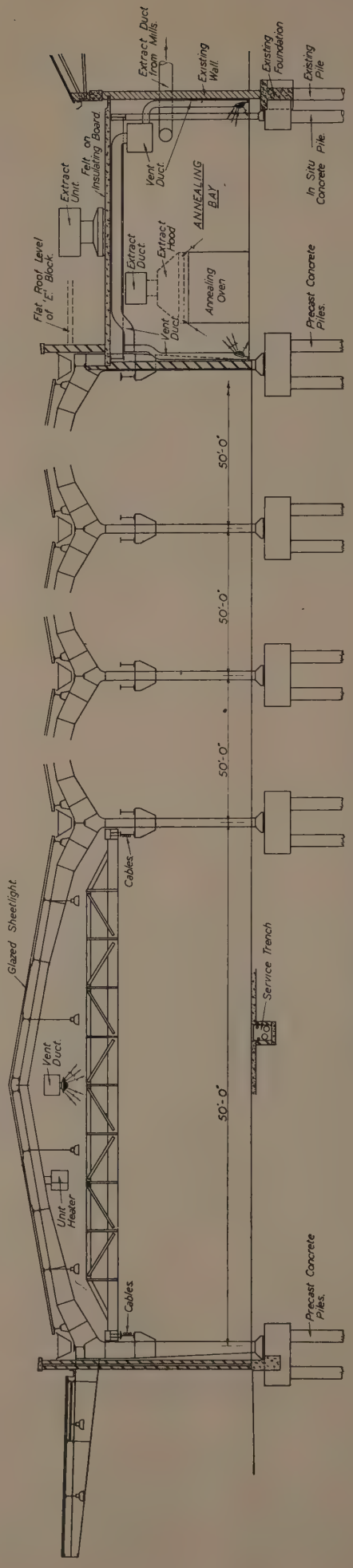
All underground services have been run under the footpaths as far as possible, with the exception of the incoming H.T. cables. A layer of ashes well rolled formed the base on which were laid 2 in. compressed concrete slabs to form paths against 10 in. × 5 in. precast concrete kerbs.

Wherever possible flower beds will be laid out, and provision has been made for a future tennis-court.

Heating

Steam at 100 lb./sq. inch serves the new factory extension for all process and heating purposes. The steam is generated in the existing boiler plant, comprising two Lancashires and two Babcock water-tube boilers.

Heating in the three main blocks is dealt with by a combined system of steam unit heaters and fresh air



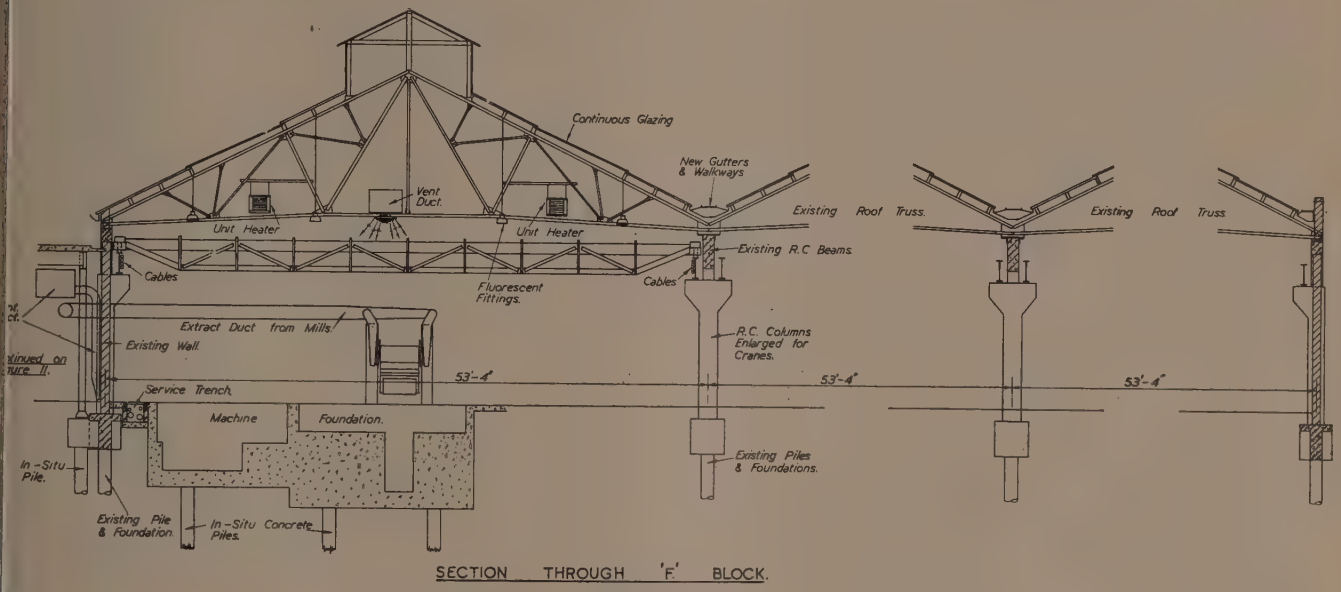
SECTION THROUGH 'J' BLOCK & ANNEALING BAY.

Fig. 11.

ntilation. The unit heaters are of the re-circulating pe mounted horizontally above the cranes and are pable of making good all the heat losses through the lding fabric. The units are thermostatically con-lled in groups to maintain a shop temperature of F. in the working plane.

The method of ventilating is by means of air ducts unted above the cranes for the length of each bay l fitted with special diffuser type outlets which spread air evenly over the working plane. Centrifugal type s draw the outside air through filter banks and liver it through steam heating batteries to the diffuser tlets. Fresh air is delivered to the shops at a tem-rature of 70°F. in sufficient quantity to maintain a e of ventilation of 1½ complete changes per hour is the air which is exhausted from the various fume moval systems.

The area under the flat roof on the east of J Block uses an annealing furnace and cooling tunnel and parate air intake and extraction is provided. Hot air awn from this area is returned to the main ventilating



SECTION THROUGH 'F' BLOCK.

Fig. 12

stems and re-circulated during the winter months, ius saving a considerable quantity of steam.

In the centre lavatory block and various small offices eating is by means of low pressure steam-heated nvectors and coils. The cloakrooms have steam coils ranged under the clothes racks for drying purposes id the moisture-laden air is exhausted through the roof.

Electricity Supply

Electricity is brought into the meter and main switch-oom at the north end of the site at a pressure of 11,000 olts. Six 750 kVA transformers step this down to 15 volts, which is distributed through the L.T. switch nd protective gear to the various sections.

The system of distribution is by paper-insulated luminium sheathed cables for all 415 volts 3-phase ervices. Screwed conduit, of a minimum diameter of in., is used for all 240 volts single-phase circuits, while ll circuits below 240 volts are in ⅝ in. conduits.

Flameproof fittings and gear have been used in places f special fire risk, such as those where solvents are eing handled. The present connected load is 5,100 kVA.

Lighting

All internal artificial lighting is by 5 feet fluorescent ubes, generally in pairs calculated to give a lighting intensity of 15 lumens per square foot in the shops on a working plane three feet above floor level, but increased to 20 lumens per square foot for special areas where colour matching is carried out.

Road lighting is by mercury discharge lamps of 25 feet mounting height to give an equivalent of Class A road lighting.

Conclusions

The work started towards the end of October, 1949, and finished in January, 1951, with a labour force which worked up to, and remained fairly constant at 150. This was about a month beyond the anticipated completion date, but quite a considerable amount of work had been completed which was additional to the original contract.

The total value of the works described is well under £400,000, of which the electrical installation accounts for some 25 per cent. of the total cost and the mechanical services about 15 per cent.

Acknowledgements

The author is indebted to Messrs. Venesta Ltd., for permission to present this paper, and to Mr. F. R. Bullen, B.Sc., M.I.C.E. (Member), Consulting Engineer, who was responsible for the design and supervision of the works and all services.

The main contractors were Messrs. J. Jarvis & Sons, Ltd., and the following is a list of the main sub-contractors :—

Messrs. Regional Heating Co.	Heating and all mechanical services.
Messrs. T. Clarke & Co., Ltd.	All electrical services.
Messrs. Moreland Hayne & Co., Ltd.,	Structural steelwork.
Messrs. Wolverhampton Corrugated Iron Co.	Roof sheeting and lining.
Messrs. Concrete, Ltd.	Precast concrete floors.
Messrs. Williams & Williams, Ltd.	Windows, lanterns and partitions.
Messrs. William Briggs, Ltd.	Felted roofs.
Messrs. Carter & Sons (London), Ltd.	Terrazzo and tiling.

Building on Fill with Special Reference to the Settlement of a Large Factory*†

By G. G. Meyerhof, Ph.D., M.Sc.(Eng.) F.G.S., A.M.I.C.E., A.M.I.Struct.E.

Synopsis

The factors to be considered in constructing fills for building purposes are briefly discussed, and the behaviour of fills in service is studied to obtain some information on the changes in strength and deformation with age and under the imposed loads. Settlement observations show that on well-compacted materials buildings can be constructed immediately after completion of the fill. For fills consolidating under their own weight a time interval of two years may generally be taken as a lower limit. Even after a long period of consolidation of poorly compacted materials special foundations or structural precautions will have to be used for larger structures so that a long time interval is of doubtful benefit, as illustrated by the settlement of an extensive factory on a mixed fill. The movements of its foundations and damage to the structure are discussed in relation to the structural and soil conditions. A site exploration and soil tests on the fill and underlying natural ground have been carried out for an estimate of the probable movements of the floor slab, and fair agreement with the observations has been obtained. A similar estimate for the piles carrying the superstructure indicates that plastic flow of the clay must have taken place and was aggravated by the drag from the consolidating fill. In conclusion, the design of foundations for buildings on fills is briefly considered.

1. Introduction

In industrial areas suitable building sites are becoming more difficult to find, so that the construction of buildings on fill or made-up ground is assuming greater importance. The control exercised in filling waste ground has, in the past, been insufficient to ensure an adequate and uniform support for structures immediately after completion of the filling. A time interval had, therefore, to be allowed between filling and building operations to enable the material to consolidate under its own weight. Although this procedure reduces subsequent movements due to the foundation load, and usually keeps these within tolerable limits in the case of small and lightly loaded buildings, it is likely to be inadequate for larger and more heavily loaded structures, which may produce substantial settlements, as shown later.

Before constructing buildings on an existing fill or made-up ground it is, therefore, necessary to assess its suitability for the support of the proposed structure by means of a site exploration and laboratory tests on samples to determine the properties of the fill and underlying natural ground. On the basis of this investigation it can be ascertained whether any subsequent treatment of the material, special foundations or structural precautions are required. Most, if not all, of these special precautions can be avoided by a careful selection of the materials and by the use of modern compaction techniques during placing of new fills so that they will provide a satisfactory support immediately after completion, and as good as virgin ground. This procedure has been used in the construction of embankments for roads and of subgrades for flexible and rigid pavements for some time, but experience in the controlled placing of fills for building purposes is very limited at present, and only general principles can be indicated.

Still less information exists on the behaviour of fills after construction and on the changes in strength and deformation with age and under the imposed loads. A summary of the principal published observations on fills in service is, therefore, given below with a brief analysis of the data. This is followed by the results of an investigation of the settlement of an extensive factory building on fill, which illustrates the main factors to be taken into account in the design of foundations for structures on this type of ground.

2. Construction of Fills

The general principles governing the construction of a new fill or the investigation of an existing one for subsequent support of structures have been considered in a recent digest of the Building Research Station (1949) and a report of the Institution of Structural Engineers (1942). The main factors involved are the nature of the materials and the method of placing of the fill, and the strength and deformation characteristics of the fill and underlying natural ground.

The materials used for fills may be divided into excavated natural deposits, such as mine and quarry wastes and all types of soil, and industrial and domestic wastes, such as ashes, slag, clinker, chemical wastes, building rubble and refuse. Materials of the first group are frequently satisfactory, particularly when properly compacted, but those of the second group are likely to give a variable support and should generally be avoided, especially if liable to decomposition or combustion. End or side tipping of the materials in thick layers, even where followed by rolling, usually results in low densities and an unsatisfactory support. But by disposing the material in a selected manner, spreading it uniformly in thin layers and compacting it properly at a controlled moisture content by heavy construction equipment or special plant (Markwick 1945, Williams and Maclean 1950), a high density is produced.

A broad assessment of the strength and deformation characteristics of a fill can be obtained by its density. Thus the bearing capacity of a loose granular material may be less than one-quarter of that of the same material in the dense state while the corresponding settlement may be nearly twice as large (Meyerhof 1950); vibration would considerably increase the latter ratio. Similar results may be obtained for the bearing capacity and settlement of cohesive materials, which also weaken much less due to entry of water when properly compacted (Skempton 1946). In general the support afforded to foundations is likely to be inadequate if the density of a cohesionless fill is less than 90 per cent. of the maximum value given by the standard compaction test, and in-situ compaction or stabilisation is then

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uired. For cohesive material a somewhat higher degree of compaction may be necessary. But until the relation between the degree of compaction and the physical properties of materials is known, it is necessary to obtain undisturbed samples for laboratory tests to determine the bearing capacity and probable settlement of foundations on fill. Such an investigation must also include the underlying natural ground so that the overall stability and the deformation of the whole construction can be ascertained.

3. Behaviour of Fills in Practice

A fill has always been expected to settle under its own weight and an allowance was made varying with the type of material used from 2½ per cent. of the height for rock fills, 5 per cent. for sandy material to about 10 per cent. of the height for cohesive fills. The rate of movement decreases gradually with time and an appreciable period was anticipated before such movements would decrease to within tolerable limits for building foundations. A survey of present practice has shown that this interval commonly varies from 5 to 20 years and is rarely less than five or more than 10 years. Although these restrictions are based on local experience, they are not always related to the actual state of the fill and underlying ground both of which vary within very wide limits. Thus buildings are reported to have been damaged on loosely tipped clay after over 50 years of previous natural consolidation has allowed, while in other cases where granular materials were used a few years' consolidation was found to be adequate. At the same time the allowable bearing pressures were found to vary from ¼ to ¾ t./ft.² according to the consolidation found on inspection, but generally a maximum bearing pressure of ½ t./ft.² is specified together with a reinforced concrete raft foundation. The following summary of observations of fills in service indicates that the above restrictions are frequently rather conservative.

3.1. Excavated Natural Deposits

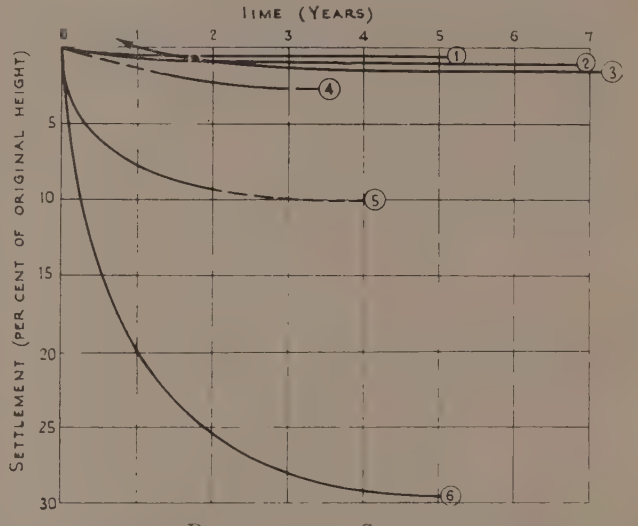
(a) Coarse-grained Soils

Where well-graded materials are available and a high degree of compaction is used (within a few per cent. of the maximum density of the standard compaction test) as is common with earth dams, the settlement of fills can be reduced to the minimum and an allowance of 1 to 2 per cent. of their height has been suggested as adequate exclusive of any settlement of the foundation strata (Justin, Hinds and Creager 1945). Extensive observations of rock fills with dry densities from 100 to 110 lb./ft.³ gave settlements varying from ½ per cent. to less than 2 per cent. of the height after five years (Steele 1942), while for rolled fills the corresponding movement rarely exceeded ½ per cent. unless the material was uncompacted, when at least four times greater settlements were observed (Plummer and Dore 1940) (Fig. 1). In most cases the larger portion of the movements occurred during the first two years after construction of the fill at a rapidly reducing rate and the settlements beyond five years were generally very small.

The high degree of compaction which can be obtained with well-graded gravel may be illustrated by the construction of the subgrade for the London airport at Heathrow, where a pond and number of gravel pits up to 22 ft. deep were drained, cleaned and then filled with gravel in 12 in. loose layers compacted by rolling at the optimum moisture content to a dry density of 130 to 140 lb./ft.³. This compared well with the density of the virgin soil of the order of 135 lb./ft.³, and no settlements of the concrete runways constructed very soon after the filling have been observed after four years (Murdock 1950).

(b) Fine-grained Soils

The only published settlement records of clay and chalk fills appear to be those given by Markwick (1946) and Lewis (1947). Although the observations were begun only one to two years after completion of the fills when considerable settlements must have occurred, they show the beneficial effect of good compaction, which is essential if fine-grained materials are to be used for the support of structures. The recorded settlements of these clay embankments after five to seven years placed on solid chalk in 2 to 3 ft. layers and lightly compacted by lorries and other construction equipment amounted from 1 to 2 per cent. of the height of the fill, while for sites where the material had been placed and compacted in thin layers using heavier equipment such



DESCRIPTION OF CURVES

No.	State of Compaction	Material	Reference
1	Well-compacted	Well-graded Soil	Plummer-Dore (1940)
2	Medium	Rock Fill	Steele (1942)
3	Lightly-compacted	Clay and Chalk	Markwick (1946) Lewis (1947)
4	Uncompacted	Sand	Plummer-Dore (1940)
5	Uncompacted	Clay	Casagrande (1949)
6	Well-compacted	Mixed Refuse	Eliassen (1942)

Fig. 1.—Observed settlement of fills

as scrapers and tractors the movement was reduced to 0.2 to 0.5 per cent. or one-quarter of the above amount.

Similarly, after five to seven years the settlement of the chalk embankments placed on solid chalk generally in 3 ft. layers and lightly compacted varied from ½ to 1 per cent., which was decreased to about one-third of that amount by better compaction such as spreading and rolling in 6 in. layers by bulldozers. About one-half of these movements, which were found to be roughly proportional to the height of the fill, occurred during the first one to two years, the rate decreasing with time (Fig. 1). It is of interest to note that where differential settlements exceeded about ½ in. in 25 ft. length, cracking of the concrete slabs was usually observed.

With a high degree of compaction the above movements can be considerably reduced. This is shown by the construction of a large clay fill up to 23 ft. high and deep cuts on a very extensive site for an atomic plant in U.S.A. (Watson and Bradley 1945). The fill was

carried out in 6 in. layers near the optimum moisture content and thoroughly compacted by rollers to a dry density of 87½ lb./ft.³, which even slightly exceeded the value of 86½ lb./ft.³ of the undisturbed soil in borrow pits and cut areas. No settlement had been discovered two years after construction of the plant, which consisted of three-storey reinforced concrete and steel-framed buildings with a basement and an allowable bearing pressure of 2¼ t./ft.².

(c) Hydraulic Sand Fills

To obviate excavation in a very thick bed of loose fine sand with a ground water table near the surface the foundations of a large rolling mill in Cheshire were constructed from marsh level upwards and subsequently embedded by raising the whole level of the ground by a hydraulic sand fill 16 ft. thick (Faber 1944). This fine to medium sand was obtained by suction dredging from the adjacent river and placed in two layers of about 8 ft. each with time to consolidate the first layer before the second one was deposited. While the structure and plant were supported by piles, the ground floor rested on the fill and its settlement was said to have been small. Quite recently a soil exploration was carried out on an adjacent site for a new steel works and the dry density of the hydraulic sand fill was found to vary from 94 lb./ft.³ at the surface to 98 lb./ft.³ near the bottom of the fill corresponding to a fairly compact state. The lower limit was similar to that obtained in the laboratory after allowing the sand to settle in water. Field loading tests on the material in its normal moist condition gave an ultimate bearing capacity of 15 t./ft.² so that the new works can be constructed on shallow foundations.

Higher and more uniform densities throughout the thickness of hydraulic fills can be obtained if the material is placed and compacted in thin layers. This was done at the site of a new steel works in Wales where about 10 ft. of material was placed on mixed soils containing a layer of peat (Lumbard and Wright 1950). The filling consisted of a uniform medium dune sand and hard fill from a slag heap. Laboratory tests carried out at the Building Research Station on the sand showed that its critical density, which should be considered as the minimum requirement for granular fills, was 96½ lb./ft.³ and that by pumping the filling a value of 98½ lb./ft.³ could be obtained representing 95 per cent. of the maximum dry density. Measurements in the field showed that this degree of compaction could readily be obtained by placing the sand hydraulically in 18 to 24 in. layers, covering it with 6 to 12 in. of slag and then compacting it by routed traffic of the heavy construction equipment available. The buildings and plant were supported by piles except for a large brick store which rested on the fill with a bearing pressure of about ½ t./ft.². A portion of it was constructed six months after placing the fill and practically the whole of the observed movements could be accounted for by consolidation of the underlying soil, thus indicating that any settlement in the fill was very small (Murdock 1950).

(d) Hydraulic Clay Fill

When a hydraulic fill is made of fine-grained material the settlements are large and continue for a long period of time. Although without additional compaction such fills are unsuitable for buildings, they have been successful for other civil engineering works. Thus the new Boston airport has been built on a 15 to 25 ft. high fill consisting of blue clay dredged hydraulically from adjacent harbour areas and deposited on an up to 100 ft. thick bed of the same pre-consolidated firm to soft clay (Casagrande 1949). The fill, which was found to consist of balls of firm clay varying from pebble to head size and embedded in a matrix of semi-fluid clay, was deposited partly continuously for its full depth in a few days and partly in stages over a period up to one month. About a day after placing when the material had adjusted itself and consolidated by about 6 in., the dry density is estimated to have been of the order of 75 lb./ft.³ compared with the original density and that of the foundation clay of nearly 85 lb./ft.³ (Gould 1949).

The average settlement observed during the subsequent 2½ years of natural consolidation is given in Fig. 1 expressed as a percentage of the average height of the fill as before. About one-third of the movement

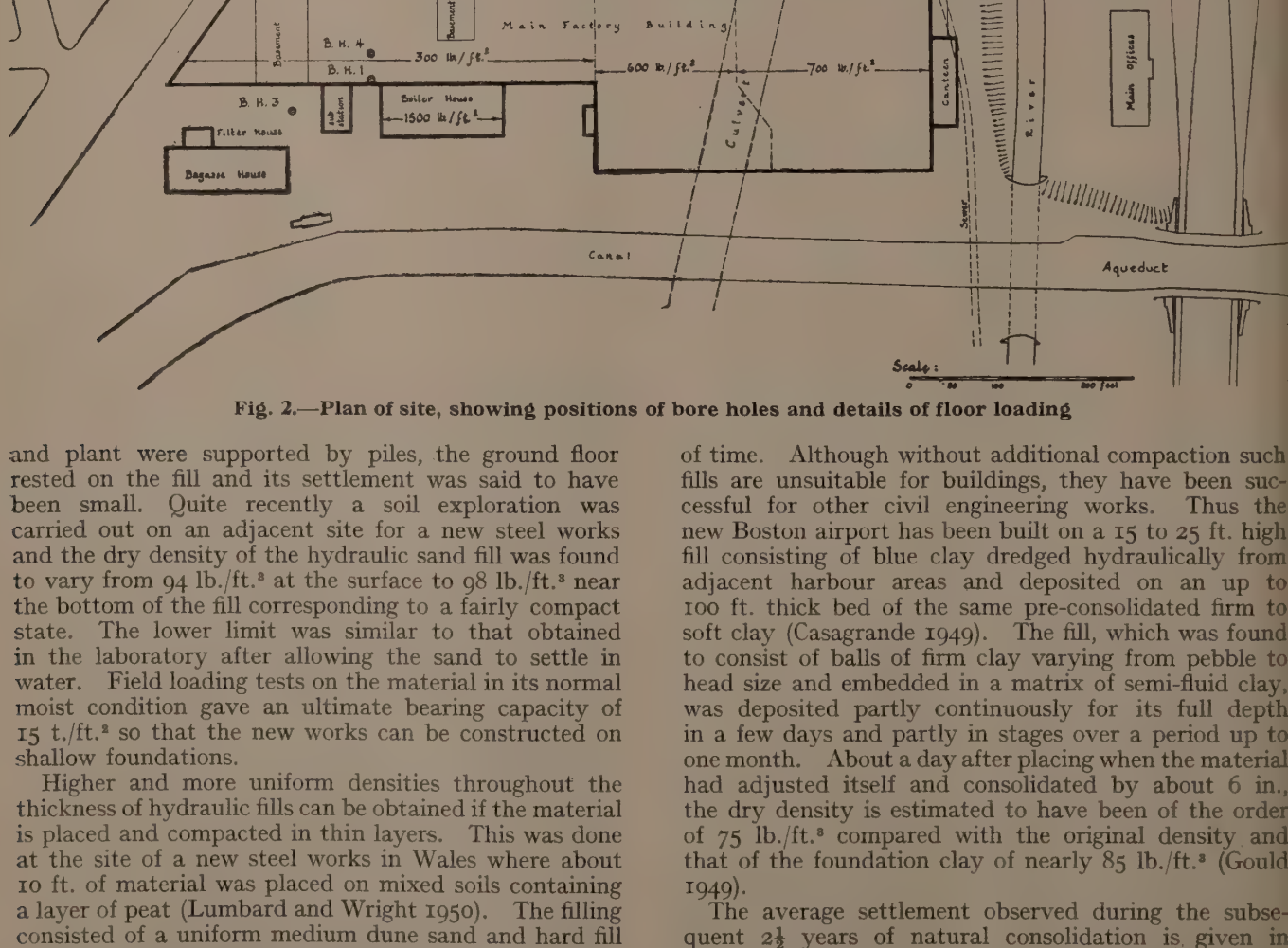


Fig. 2.—Plan of site, showing positions of bore holes and details of floor loading

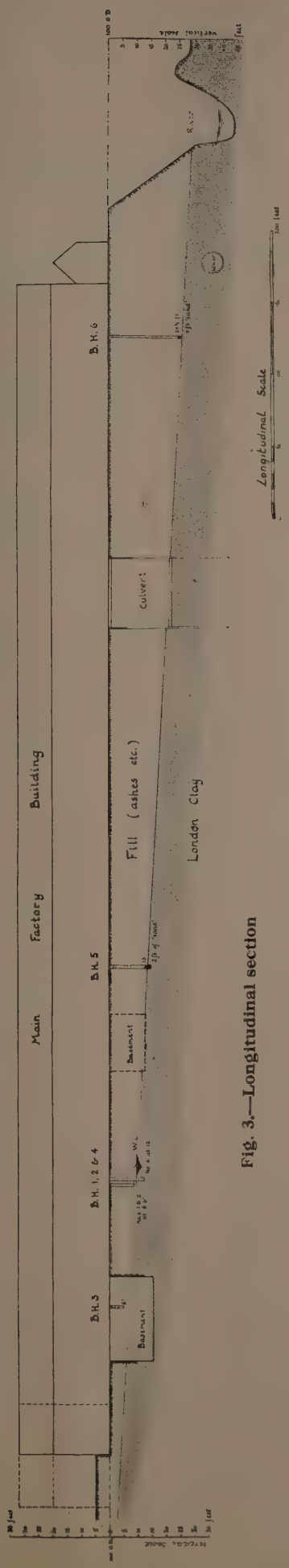


Fig. 3.—Longitudinal section

is due to the consolidation of the underlying very soft silt-clay while that of the foundation clay is very small by comparison. It will be noted that most of the settlement developed during the first year after placing of the fill and that after about two years the rate of movement was relatively small. Subsequent observations of the runways indicated the beneficial effect which is obtained by preloading the fill for one to two years depending on the weight of the ballast ; the efficiency of this method can be increased considerably by providing deep drainage to accelerate the consolidation.

3.2. Industrial and Domestic Wastes

(a) Ashes

At Flushing Meadow Park, New York, ashes from the city had been dumped for some 30 years to various heights up to 125 ft. on very soft silt and clay, which had been consolidated and displaced to such an extent that the fill penetrated up to 40 ft. below the original ground surface (Foster and Glick 1938). A few tests were carried out on the fill indicating that its dry density varied from about 55 lb./ft.^{3} near the surface to about 75 lb./ft.^{3} at a depth of 40 ft., which was of the same order as that found by tamping or by compaction under road traffic (Freeman 1936). From these densities the corresponding settlement at the surface is estimated to have been about 10 per cent. of the height of the fill and could have been largely avoided by the compaction mentioned.

In the preparation of this site for the World's Fair, 1939, the ash piles were levelled and the material spread and compacted in the low areas up to a height of 10 ft. with a dry density of about 80 lb./ft.^{3}. Where the fill had been in place many years, only very small movements were recorded while in the newly-filled areas the settlements were up to $1\frac{1}{2}$ in. per month during the early stages, mainly due to consolidation of the underlying soil. Field loading tests indicated an ultimate bearing capacity of the ash of about 1 t./ft.^{2} so that bearing pressures up to $\frac{1}{2}$ t./ft.^{2} were allowed largely in view of the temporary nature of most of the buildings. They were generally of light steel frame construction sufficiently flexible to permit differential movements. In one of the more important structures provision was made to enable the columns to be jacked up in case of excessive differential settlements. During the first six months after completion of the larger buildings total settlements varying from 1 to nearly 6 in. were recorded with differential movements between adjacent columns up to about 1 in., which in the worst cases required correction by jacking up to avoid excessive secondary stresses in the steel frame. Some of the buildings, especially those with long span roofs or heavy loading, and all permanent structures, were supported by long timber piles with the ground floor resting on the fill, or on pile foundations in areas where large settlements were expected. The services were arranged in common trenches in the fill ; where the soil was soft they rested on grillages supported by piles.

(b) Mixed Wastes

At a building site in Kent mixed wastes from a railway engineering works (mainly ashes, clinker and brick rubble) were tipped in an old gravel pit up to 25 ft. deep, the loose dry density being about 70 lb./ft.^{3}. After some 10 years' natural consolidation the density at a depth of 3 to 6 ft. below the surface varied from less than 80 to about 100 lb./ft.^{3} compared with the maximum standard compaction test value of 112 lb./ft.^{3}. In view of the low relative compaction and variability

of the fill it was suggested that the site would only be suitable for independent bungalows on reinforced concrete strip or raft foundations.

(c) Refuse

Several extensive and unusable and low-lying areas in New York have recently been filled with mixed industrial and domestic refuse about one-half of which by weight was ashes and the remainder paper, garbage, etc., from which all tins and other metal had been salvaged to aid compaction (Eliassen 1942). The material had been spread and compacted in layers by heavy bulldozers and lorries to a height up to 20 ft. so that its original weight of 15 lb./ft.^{3} was increased to 50 lb./ft.^{3}. During the subsequent consolidation under its own weight and of the underlying ground a large number of settlement observations were taken. Their mean result (Fig. 1) indicates that most of the movement occurred during the first few years. Experience showed that after two years the surface as a whole was relatively stable and subsequent settlement was small and fairly uniform over the whole area. On the other hand loading tests up to 4 t./ft.^{2} carried out at various intervals of time indicated that the compression modulus increased for a considerably longer period. On the basis of these results raft and wide strip foundations with bearing pressures up to 1 t./ft.^{2} after two years of consolidation have been recommended with relatively small and fairly uniform settlements.

3.3 Analysis of Settlement of Fills

The various settlement-time curves of fills (Fig. 1) show that the rate of movement decreases rapidly with time. Within the period of observations the largest portion of the settlement takes place during the first year after placing of the fill, and beyond two years the movement is relatively small. A time interval of two years may, therefore, be taken as the lower limit before buildings are constructed on fills consolidating under their own weight. If the fill is of such a nature that substantial movements continue for a very long time, then special foundations will probably be necessary in any case and the sterilisation of the ground for long periods would not appear to be justified. When fills are well compacted in thin layers, however, there appears to be no reason why building could not start immediately after completion of the fill.

The results of loading tests show that the bearing capacity of fills varies within wide limits, and settlement observations on structures indicate that a bearing pressure of $\frac{1}{2}$ t./ft.^{2} is very conservative except on poorly compacted fine-grained soils and on industrial or domestic wastes consolidating under their own weight. For well-compacted soils the bearing capacity customary for similar virgin ground can safely be taken in practice while the safe pressure on well compacted wastes is best determined from tests.

4. Investigation of the Settlement of an Extensive Factory Building on Fill

Since the above review of the behaviour of fills showed that if well compacted they differ little if at all from similar virgin ground, the design of satisfactory foundations can be based on standard methods. For poorly compacted fills, however, and those which have been allowed to consolidate under their own weight special foundations and structural precautions are required. In the absence of such measures serious damage to buildings can occur, which may be illustrated by the settlement of an extensive factory building on fill. This investigation has also provided some evidence that soil mechanics principles can be applied to problems of foundations on poor filling for which experience is as yet very limited. The factory concerned is in an area in North-West London where free end tipping of industrial and domestic waste products has been in progress since the middle of the last century and is still continuing. The bulk of the fill covering the site was placed before 1914, and the remainder during the following few years. The fill had then been left to consolidate under its own weight for about 20 years when the construction of the works began in 1937. Soon after completion of the factory, cracking of the floors and walls was noticeable. Substantial maintenance had been required since 1941, when some boiler plant had also to be rebuilt and underpinned by piles.

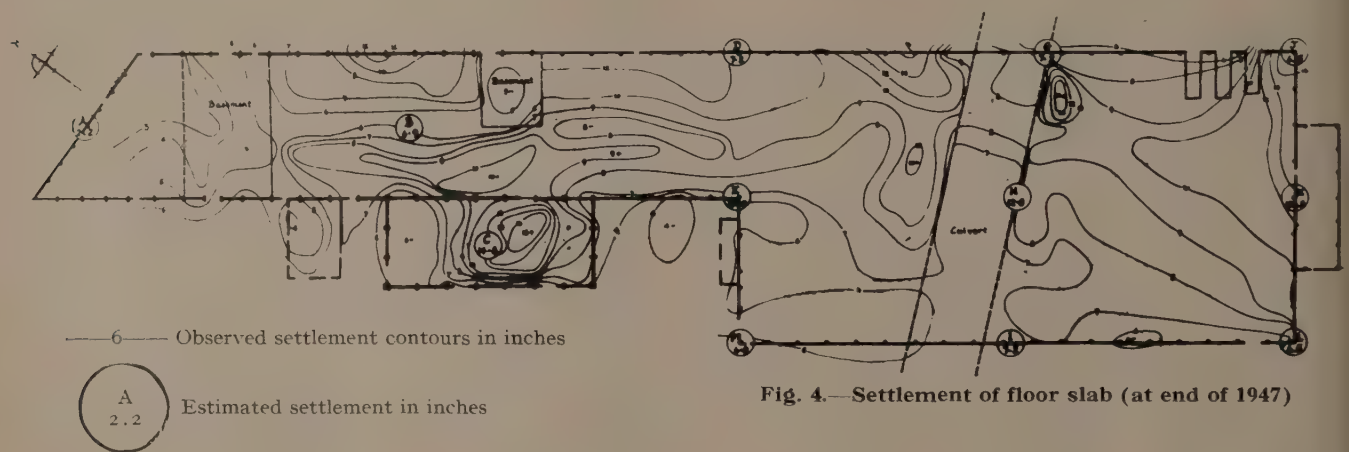


Fig. 4.—Settlement of floor slab (at end of 1947)

Progressive cracking of the buildings and distortion of the plant became so serious that in 1947 consulting engineers were called in to make a survey of the structural movements and damage and to advise on remedial measures. On the result of this survey it was decided among other things to underpin some plant which was very sensitive to differential settlement. Since bored piles were to be used for this operation, the composition and thickness of the fill could readily be ascertained. Further exploration was carried out by the Building Research Station to obtain samples of the fill and underlying natural soil for laboratory tests. On the basis of this investigation an estimate of the stability and probable settlement of the foundations since their construction was made for comparison with the present observations.

4.1. Site Conditions and Foundation Movements

(a) Site and Structure

A general plan of the site of the works is given in Fig. 2. The original level of the (underlying) ground varied from about 95 ft. O.D. in the North to about 70 ft. O.D. in the South, representing a slope of about 1 in 35. The site, which was underlain by London clay, had been filled with a mixture of industrial and some domestic refuse to a constant height at somewhat over 100 ft. O.D. The fill had consolidated to this level when the factory was built and increased thus in thickness from the North to South (Fig. 3).

The factory buildings consist mainly of single-storey steel frames, except at the extreme North and South ends where two-storey office blocks are located. The frames are spaced at 20 ft. centres and rest generally on 13½ in. dia. bored piles which increase in length from North to South and extend through the fill for some 5 to 15 ft. into the underlying natural ground. While the cavity brick walls are carried by reinforced concrete wall beams spanning between the piles, the 6 in. thick reinforced concrete floor slab is usually resting directly on the surface of the fill. This does not apply to two small areas in the northern portion of the main building where basements are provided and an area in the southern portion below which passes a subway carrying some water mains; these areas extend to various depths in the underlying clay as shown in the longitudinal section through the main building (Fig. 3).

(b) Floor Movements and Damage

Apart from some boilers which had recently to be underpinned by bored piles, the whole of the plant is resting on the floor slab, which also supports considerable quantities of manufactured products. This heavy loading (indicated in Fig. 2) led to substantial movements of the floor as shown by the settlement contours at the end of 1947 in relation to the original floor level of about 100.5 ft. O.D. (Fig. 4). The settlement increases from somewhat less than 3 in. at the northern end, where the loading and thickness of the fill are relatively small, to about 15 in. at the southern end where both are fairly large. Superimposed on this general settlement distribution are the effects of variations in loading conditions and soil properties. Thus heavy plant, vibrating machinery and possibly also higher temperatures below boilers and drying plant have increased the settlements locally by dishing of the floor; this is particularly severe in the boiler house, where movements exceeding 16 in. have been recorded. On the other hand the presence of basements and subway resting on the natural soil have reduced the general movements in their neighbourhood, as would be expected. In addition, local non-uniformity of the compressibility and thickness of the fill has modified the settlement distribution further, particularly in the south-eastern portion; as the eastern side of the floor has generally settled more than the western side, it would appear that the thickness of the fill tends to increase in that direction.

The pattern of the cracks in the floor slab reflects the variations of its settlement and most of the cracks run, therefore, in the East-West direction. This tendency is modified, as before, by the effects of heavy plant and the variability of structural and ground support, especially around the piles along the Southern basement and subway; here large differential movements have led to extensive local cracking. The width of the cracks varies from fine hair cracks to about 6 in. in the worst case, and the relative displacement of adjacent slab levels is up to 6 in. in the vertical direction.

(c) Pile Movements and Damage to Structure

Although the brick walls of the factory are not carried by the fill but by wall beams supported on piles, large settlements have occurred. The absolute settlement of the piles cannot be ascertained accurately since the original beam levels are not known and the levelling survey had, therefore, to be carried out in relation to two groups of reference piles on the side walls. From the observed absolute levels of the sill and floor, and the differential movements between floor and walls and between floor and piles where the latter are known to be very small (as in the northern portion and over the subway) the reference piles appear to have settled about 4 in. On this basis the settlement contours of the piles at the end of 1947 have been obtained (Fig. 5) and indicate a variation of settlement from rather less than 4 in. in the northern portion to about 12 in. at the southern wall of the main building. These large movements show that the bearing capacity of the piles must have been exceeded in most if not all cases. Comparison of the distribution of floor and pile settlement shows a striking similarity, which indicates that the filling had an appreciable effect by dragging on the piles and consolidating the underlying clay. Owing to the arching action, however, the local variations of loading, ground support and properties are less important than before and only the main trends are reproduced.

The large movement of the piles is accompanied by considerable differential settlements between walls and floor and between the piles themselves. The former could be noticed along the inside of the walls where the original floor level was shown on the brickwork, and the damp-proof course was frequently exposed above the present slab level (Fig. 6), and usually the outside

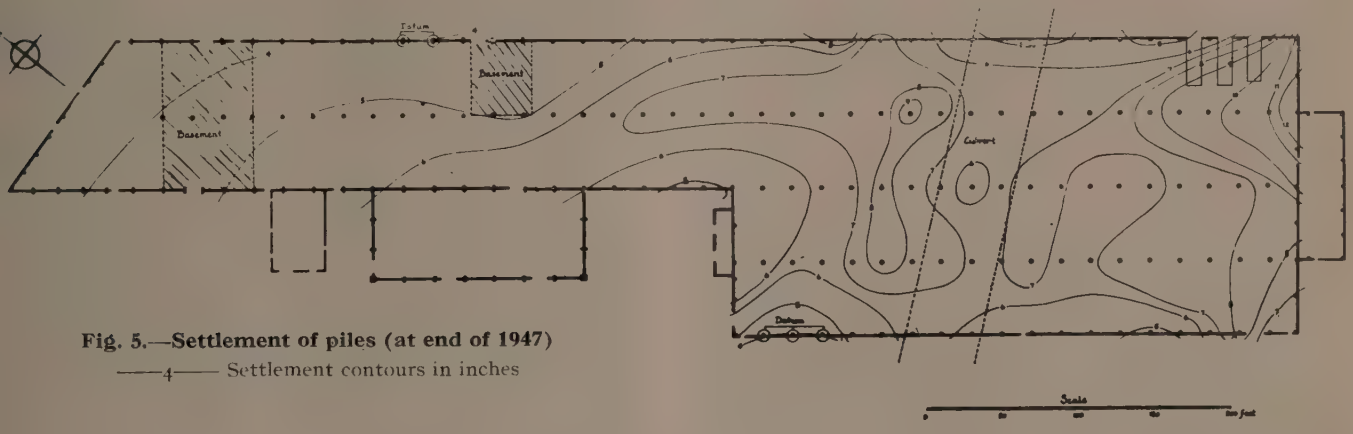


Fig. 5.—Settlement of piles (at end of 1947)
——4—— Settlement contours in inches

pavement was also similarly displaced. The differential movement of the piles has led to appreciable cracking of the outside walls ; this is generally due to excessive shearing (diagonal tension) stresses and occurs, therefore, in a diagonal direction and is accompanied by

Fig. 6.—Differential settlement between floor and wall

local bulging of the walls (Fig. 7). Where heavy plant causes high local loading, bending stresses predominate and result in vertical cracks which sometimes pass along the encased columns as at the boiler house. The cracks, which at the worst places were nearly 2 in. wide, and distortion of the walls was particularly severe at sections

Fig. 7.—Typical damage of wall

of variable height and support, especially in the neighbourhood of the basements and subway, where the building and slabs had been warped considerably (Fig. 8). This warping was accompanied by horizontal movement of the walls ; the East wall has moved outward at its top by an average of ½ in. with a maximum of 2½ in., while the top of the West wall has moved inward by nearly the same amount. Apart from causing large displacements of the roof supports up to 6 in. and of gantry rails due to lack of bracing, the wall movements have resulted in deleterious cracking at lintel bearings.

Beyond the south-eastern corner of the site where the fill ends in a 30 ft. high free slope at about 1 : 1½ a canal runs over the road in an aqueduct (Fig. 2). The main structure is supported on natural ground and the approach wing walls on the filling of the canal bank. Although the aqueduct was built about 70 years after construction of the bank, further consolidation of the material displaced the walls to such an extent that gaps of about 6 in. at the top and 3 in. at the bottom were formed between the approach and main parapets. The joints had recently been grouted, but new cracks were observed soon afterwards, showing that consolidation is still progressing (Fig. 9).

4.2. Soil Investigation

A site exploration was carried out to determine the thickness of the fill and bore holes were sunk (Fig. 3) to

Fig. 8.—Distortion of building and pavement by variable support

obtain samples of the fill and underlying soil for laboratory determination of their index properties, strength and consolidation characteristics.

(a) Index Properties

The filling was found to consist of industrial wastes, mainly ashes with some broken bricks and domestic refuse ; no chemical wastes, clinker or slag were detected. The material had a crumb structure and was well graded from about 1½ cm. (½ in.) down to .001 mm. (Fig. 10) ; it had a plastic limit of about 30 per cent. and a mean specific gravity of 2.25. The organic matter and carbonates did not exceed about 8 per cent. each, and since the combustible content was about 10 per cent., there appears to be little possibility of internal combustion in this case. This is supported by temperature measurements in the fill at various depths which indicate that the temperature was generally 60 to 70°F. except near the drier, where 90°F. was reached.

The natural water content of the fill varied in the two main bore holes between 30 and 70 per cent., but was fairly consistent in the underlying firm brown London clay, near whose interface the ground water level was found (Fig. 11). In spite of this variation, the degree

f saturation was similar in the two holes and increased rom about 70 per cent. at a few feet below the surface o full saturation at the bottom of the fill. The wet nd dry densities also increased with depth of the over- urden, as would be expected. The dry density near he surface of the fill was only slightly greater than the inimum value obtained in the laboratory by loose ouring of the material. Even near the base of the fill he maximum dry density was only about 75 per cent. f that obtained by the standard compaction test 95 lb./ft.³ with an optimum moisture content of about o per cent.). The natural field compaction must thus e regarded as very poor and insufficient by present tandards.

Within the depth of exploration the natural water ontent of the brown London clay did not vary much rom the average of 25 per cent. ; the plastic and liquid mits were found to be 21 and 54 per cent. respectively.

(b) Compression Tests

Undrained triaxial compression tests were carried out n some samples of the fill under a small lateral pressure o that the density of the samples was not changed

Fig. 9.—Displacement of aqueduct parapet

an average value of 25° may be taken as representative. The modulus of instantaneous compression varied considerably from the mean of 50 t./ft.² due to differences in the material. Unconfined compression tests on the brown London clay gave an average shear strength of 0.5 t./ft.².

(c) Consolidation Tests

Consolidation tests were carried out on two samples of the fill, one from the middle and the other from the base, and on two samples of the firm brown clay. The fill was tested at the natural water content and at the actual degree of saturation which varied from 50 to 70 per cent. at the beginning of the test. Consolidation-time curves for effective pressures of $\frac{1}{2}$ to 2 t./ft.² show that the degree of consolidation is directly proportional to the logarithm of the time. Even at later stages and higher pressures when the degree of saturation was slightly above 80 per cent. no primary consolidation was found and secondary compression occurred throughout the period of test due to frictional lag between the particles. For complete saturation, however, the consolidation-time relation is likely to be similar to that for a typical test on the clay. While a coefficient of consolidation of the fill cannot, therefore, be determined, the initial rates correspond to a coefficient of the order of 1000 ft.²/year ; by comparison, the average coefficient of the brown clay was 5 ft.²/year.

Average void radio-pressure curves (Fig. 13) indicate that while loose fill is very compressible, especially at low pressures, at greater densities and pressures the compressibility approaches that of the brown London clay, which gave an average modulus of volume change of 37 t./ft.². It is of interest to note that for the fill good agreement existed with the preconsolidation pressures determined from the test results and those corresponding to the superimposed load and depth of the fill. The tests have, therefore, been used to estimate the dry density of the fill for various depths ; comparison with the observed values (Fig. 11) shows that the estimates are of the same order considering the variability of the material.

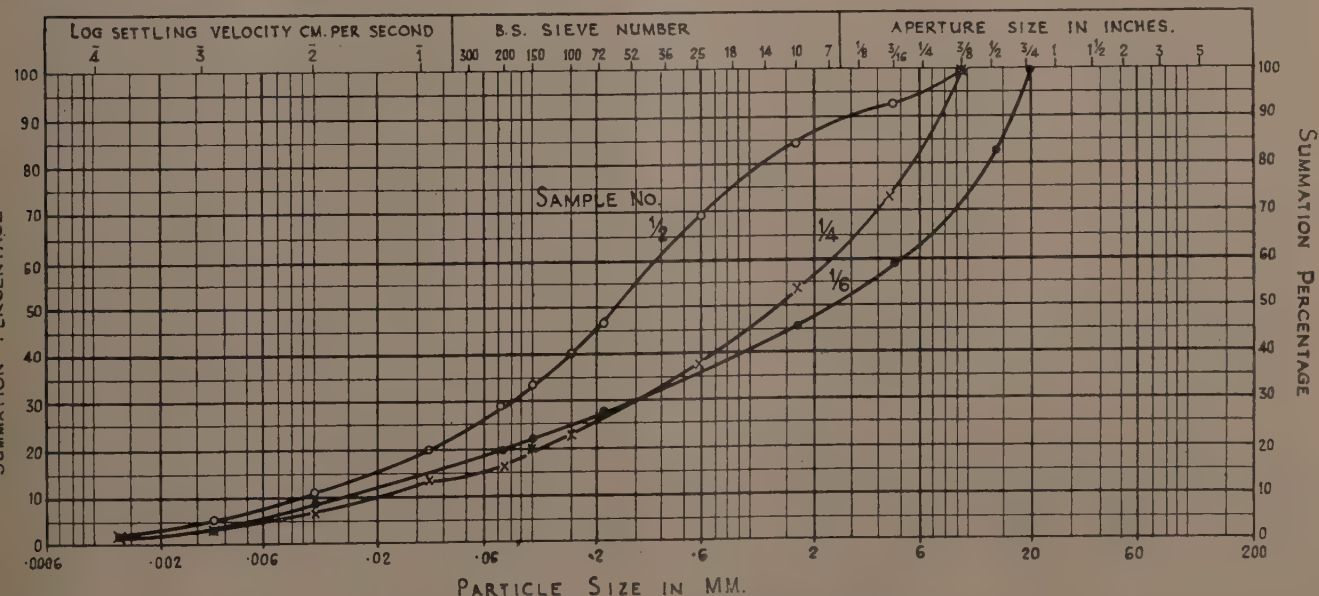


Fig. 10.—Particle size distribution of fill

gnificantly from the field conditions. The results Table 1) show that there is some increase in the angle f internal friction with depth due to greater dry density. n account of the variability of the material, however,

4.3. Settlement and Stability Estimates

(a) Floor Slab

For the purpose of estimating the movements of the main factory floor slab, the average superimposed net

loads from the plant and goods were taken as uniformly distributed over the areas shown in Fig. 2.

The general variation of the thickness of the fill was determined from the site exploration data and from the height of the subway below the Southern portion of the factory. This information showed that in a longitudinal section through the main building (Fig. 3), the thickness of the filling increases fairly uniformly from about 5 ft. in the North to about 30 ft. in the South where it slopes towards the river. The general ground water level has been taken at the interface of fill and clay where it was generally found in the bore holes, and would be expected in view of the high permeability of the fill material in relation to the clay.

The deepest bore hole (B.H.4, Fig. 12) indicates that the London clay consists of about 20 ft. of brown clay, followed by blue clay extending to about —80 ft. O.D., according to geological records. This stratum rests on the Woolwich and Reading beds whose upper 40 ft. consist of mottled clay, so that the average thickness of the compressible clay stratum may be taken as 200 ft. for the purpose of the analysis.

The total settlement of any point on the surface of the fill consists of the immediate and consolidation settlements of the fill and clay. They are estimated below for a period of ten years after construction of the factory when the present survey was carried out.

The immediate settlement is considered to occur on application of the load and has, therefore, been determined for the superimposed loads only, since the corresponding deformation of fill and clay due to the weight of the former are assumed to have occurred during its placing. An average compression modulus of 50 t/ft.^{2} was used for the fill and of 600 t/ft.^{2} for the clay based on the above and previous (Cooling and Skempton 1942) test results.

TABLE 1.—Results of Compression Tests

Sample No.	Natural Water Content (per cent.)	Dry Density (lb./ft.^{3})	Lateral Pressure (lb./in.^{2})	Maximum Vertical Pressure (lb./in.^{2})	Shear Strength (lb./in.^{2})	Angle of Internal Friction (°)	Instantaneous compression Modulus (lb./in.^{2})
1/3	40.0	53	5	4.9	—	19	1100
1/5	42.8	55	10	17.0	—	27	400
1/8a	64.2	57	10	19.2	—	28	950
1/8b	66.2	60	10	15.8	—	28	700
1/10	26.0	98	0	17.6	8.8	—	——
1/12	23.1	103	0	14.1	7.1	—	——
2/4	24.9	100	0	16.4	8.2	—	——
				average	8.0 = 0.5 t/ft.^{2}	25°	800 = 50 t/ft.^{2}

Since the factory was built some 20 years after completion of the fill, the consolidation of the latter under its own weight was assumed to have been complete at that stage and only its consolidation due to foundation loads has been taken into account. While primary consolidation need not be considered, as mentioned earlier, the corresponding secondary compression rate cannot be estimated at present. After the period of ten years, however, as considered here, secondary compression is likely to be practically complete and the total consolidation can be estimated from the usual theory. This is supported by a levelling survey from 1947 to 1950 on the boiler house, which is the most heavily loaded and settled portion of the factory; during this period the settlement had increased by an average of $\frac{3}{8}$ in. with a maximum of about 1 in., which is small compared with the total of over 16 in. recorded there at the end of 1947 and is largely due to deformation of the underlying clay. In estimating the consolidation of the clay both floor and fill loads have been taken into account with an average coefficient of consolidation of 3.5 ft.^{2}/yr. based on 5 ft.^{2}/yr. for the brown clay and 3 ft.^{2}/yr. for the blue clay; the moduli of volume change were taken at 40 and 100 t/ft.^{2}, respectively, observed

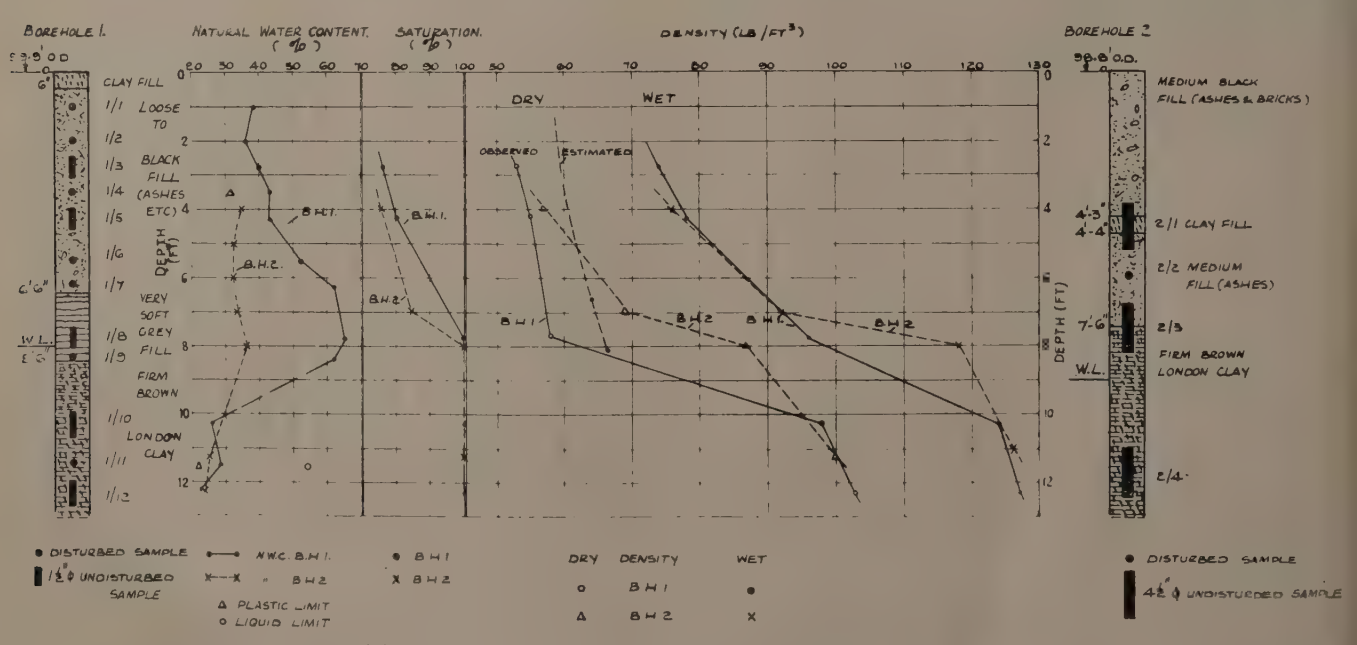


Fig. 11.—Borehole records 1 and 2, and index properties

m the above and previous (Cooling and Skempton 42) test results.

The estimated settlements of a number of typical ints of the floor slab are summarised in Table 2. mparison of these results with the observed values ig. 4) shows fair agreement, particularly in the rthern portion. At the middle of the Southern ction the subway has reduced the movements to about ree-quarters of their estimated magnitudes. On the ner hand, at the extreme Southern end, the observed ttlements are somewhat greater than predicted ; this ay be explained by its proximity to the free slope of e fill, which prevents the lateral restraint assumed the consolidation estimates and where the com-essibility is likely to be more than at other sections e to the reduced overburden. At the same time some astic flow may have occurred in the underlying brown ndon clay since the maximum shear stress below the pe due to the weight of the fill and its superimposed ad is estimated to be same as the shear strength of 5 t/ft.^{2} of the clay. But the factor of safety against lure of the fill under the building load is adequate.

(b) Piles

The large settlement of the piles and its distribution er the loaded area indicated, as mentioned previously, at the loads on the piles were excessive in relation to e bearing capacity of the ground and that they were fected by the weight of the fill. In the design of the es the loads were assumed to be resisted by skin ction on the fill and on the underlying clay. It was own, however, that under the superimposed floor ads the fill must be expected to consolidate, and hence, stead of supporting the piles, it exerts a drag or negative " skin friction on them so that the pile loads well as the forces on the clay were considerably larger an had been anticipated.

Except near the Northern end where overhead ntries caused greater loads requiring larger piles and eater penetrations, the inside piles carry about 15 t. m the roof, while the outside piles carry normally out 25 t. from the roof and outside walls, apart from e Southern gable end, where this load is some 35 t. ese loads have to be increased by the drag from the , which varies with its thickness. On the assumption at during consolidation of the fill a pressure at rest of e-half the vertical pressure is acting on the piles with coefficient of friction corresponding to the average gle of internal friction of 25° found from the above st results, the estimated drag increases from about . in the Northern portion to about 20 t. at the Southern d of the main building.

In accordance with the design assumptions the length of the 13½ in. dia. bored piles used and their penetration into the clay varies, except near the Northern end, by about ±5 ft. throughout the building where the thickness of the fill increases from some 10 to 30 ft. On the average, the inside piles were 25 ft. long, penetrating 5 ft. into the clay, while the outside piles were 30 ft. long with 10 ft. penetration, based on the mean fill thickness of 20 ft.

If the loads on the piles do not cause plastic flow of the clay, their total settlement consists of the general settlement of the clay due to the floor and fill loads, which varies from about 1 in. in the Northern portion to some 3 in. at the Southern end (Table 2), and the settlement of the pile loads, which is of the order of 1 in. Comparison of these total settlements ranging from 2 to 4 in. throughout the main building with the observed values of 4 to 12 in. as well as their distribution over the loaded area indicate that considerable plastic flow of the clay must have occurred.

While the amount of plastic flow cannot be estimated at present, it is of interest to compare the bearing capacity of the piles with the loads that they carry. The ultimate bearing capacity Q of a 13½ in. dia. pile with a penetration d (ft.) into the brown London clay having an adhesion equal to the average shear strength of 0.5 t./ft.^{2} based on the above test results may be expressed as $Q = 4.5 + 1.75d$ tons (Meyerhof 1949). For the average conditions mentioned above the estimated bearing capacities are thus 13 t. for the inside piles and 22 t. for the outside piles. The average factor of safety by the piles is, therefore, only 0.9 with respect to the net loads or 0.6 when a mean drag of the fill of 10 t. is added. Even under the most favourable conditions in the Northern portion the factor of safety is only slightly above 1.0 for the total loads whereas under the worst conditions at the Southern wall the corresponding factor is only about 0.2. It can thus be concluded that the ultimate bearing capacity of the piles has been exceeded in all cases and that large settlements due to plastic flow must, therefore, be expected.

TABLE 2.—Estimated Settlement of Floor

MATERIAL		FILL		CLAY		TOTAL	
Location	Point	Immediate Settlement (in.)	Consolidation (in.)	Immediate Settlement (in.)	Consolidation (in.)	Estimated (in.)	Observed (in.)
	A	0.1	1.3	0.2	0.6	2.2	2
Northern	B	0.3	5.3	0.4	0.9	6.9	6.5
	C	1.5	10.0	1.5	1.8	14.8	12
Portion	D	0.4	4.2	0.3	2.3	7.2	11
	E	0.6	6.4	0.6	2.4	10.0	7
	F	0.3	4.3	0.2	1.2	6.0	6
Southern	G	0.7	4.4	0.5	1.6	7.2	7
	H	1.4	8.7	0.9	1.8	12.8	8.5
Portion	I	0.7	4.4	0.5	1.6	7.2	7
	J	0.6	3.4	0.4	2.1	6.5	16
	K	1.3	7.7	0.8	2.4	12.2	13
	L	0.6	3.4	0.4	2.1	6.5	7

5. Conclusions with Special Reference to the Design of Foundations

The investigation of the settlement of an extensive factory building shows that fills of domestic and industrial waste products should be very carefully considered before they are accepted as building sites. They are usually very mixed in composition, variable in compressibility and may be subject to decomposition and combustion unless they consist principally of inert materials. Since the density of the fill governs to a

large extent the stability and settlement of the foundations carried, random end tipping in thick layers is unsatisfactory. Not only does appreciable consolidation under the weight of the fill occur for a considerable time, making the site unsuitable for building, but also further excessive and non-uniform settlements are likely to take place under the foundation loads, particularly when they are heavy and variable as in factory buildings. Since natural consolidation always leaves the upper part of fills in a loose state in-situ compaction or stabilisation is generally required.

The material of new fills should be spread uniformly in thin layers and be compacted at the optimum moisture content required to produce the maximum density for which heavy construction equipment is frequently adequate. Special care is necessary in placing and

A study of the behaviour of fills in practice shows that on well-compacted materials buildings can be constructed immediately after completion of the fill where this consists of excavated natural deposits, the design of foundations can be based on the same principles as used for similar virgin ground. Well-compacted waste products usually necessitate special investigations. On poorly compacted materials or fills consolidating under their own weight building can frequently begin two years after placing of the material, and special foundations or structural are generally required.

In view of the relatively high and variable compressibility of most fills the settlement distribution over the loaded area is very irregular even under uniform loading conditions. It is, therefore, necessary to design the structure and its foundations so that the total as well as

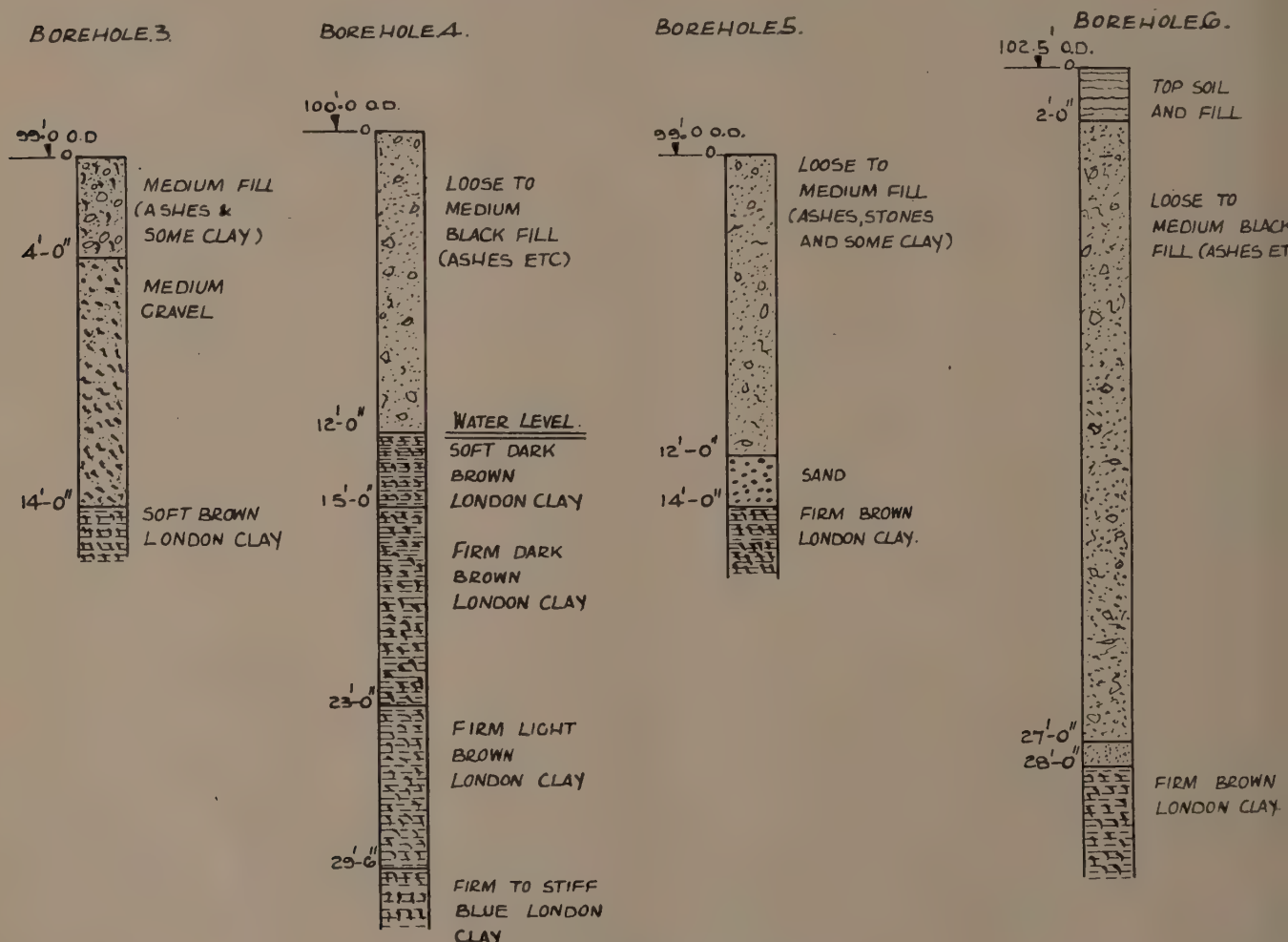


Fig. 12.—Borehole records 3 to 6

consolidating the fill around culverts and foundations and behind retaining walls without endangering their stability.

Apart from the density, the thickness of the fill and the site conditions affect the magnitude and time rate of the settlement under otherwise similar circumstances. A greater thickness of the fill causes larger deformations and increases the period of time during which they occur. Superimposed on these movements are the general and frequently progressive settlement of the underlying natural ground, particularly where this consists of clays or similar compressible soils. These soils consolidate considerably and for a long time under the foundation loads and weight of the fill. On sloping sites special precautions may have to be taken to ensure general stability.

differential movements are restricted or can be withstood without damage. To achieve this the superstructure acting in conjunction with the foundations should either be sufficiently rigid to redistribute the loads and thereby reduce the relative settlement of the structure, which is particularly economical for compact and high buildings, or they should be relatively flexible to accommodate themselves without cracking which is more economic for extensive and low buildings as in the present investigation. In such cases long continuous structures should be avoided. They should be divided into compact sections especially where loading conditions vary or where parts of buildings differ in height or are irregular in plan. Ordinary pad and strip footings are rarely adequate. Wide strip footings reinforced in both directions may be sufficient where good bearing

n be obtained and, if this is not available, a concrete ft reinforced top and bottom in both directions should e provided. If the fill is relatively thin and overlies a m natural stratum, a beam and pier or beam and pile undation may be preferable. This is particularly eful where the fill is loose and of poor bearing value ace the structure can then be carried on reinforced ncrete or encased steel beams spanning between the ers or piles to which they should be well bonded. The tter have, however, to be designed to carry not only e building loads but also the drag from the consoli- ating material, which may be considerable for thicker ls and where the underlying ground is compressible. ored piles may be preferable to driven ones to avoid sturbance of the fill by vibration, particularly where is is relatively loose as in the present case.

Foundations should be located at the same level as r as possible to reduce differential movements. In neral, buildings should not be constructed partly on

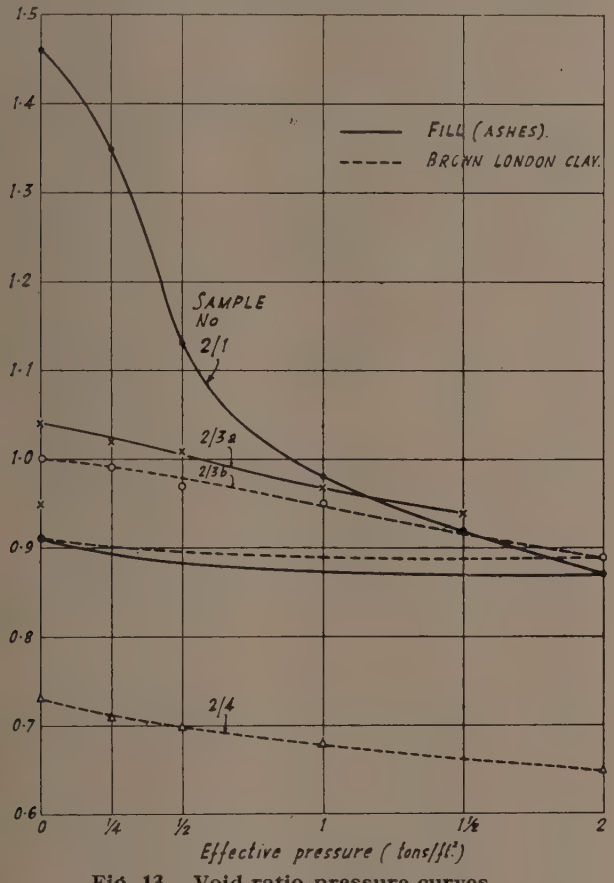


Fig. 13.—Void ratio-pressure curves

l and partly on natural ground unless the former is mpacted to the same density as the latter, nor should ey be constructed near the edge of made-up ground, pecially where this ends in a free slope, which is nerally in a loose state. To prevent displacement d increase stability the whole foundations should be terconnected and well tied in two directions by ranging them as grid footings or beams or a continuous ft. In addition the superstructure should be con- cted to the foundation and framed constructions be aced in two directions at floor and roof levels. The adding should be adequately fixed to the main struc- ral members and brick walls, for which weak mortar frequently advisable, may have to be reinforced, rticularly near openings. This also applies to the of construction and its components, which should be fficiently braced and secured to the bearing members prevent them from being moved or collapsing if the pports should be displaced through differential settle- ment of the foundations. Jacking-up positions may be introduced in flexible steel structures to realign them when necessary. The design of services should ensure that relative movements between various sections and the structure do not interfere with their function ; all concrete work below ground level should be resistant to deterioration. In conclusion, it may be said that the overall cost of completed works is likely to be much less if fills have been properly compacted since the latter expense is more than offset by immediate possession of the site and the reduced initial and maintenance cost of structure, foundations and services.

Acknowledgements

The survey of the structural movements and damage of the extensive factory building on fill was carried out by Messrs. Travers Morgan & Partners, Consulting Engineers, who were called in by the owners to advise on remedial measures. The author is also indebted to his colleagues, in particular Mr. T. K. Chaplin, B.A., for assistance in the field and laboratory work in connection with this investigation.

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Discussion on

The Fabrication of a 90-ton Welded Crane Girder*

By E. Ibbotson, M.I.Struct.E., and

An Investigation of the Stress Distribution in a Welded Plate Girder*

By S. Mackey, M.E., B.Sc., A.M.I.C.E., A.M.I.Struct.E., and D. Brotton, B.Sc.

The PRESIDENT, introducing the authors of the papers, recalled that the Civil Engineering Department of Leeds University, under the direction of Professor R. H. Evans, had collaborated for many years with the late Mr. P. Mitchell, of Messrs. Dorman Long & Co., Ltd., on an extensive investigation concerning the stress distribution in a large welded girder, and the papers to be presented that evening were the outcome of that work. Mr. Mitchell had died early in 1949, and Mr. Ibbotson, who had been working with him, had continued the preparation of the paper.

(The authors presented their papers.)

Discussion

Professor R. H. EVANS (Past Chairman, Yorkshire Branch), proposing a hearty vote of thanks to the authors of the papers, said that one of the most outstanding developments in the realm of structural engineering during the present century was the introduction of arc welding in lieu of riveting. It was used on a wide variety of structures on grounds of economy and improved characteristics. That it was superior to riveting there could be no doubt at all ; and he was quite sure that, when the technique and theory, and more particularly the plastic theory, had been finalised, arc welding would become more economical than riveting. At the moment this question of greater economy on all occasions may be doubtful but it was a matter of gaining further experience, and he had no doubt that it had a tremendous future.

It was very gratifying to him to be present to hear his colleagues present their two papers describing a piece of work which he had seen grow from the very beginning, and he was very grateful to the President for his reference to the late Mr. P. Mitchell, for it was he who was initially responsible for the work. Since his death Mr. Ibbotson and Mr. Barlow had carried on the tradition, and Professor Evans was extremely grateful to them.

With regard to the question of distortion mentioned by Mr. Ibbotson, it was a fact that high locked-up stresses in welded structures had very little effect on the ultimate load of structural members when subjected to static and repeated load providing the material was ductile, without notches and free from residual stresses in more than two directions. It had been also stated in the paper by Mr. Ibbotson that the butt ends of the flanges were first welded, then the web, the flitch plates, etc., to allow the structural parts every freedom to shrink. But he wondered whether in certain cases it would not be better if those plates were held rigidly in a manipulator or a jig. As to the testing of welds, many of those engaged in engineering testing were now interested in the development of the supersonic method. He believed it was developed originally for the testing of concrete. For years engineers had criticised strength tests on test cubes at ages of 7, 14 and 28 days after casting as being too late. At the Road Research Laboratory, Dr. Jones and his colleagues were developing this non-destructive supersonic method, and it certainly had a promising future. He knew that it had been applied with success in detecting cracks in welds, but he was not sure whether or not it was successful in detecting differences in the character or quality of welds i.e., whether it was capable of detecting the difference between a bad and a good weld. One would be very interested if Mr. Ibbotson would give his experience in this direction.

Mr. Mackey had indicated that one of the objects of the investigation was to determine the distribution of stress in the flanges and in the flitch plates. The flitch plates were introduced through necessity and it was obvious that they had been a great success by giving the flanges greater stability. Further, the stiffeners via the flitch plates, were giving a better distribution of stress throughout the web. He invited Mr. Mackey's views concerning the use of flitch plates in future designs, and wondered whether he would agree that flitch plates seemed almost essential in similar designs.

Another striking feature was the difference in behaviour as between series 1 (a) and 1 (b). Series 1 (a) was repeated, and there was a difference in deflection amounting to something like 7 per cent., and it was gratifying to note that there was a good measure of agreement between theory and experiment.

When one considered the tests as a whole one found slight discrepancies here and there. For example, in connection with the distribution of stress in the flanges it could be shown theoretically and from other experimental work that the stress at the centre of a flange was higher than that at the two edges. But it was extremely difficult to guarantee that a girder was properly strained, and his experience with, say, a reinforced concrete T-beam had been that the stress on one edge of the flange was often less than that on the other edge. Dr. J. M. Hawkes, A.M.I.Struct.E., who was present at the meeting, appreciated as well as he did how difficult it was to load anything correctly. The results showed clearly that the influence of the points of concentration of load was to change the shape of the shear stress distribution curve so that the maximum shear stress did not occur at the neutral axis, but between the loading point and the neutral axis. Also the authors had checked the accuracy of their experimental work by balancing the internal and external forces.

In a comment on Codes of Practice, Professor Evans said that Codes, on the whole, were and must be somewhat conservative, and recently his attention was drawn by Mr. S. M. Reisser, B.Sc., A.M.I.C.E., M.I.Struct.E., to a new French practice in which the stiffeners were not welded to the compression flange. He had always considered it essential to weld a stiffener to the compression flange, but not to the tension flange, and he would be grateful to have the views of Mr. Reisser and Mr. Mackey on this point.

With regard to stiffeners and B.S.S. 449, it was striking from the graphs in the second paper that the stiffeners were very lightly stressed, and he asked

*Papers read before the Institution of Structural Engineers at 11, Upper Belgrave Street, London, S.W.1. on Friday, February 24th, 1950. Mr. Leslie Turner, B.Sc., M.I.C.E., M.I.Struct.E. (President), in the Chair. Published in THE STRUCTURAL ENGINEER, Vol. XXVIII, No. 2, pp. 23–42.

hether Mr. Mackey considered that any modifications ere necessary in that direction. On the whole the esign of stiffeners was empirical and there was plenty scope for further research. It was certainly a case here research, theory and practice should move rward together, and it would be a step in the right irection if we in this country followed the practice dopted on the Continent and in America, namely, that contractors and consulting engineers and the uni-ersities working very closely together to the mutual enefit of all.

It was therefore a great pleasure to propose a very earty vote of thanks to the three authors for their nformative and interesting papers.

Professor W. Fisher Cassie (Hon. Librarian), econding the vote of thanks to the authors, said he as voicing the feelings of the members of the Northern ounties' Branch, some of whom were present at the eeting that evening, in saying that they were very leased to see the fruition of the work Mr. Mitchell stigated and encouraged. Mr. Pullan Mitchell was the econd Chairman of the Northern Counties' Branch, and his ss was felt very much by all members of theInstitution.

It was very valuable to have, in the papers, first an lustration of the development of welding technique, nd secondly the correlation of that with theoretical ork and with testing. His interest had been particu-rly aroused by the experiment described in the paper y Mr. Mackey and Mr. Brotton (on page 31), where the orsion test was mentioned, chiefly because work on the orsion of structural sections had been going on for me years in his laboratory. He had, with W. B. obie, read a paper to the Institution in 1948* on the bject ; and they were very interested indeed to read the full-scale torsion test. They had stated in their aper that full-scale tests were necessary and that xtrapolation from model tests might not be satis-ctory, and had tested a beam 24 in. × 7½ in. The rger one discussed in the paper provided a very teresting corroboration of the effects of torsion on on-circular sections.

Using methods similar to those described in their aper and based on work which had been going on, and hich he hoped would be reported in due course, W. B. obie had worked out the torsion constant for the beam escribed in the present papers, and had arrived at the gure of 3.424. The experimental figure was 3.23, and he theoretical figure calculated by Mr. Mackey and r. Brotton was 2.34. The difference between the xperimental value (3.23) and Cassie and Dobie's value 3.424) was within 6 per cent. One would be interested o know what method Mr. Mackey and Mr. Brotton had sed to calculate the torsional constant ; presumably was the usual one-third of length times thickness bed. This did not take into account the " junction ffect," which could be very important indeed. *Professor Cassie then showed a model of an inflated embrane over the cross section of an I-beam. On this odel the addition to the torsion constant which is provided the junction could be clearly observed.*)

The reason given in the paper by Mr. Mackey and r. Brotton for the difference between the experimental nd the theoretical figures was a very modest one, and fact the authors' experimental figure was very much oser to the theoretical figure than they had indicated. s a corollary it could be argued that the stiffeners, though giving resistance to diagonal tension and ompression, do not in fact give so much stiffness in orsion as was suggested in the paper.

**The Torsional Stiffness of Structural Sections.* STRUCT. ENG., *ol. XXVI. No. 3. pp.* 154–182.

Mr. Ibbotson, after expressing his appreciation, commented on the point raised by Professor Evans concerning freedom in welding as against fixed welding. In the girder as he had illustrated it, each butt had shrunk 1/16-inch ; there were two butts in the top flange, giving a total shrinkage of ⅛-inch. The total shrinkage of the girder was 5/16-inch. He suggested that, by allowing the flange to shrink freely before welding, the final shrinkage of the girder was reduced to something like 3/16-inch. He had feared that, by welding the longitudinal joints before the butts, cracking might be experienced, and it was found that by allowing the material to shrink freely the work was quite successful, and he suggested that, while that method was successful, one should continue to use it.

With regard to the supersonic apparatus, he said that definitely it would indicate any flaws and breaks and laminations in the steel or cracks in the welds, but he could not speak concerning its success in detecting slag inclusions ; he imagined that it would show them up, but he did not know to what extent.

Dealing with the question concerning the depth of the web, he said it was chosen as a matter of convenience. To transport from the mills plates of more than 9 feet in depth was extremely difficult.

Mr. Mackey, who also thanked Professor Evans and Professor Fisher Cassie for their kind remarks, said that it had afforded him and his colleagues very great pleasure to address the Institution.

Replying to Professor Evans' remarks concerning deflection, he said that under the Series I (a) type of loading the discrepancy between the measured and theoretical deflections amount to 8.5 per cent. A simple inclined plane test was carried out to determine the coefficient of friction at the end bearings and a value of 0.3 was obtained for the coefficient. Assuming that a horizontal compressive force equal to 0.3 times the end reaction acted inwardly towards the centre of the girder, at each bearing plate the effect of this eccentric force would be to reduce the control deflection by a little over 7 per cent. This would reduce the difference between the experimental and theoretical deflections to 1.4 per cent.

As to the effect of the flitch plates, they were carrying out some tests on girders with uniform webs and from these it was hoped to obtain the relative value of the flitch plates in distributing the stress to the flanges. From the results of the tests already carried out it seemed that complete success had been achieved in securing uniformity of stress throughout the flange plate ; but whether similar uniformity of stress would be obtained without the use of flitch plates remained to be seen.

Looking forward to hearing Mr. Reisser's remarks concerning the welding of stiffeners to the flanges, Mr. Mackey said that whilst he was of the opinion that stiffeners should not be welded to the tension flange, his mind was rather fixed on having them welded to the compression flange.

In referring to the spacing of stiffeners, he pointed out that the girder was tested as a simply supported span, whereas the stiffener spacings at the ends approximated to those in the full-size girder which was continuously supported. He agreed with Professor Evans that a considerable amount of work remained to be done in endeavouring to solve the problem of spacing of intermediate stiffeners.

Mr. Mackey was very pleased that Professor Fisher Cassie had gone to the trouble of making his torsion calculation ; he agreed that the authors had calculated on one-third × length × thickness cubed, and that that was probably the reason for the differences between the experimental and theoretical values. He would be very

pleased if Professor Fisher Cassie would at some time discuss his experience of the elastic membrane treatment.

On the question of stiffeners adding to the torsional stability, he said that, on the face of it, it would appear that, the more one stiffened up a girder, the greater would be the torsional stability. But in his view, in dealing with welded girders, the question as to how the stiffeners were welded on was of fairly considerable importance. If there was any draw on the flanges and one forced the stiffeners in, there might be a certain amount of local and residual stressing beyond the yield point over a section of the girder which would influence its torsional rigidity, but he was afraid the results of his tests did not confirm that.

Dr. J. M. HAWKES (Associate Member), said he had listened with great interest to the presentation of the papers, and was particularly grateful to notice how what might be called operational research was following practice. The research described in the papers was partly academic and partly what the present generation called operational research, i.e., it was not completely divorced from practice.

Putting questions to Mr. Mackey and Mr. Brotton, he referred to loading No. 1, where the loading was made to simulate the movement of the wheels of a crane over the span, and asked whether the authors knew, or whether they could say from their reading, anything about the stress distribution in the web when the wheel loads were not over a stiffener. The particular girder discussed was one carrying an overhead travelling crane, so that it differed from an ordinary girder carrying static loads, where the stiffeners were placed underneath each load.

At the end of the section of the paper headed " Web Stresses " (page 37), the authors had stated that by actually taking the area underneath the shear stress diagram given on page 38, they had found the total shearing force as shown by the electrical resistance strain gauges to be 36.2 tons in the web, and the shear stress as shown by the Huggenburger extensometers was 38.5 tons in the web, giving a mean value of 37.35 tons, which compared very well with the theoretical value of 37.5 tons. If they had integrated the right-hand diagram in Fig. 9 to obtain those shear loads it seemed that the measured value was always above the theoretical.

Mr. S. M. REISSER (Member), said that Professor Evans' remarks had put him on the spot, as he had not read the book which he had recommended ! A brief perusal of this publication, however, had been sufficient to show him that some recent research was apparently in contradiction to some of the details which had been regarded in the past as good practice, including the particular case of stiffener welding. Nevertheless, he was still inclined to agree with Mr. Mackey that stiffeners should be welded to the compression flange.

Adding his personal thanks to the authors, he said that he had particularly enjoyed the papers, as he believed them to have been the first to deal with welded steelwork in the construction of which welding was regarded as a normal method of fabrication and not as something miraculous.

Commenting on the reference to " arcing time " in connection with the assessment of the economy of fabrication, Mr. Reisser took it that the term meant the net time during which the arc was established. With regard to the chipping of slag, he asked whether this had been done by labourers or whether the welders were expected to do it themselves.

Finally, he asked if the authors could give a figure representing the ratio of the total welding time to the arcing time, as such a figure would be particularly helpful for estimating purposes.

Mr. IBBOTSON replied that the term " arcing time " covered burning time only. Separate labour was available for de-slagging, so that the welders themselves had not to do that. The operating factor was approximately 4, which meant that the " actual welding time " was four times the arcing or burning time.

Mr. J. S. TERRINGTON (Associate Member), referring to the welding of the flanges, commented that there were as many as eight runs of welding on each face, due to the thickness of the flanges, etc., and he asked whether larger electrodes could have been used, so reducing the number of runs.

As to the straightening up of the girder, he asked what methods were adopted to ensure that the web plate was truly straight ; it seemed to him that over a length of 110 feet it was quite a problem to ensure that the web plate was perfectly straight.

Discussing the bogies on which the girder was run out, at 8 ft. 6 in. centres, he asked what determined the spacing of the wheels. Having seen the girders on the site, he wondered how the spacing of the wheels was arrived at, and, in spite of tying down with chains, to what extent the girders on the bogies were stable.

Finally, Mr. Terrington commented that the sort of work described was usually associated only with Germany, and one had in mind the work of men such as Dr. Dörnen. It was, therefore, a matter for congratulation to the Steel Company of Wales, to Messrs. W. S. Atkins & Partners, to Messrs. Dorman Long & Company, Ltd., and to all concerned that they had made use of welding on such an extensive scale.

Mr. STANLEY VAUGHAN (Hon. Treasurer), congratulated not only the authors of the very interesting papers presented, but also those who were responsible for the excellent and simple design of the unusually large girder described, and also for the carefully thought-out arrangements for the welding of the girder, including the " manipulators," which so much facilitated the construction.

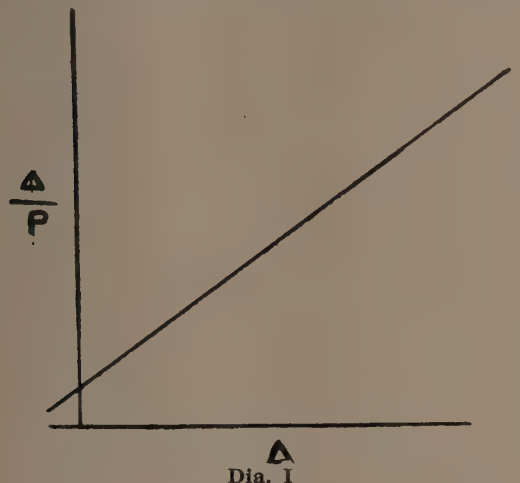
One was delighted to note that, in general, the stresses ascertained by the very comprehensive tests that were made accorded extraordinarily well with the values obtained by calculation. There appeared, however, to be two cases where there was quite definite deviation between the observed behaviour of the girder and what might have been expected from theoretical considerations.

The first related to the distribution of the stresses along the girder flanges. The results were shown in Fig. 8 of the paper by Messrs. Mackey and Brotton (page 37). What had impressed Mr. Vaughan was that in each of the three tests the compressive flange stress at mid-span had dropped appreciably below its value at the applied loads, whereas the tension stress had continued to increase up to mid-span. It appeared that this deviation from the calculated behaviour was not haphazard, and it seemed to be too considerable and too consistent to be ascribed to experimental errors. Mr. Vaughan would therefore be very interested to hear if the authors could offer any explanation of this phenomenon, and he furthermore asked whether, in the authors' opinion, the reverse effect would have occurred if the loads had been applied to the bottom flange and not to the top flange.

The second deviation from expected results was shown by Fig. 11 in the paper, and concerned the distribution of stress across the longitudinal legs of the stiffeners. Whereas one would expect approximately uniform distribution the tests showed very considerable variations, this variation in one case amounting to

ons per sq. in. (i.e., from 6 tons per sq. in. at one edge 12 tons per sq. in. at the other edge. He would lcome any further information or any views which e authors might be able to give as to the reasons for is variation, and he asked whether it might have been e to the procedure adopted for the welding of the ffeners to the flanges of the girder. Perhaps this riation could have been avoided if the welding had en carried out by starting at the centre point and rking outwards. Mr. Vaughan suggested however at this variation in the distribution of stress across the ffeners was not very important, and would not be ely to affect the ultimate strength of the girder, as the n-uniformity would die out when the plastic stage is reached.

Mr. R. J. Fowler (Associate-Member), discussing the lding of the flitch plate to the flange plate, said that -in. rod was used for the first run, and for the second n a $\frac{3}{8}$-in. diameter rod. He asked if there were not a ndency to cracking of the weld laid with the $\frac{1}{4}$-in. rod, it seemed to him that they were very thick plates on ich to lay the first run as a $\frac{1}{4}$-in. fillet ; he would have ought that a 5/16-in. fillet would have been the



Dia. I

solute minimum that should have been put down as e first run.

As to the welding of the stiffeners to the flanges, r. Fowler said he was rather old-fashioned in that atter, and he liked to see stiffeners welded to flanges, en if only to protect the flanges against corrosion. In at connection he referred to a bridge situated not ry far from the works of Messrs. Dorman Long & Co., d about which they knew quite a lot ; it was erected me 15 years ago, and the stiffeners were welded to the nges. It carried some of the heaviest traffic in the untry, and the road over it was quite bumpy, and only the previous day he had heard, on very good report, at there was no sign of failure in that welded bridge. at experience rather confirmed his view that it was ore practical to weld the stiffeners to the flanges than t to do so.

Coming to the distribution of stress across the flange the stiffeners, he said he had noticed that on the l size girder described in the papers automatic welding as used in welding the stiffeners. Probably they were lded by starting at one end and working to the other, en turning in the machine and starting from the cond end and working back to the first end.

If a similar sequence and direction of welding were ed on the test girder stiffeners it might account for the even distribution of stress across the flanges.

Finally, in offering his compliments to the authors on eir very fine papers, Mr. Fowler commented, in reference to the paper on the testing of the girder, that he would like to suggest similar tests made on a comparable riveted girder.

Mr. E. M. Lewis had been concerned with the design of the full-scale girders in his capacity as Assistant Engineer in the Consulting Engineer's office. He concerned himself with the part of the paper by Messrs. Mackey and Brotton in which they tried to link classical theory with experimental observations, and there were two matters about which he was in some doubt. First, he considered the buckling of web panels Nos. 5 and 6 under the action of predominantly bending stresses. The authors had observed the lateral deflections of these

Dia. II

panels and had plotted the contours shown at Fig. 4 in the paper and the curves shown at Fig. 6. They stated that they had then used the Southwell method to determine the crippling load due to elastic instability. He observed, parenthetically, that some authorities did not consider the Southwell method applicable to web panels of this kind, for instance, he believed Dr. S. R. Sparkes had arrived at this conclusion during his recent work on the behaviour of plate girder webs. However, the authors had plotted Δ/P against Δ as indicated in the *diagram* (I) and had, presumably, deduced the buckling load of 114 tons for panel No. 6 from the slope of the curve. In another part of the paper, however, they stated clearly that the girder had finally failed by yielding of the compression flange in panel No. 6. It seemed to him that such yielding of a boundary would cause a redistribution of the stresses in the panel which would make it invalid to compare the elastic crippling load of 114 tons with the failing load of the girder, i.e., 115 tons. If there were other arguments which justified this comparison he would be glad if the authors could enlarge on the subject.

The second point concerned the shear buckling of No. 1 panel. The contours shown at Fig. 4 were not of the shape which he would expect under shear loading. He would have expected the buckles to run the other way as indicated in *diagram* (II)—a simplified illustration of experimental observations by Bergman and Wastlund. It therefore seemed to him that the lateral deflections were not due to shear buckling, but might be accounted

for by initial lack of flatness of the web plate. If this was the case, it was surely wrong to relate the slope of the nodal lines to the wavelength of the buckles using the classical relation quoted from Timoshenko's work.

A previous speaker having asked Mr. Ibbotson to what extent flaws in welds could be differentiated by the ultrasonic testing technique, Mr. Lewis took the opportunity to amplify Mr. Ibbotson's reply. Mr. Lewis had been privileged to assist in a demonstration of the technique at the recent exhibition organised by the British Welding Research Association. The Association had prepared seven specimen welds each containing a classical defect which was known to the Association but not to the operator. These were examined at the end of the lecture and so far as he could remember the results were as follows :—

The first three welds had severe lack of penetration, small slag inclusions strung out and a hot crack in the first run, respectively. These predictions were confirmed at the meeting by breaking open the specimens. Unfortunately, it was not possible to break open the remaining four, but it was learnt subsequently from the British Welding Research Association that the predictions were correct. The fourth weld was in high alloy steel and " cold " cracks were present in the fusion zone. The fifth weld was almost perfect. The sixth contained one large slag pocket at the end of a weld run and the seventh was described by the Association as having medium porosity. It was interesting to note that in the case of the second weld, by using maximum sensitivity, definite reflections were observed from the welded zone, but these could not be interpreted as yet. This and other similar observations however held promise that in the future it might be possible to inspect the structure of the weld metal by ultrasonics.

Mr. Lewis concluded by saying that in his opinion this demonstration gave a fair indication of the degree of interpretation attainable at the present time.

Mr. Ibbotson, replying to Mr. Torrington's reference to the welding of the flanges, said the welding was carried out vertically, using 8-gauge electrodes for the first run and 6-gauge electrodes for subsequent runs. The 6-gauge electrode was the largest size of rod to be used for vertical welding.

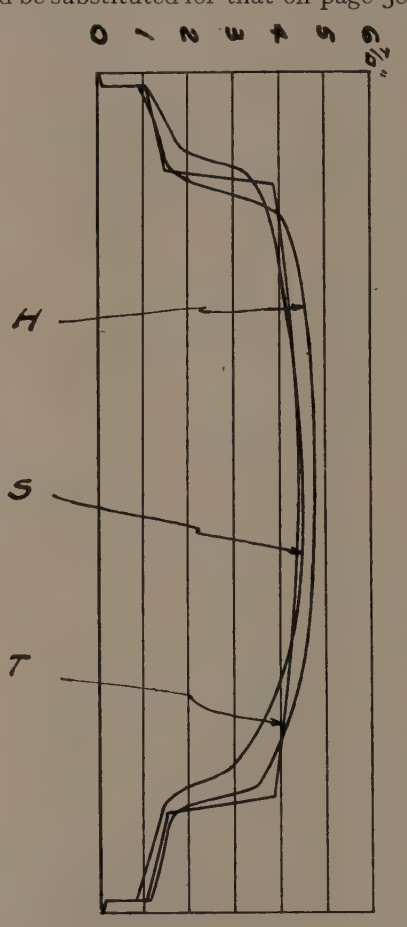
As to the spacing of the wheels on which the girder was run out, by which presumably Mr. Terrington meant the gauge of the wheels, he said the assessment was not very scientific ; the 8 ft. 6 in. gauge wheels and axles used just happened to be in stock !

The design of the girder was by Messrs. W. S. Atkins and Partners ; and Messrs. Dorman Long & Company, Ltd., as contractors, were responsible for the practical interpretation. The manipulators were designed and fabricated by the company at Middlesbrough.

Coming to Mr. Fowler's remarks on the welding of the flitch plates to the flange, he said there was no difficulty in the welding of the $\frac{1}{4}$-in. fillet ; but there was no attempt to maintain the $\frac{1}{4}$-in. fillet so long as the subsequent run would complete the $\frac{5}{8}$-in. fillet. The point was that the $\frac{3}{8}$-in. electrode, with its coating, was quite a large rod, and if it were used for the first run it would be difficult to get down to the bottom of the " V," without excessive arc length.

Mr. Brotton, replying to Dr. Hawkes, referred him to the section of the paper headed " Under Crane Wheel Loads " (page 39). The maximum principal stress under the wheel load in web panel 5 which was approximately equal to the bending stress at that section, was of the order of 10 tons per sq. in. at 75 tons applied load ; near the stiffener in web panel 6 the value was approximately 8 tons per sq. in. The minimum principal stresses under the wheel loads were compressive, whereas in the absence of radial stresses they would have been tensile. Their values were, however, quite small, being 5 tons per sq. in. in web panel 5 and 1.5 tons per sq. in. near the stiffener in web panel 6. The authors had plotted the stress distribution curves but had not included them in the paper in view of the number of diagrams involved.

He was glad to have his attention drawn to the anomaly in the shear distribution diagram in Fig. 9. Shear distribution diagrams were drawn for each load increment, and it was found subsequently that the theoretical shear distribution diagram for a load of 85 tons had been inadvertently drawn on Fig. 9 in preparing the separate diagrams for reproduction. The corrected figure, which should be substituted for that on page 38, is given below.



Shear stress in panel 2 at 75T load

H = Huggenburgers
S = Electric strain gauges
T = Theoretical

Fig. 1

It should be noted in examining this diagram that, due to non-uniformity of the web section, any differences in the areas of the three diagrams over the length of the flitch plates will be more effective than corresponding differences over the central web portion where the web thickness is reduced.

Concerning Mr. Vaughan's remarks on the distribution of stress along the flanges towards the centre of the girder (Fig. 8), he said that probably the reduction in compressive stress in Series 1 (a) and 1 (b) tests was due to restraint between the wheel loads. The loads were applied as shown in Fig. 8 through a series of beams to a loading rail bearing directly on the compression flange of the girder ; this induced frictional forces as shown in diagram below which caused the reduction in stress in the compression flange.

If the loads were applied to the bottom flange as indicated by the dotted lines above, the tensile stress would have dropped between the loading points, whereas the compressive stress would have continued to increase up to mid-span.

With regard to the variation of stress across the stiffeners, Mr. Brotton said that no record is now available of the direction in which the welding of the stiffeners to the flanges had been carried out. As pointed out in the section of the paper under the heading " Intermediate

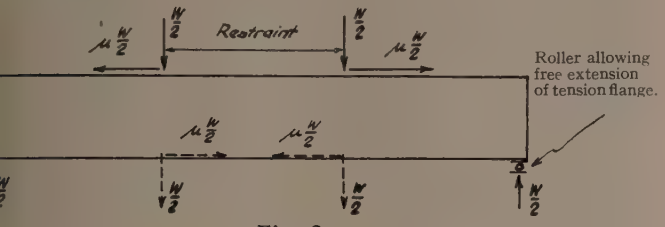


Fig. 2

Stiffeners," the stress variation might be induced either by differential longitudinal shrinkage, between the web and flitch plates, shear deflection, or by a combination of both these effects. It is therefore probable that similar variation would still have occurred if the welding had been carried out from the centre towards the edges. The electric strain gauge readings from which the stress values were obtained represented the change in strain of the stiffeners from zero load to 75 tons load, consequently strain or distortion due to welding was not measured. Mr. Vaughan was quite right in his suggestion that these variations would be evened out when the plastic stage was reached, and consequently they would not be critical in determining the ultimate strength of the girder.

On Mr. Lewis's remarks concerning the buckling curves for web panels Nos. 5 and 6, he said the estimated crippling load for panel No. 6, of 114 tons, was brought about by the failure of the compression flange ; that had caused the panel to be supported only on three sides, so that it was not fair to compare it with the theoretical buckling load for a panel supported along the four edges.

Mr. Lewis said he had thought the authors had made the comparison in that way.

Mr. Brotton replied that that was not so. With regard to shear panels, he said one would expect the lines to go in the directions in which Mr. Lewis had drawn them ; but probably the results obtained were due to the initial deflections of the web.

Mr. Mackey, replying to Mr. Fowler's suggestion concerning tests on a riveted girder, said that this work was under way and it was hoped in the very near future to carry out tests on similar riveted and welded girders, without flitch plates.

With regard to initial out-of-straightness, he said that the lateral deflection diagrams had been corrected for this. The maximum value of the initial out-of-straightness, for any one panel, did not exceed 0.050 in.

As to reducing the scale of the model work, he said that the effect of initial out-of-straightness masked most of the lateral deflection results unless in the manufacture of the girder very accurate workmanship, careful beating out of the panels and careful control generally was involved. To deal with a girder size very much smaller than that tested would lead to erroneous results.

Finally, Mr. Mackey thanked the meeting for so patient a hearing, bearing in mind the limitations that were bound to apply in trying to derive conclusions from one particular test. Those who were working in Leeds, he said, could not control the design and fabrication of the girder from its initial stages, although that was a matter of prime importance in carrying out of any elaborate series of tests ; as Professor Evans had stated, the girder was already fabricated when they had come into the picture, and they had had to do their best with a girder which they could not control from the outset.

The President complimented the authors on their clear replies to the questions raised in the discussion, and thanked all who had contributed to the discussion. Moreover, he complimented the consulting engineers and the contractors concerned, as well as the Leeds University for having seized the opportunity presented to carry out a most interesting series of tests, and thanked them for having provided the Institution with the technical information and test results which had accrued.

Written Contributions

Mr. R. J. Fowler (Associate-Member) writes : I should like to add to my rather brief remarks made in the discussion at the meeting on February 24th, concerning the welding of the flitch plates to the flange plates.

To my mind, the welds warranting major attention in these girders (considered from the welding deposition and not the design loading point of view), are the fillet welds between the heavy plates, to which I referred in the discussion.

In depositing the welds in the tilted or gravity position in any tee joint, one fillet, or first run of that fillet must necessarily be deposited after the other. In this case, according to the paper, a length of some 30 ft. of the full $\frac{5}{8}$ in. fillet weld is completed before turning the girder and commencing on the first run of second fillet weld.

Thus the first run of the second weld is made in a condition of maximum restraint firstly due to the inherent rigidity of the clamped thick plates, and secondly due to the tying effect of the first fillet weld.

It is because of this excessive restraint that I consider that a $\frac{1}{4}$-in. fillet weld for the first run of these welds, made with a $\frac{1}{4}$-in. dia. electrode may be a dangerous procedure, and such a weld is very likely to suffer from cracking before it is covered by the remainder of the weld made with the $\frac{3}{8}$-in. dia. electrodes.

Experience on the welding of heavy machine frames, and during the war on the welding of heavy armour plate, has demonstrated this to be so, and the absence of any sign of cracking when the second run is complete is no assurance that the initial crack is not there.

I would go further than this and suggest that by building up the second run to the full $\frac{5}{8}$-in. fillet weld profile (in one pass) is less than likely to result in the complete fusing down or penetration of the first run, so that any cracking of the first run is not at all likely to be fused away.

My experience is that such welds on heavy plates should be made with as heavy a first run as is reasonably practicable, and that such first runs are best made as $\frac{3}{8}$-in.-7/16-in. leg length fillet welds using a 5/16 in. or $\frac{3}{8}$ in. dia. electrode—the larger the better—and thereby incidentally considerably speeding up the welding with less heat input and so less residual stress and contraction or distortion.

As an extension to this I wonder if the author has considered the use of even larger electrodes and making each fillet weld in one run ? I think the job is of such a size as to warrant this practice.

This brings me on to the weld testing by the supersonic process. It just so happens that the butt welds are the easiest welds to inspect by this or any similar means. I understand that it was not used for the fillet welding on this job, nor is it easy to do so. I would like to suggest, at the risk of being corrected, that it is

the fillet welding of the type I have commented on above that most needs such inspection—if it is required to be used.

Apart from this comment, I should like to say how much I have appreciated the author's description of this work, his mention of the necessity of the proper use of adequate welding procedure, which also of course applies to small welded jobs and to the conception of the very fine welding manipulator which he may be interested to know is similar in some respects to those used for the repetition welding of heavy tank hulls during the war.

Mr. IBBOTSON has replied as follows: In reply to Mr. Fowler's written contribution, I would like to make further comments respecting the welding of the $2\frac{1}{2}$-in. thick flange plates to the $2\frac{1}{2}$-in. thick flitch plates.

I agree that this weld warranted and, in fact, received major attention and it was only after considerable experiment and experience on similar classes of welding that the method explained was adopted. As stated, the first run was put down to reduce the arc length of the $\frac{3}{8}$-in. electrode and to enable the $\frac{5}{8}$-in. fillet to be completed with a further single run. The welder using the large electrodes followed immediately behind the first welder, so that the second run of weld was laid whilst the first run was still hot. I agree that should the first weld crack, the second run would not result in complete fusion and making good of the crack, in fact, inevitable cracking of the second run would result.

With large electrodes, the diameter over the coating is such that even when used as a "touch rod," i.e., allowing the coating of the rod to contact each side of the "V," the arc length can be excessive, giving incomplete penetration under the rod and adding to the danger of trapping the slag. I consider that a $\frac{1}{2}$-in. fillet weld carried out in a down hand position, using a $\frac{3}{8}$-in. electrode, is the largest weld that should be made in one run. Experiments carried out have shown that the welder has considerable difficulty in controlling the slag when larger welds are attempted, and although satisfactory welds have been produced, the risk of trapping the slag with the inevitable cracking of the weld is always present.

The author has considered using $\frac{1}{2}$-in. diameter electrodes, but due to the difficulty of providing a suitable electrode holder and the fatigue to the welder in handling the heavy cable necessary for the large amperage required, the venture has been postponed.

I would like to thank Mr. Fowler for his interesting criticism, and for bringing forward a problem to which I have given considerable thought in the past. I entirely agree that when welding could be used, laying as heavy a run as is practicable.

Written Contributions to the Discussion

Mr. K. C. ROCKEY, M.Sc.(Eng.), A.M.I.Mech.E., writes: The authors are to be congratulated on their fine paper, which is sure to provoke lengthy discussion amongst interested engineers.

The writer is particularly interested in the details concerning the behaviour of the vertical intermediate stiffeners. The function of such intermediate stiffeners is to stiffen the plate, so that the plate buckles in a series of panels between the stiffeners, which remain straight. In 1936, Stewart Way[1,2] determined the minimum value of the flexural rigidity which intermediate stiffeners should possess, if they are to remain straight under such conditions. On comparing the flexural rigidity of the intermediate stiffeners used on the model girder and the theoretical values which are required by Way's analysis, it is found that the ratio of the former to the latter is very large indeed. It would thus appear that the intermediate stiffeners on the model girder are theoretically too large, a point which is verified by the fact that the stresses in the intermediate stiffeners were found to be low in the tests. It is appreciated that in this particular case the intermediate stiffeners will, in practice, be subjected to different loading conditions than those used on the model. It would appear, however, that in cases where the intermediate stiffeners are not subjected to any direct loading, and are used solely to prevent buckling, that their design could safely be revised in order that the dimensions of the stiffeners be reduced. Since the writer is investigating this problem, he would be glad to learn of the authors' comments.

With reference to web panels 5 and 6 (see Fig. 6), the authors have commented on the increase in the rate of lateral deflection of web with load at the load of 85 tons. Many of the previous research workers, mentioned in the authors' bibliography, have considered such occurrences as an indication of the buckling load of the panel. Do the authors consider this load (85 tons) the buckling load of the panels?

Mr. F. MICKLETHWAITE writes: Having been concerned with Mr. Lewis in the design of the full-scale girders, I also am interested in the correlation of the theoretical analysis with the experimental results given by Messrs. Mackey and Brotton in their paper. I am particularly interested in the stress distributions and there are certain points which are not clear to me on which I would be glad if the authors would enlarge.

In Fig. 10 of the paper, the shear stress distribution adjacent to the end stiffener of the girder is given. If the behaviour was purely elastic as indicated by the paper I would have expected the maximum shear stress to occur at a point nearer to the lower flange than section XX. The authors state that readings from the strain gauges in the middle of the first panel have been used in deriving the stresses adjacent to the end stiffener and it is difficult to see how this could be done unless the stress distribution in the panel was already known. Perhaps the authors would enlarge on their method of obtaining this curve.

From the published results it would seem that the behaviour of the girder was elastic at all times. Recent work shown that this country and in America has, however, certain to be any welded construction there are almost the yield point va where "locked-in" stresses exceed loading applied to the of the steel. Under the first therefore behave plastica structure, some areas would subsequent loading in the , although on unloading and would be truly elastic as in ame range, the behaviour diagram in Fig. "A" below. ated by the stress strain it seems to me likely that such plas In this particular case alongside the continuously welde ic zones might appear row of strain gauges was positioned c stiffener. Since a I would ask the authors whether, on fi e to the stiffener, the load, any observations were made wh application of to the conclusion that plasticity occurred. ch might lead curve in Fig. 10 is based on purely elastic b Clearly, the otherwise, if plastic strains were interpret ehaviour as, of the corresponding elastic stresses the total d in terms across the section would be in excess of the ap shear force

Apart from this particular point where, plied force. elasticity might have been recorded on first l I think, should like to ask the authors if, at first, any oading, I signs of

[1] Way, Stewart. "Stability of Rectangular Plates under Shear and Bending Forces" JOURNAL OF APPLIED MECHANICS (A.S.M.E.) December, 1936.

[2] Way, Stewart, "Stability of Rectangular Plates under Shear and Bending Forces." Final Report. Second Congress of the International Association for Bridge and Structural Engineering. Berlin, 1936.

plastic behaviour were observed. If plasticity did occur, it would be indicated by the variations of stress or deflections with the applied load as shown by the idealised curves in Fig. " B." Although the authors did not apply reversed loadings, the effect of plasticity may have been observed in the permanent strains after the first loading and during the higher loadings.

I should find it very interesting to compare any such observations with the results of work now being carried out at Brown University on this important subject of " shake-down " and behaviour in the plastic range.

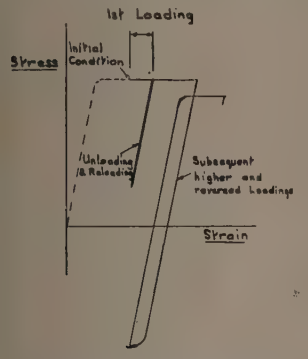


Fig. A.—Idealised variation of stress with strain for a point with " plastic locked-in " stress

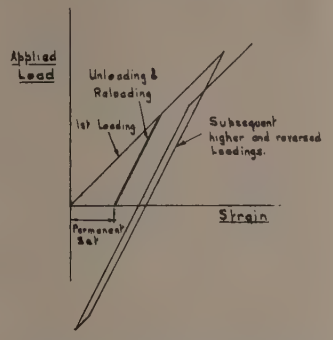


Fig. B.—Idealised variation of strain with applied load

Reply by the Authors

Further to Mr. Lewis's remarks on the application of the Southwell method to determine the crippling load of the panels due to elastic instability, the authors wish to state that the Southwell theory is applied purely to experimental deflection results independently of the type of edge support and the manner in which the actual failure is brought about.

With regard to the rejection of Southwell's method by other research workers, it should be pointed out that initial curvature of web panels which is inherent in all plate-girder work, very often masks the true deflection readings under low loads and one must therefore carry the readings to a sufficiently high stage of loading to eliminate the effect of this curvature. In certain cases it may not be possible to obtain a hyperbolic relationship between load and deflection before the plastic stage is reached, and in such cases the method cannot be applied.

In justification of its use in this particular instance, the graph of $\frac{\text{defln.}}{\text{load}}$ against defln. is perfectly linear, as in Fig. 3.

In reply to Mr. Rockey's written contribution, the authors were interested to learn that he is investigating the design of intermediate stiffeners. They are of the opinion that the design of these stiffeners in cases where they are not subjected to any direct loading could with safety be modified to give lighter sections.

With reference to buckling of web panels 5 and 6, the authors consider the load of 85 tons to be the safe limit for design purposes for web panel 6.

Mr. Micklethwaite is not correct in assuming that the maximum shear stress occurs at section XX in Fig. 10, as this section was chosen at random to illustrate the construction of the diagram. Regarding the method of obtaining the curve shown in Fig. 10, the shear stress distributions in the lower half of the web panel No. 1 and the web extension beyond the stiffener, were obtained from gauges in bearing group G, shown in Fig. 3. The shear stress distribution in the upper half of the panel near the stiffener was obtained from strain gauge readings adjacent to the stiffener and not shown on this figure. The values obtained in this case were found to be similar to the distribution in the centre of the panel.

Certain plastic zones did undoubtedly develop in the girder adjacent to the end stiffener during the first run of load, and evidence of this was shown in the strain gauge readings at that section. At that stage of the testing only seven pairs of shear gauges were available from which the experimental shear stress diagram could

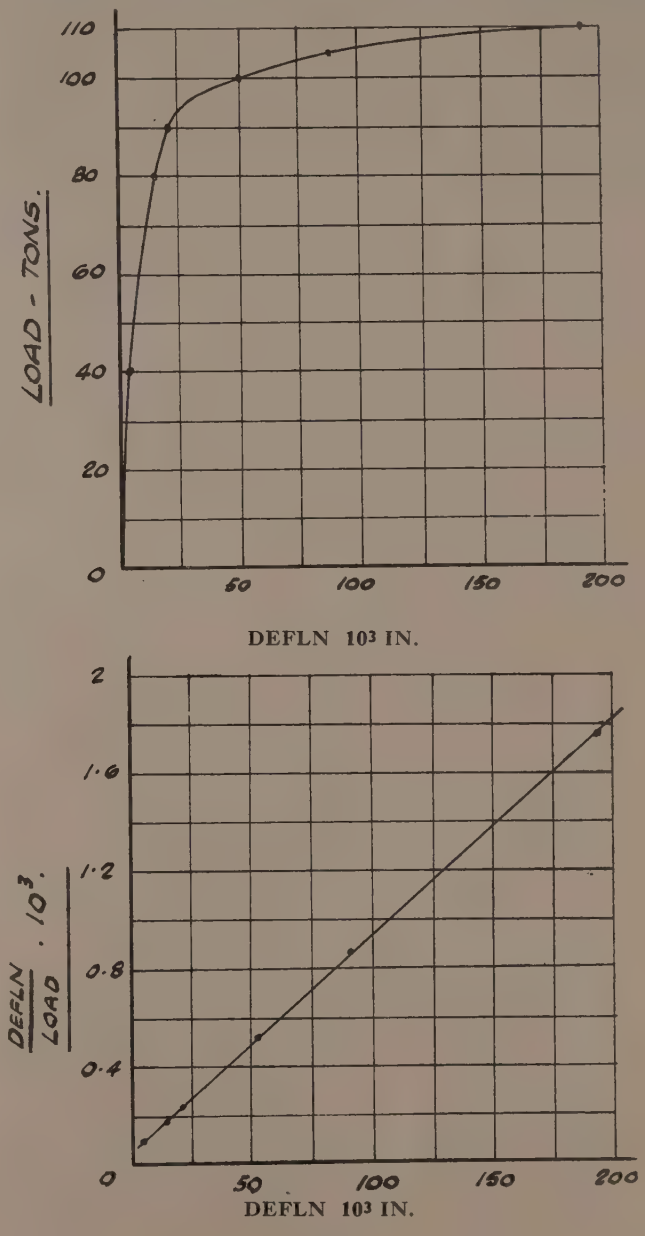


Fig. 3

be plotted. It was not possible therefore to integrate this diagram to give the measured shear force with any degree of accuracy, and while this work was done, giving a measured shear force very much in excess of the theoretical value, it was felt that the collective influence of the friction at the supports and loading points together with the slight zero lag in the test machine itself rendered the results of the readings of little practical use. In the subsequent testing, additional gauges were incorporated and every endeavour was made to eliminate friction at the supports, but of course, as Mr. Micklethwaite rightly mentioned, only elastic behaviour was recorded. As far as actual design is concerned, local yielding is inevitable in most built-up members, but the effect of this is not noticeable on the ultimate carrying capacity of the member concerned.

Institution Notices and Proceedings

ORDINARY GENERAL MEETING

An Ordinary General Meeting of the Institution of Structural Engineers was held at 11, Upper Belgrave Street, London, S.W.1, on Thursday, December 14th, 1950, at 5.55 p.m., Mr. Walter C. Andrews, O.B.E., M.I.C.E., M.I.Struct.E. (Vice-President), in the Chair.

The Minutes of the Ordinary General Meeting held on October 26th, 1950, as published in the Journal in December, 1950, were taken as read, were confirmed and signed.

The following members were elected in accordance with the Bye-Laws. Will members kindly note that the elections, as tabulated below, should be referred to when consulting the Year Book for evidence of membership ?

Students

Bakala, Eugeniusz, 119, Ashmore Road, W.9.
Clark, Tony Allen, 4, Chigwell Rise, Chigwell, Essex.
Hall, Brian, 37, Hall Croft, Beeston, Nottingham.
Harris, David William, 42, Gap Road, Wimbledon, S.W.19.
Mercer, Horace John Gerald, 27, Ward Road, Goldthorn Park, Wolverhampton.
Michau, Peter St. Cyr, P.O. Box 848, Bulawayo, Southern Rhodesia.
Robson, Ian Dudley, 9, Hawtrey Drive, Ruislip, Middlesex.
Sacchett, Richard, 26, El Yemen Street, Sidi Gaber, Alexandria, Egypt.
Scott, John Rodger, 15, Tweed Crescent, Riddrie, Glasgow, E.1.
Tear, Raymond George, 42, Albert Street, Brigg, Lincs.
Twaddle, Ian Edward, 8, Tilia Drive, Wychwood, Germiston, Transvaal, S. Africa.
Webb, Ronald Edward, The Flats, Guy's Hospital, S.E.1

Graduates

Agrawal, Ramesh Chandra, B.Sc.(Eng.), London, 307-D Vinya Nagar, New Delhi, India.
Cantlay, William Gordon, B.Sc.(Eng.)Hons., Glasgow, 53, Queen Square, Strathbungo, Glasgow, S.1.
Carlile, James Sim, B.Sc.(Eng.), Edinburgh, c/o "Lauriston." 37, Queensville, Stafford.
Carter, Raymond Leslie, 26, Pages Mead, Avonmouth, Bristol.
Deakin, Brian, 56, Church Street, Wincham, Northwich, Cheshire.
De Souza, Terence Anthony, 167, Newport Road, Cardiff.
Evans, Arthur, B.Sc., Hons., Wales, 9, Rosehill Terrace, Swansea.
Forcer, Thomas Austin, 15, Roxborough Avenue, Harrow, Middlesex.
Green, Peter Royston, B.Sc.(Tech.), Manchester, 117, Market Street, Chapel-en-le-Frith, Via Stockport, Cheshire.
Guthrie, John Lawson, B.Sc.(Eng.), Edinburgh, 8G, Bay View, Kinghorn, Fife, Scotland.
Hall, William Roy, 48, Barnet Road, Birmingham, 23.
Herbert, Robin William, Harleigh Hotel, 30, St. Andrews Street, Durban, South Africa.
James, Bryan William, Stewart Street, Whakatane, Bay of Plenty, New Zealand.
Kellaway, Thomas Henry, 10A, Christchurch Street West, Frome, Somerset.
Langfield, Ronald Alexander, M.Eng., Liverpool, "Beechwood," 11, Rock Park, Rock Ferry, Cheshire.
Lewis, John Charles, B.Sc.(Tech.), Manchester, "Caldicot," Lime Avenue, Northwich, Cheshire.
Lowe, John Robert, B.Sc.(Eng.), Hons., London, 36, St. George's Square, S.W.1.
Molyneux, John Stanley, 38, Gateacre Brow, Gateacre, Liverpool.
Palmer, Ronald William, A.M.I.Mun.E., Chief Engineer's Dept., Bracknell Development Corporation, Farley Hall, Bracknell, Berks.
Porter, Arthur James, B.Sc.(Tech.), Hons., Manchester, 123, Butterstile Lane, Prestwich, Manchester.
Shah, Navinchandra Ambalal, B.E. Civil, Bombay, Arun Building, Shanker Tekri, Baroda, India.
Shapiro, Myer Jacob, 39, Baytree Road, Brixton, S.W.2.
Smith, Keith Pegden, A.R.I.B.A., 20, Chalvey Park, Slough, Bucks.
Wakefield, Wilfred Eric, 2, Fford Maelor, Maes-y-dre, Wrexham, Denbighshire.

Associate-Members

Leong Kee Yap, c/o Mr. Wong Swi Vong, Cash Dept., Chartered Bank, Kuala Lumpur.
Singh, Kishen, 8, Rama Kotir, Began Bagh, Meerut, U.P., India.
Wong Gai Hong, Singapore Improvement Trust, Municipal Office Building, Singapore.

Transfers

Students to Graduates

Edwards, Paul Harper Dickinson, c/o Central Baptist Youth Hostel, 58, Boulcott Street, Wellington, New Zealand.
Dale, John Granton, 59, Mossley Hill Road, Liverpool, 19.
Modlin, Sydney, 12, Baytree Road, Brixton, S.W.2.
Morrison, Albert Duncan Constantine, 17, Prospect Hill, London, E.17.
Reed, Robert William, 562, Bartle Road, Umbilo, Durban, South Africa.
Rudd, Arthur Claude, 47, Office Road, Merivale, Christchurch, N.W.1, New Zealand.
Simms, George Alfred, 114, Crestway, Roehampton, S.W.15.
Wylde, Gerald Alfred, 366, Burmah Road, Penang, Malaya.

Graduates to Associate Members

Armstrong, Stanley George, B.Sc.(Eng.), South Africa, c/o H.L. & H. Concrete Engineers (Pty.), Ltd., P.O. Box 47, Johannesburg, South Africa.
Chelvadurai, Kanakenthiram, B.Sc.(Eng.), London, Assistant Industrial Engineer, Dept. of Industries, Colombo, Ceylon.
Cliff, Derick Rhydal, B.Sc.(Eng.), South Africa, 451D, Apsley Court, Musgrave Road, Durban, Natal, South Africa.
Crossley, Bernard Trevelyan, 128, Hurstmere Road, Takapuna, Auckland, N.2, New Zealand.
Dapling, Ronald Hawkins, 41, St. Louis Road, West Norwood, S.E.27.
Dobson, Patrick Arthur, B.Sc.(Eng.), Rand, c/o. H.L. & H. Concrete Engineers (Pty.), Ltd., 350, Victoria Road, Pietermaritzburg, South Africa.
Dubash, Ruton Jamshedi, B.E. Civil Hons., Bombay, Meher Villa, 631, Parsi Colony, Bombay 14, India.
Ehrlich, Robert, B.Sc.(Eng.), Rand, 8, Highlands Heights, Louis Botha Avenue, Highlands North, Johannesburg, South Africa.

GEDYE, Noel Trevor, c/o Auckland Metropolitan Drainage Board, 28, Quay Street, Auckland, New Zealand.
GOLDIE, John Barkley, District Design Office, Ministry of Works, Sydney Street, Wellington, New Zealand.
KHARE, Shrikrishna Tryambak, B.E. Civil, Bombay, 164D, Vincent Road, Dadar, Bombay, 14, India.
MATTHEWS, Samuel Wauchope Armour Charles, B.Sc. (Eng.), South Africa, 97, Gray Park Road, Brighton Beach, Durban, South Africa.
PITKIN, Kenneth John, A.M.I.C.E., P.O. Box 604, Bulawayo, Southern Rhodesia.
TARLTON, Roy, 33, Wolfa Street, Derby.
VAN TINTEREN, Boudewijn, B.Sc.(Eng.), Rand, 1124, Burnett Street, Hatfield, Pretoria, South Africa.

Associate-Members to Members

CLARKE, Arthur Thomas, 15, Old Croft Road, Walton, Stafford.
FOWLE, Frederick Edward, A.M.I.C.E., The Chalet, St. Mary's Road, Leatherhead, Surrey.
MATHESON, James Adam Louis, M.B.E., Ph.D., M.Sc., M.C.E., M.I.C.E., M.I.E.(Australia), Department of Engineering, University of Manchester, Manchester, 13.

Members to Retired Members

AMBROSE, Frederick Arthur, "Hedgerow," Moat Lane, Sedlescombe, Sussex.
BOOTH, Arthur, F.R.I.B.A., M.I.Mun.E., 2, Mount Pleasant, Bilston, Staffs.
HEATHCOTE, Charles Harold, Major, T.D., M.Eng., L.R.I.B.A., 46, Waldeck Road, Ealing Broadway, W.13.
LEA, Frederick Charles, O.B.E., D.Sc., M.I.C.E., M.I.Mech.E., Wayside, Church Lane, Dore, Sheffield.
NORTH, Harold Napier, Brigadier, D.S.O., Calcot Golf Club, Nr. Reading, Berks.
PEARSON-POWIS, Leonard John, "Hazelwood," Coonoor, Nilgiri Hills, S. India.
SCOTT, Archibald, M.B.E., F.R.I.B.A., Ministry of Health, Whitehall, S.W.1.
SPENCER, Albert S., 29, Cloister Road, Heaton Mersey, Stockport, Cheshire.

RE-ADMISSIONS

Associate-Membership

STANTAN, Henry James, 68, Brooklands Park, Blackheath, S.E.3.
THON, J. George, M.Sc.(Eng.), London, D.I.C., A.M.I.C.E., 1655, East 67th Street, Chicago, 49, Illinois, U.S.A.

MEMBERSHIP

LEVENSPIEL, Abraham, c/o Sino-Chemica, P.O. Box 392, Shanghai, China.

OBITUARY

The Council regret to announce the deaths of Evan Lovell Widdrington BYRNE, David CHATTAWAY, Colin Mardall KAY, Jeremiah Leask MANSON (Members); Frank BRITTAIN, Major James Ponsonby GALBRAITH, James Hepburn RICKIE (Retired Members); James MARSHALL, Edward Inwood SEALY (Associate Members).

RESIGNATIONS

Notification was given that the Council had accepted with regret the resignations of Sidney Norman BARRON, Professor Cyril BATHO, Emile A. BRIZAY, Thomas HORROCKS, Thomas Stanley HOSKING, Sydney ROBERTS, Robert Henry ROWCROFT, James Herbert WOOD (Members); George Bruce JOHNSTON, Edmund Norman Charles MARSHALL (Retired Members); Charles Donald ALLDERIDGE, Frank Freeman BAYNE, Richard Goulden BRICKELL, Donald Frank COOCH, Augustus William de Rohan GALBRAITH, Alfred George GROSS, Philip KERMEEN, Frederick Henry MARTIN, Herbert Jones NICHOLS, Thomas Emslie YOUNG (Associate Member); Arthur Edgar TYDEMAN (Associate); Steven Sandford COOKE-YARBOROUGH, Gordon Hugh DABELL, Bernard James DEAN, Kaikhusru Dinshah KAPADIA, Kashinath Vinayak NATU, Vane SUTTON-VANE, John Herbert WAINWRIGHT, Harry Charles YOUNG (Graduates); Adam CIUPKA, William Frederick CLARK, John Alfred GOODYEAR, Ian Isles GUTHRIE, John Robert HARVEY, John Truman HASLAM, Robert Weston HUGHES, Clifford Sydney JAQUES, Stanley Joseph NEALE, Raymond Cecil PARKES, Kenneth WALLER (Students).

FORTHCOMING MEETINGS

The following meetings will be held at 11, Upper Belgrave Street, London, S.W.1:—

Thursday, February 8th, 1951

Ordinary Meeting at 6 p.m., when Mr. F. G. Etches, B.Sc.(Eng.), A.M.I.C.E. (Member), will present a paper on "The Construction of an Extension to a Factory at Silvertown."

Wednesday, February 14th, 1951

Joint Meeting with the Reinforced Concrete Association, 6 p.m., when Professor A. L. L. Baker, B.Sc., M.I.C.E. (Member), will present a paper on "The Ultimate Strength of Prestressed Concrete." Advance copies of this paper may be obtained on application to the Secretary.

Thursday, February 22nd, 1951

Ordinary General Meeting for the election of members, 5.55 p.m., followed by Ordinary Meeting at 6 p.m., when Dr. G. G. Meyerhof, M.Sc., A.M.I.C.E. (Associate Member), will present a paper on "Building on Fill with Special Reference to the Settlement of a Large Factory."

Thursday, March 1st, 1951

The MacLachlan Lecture. Mr. W. Shearer Smith, A.M.I.C.E. (Associate-Member), on "Cold Formed Sections in Structural Practice with a Proposed Design Specification." 6 p.m.

Thursday, March 8th, 1951

Ordinary Meeting at 6 p.m., when Mr. C. L. a'Court, B.Sc., M.I.C.E., will present a paper on "Special Treatments for Portland Cement Concrete."

Thursday, March 29th, 1951

Ordinary General Meeting for the election of members, 5.55 p.m., followed by Ordinary Meeting at 6 p.m., when Mr. E. Roland Hole (Member), and Mr. F. C. Eales (Associate Member), will present a paper on "The Construction of the Royal Festival Hall."

Thursday, April 12th, 1951

Ordinary Meeting at 6 p.m., when Lt.-Col. G. W. Kirkland, M.B.E. (Member), and Mr. A. Goldstein, B.Sc.(Eng.), A.C.G.I., D.I.C. (Graduate), will present a paper on "The Design and Construction of a Large Span Prestressed Concrete Shell Roof."

Thursday, April 26th, 1951

Ordinary General Meeting for the election of members, 5.55 p.m., followed by an Ordinary Meeting at 6 p.m., when Dr. K. Hajnal-Konyi (Member) will present a paper on "Comparative Tests on Various Types of Bars as Reinforcement of Concrete Beams."

Members wishing to bring guests to the Ordinary Meeting announced above and to the MacLachlan Lecture are requested to apply to the Secretary for tickets of admission.

LONDON GRADUATES' AND STUDENTS' SECTION

The next meeting of the Section will be held at 11, Upper Belgrave Street, London, S.W.1, on Tuesday, February 13th, 1951, at 6 p.m., when a lecture on "Tunnelling" will be given by a representative of Messrs. Mott, Hay and Anderson.

Mr. R. E. Foot has been co-opted a member of the Committee in place of Mr. R. W. C. Steel, who has resigned.

Hon. Secretary : W. S. Twelvetrees, 6, Dell Road, Enfield Wash, Middlesex.

THE MACLACHLAN LECTURE

General Conditions

Through the generosity of Mr. John MacLachlan (Retired Member), the Council was able in 1948 to institute an Annual Lecture to be competed for by Associate-Members. The conditions of the presentation are as follows :—

1. The Institution of Structural Engineers shall institute a written lecture to be known as the MacLachlan Lecture, and to be held annually.

2. The subject of the Lecture may be on any aspect of Structural Engineering so long as in every second year the subject shall be confined to steel structures.

3. Entrance into the competition for the Lecture shall be confined to Associate-Members of the Institution.

4. All papers entered for the competition shall be submitted to assessors to be appointed by the Council of the Institution and all such papers (including the prize-winning Lecture) shall be available for publication in the Journal of the Institution at the discretion of the Council.

5. The winner of the competition shall be required to present the Lecture to a meeting of the Institution at which he will be presented with the sum of £17 10s.

6. In the event of there being no winner of the competition in any one or more years, whether because no lecture is submitted or because no lecture submitted is considered to be of sufficient merit to warrant an award, or for any other reason, the Institution shall transfer this sum to the Research Fund of the Institution.

Particulars of the Competition for 1951

1. The MacLachlan Lecture will be given at a meeting of the Institution to be arranged towards the end of 1951.

2. The subject of the Lecture shall be confined to Steel Structures.

3. The work should be submitted as the script of a lecture which the author, if successful in the competition, will deliver before an audience in the course of about one hour. The development of mathematical formulæ and detailed calculations should be avoided as far as possible in the text ; if they are essential they should be embodied in appendices. Photographs, drawings, graphs, etc., which would appear as illustrations to the Lecture in published form, should accompany the script. If additional illustrations would be shown as slides, a list of these should be included.

4. Six copies of each Lecture should be submitted and should be addressed to the Secretary of the Institution.

5. The closing date for the receipt of entries by the Institution is Friday, March 30th, 1951.

THE MACLACHLAN LECTURE, 1950

The MacLachlan Lecture for 1950, which was unavoidably postponed, will now be given by Mr. W. Shearer Smith, A.M.I.C.E. (Associate-Member), on Thursday, March 1st, 1951, at 6 p.m. The meeting will be held at 11, Upper Belgrave Street, London, S.W.1, and the subject of the Lecture will be "Cold Formed Sections in Structural Practice with a Proposed Design Specification."

ANNUAL DINNER, 1951

The Annual Dinner of the Institution will be held at the Dorchester Hotel on Wednesday, March 28th, 1951. Vice-Admiral The Earl Mountbatten of Burma has graciously consented to attend as Principal Guest.

KOKINE REST HOUSE

As a result of the sale of Kokine Rest House, the Board of Trustees has been enabled to present to the Institution of Structural Engineers' Benevolent Fund the sum of £2,500. This action was taken in accordance with the terms of the most generous gift of Mr. E. J. Buckton, to whom the Committee of Management of the Benevolent Fund is deeply indebted.

REPRESENTATION OF THE INSTITUTION

The Council have nominated Mr. John Perrins (Associate-Member) to serve on the City of Liverpool Building Professional Advisory Committee.

AWARD OF LONDON PRIZE

The Council have awarded the London Prize for the best paper read at Headquarters during the Session 1949-1950 to Professor J. F. Baker, O.B.E., Sc.D., M.I.C.E. (Member), for his paper entitled "The Design of Steel Frames," which was read before the Institution on October 27th, 1949.

SESSIONAL PROGRAMME, 1951-1952

The Literature Committee are now considering and selecting papers for inclusion in the Sessional Programme for 1951-1952. Members who may wish to offer papers during the coming Session are invited to communicate with the Secretary.

EXAMINATIONS, JULY, 1951

The Examinations of the Institution will be held at centres in the United Kingdom and Overseas on July 17th and 18th (Graduateship), and July 19th and 20th (Associate-Membership).

REPRINTS FROM THE JOURNAL

Authors who wish to obtain reprints of their papers published in the Journal, are requested to communicate with the publishers at least one calendar month before the date of publication of the paper in question.

JOURNAL BINDING, 1950

A binding case can be supplied for the twelve issues of the Journal, January-December, 1950 (Volume 28), price 10s., post free.

The price for binding volumes is 24s. per volume, inclusive. This price is for the half-leather binding which has been in use for some years.

It is requested that all parcels and Journals forwarded for binding should bear the name, address and rank of the member concerned. All volumes for binding must be despatched to the Institution by April 1st, 1951.

An Index will be included in all volumes bound. This index will not be generally distributed, but members and others wishing to have a copy should apply to the Secretary.

BRANCH NOTICES

LANCASHIRE AND CHESHIRE BRANCH

The following meetings have been arranged :—

Thursday, February 15th, 1951

Fourth Ordinary Meeting. Joint Meeting with the Institution of Civil Engineers, North-Western Association. Mr. J. Cunningham, B.Sc., A.M.I.C.E., on " The Britannia Tubular Bridge over the Menai Straits," at the Engineers' Club, Albert Square, Manchester, 6.30 p.m.

Friday, March 16th, 1951

Fifth Ordinary Meeting. Mr. L. W. Elliott, A.R.I.B.A., A.M.I.C.E., on " Prestressed Concrete in Architecture."

Thursday, April 19th, 1951

Annual Business Meeting.

All meetings will be held in the Reynolds Hall, College of Technology, Manchester, at 7 p.m., unless otherwise stated.

Hon. Secretary : G. Greenlees, 16, Mayfield Avenue, Stretford, Lancs.

GRADUATES' AND STUDENTS' SECTION

The following meeting has been arranged :—

Friday, March 30th, 1951

Annual General Meeting. Details to be announced later. At the Reynolds Hall, College of Technology, Manchester, 7 p.m.

Hon. Secretary : F. H. Turner, 11, Colwell Avenue, Stretford, Lancs.

MIDLAND COUNTIES' BRANCH

The following meetings have been arranged :—

Friday, February 23rd, 1951

Mr. A. J. Francis, M.Sc.(Eng.), Ph.D., A.M.I.C.E. (Associate Member), on " Distribution Methods of Structural Analysis."

Friday, March 30th, 1951

Mr. W. P. Andrews and Mr. P. G. Bowie, A.M.I.C.E. (Member), on " The Solution of the Car Parking Problem here and in the U.S.A."

Friday, April 27th, 1951

Annual General Meeting followed by a paper or a film.

The meetings will be held at the James Watt Memorial Institute, Birmingham, 6 p.m.

Hon. Secretary : E. R. Deeley, Arranmoor, Adshead Road, Dudley, Worcester.

GRADUATES' AND STUDENTS' SECTION

The following meetings have been arranged :—

Saturday, February 24th, 1951

Visit to the Round Oak Steel Works, at 10 a.m.

Tuesday, March 20th, 1951

Annual General Meeting. Films : " The Failure of the Tacoma Narrows Bridge," and " The Collapse of the Remagen Bridge," at the James Watt Memorial Institute, Birmingham, 7 p.m.

Hon. Secretary : M. H. Evans, B.Sc., 107, Metchley Lane, Harborne, Birmingham, 17.

NORTHERN COUNTIES' BRANCH

The following meetings have been arranged :—

Tuesday, February 6th, 1951

Mr. N. J. Ruffle (Graduate), on " Concrete Control for Small Works," at Middlesbrough.

Wednesday, February 7th, 1951

Mr. N. J. Ruffle will present the same paper at Newcastle.

Tuesday, March 6th, 1951

Mr. P. L. Capper, T.D., M.Sc., A.M.I.C.E. (Associate-Member), on " Testing Methods in Structural Engineering," at Middlesbrough.

Wednesday, March 7th, 1951

Mr. P. L. Capper will present the same paper at Newcastle.

Tuesday, April 3rd, 1951

Annual General Meeting.

All meetings commence at 6.30 p.m., preceded by tea at 6 p.m., the Middlesbrough meetings being held at the Cleveland Scientific and Technical Institution, Corporation Road, and those at Newcastle in the Neville Hall, near the Central Station.

Hon. Secretary : J. A. Williams, A.M.I.C.E., 57, Reid Park Road, Jesmond, Newcastle-upon-Tyne, 2.

NORTHERN IRELAND BRANCH

The following meetings have been arranged :—

Tuesday, February 6th, 1951

Mr. W. Shearer Smith, A.M.I.C.E. (Associate-Member) on " Cold Formed Sections in Structural Practice with a Proposed Design Specification."

Tuesday, March 6th, 1951

Film : " Military Pile Driving."

Tuesday, April 3rd, 1951

Annual General Meeting.

The meetings will be held in the College of Technology, Belfast, 7.15 p.m.

Hon. Secretary : S. G. Duckworth, " Lisleen," 13, Finaghy Road North, Belfast.

SCOTTISH BRANCH

The following meetings have been arranged :—

Tuesday, February 20th, 1951

Mr. H. B. Sutherland, S.M., A.M.I.C.E. (Associate-Member), will present a paper at Ca'doro Restaurant, Glasgow, 6 p.m. (Title to be announced later.)

Tuesday, April 17th, 1951

Annual General Meeting at Ca'doro Restaurant, Glasgow, 6 p.m.

Hon. Secretary : D. G. Drummond, B.Sc., A.M.I.C.E., 11, Woodside Terrace, Glasgow, C.3.

SOUTH-WESTERN COUNTIES' BRANCH

Hon. Secretary : E. W. Howells, 10-12, Market Street, Torquay.

WALES AND MONMOUTHSHIRE BRANCH

The following meetings have been arranged :—

Tuesday, February 6th, 1951

Mr. W. S. Atkins, B.Sc., M.I.C.E., on " Modern Trends in Structural Design." at Cardiff.

Tuesday, February 13th, 1951

Mr. Norman Thomas, M.B.E., A.R.I.B.A., on " Planning Factories for Light Industry," at Swansea.

Friday, February 24th, 1951

Mr. A. J. Harris, B.Sc., A.M.I.C.E., on " Prestressed Concrete in Civil Engineering Works," at Colwyn Bay.

Tuesday, March 6th, 1951

Mr. G. R. Brueton, A.M.I.C.E. (Associate Member), on "The Fatigue of Materials related to Structural Design," at Swansea.

Thursday, March 22nd, 1951

Mr. G. R. Brueton will present the same paper at Cardiff.

Friday, March 30th, 1951

Mr. A. V. Hooker, A.M.I.C.E. (Associate-Member), on "Structural Engineering at Abbey Works," at Colwyn Bay.

Wednesday, April 11th, 1951

Visit to Abbey Works site, Margam, to inspect the buildings for the New Strip Mills for the Steel Company of Wales.

Annual Dinner, at Porthcawl.

The meetings in Cardiff will be held at the South Wales Institute of Engineers, Park Place, at 6.30 p.m.

The meetings in Swansea will be held at the Mackworth Hotel, at 6.30 p.m.

The meetings at Colwyn Bay will be held at the County Buildings, at 6 p.m.

Hon. Secretary : E. R. Steward, Edrom, Ashleigh Road, Blackpill, Sawnsea.

WESTERN COUNTIES' BRANCH

The Second Meeting of the Session was held at the Grand Hotel, Bristol, on Friday, November 3rd. Mr. P. C. Girdlestone, M.A., M.I.Struct.E., M.I.Mech.E., A.M.I.C.E., was in the Chair. Dr. A. J. Francis, M.Sc., A.M.I.C.E. (Associate-Member), gave a lecture entitled "The Behaviour of Riveted Joints under Load," explaining extensive research work in the stress distribution in riveted joints, particularly in light alloys. A keen discussion followed the lecture, and a vote of thanks was proposed by Mr. N. G. T. Ball (Associate-Member).

The Third Meeting was held at the Grand Hotel, Bristol, on Friday, December 1st, when Mr. F. S. Snow, M.I.C.E., M.I.Mech.E. (Past-President), presented a paper entitled "Some Recent Developments and Further Applications of Shell Concrete Construction." An interesting discussion followed, and a vote of thanks was proposed by Mr. H. C. J. Cornelius (Associate), of Exeter (South-Western Counties' Branch).

The following meetings have been arranged :—

Friday, February 2nd, 1951

Joint Meeting with the Institution of Civil Engineers, Professor Andrew Robertson, D.Sc., F.R.S., M.I.C.E., on "A Chapter of Engineering a Century Ago," at College of Technology, Bristol.

Friday, March 2nd, 1951

Mr. F. W. L. Heathcote and Mr. A. H. Chilver, B.Sc. (Student), on "Light Gauge Structural Steel Sections," at College of Technology, Bristol.

Friday, April 6th, 1951

Annual General Meeting, followed by a talk by an Architect, at Grand Hotel, Bristol.

Thursday, April 26th, 1951

Joint Meeting with the Institution of Civil Engineers. Mr. Gilbert Roberts, B.Sc., A.M.I.C.E., on "The Dome of Discovery—Festival of Britain Site," at Reception Room, Bristol University, Queen's Road, at 5.30 p.m., preceded by tea at 5 p.m.

All meetings, except the last, will be held at 6.15 p.m., preceded by tea at 5.30 p.m.

Hon. Secretary : C. E. Saunders, Dunkery, Edward Road, Walton St. Mary, Clevedon, Somerset.

YORKSHIRE BRANCH

The Third Ordinary Meeting was held on Wednesday, December 13th, 1950, at 6.30 p.m., at the Great Northern Hotel, Leeds, when Mr. H. C. Husband, B.Eng., M.I.C.E., M.I.Mech.E. (Member), was in the Chair, and thirty members and visitors were present. A lecture, which was illustrated by lantern slides and models, entitled, "Concrete Surface Finishes," was given by Mr. S. C. Boakes (Associate-Member), of the Cement and Concrete Association. A discussion followed, and Mr. Boakes answered questions. A vote of thanks, proposed by Mr. D. R. S. Wilson (Member), and seconded by Mr. J. Dossor, A.M.I.C.E., A.M.I.Mech.E. (Member), was warmly accorded.

The following meetings have been arranged :—

Wednesday, February 21st, 1951

Junior Branch Prize Paper.

Friday, February 23rd, 1951

Annual Dinner and Dance, at Castle Grove Masonic Hall, Headingley, Leeds, at 7 p.m.

Wednesday, March 21st, 1951

Joint Meeting with the Yorkshire Association of the Institution of Civil Engineers. Mr. J. Guthrie Brown, M.I.C.E. (Member), on "Hydro-Electric Developments, Pitlochry," at Leeds University, at 7 p.m.

Wednesday, April 18th, 1951

Professor R. H. Evans, D.Sc., Ph.D., M.I.C.E., M.I.Mech.E. (Member), on The Institution of Structural Engineers' "Report on Prestressed Concrete."

All meetings, with the exception of the Joint Meeting, on March 21st, will be held at the Great Northern Hotel, Leeds, at 6.30 p.m.

Hon. Secretary : E. Wrigley, City Engineer's Dept., Civic Hall, Leeds.

UNION OF SOUTH AFRICA BRANCH

A meeting of the Branch was held at Barclays Bank Building, Johannesburg, on Monday, November 13th, 1950, at 8 p.m., when Mr. F. B. Thompson (Associate-Member), presented a paper on "Some problems encountered in the Design of the Steelwork and Steelwork Foundations of the Steel Melting Plant, new Van der byl Steelworks," and Mr. J. C. Panton (Associate-Member) gave a paper on "Problems encountered during the Erection of the Steelwork."

A visit to the Steelworks was arranged for Saturday, November 18th, when members had an opportunity of viewing the work described in the above papers.

On December 2nd, members of the Branch visited the works of the Rand Water Board at Vereeniging.

Branch Hon. Secretary : A. E. Tait, B.Sc., A.M.I.C.E., P.O. Box No. 3306, Johannesburg. During week-days Mr. Tait can be contacted in the City Engineer's Department, City Hall, Johannesburg, 'Phone 34-1111 ext. 257.

Natal Hon. Secretary : E. G. Bennett, c/o Reinforcing Steel Co., Ltd., P.O. Box 478, Durban.

Cape Section Hon. Secretary : R. Stubbs, P.O. Box 1692, Cape Town.

Special Treatments for Portland Cement Concrete*

By C. L. a'Court B.Sc., M.I.C.E.

Synopsis

Portland Cement has proved itself to be so useful that there is a great desire to treat it as a general purpose material. Naturally, however, its physical and chemical characteristics impose certain limitations. These limitations can be modified by manufacturing adjustments of the cement itself or by suitable treatments of the concrete.

In this Paper are described methods of concrete treatment to modify setting and hardening times, increase workability and improve durability. Improvement of workability is an indirect means of improving strength, density and durability, since its object is, in effect, to render possible maximum values in these for given conditions of placing. Reference is made to wetting agents, air-entraining agents, waterproofing agents, pozzolanas and other additions such as inert pore-fillers. Surface treatments of set concrete are also dealt with.

In conclusion, an attempt is made to estimate the way in which such techniques may be expected to develop or expand.

Introduction

Binders having some of the characteristics of Portland Cement have been known at least since classical times. The material as we know it is, however, little more than 100 years old. Furthermore, Fig. 1 shows the very large growth of its use that has taken place during the last 20 or 30 years. This growth has been out of all proportion to any increase in the amount of constructional activity during this period. It must be, therefore, that Portland Cement, or, more particularly, Portland Cement concrete, has taken the place of other constructional materials. There are a number of obvious reasons for this, such as :

(a) The steady improvement in the quality of the material and the overcoming of the natural prejudice against new materials.

(b) The progress in design and constructional methods, notably for reinforced concrete.

(c) Cost advantage.

(d) Ease of use.

It is probably the last of these factors which has been the most significant in the almost phenomenal rate of growth of the use of the material, and which leads most directly to the subject of the present discussion.

Portland Cement concrete is, relatively speaking, such an easy material to use and so readily adaptable to great varieties of structures and site conditions, including prefabrication, that there arises almost unnoticed a tremendous desire to press its use to the limit and beyond. If, therefore, we suggest means of modifying its natural qualities, this must be taken as deriving from the great virtues rather than from the vices of the material.

Paper to be read before the Institution of Structural Engineers at 11, Upper Belgrave Street, London, S.W.1, on Thursday, March, 8th 1951, at 6 p.m.

The growth in use has been somewhat indiscriminate in the past, but there is, now, also a great deal of scientific knowledge as to suitable and unsuitable uses, these being determined by the physical and chemical characteristics of the material. It is upon this knowledge that we must base our attempts to achieve modification. It is as well at this point to emphasise the fact that additive treatments to concrete are additions to and not in replacement of the production of good concrete. Thus, in approaching any problem, the first requirement is to ensure that the normal materials are being used to the best of their advantage. Having done this, the use of further media may be considered. This must be clearly understood as underlying what follows. It is, for example, quite possible to produce a watertight concrete without the use of any water repellent media ; it is also very easy to produce a permeable concrete with such media incorporated.

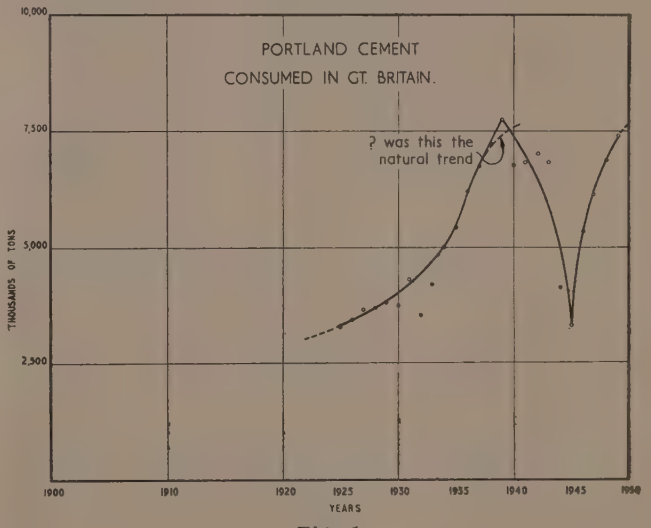


Fig. 1

The basis on which this Paper is written is, therefore, that we have come to regard Portland Cement as virtually a *sine qua non* of constructional activity ; that we recognise that, naturally, it has its limitations ; that we seek to overcome these limitations by any means that are reasonable and which give the required results. With this in mind, it is clearly desirable to understand firstly the nature of the limitations, and secondly the nature and effects of the materials with which we seek to overcome these limitations : presupposing, of course, that we have a fair idea of our objectives. The methods adopted in the past have been largely empirical in character. That is to say, it has been found that the use of certain materials results in certain effects. It has, then, often been left to independent investigators to determine the causes. Such procedure is natural to engineering practice and will continue but it is likely to be increasingly reinforced by a more inductive method, whereby the nature of the problem is first explored and a consequent solution hypothesised. In this connection it is well to remember, however,

that Portland Cement, although easy to use, is not simple in chemical constitution or reaction.

Physical and Chemical Background

As has just been indicated, the approach to this problem must be by way of modifying the physical and chemical characteristics of the concrete, with particular reference to the part played in these by the cement.

There are two physical characteristics which are of particular concern :

(a) the overall grading of the concrete mix ;
(b) the pore structure of the resultant concrete.

The importance of proper grading of aggregates is now well understood. In a previous publication by the author[1] attention was drawn to the recurrent factor of poor aggregate grading throughout a whole series of concrete failures, although these appeared to take various forms. This defect could be illustrated in a diagram of the form shown in Fig. 2, the flat portion of the curve indicating a gap in the grading. It is probable that this principle can be carried to still further limits. The finest material in the aggregates is generally limited by such a requirement as that there shall be not more than about 4 per cent. passing a 100-mesh sieve. Normal Portland Cement is ground to a fineness showing a maximum of a few per cent. retained on a 170-mesh sieve. There is, therefore, a virtual gap between the 100-mesh and the 170-mesh, with a possible decreasing extension on either side. The filling of this gap may offer advantages.

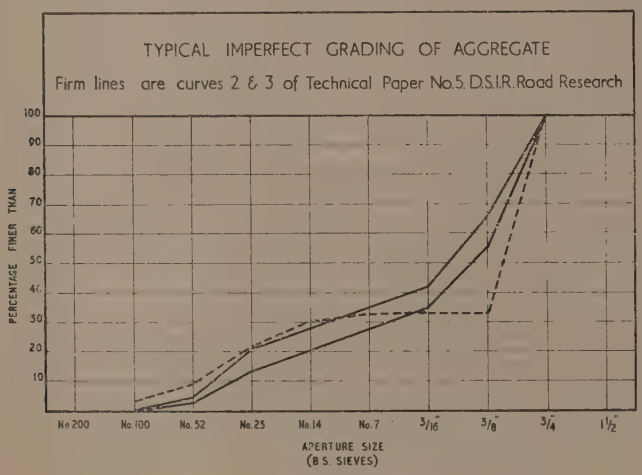


Fig. 2

Even well consolidated concrete contains pores distributed through it : these arise partly from the air which is entrapped during mixing and placing and has not been able to escape, and partly by virtue of the volume left by the drying out of the uncombined mixing water.

The major components of Portland Cement are :—

1. Tricalcium silicate (C_3S)
2. Dicalcium silicate (C_2S)
3. Tricalcium aluminate (C_3A)
4. Tetracalcium aluminoferrite (C_4AF)
5. Gypsum

The tricalcium silicate reacts rapidly with water to form calcium hydroxide and a hydrated calcium silicate, probably 3 $CaO.2SiO_2$ Aq.

The dicalcium silicate also liberates some lime and forms a hydrated calcium silicate, probably 3 $CaO.2SiO_2$ Aq. but this reaction is relatively slow.

The tricalcium aluminate and gypsum react to give calcium sulphoaluminate 3 $CaO.Al_2O_3.3\,CaSO_4 31\,H_2O$; the remainder of the tricalcium aluminate forms a hydrate 3 $CaO.Al_2O_3$ Aq.

Tetracalcium aluminoferrite forms hydrated calcium aluminate 3 $CaO.Al_2O_3$ Aq. and hydrated monocalcium ferrite $CaO.Fe_2O_3$ Aq. (or possibly 4 $CaO\ Al_2O_3$ Aq. and Fe_2O_3 Aq.).

It is the calcium hydroxide and the hydrated tricalcium aluminate which are the most vulnerable constituents of set Portland Cement. Indeed, when anything but the calcium hydroxide is attacked, it may be said that it is time to look for some other material. Although the tricalcium aluminate is partially affected by some additives, as for example calcium chloride, it is to the neutralisation of the free lime content that any chemical modification of the set cement is primarily directed.

The site engineer would naturally desire that the modification should be carried out in the cement itself. Within limits this can be done. An obvious limit is the scale of manufacture. A requirement which results in consistent large-scale demand can obviously be met, whereas requirements which are variable and intermittent can best be dealt with by site treatment. Thus, in this country we have had for a long time, normal and rapid hardening Portland Cement, coloured and water repellent cements and, more recently, low heat and sulphate resistant types. Some of these variations are brought about by a fundamental change in the cement, but others are due to simple additions. In dealing with this particular aspect of the problem it is worth noticing that, apart from manufacturing considerations, there are fields in which the preponderance of the argument is in favour of works addition and some in which site addition is preferable. For example, pigments can be intermixed intimately in the cement bulk more easily in the manufacturers' works than on a normal site : consequently, consistency of tone is more likely to be achieved with a ready mixed cement. Other kinds of additions may, however, need to be adjusted to specific site conditions. This feature has been emphasised in the United States in relation to air-entrainment.

Some over-all modifications of the concrete can be achieved by variation in the type of aggregate used ; thus, if fire resistance is a predominating consideration, a limestone aggregate could be preferable to a gravel aggregate ; if lightness in weight is necessary, foamed slag or other appropriate aggregates may be used.

The objectives of site treatments which take the form of additions to the normal concrete can be summarised under the following general headings :—

(a) Modifications of setting time.
(b) Modifications of hardening time.
(c) Increased ease of placing.
(d) Improvement in durability.

They may be achieved by integral treatments, by addition to the concrete during mixing ; and, in the case of item (d), by superficial treatment of the finished concrete.

No mention is made in this list of strength or cost, the reason being that these are either the ultimate objectives or incidental to the items listed. For example, when we speak of greater workability, we mean, in fact, the maximum workability which can be achieved for the lowest tolerable strength and at an economic cost. Water is the cheapest workability agent, and the one most readily available as we all know only too well, but by such use, strength, density and durability all suffer. Cement may be used indirectly for the same purpose, but at the expense of cost and enhanced movement characteristics.

Modifications of Setting Time

The setting time may, broadly speaking, be defined as the time during which the concrete remains plastic; it can be shortened by the use of a number of materials such as the caustic alkalies and, to a lesser degree, by other mineral substances less alkaline in character. The setting time can be lengthened by various substances, notably those of an organic nature such as glue and sugar. There are a number of proprietary articles on the market for such modifications and this is a case where it is probably wise for the general practitioner to use these, as all such treatments tend to weaken the resultant concrete to a significant degree. Furthermore, results may be very variable. It is to be expected that a reputable manufacturer will have studied the problem in some detail and be in a position to give appropriate warnings as well as his positive advice.

Modifications of Rate of Hardening

Modifications in hardening are generally required in the direction of increased rate of hardening and, consequently, of early strength, notably at one and three days. To meet this requirement, the best known material for site usage is calcium chloride. The general recommendation is 2 to $2\frac{1}{2}$ per cent. of commercial grade calcium chloride, based on the weight of cement in the mix for ordinary Portland Cement, and a little less for rapid hardening. In such use, it must be borne in mind that an increase in the rate of hardening means that there is a speeding up in the chemical reactions which are taking place: as these are exothermic in character, attention must be given to the effects of

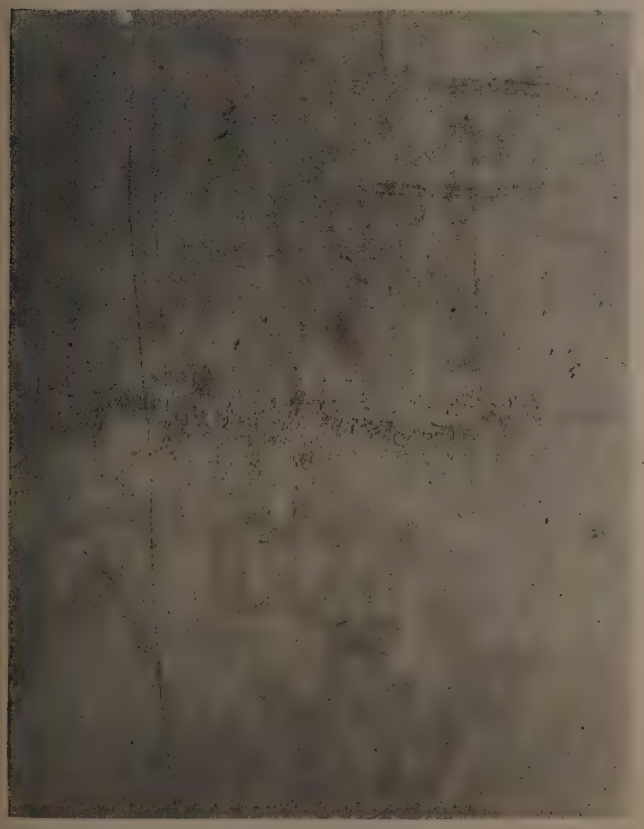


Fig. 3

the increased temperature rise of the concrete. In winter this is generally an advantage, but in summer, or at any time in relatively massive constructions, care must be taken to mitigate the effects of this rise. At the recommended addition, the evidence is that there is no significant effect on reinforcement and no noticeable alteration in the eventual long-period strengths of the concrete, but if the amount added is substantially greater, then such effects may have to be taken into account.

Much information on this subject has already been published, so that it is hardly necessary to deal with it in detail here. Tables 1, 2 and 3 are included as giving a summary of the position. Table 2 is of interest as indicating that the effect of calcium chloride can be affected by the constitution of the cement—the proportions of the chemical constituents mentioned earlier are not necessarily the same in all brands of cement. With the normal and rapid-hardening cements on the market in this country, this will not be of any serious concern for the practising engineer, but it would be well to make enquiries if special cements were being used.

TABLE 1

Age	Effect of Addition of 2 per cent. Commercial Calcium Chloride (70/72 per cent. $CaCl_2$) = 1.4 per cent. $CaCl_2$
1 day	120 per cent. increase in strength
3 days	50 ,, ,, ,, ,, ,,
7 ,,	25 ,, ,, ,, ,, ,,
1 month	14 ,, ,, ,, ,, ,,
1 year	6 — 10 ,, ,, ,, ,, ,,

TABLE 2

	Type of Cement		
	White Portland	Normal Portland	Rapid Hardening
C_3S content	45 per cent.	50 per cent.	66 per cent.
Normal mix (by volume)	1 : 2 : 4	1 : 2 : 4	1 : 2 : 4
Water : cement ratio	0.58	0.58	0.58
3-day Strength (lb/sq. in.)			
Without Calcium Chloride	1160	1210	2820
With 2 per cent. Calcium Chloride	1160	1960	3650

TABLE 3

Time required for 1 : 2 : 4 concrete to reach a strength of 2,000 lb. per square inch

Temperature °F.	Without Calcium Chloride days	With Calcium Chloride days
32	13	5
40	$6\frac{1}{2}$	$2\frac{1}{2}$
70	3	$1\frac{1}{2}$

The main purpose in using calcium chloride is to increase the rate of hardening. Owing to the nature of the reaction, the natural corollary to this is the increase in temperature rise, a feature which is of value in cold weather; but if proper advantage is to be taken of this, the concrete must be adequately insulated to prevent dissipation of the heat. Calcium chloride seems also to have some effect in improving the workability of the mix. A further effect which can be demonstrated is the improvement which can be obtained in the abrasion resistance of floors laid with the normally recommended inclusion. This is shown quite significantly in the accompanying photographs. Adjacent bays were laid in normal sequence, under the same supervision and, as far as humanly possible, with the same characteristics, apart from the inclusion in one of the calcium chloride. Figs. 3 and 4 show the results as laid and after two years of average industrial wear respectively.

There are two suggested reasons for this : one is that the slight increase in early stiffening time reduces bleeding and the amount of fine material brought to the surface under the trowel ; the other is that calcium chloride holds more water in the surface and so improves curing.

Wetting Agents

The strength of concrete is related to the water/cement ratio. Its ease of placing is also related to the water/cement ratio, but in the contrary sense. The natural desires of the designer and the man on the job are, therefore, apt to be at variance on this matter. The first means of bringing them together is by proper choice of aggregates. Increased cement content has the effect of reducing the water/cement ratio for the same workability, but at the expense of economy and, probably, with increased shrinkage characteristics in the resultant concrete, as has been mentioned earlier. Over-

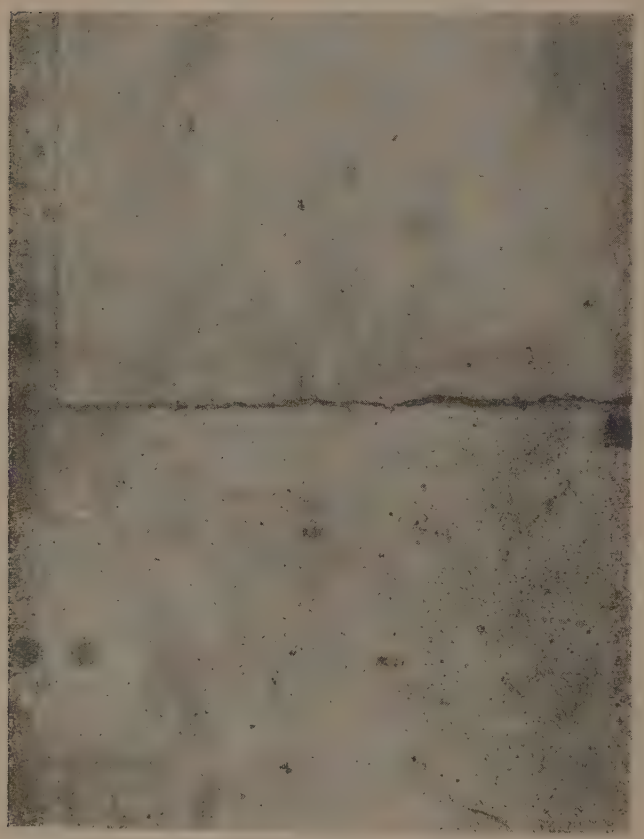


Fig. 4

sanded mixes inevitably result in high water/cement ratios.

To meet this requirement, the idea has developed of adapting to concrete practice a procedure which is well known in other industries. This is to use surface-active or dispersing agents. The object of these is to modify the characteristics at the liquid/solid interface, thereby increasing the rate of adsorption of water to the cement and aggregate particles, and so also improving the dispersion of the cement through the mix.

Four inter-related factors in concrete are : strength ; workability ; water/cement ratio ; cement content. Assuming proper compaction, a low water/cement ratio increases strength but reduces workability ; reduced cement content reduces strength for the same workability or reduces workability for the same strength, principally because of its effect on the water/cement ratio. A surface active agent may, therefore, be expected to affect all of these characteristics. It may increase strength for a given workability or increase workability for a given strength ; it may be used to reduce the cement content, maintaining the same strength and workability, and so reduce costs. Or, an attempt may be made to compromise between these various factors.

Our knowledge of the precise behaviour of these materials is, as yet, very incomplete. Some early lack of success in their use was due to the unsuitability of the agents used. It is, obviously, highly desirable that the material shall be inert in the presence of the other compounds formed in the cement/water reaction. Some agents are unstable in the presence of the soluble calcium salts formed in this reaction : they are thereby precipitated and so may be unable to perform their required function. Associated with this, too, is the desirability that they shall be unreactive with calcium chloride in view of the widespread use of this material in concrete. This particular problem may be fairly readily resolved in the laboratory, but the effective physical results of suitable additions are not so easily demonstrated. It is, indeed, a feature of much laboratory work on concrete additions that site observations are not readily reproduced. This is due very largely to the fact that it is easy, indeed easier than not, to produce really good concrete under laboratory conditions : the result is that, although a substantial effect may be produced in a moderate concrete, that is to say in average field concrete, possible differences may become scarcely observable in relation to perfect specimens. In the present instance, this question is further complicated by the difficulty of measuring in precise terms the factor of workability. This problem has itself been receiving attention in very recent times and, in attempting to measure effects of workability, it has become very apparent that it is necessary first to evaluate the methods of measurement. At the time of writing, the main evidence of the utility of surface active agents is that which has been accumulated on sites. There is a growing body of responsible evidence of this nature, noticeably in relation to concrete which is being vibrated or pumped. There is also evidence that precast concrete products show less breakage when treated in this way. Reference may also be made to the effectiveness of these additions in certain grouting operations.

Air-Entraining Agents

The consideration of air-entraining agents follows naturally upon that of surface active agents, since some materials show both properties, according to the quantities in which they are used. It may, indeed, be that, when used in smaller quantities for their wetting properties, there is already a small measure of air-entrainment which aids flow under movement.

The main study of this problem has been in the United States : in that country there is now a great deal of published literature on the subject. This is probably due to the fact that one of the main objects of air-entrainment is to resist the action of frost ; in this country, of course, we do not experience such extremes of temperature as are common in America.

In summary, the evidence to date seems to be as follows :

1. Air can be deliberately and permanently incorporated in a concrete mix by the addition of special agents.

2. The amount of air entrained is determined by a number of factors, in addition to the properties and quantity of the agent added. Such factors, for example,

are the proportions and gradings of the other ingredients. The sand is said to be quite critical in this respect.

3. The air is now regarded as an additional aggregate component, replacing an ascertainable proportion of fine aggregate. In some accounts the proportions are said to be equal.

4. The eventual air component of the concrete may be materially affected by the degree of compaction, particularly if this is achieved by means of vibration.

5. Concrete with entrained air is lower in strength than corresponding concrete without. The Road Research Laboratory in this country have obtained an indication that the compressive strength of the concrete is reduced by 5·5 per cent. for each 1 per cent. of entrained air.

6. The use of air-entraining agents enhances workability in the concrete in proportion to the amount of air entrained.

7. Air-entrainment tends to produce less bleeding and segregation in the concrete mix.

8. As the quantity of air entrained is specific to a number of site factors, the preponderance of argument now seems to be in favour of a site addition of the agent rather than its being premixed in the cement.

The virtues of air-entrainment are generally considered to be related to the dispersed nature of the air bubbles and the reduction in capillary paths. This has the effect, on the one hand, of providing room for expansion of water on freezing, and, on the other, a certain reduction in permeability, which may contribute to increased durability.

A brief mention should perhaps be made of another type of aerating medium. With this type there is reaction with the products of the cement/water reaction, yielding a gas, which is held within the concrete mass. The best known of such media is aluminium powder, which gives rise to the evolution of hydrogen when incorporated in a concrete mix.

An appreciable amount of experience is obviously necessary in the handling of all air-entraining or foaming agents, together with a measure of laboratory control on site.

Water-Proofing Agents

Agents for incorporation in cement mixes which have, or are claimed to have, water-proofing properties, are very varied in composition. Correspondingly, the mechanism by which they act is also varied. In summary, they are generally included in the following categories :—

1. Inert materials which act as pore fillers. These are generally in the category of materials which supply a possible missing fraction in the over-all grading constitution of the concrete mix. If this hypothesis is correct, it might be assumed that almost any inert material ground to a suitable range of particle size would perform the desired function, but it may be that there are other desirable characteristics for the shape and surface qualities of the particles. Various forms of clay and silica have been particularly advocated. Calcite seems recently to have been used in Scandinavia, although in this case there have been additional claims that it has proved reactive in the cement mix. Hydrated lime has also been used for this purpose.

2. Other forms of pore fillers which react in some way in a concrete mix. Thus iron fillings, generally in combination with ammonium chloride, may oxidise and fill the voids by virtue of the expansion which takes place during this process. Certain silicates fall into this category as well as reactive silica—such as diatomaceous earth.

3. Pore greasing agents, which have the effect of altering the surface characteristics of the pores so that water is not able so readily to travel along them by capillary action. There is a wide range of such materials, of very varied chemical composition. Some are inert while others react with the cement. They include fatty acids, soaps of various metals, petroleum and bituminous products and other organic materials.

There are a large number of proprietary articles on the market which may be solutions of simple salts or various combinations of materials such as those indicated above.

There are many difficulties in commenting upon this particular type of agent. In theory, there should be no need to use any of them. In practice, we know that it is not at all easy to achieve completely water-tight concrete. A water-prcofing or water-repellent agent, therefore, offers a margin of safety for awkward cases and should be regarded in this light.

" Pozzolanas "

The name " Pozzolana " is given to a range of materials which have the capacity of combining with calcium hydroxide in the presence of water, to give cementitious materials. They were well known in classical times, and their name derives from a place in the South of Italy. Natural pozzolanas are generally of volcanic origin and contain mixtures of silicates. They have been found and used in various other parts of the world besides Italy, notably in France, Germany and the United States. As there are no such natural deposits in this country, their use here has not been extensive. Consequently, our own experience with them is slight. Some diatomaceous earths are said to have pozzolanic properties, but such materials are generally improved by heat treatment. Pozzolanas may be produced by the roasting of suitable clays, while Fly ash is another artificial material having similar properties. In Europe, pozzolanas have tended to be used more in the Mediterranean regions than further north. Apart from the early history and experience, there may be two other reasons for this. They have been used for constructional work very largely in seawater on account of their greater resistance to sulphate attack : such attack is enhanced by rise in temperature. The second possible reason is that the pozzolana mixtures tend to be slower hardening than normal Portland Cement mixes : this would be disadvantageous in cold climates, but certainly less so in warmer regions.

The advantage of pozzolanic additions to Portland Cement mixes is that these materials combine with the calcium hydroxide formed in the cement/water reaction. The quantity to be added will depend upon the quantity of free lime and also upon the amount of correspondingly reactive material in the pozzolanic substance. The degree of reactivity is also a function of the fineness of grinding. As has been noted above, such additions lower the early strengths of Portland Cement concretes but, up to their optimum addition their tendency is to increase the long period strengths. Quantities in excess of that required for the complete reaction of the available free lime will, presumably, act as a filler and consequently tend to reduce strength if the other aggregate quantities are maintained constant ; such effect being enhanced by the fact that, being finely ground, this material will require a proportionately large water content for wetting purposes.

The elimination of the free lime in the set concrete removes the most vulnerable constituent : consequently, concretes with suitable pozzolanic addition are less susceptible to attack by certain types of aggressive

agents, notably sulphates. Used in suitable quantities, they tend also to produce a denser concrete : this, in turn, would promote resistance to attack. It is claimed also that concretes with pozzolanic addition are more workable and have less tendency to segregate and to bleed. In view of the nature of the reaction, it might be expected that the use of these materials on concrete structures and renderings would tend to reduce the amount of that form of so-called " efflorescence " which is due to the carbonation of free lime on the surface, but the writer is not aware of any published evidence on this subject.

Surface Treatment

Up to this point, we have been concerned with materials which are added to the normal constituents of concrete during the mixing process. It remains to give a note of materials which can be applied to set concrete. Obviously, this kind of treatment can affect only the characteristics of the set material such as permeability, durability and abrasion resistance. It is clearly necessary to give some definition of the term " surface treatment." A tar macadam carpet applied to a concrete road or a lead lining to a concrete tank could be described as such, but clearly they are somewhat remote from the general intent. Under the heading of " Concrete surface treatment," one has in mind applications which result in a thin film covering, or a filling up of the surface pores. Of this type also, it is not intended to do more than make a passing reference to paints adapted for application to concrete surfaces. These are materials of special formulation and may be considered to have crossed the line between concrete technology and paint technology.

The most commonly used of the remaining materials may be conveniently summarised as follows :—

1. Silica containing media.
2. Drying oils such as linseed oil.
3. Various forms of bituminous or asphaltic compositions.
4. Miscellaneous.

The silica-containing media which are used include sodium silicate, magnesium silico-fluoride and silicon esters. They depend for their effectiveness upon the deposition of silica gel within the pores of the concrete, and probably to some extent upon a reaction of this silica with the available lime to form calcium silicate.

The materials which come under items 2 and 3 depend upon the impenetrability of the film which they provide, when they have been spread over the concrete surface in a normal manner and have then oxidised or dried.

The miscellaneous items cover a wide range of materials, among which may be noted metallic soaps in various solvents and sundry other organic compounds, some reactive and some unreactive with the concrete. These are more likely to come within the province of practising engineers as constituents of proprietary materials than as specifically designated compounds.

Surface treatments may give protection against water penetration, chemical attack or simple wear. Insofar as any of them prevent penetration by liquids, they will obviously provide a corresponding measure of protection against the aggressive action of such liquids : this refers particularly to aggressive salts which may be carried in aqueous solution. The resistance to the penetration of any given chemical will clearly depend upon the resistance of the surface coating itself to attack by such chemical. No material can, therefore, be held to be of general utility in this matter, so that treatments must be specific to the conditions. Documents have been published by the Portland Cement Association of America and circulated by the Cement and Concrete Association in this country which give general guidance, but in cases where serious attack is to be expected specialist advice is obviously necessary.

For abrasion resistance, materials coming within the two first categories of those mentioned are generally used. The author has already published a Paper on an investigation into the effects of these materials[3]. In summary the conclusions reached were that :—

1. If really excellent concrete floors are laid, then no such treatments are necessary ;
2. If the floors are extremely bad, no such treatments do any good ;
3. In between these limits there is the great bulk of work which is carried out in practice ; in such work, definitely beneficial results are possible ;
4. Treatment will probably have to be renewed at intervals depending upon the circumstances of the case. Such intervals may be of the order of from one to two years, but there is evidence to suggest that, in reasonable conditions, one or two such subsequent treatments may be all that is necessary. Recent work by the author gives evidence that, to obtain the very best results, there is an optimum age for the concrete at the time of application. For sodium silicate, this seems to be from about 7—14 days with normal cement under average conditions. If abrasion resistance is the object in view, it is inadvisable to apply this material at a concrete age of less than 24—28 hours. At any time after this, there are advantages to be gained, with the optimum conditions just mentioned. This is typified in Fig. 5.

Conclusion

This Paper has set out to discuss the reasons for making additions to normal concrete mixes and to summarise the particular purposes for which this may be done, the types of materials which may be used for such purposes, and the present state of our knowledge on the subject. It may be well to conclude by attempting some estimate of the way in which such techniques may be expected to develop or expand.

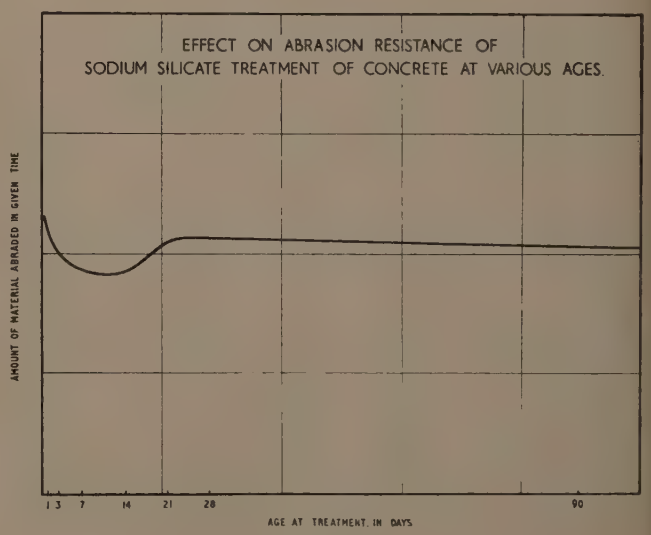

EFFECT ON ABRASION RESISTANCE OF
SODIUM SILICATE TREATMENT OF CONCRETE AT VARIOUS AGES.
AMOUNT OF MATERIAL ABRADED IN GIVEN TIME
1 3 7 14 21 28 90
AGE AT TREATMENT, IN DAYS

Fig. 5

Knowledge of concrete technology has advanced very considerably within the last twenty years, that is to say, within the working life of practising engineers who would still consider themselves relatively young. The main line of development to date has, very properly, been in the direction of using the normal constituent materials to their best advantage. While it is certain that much will yet be done in this direction, it would not be unduly rash to suggest that the curve of such rate of progress is now in the process of flattening out : the important thing in the future will be to disseminate knowledge so that average practice comes more into line with the best.

It is equally certain that more and more demands will be made upon Portland Cement concrete, not only in the matter of quantity, but also of quality. One needs only to instance the impetus towards high strength concrete which has been given by prestressing techniques. To achieve the next set of objectives, it seems likely that further serious scientific consideration will need to be given to additions to normal concrete as one of the means to be employed.

Materials which affect the setting time have a restricted field. Accelerating media are used mostly to speed up the set in order to deal with water penetration under hydrostatic head ; or for work between tides in tidal waters, where the size of the job does not justify the construction of coffer dams or the like. One cannot foresee any further great developments in this direction, or in connection with retarding agents—the most publicised use of which is to obtain certain surface effects by removal of the top skin after the striking of the formwork.

The use of calcium chloride as a rapid hardening medium is now well established : instructions in its use have, for example, been incorporated in various official documents such as the National Building Studies. Furthermore, there is now a special cement on the market, of which an essential ingredient is calcium chloride. This material may, therefore, be considered to have passed into the realm of bulk use.

There is little to add to what has already been said on the subject of waterproofing or water-repellent media. Their use has been very considerably popularised and, although no doubt fresh materials will be put on the market from time to time, the general principles for achieving watertight concrete are hardly likely to be affected. The same can be said of surface treatments.

Further development is therefore to be looked for, more particularly in the use of those integral agents which affect the plasticity, pore structure and durability of concrete, including those which achieve this by chemical reaction during the setting of the concrete.

Pozzolanas, if used in this country, are likely to be of the artificial variety. These have the advantage that they can be developed with specific characteristics, as compared with the somewhat fortuitous properties of the natural materials. It would appear also that they need not be very expensive. Thus, although little interest has been taken in them here so far, they are an obvious type of material upon which to base developments.

Wetting agents offer interesting possibilities, if their potentialities can be substantiated.

The use of air-entraining agents seems hardly likely to develop extensively in this country for their more obvious properties of promoting frost resistance in the treated concrete, but allied properties induced by materials of this sort can prove significant.

The problem of segregation, for example, is one which might well receive greater attention. Every user of concrete is familiar with this phenomenon, and specifications commonly require precautions to be taken against it. It is, no doubt, accepted that such attempts cannot be completely successful, but the degree of this lack of success is perhaps only now becoming appreciated, even by scientific observers. It has long been known to laboratory staffs that a degree of segregation can be observed in 6-in. cubes prepared in a laboratory under standard conditions of compaction. The Road Research Laboratory have recently made known some results obtained by means of their non-destructive methods of testing which indicate very marked difference of strength between the top and bottom, even of quite low lifts of concrete as normally poured. We are all well aware that the concrete cube taken on the site will certainly not be exactly illustrative of the concrete which is in the structure. This has had to be accepted, and it has been felt that there is a sufficient approximation for all practical purposes, but such consideration has not taken into account any radical differences which might occur in various parts of the structure due to segregation. With the best intent, it may well be that a mix of which an average sample is well up to specification, may, by such segregation, result in parts of the structure not being capable of resisting stresses much above the design stresses ; the factor of safety being thereby almost wholly absorbed by this feature alone. Therefore, in view of the desire for still higher design stresses in concrete, it does not seem out of place to emphasise the value of research into this question and into means of overcoming it : additives or special treatments may well be a feature of such a development.

Another line of thought is suggested by the question of the future of aggregate supplies. The great developments which have taken place in the control of concrete quality in the last few years have been based upon the availability of well graded aggregates. This is immediately achieved by the co-operation of quarry owners through the installation of suitable washing and screening plant. But a very important factor is also the existence of natural supplies which offer approximations to the desired result. Failing this, there must be a considerable quantity of rejects. This refers more specifically to gravel aggregates, but there are somewhat analogous considerations in the case of crushed stone. Quarries are wasting assets : their incidence is not unlimited, and the working of potential deposits will be further affected by the requirements of town and country planning and other such restrictive influences. It seems to be a serious possibility, therefore, that the growing desire for precise gradings of aggregate may have some check put upon it within the foreseeable future. If this should prove to be the case, then other means of achieving the required degree of workability with low water content may have to be proved. The alternative would be a return to lower strength concrete.

These various considerations are put forward as thoughts arising out of the natural development of concrete practice. If they are valid, they suggest that the question of concrete additives and similar special treatments is one which will necessarily be the subject of increasing scientific investigation in the years that are immediately ahead of us.

References

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[2] "Dust Nuisance of Concrete Floors." ENGINEERING, December 2nd, 1949.

The Construction of the Royal Festival Hall*

By E. Roland Hole, M.I.Struct.E., and F. C. Eales, A.M.I.Struct.E.

The main Sub-Contractors taking part in the work described in the paper include the following :—

Willment Bros. Ltd. (Excavation and demolition).
Scaffolding (Great Britain) Ltd. (Scaffolding).
Rom River Co., Ltd. (Steel reinforcement).
Blaw-Knox Ltd. (Dewatering system).
Matthew Hall & Co., Ltd. (Heating and Ventilating, Plumbing).
Redpath Brown & Co., Ltd. (Structural Steelwork).
David Esdaile & Co., Ltd. (Fibrous Plasterwork).

Introduction

The volume of construction work carried out on the South Bank of the Thames between Waterloo Bridge and the County Hall has attracted much attention and has claimed the keen interest of both visitors and Londoners. This is undoubtedly due to the general feeling of appreciation that at last this area of ugly and dilapidated property was being energetically cleared and dealt with, and that it is the main site of the Festival of Britain preparations.

The Royal Festival Hall, the subject of this Paper, occupies a very prominent position and is the only permanent building, it being one of the main features visualised in the Abercrombie County of London Plan for this area.

Forming, as it does, such an important part of the Festival of Britain site, meant that it had to be completed to coincide with this programme and this factor adds very considerable interest from the structural engineer's point of view.

The site provided most of the usual difficulties of building on the banks of the River Thames. It had previously been occupied by massive industrial buildings and in addition to these problems, those peculiar to the involved and complicated requirements of a modern Concert Hall combined to form a heavy task for completion in the allotted time of nineteen months.

In this Paper, we are mainly interested in the methods and materials employed to satisfy those requirements.

This time factor had an important influence upon the work of construction of the Royal Festival Hall. A building of this nature, calling as it does for such varied and special consideration by the Architects, presents problems in conception, design, and layout, which are intensified enormously if the time available is limited, and limited it certainly was.

We, in the Building and Engineering Industry, are familiar by now with the reputation our American cousins have for fast and speedy production in building. We have been made familiar by the Reports of Working Parties, etc., of what we ought to do to gain similar results in this country. Probably no one appreciates more than Contractors what would be possible if all drawings and information were available on the signing of the contract : this state of affairs did not, however, apply in this instance.

It is important to appreciate that the London County Council's Architect—Mr. R. H. Matthew, his Deputy—Dr. J. C. Martin, and their colleagues, had to work under exceedingly great pressure from the date of their first sketch design to the issue of their final drawings, and a study of the building itself is necessary really to visualise their immense task. It is impossible within the limits of this Paper to describe the building adequately from this point of view but the following brief particulars may give a very general picture.

The site occupied by the Concert Hall lies between Hungerford Bridge and the well-known Shot Tower. The Hall fills this site almost completely, as will be seen on the outline drawing given in Fig. 1. The over-all dimensions of the building are 240 ft. × 220 ft. and the greater part of this area is used for the auditorium which is 190 ft. × 120 ft.

Into the remaining area on the various levels the entrances, foyers, meeting rooms, restaurants, service rooms are accommodated. Figs. 2 and 3 show the levels of the various floors. Although the site is restricted, spaciousness has been obtained by the ingenious arrangement of the floors at their different levels and by the use of large areas of glass screens and windows which will, in addition, undoubtedly provide the patrons of the Concert Hall with the pleasure of inspiring views of the river and beyond, ranging from Westminster Bridge to well beyond Waterloo Bridge, and to the masses of towering buildings lying over the river to the north.

The height of the building above the original ground level to roof top is 108 ft. and from original ground level to basement 23 ft.

In the main Concert Hall 2,900 seats are provided for ; in addition to this, seating accommodation for the choir and standing space brings the total accommodation to 3,450.

The present structure represents about 80 per cent. of the complete project ; the section remaining to be constructed consisting of a smaller hall, musicians' accommodations, etc. This extension will ultimately take place at the south end of the present building and is bound up with the proposed scheme for the replanning of the Belvedere Road area, the ultimate building line extending to about the centre of the present roadway.

Provision has been made so that the main Concert Hall with its ancillary rooms, etc., as now existing can continue in use while this extension is carried out.

On the north side on the river frontage a terrace 300 ft. long by 85 ft. wide extends the full width of the site and under this a car park is provided.

This very brief description, limited as it is, will, it is hoped, at least convey some of the problems confronting the designers, and give some assistance in appreciating their undertaking.

The Consulting Engineers, Messrs. Scott & Wilson, were faced with a task no less formidable than the Architects and probably no one appreciated the task they had more clearly than they did. The following description of the structure itself is not intended to describe the problems of design involved but is given

*Paper to be read before the Institution of Structural Engineers at 11, Upper Belgrave Street, London, S.W.1, on Thursday, March 29th, 1951, at 6 p.m.

to enable the problems involved in the construction to be dealt with in some detail, as this, it is felt, is the real purpose of this Paper, as the title indicates.

The Type of Structure

The structure is in reinforced concrete construction, with the exception of the roof over the auditorium, which is in structural steel.

All construction up to auditorium floor, the lowest level of which is some 30 ft. above pavement level, is in beam and column construction, with the exception of the four staircase blocks at each corner of the site, which consist of load carrying walls, and advantage has been taken of their stiffness and box-like shape on plan to act as buttresses for the whole structure.

The carcass of the auditorium consists of 32 in. thick cavity walls composed of two 10 in. reinforced concrete load carrying walls with 12 in. cavity between and a 1 in. layer of wood-wool on the cavity side of the outer skin wall. This cavity wall is continuous around the whole of the auditorium, apart from the necessary access openings, and is supported on columns at the sloping auditorium floor level. The inner 10 in. thick skin is stiffened vertically by small projecting buttresses at about 25 ft. centres and horizontally at the various levels by slabs giving access to the boxes.

The steel roof trusses are supported on the inner skin which is thicknessed out at this point to accommodate the bearing plates. The outer 10 in. thick skin is stiffened by, and in addition supports, the various floors occurring around the auditorium itself, this floor area being used to accommodate foyers, meeting halls, refreshment bars, and service rooms.

The auditorium floor steppings are raised above the structural supporting slab to provide space for ventilation and other services.

The balcony is carried on a main beam spanning between the buttresses. Its structure is in the form of a box, one face of which constitutes the main girder carrying the vertical loads, whilst the box itself takes up the torque due to the eccentricity of these loads, and is not connected to the back wall.

The roof of the auditorium consists of two slabs with a space between, the upper slab being supported on dwarf concrete walls, a layer of glass fibre insulation between.

The lower slab of the concrete roof rests on and forms part of the compression flange of a series of steel bowstring girders, which span between the longitudinal walls.

Suspended from the steel roof trusses is the heavy plaster slab ceiling which, in addition to forming the ceiling of the auditorium, hides and also accommodates certain parts of the ventilation and lighting systems, as well as assisting in the general provision of acoustical treatment.

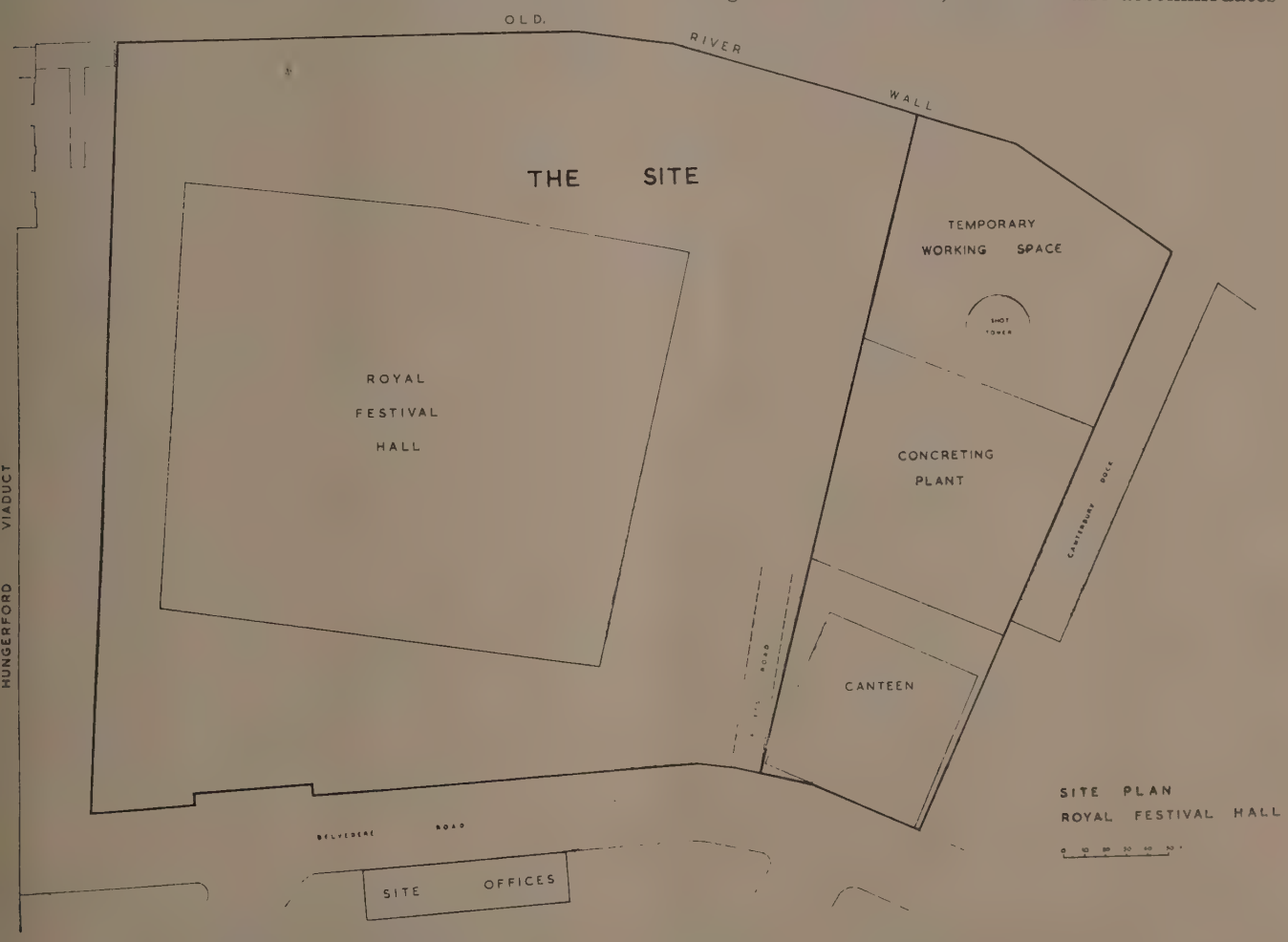


Fig. 1.—Plan showing the site in relation to its surroundings and the available working space

The demands of the acoustic problems have been dealt with in many ways but chiefly in the structure by the cavity auditorium wall and roof, and by the use of sleeved insulated columns used to prevent sound penetration to the future small Concert Hall.

Planning Consideration

This structural framework or carcass of the building, whilst representing only approximately 25 per cent. of the total cost of the scheme, as is customary, held the

key to the whole construction programme, and it was appreciated in the very early days that only by the most careful and detailed planning of operations could the completion of the scheme be achieved within the time available.

The Contractors invited to tender were called upon to submit with their tender their plan of construction and progress schedules. Whilst careful study of the drawings then available made it possible to appreciate many of the major problems involved, Architects' $\frac{1}{8}$ in. scale drawings have, as we all know, qualities of their own and most of the problems involved were as obscure as the outlook on New Year's Day, and it was felt quite certain that Mr. Churchill's famous prescription of "Blood, Sweat and Tears" would be in great demand in the months ahead. It is certain that the site staff would confirm that at least this was an accurate forecast.

The time available and the realisation of the nature of the work likely to be involved in a building of this type, at least made it obvious that the auditorium space with a roof over, was a very desirable stage to reach at the earliest possible moment. Scheming, planning and ultimately guessing decided when this could be. If information and drawings and materials were to be available to meet the requirements, this date could be fairly accurately fixed, but there were a few doubts about the drawings.

The site in itself was also a serious problem, as already stated. It was very restricted: access was from two sides only and even on these sides the Festival of Britain construction was already beginning to encroach. Of the remaining sides on the west, Hungerford Bridge was depressingly close, but we enjoyed a wonderful view over the river on the north. Fig. 4 shows the site as it existed on May 5th, 1949—the day we commenced work.

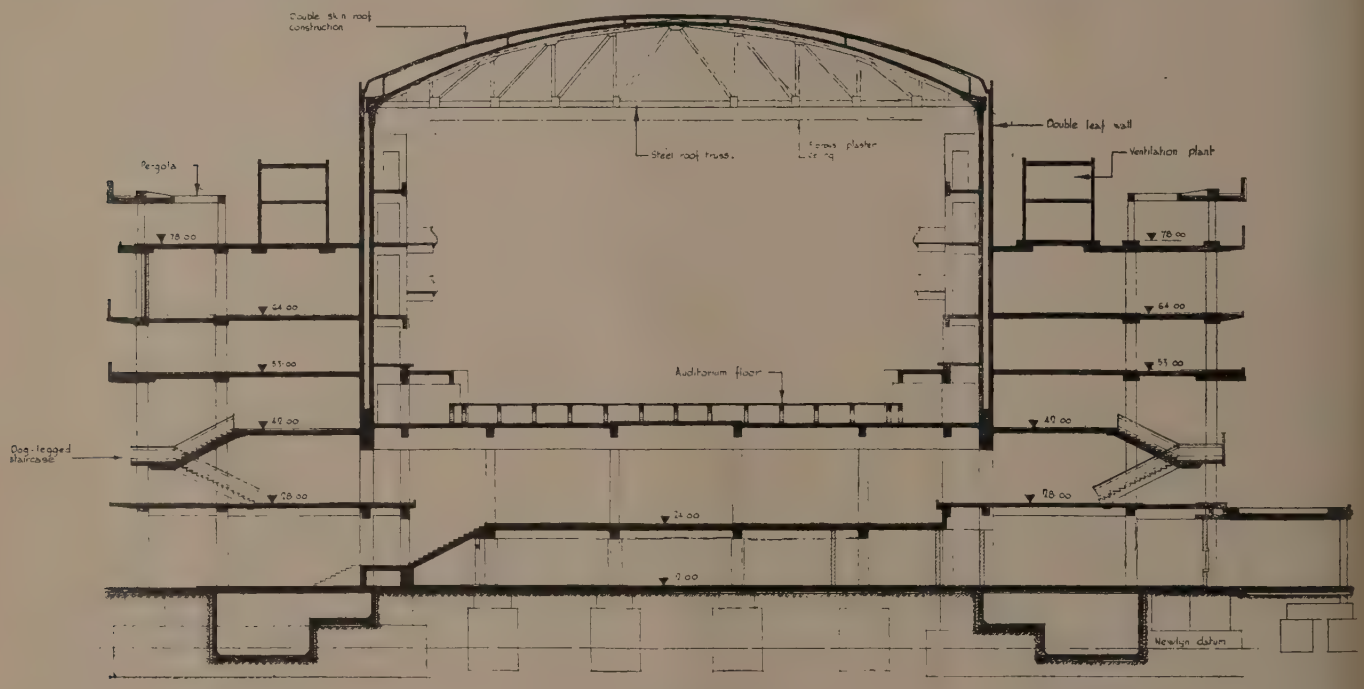
The problem was of course excavation. The site had been in occupation by various people for a long time and it was felt fairly certain that our predecessors had buried many of their troubles there. These predecessors included the Lambeth Water Works, who had flourished, we understand, from 1775 to 1837, and undoubtedly by a strange coincidence the famous Lion Brewery occupied the adjoining north-west area from 1837 to 1924, when it ceased to be a brewery but remained as storage accommodation until demolished to clear the site for the Concert Hall in 1949. The restaurants and bars will, we hope, provide appropriate reminders of those far off days.

A number of trial holes were excavated before work commenced and from them it was estimated that at the deepest point excavation would have to be carried down to — 11.00 level and averaged 11.50 ft. below mean normal ground water level.

The original ground level was approximately 14.50 O.D. and the subsoil consisted of silt, peat and the usual miscellaneous fill down to 10 ft. below ordnance datum. Below that there was a layer of compact gravel extending to the London clay at about 20 ft.

Ground water rising and falling with the tide varied from 2 ft. below ordnance datum to a maximum of 3 ft. above.

From a study of the percentage of the whole area occupied by basements and isolated bases outside the basements which were founded in the ballast and



CROSS SECTION.

Fig. 2.—Royal Festival Hall, South Bank

designed to limit the pressure to $3\frac{1}{2}$ tons per sq. ft., it was proposed to excavate in one mass over the whole site with steeply battered sides rather than excavate in isolated pits where required. By this means the maximum use could be made of machinery: heavy timbering would be avoided; the breaking out of large obstructions facilitated, and the whole area made available for construction work at the earliest possible date.

Excavation

This suggestion was approved by the Consulting Engineers, who stipulated that mechanical excavation could extend to within 6 in. from the level of the final formation, providing that effective steps were taken by the use of sleepers on the machine tracks to avoid disturbance of the virgin ballast. This remaining 6 in. was later excavated by hand labour.

Before commencing with this plan, it was calculated that to remove the unwanted excavated material to the nearest available shoot would require lorries leaving the site (and, of course, we hoped returning) at the rate of one every three minutes.

In the congested area on the site itself and in the congested traffic on the roads leading from the site to the shoot, this was obviously an unsatisfactory arrangement on which to obtain the maximum output from the mechanical plant it was desired to install, which included:

1 54 R.B. dragline
1 19 ,,
1 33 R.B. face shovel
1 17 ,, ,,
1 D.8 bulldozer
1 D.4 ,, ,,
2 Medium hammers

It was therefore decided to make a temporary dump on an adjoining space, but the use of this space was strictly limited by the requirements of the Festival of Britain construction programme. It was also necessary to provide space off the site in order to give reasonable working conditions to store the selected excavated material later to be returned as fill.

Five wells had early on been located on the site, but because of the former waterworks, or because of our post war experience of breweries it was thought that other wells probably existed and extra care had to be employed in moving the heavy plant around.

The actual sequence of excavation differed from that originally planned, and it was completed down to the de-watering shelf at 03.00 level, which was situated 1 ft. above normal subsoil level.

The excavation was continued down to the final formation level after the water level had been lowered. To do this, the site was entirely encompassed by a system of well points at 3 ft. centres linked by a 6 in. header pipe which discharged into the river. This installation is described in greater detail later.

It was originally estimated that about 35,000 yds. cube would have to be excavated at a rate of some 600 yds. cube per day. Due, however, to a variation in the site conditions from that disclosed by the original trial holes, the total quantity eventually excavated amounted to 63,000 yds. cube and this was actually completed in 70 working days, giving an average of 900 yds. cube per day.

The actual plant used for the excavation was as follows:

1 Dragline—bucket capacity 67 ft. cube
,, ,, ,, 19 ,, ,,
1 Face shovel— ,, ,, 33 ,, ,,
,, ,, ,, ,, 17 ,, ,,

A careful investigation of the existing foundations to the Hungerford Bridge disclosed a standard of construction far below that which could be reasonably expected for an important structure of this kind. In view of the possible effect of the removal of water from beneath these foundations, it was decided to drive a length of sheet piling along this side of the excavation and with a 25 ft. length returning at the ends.

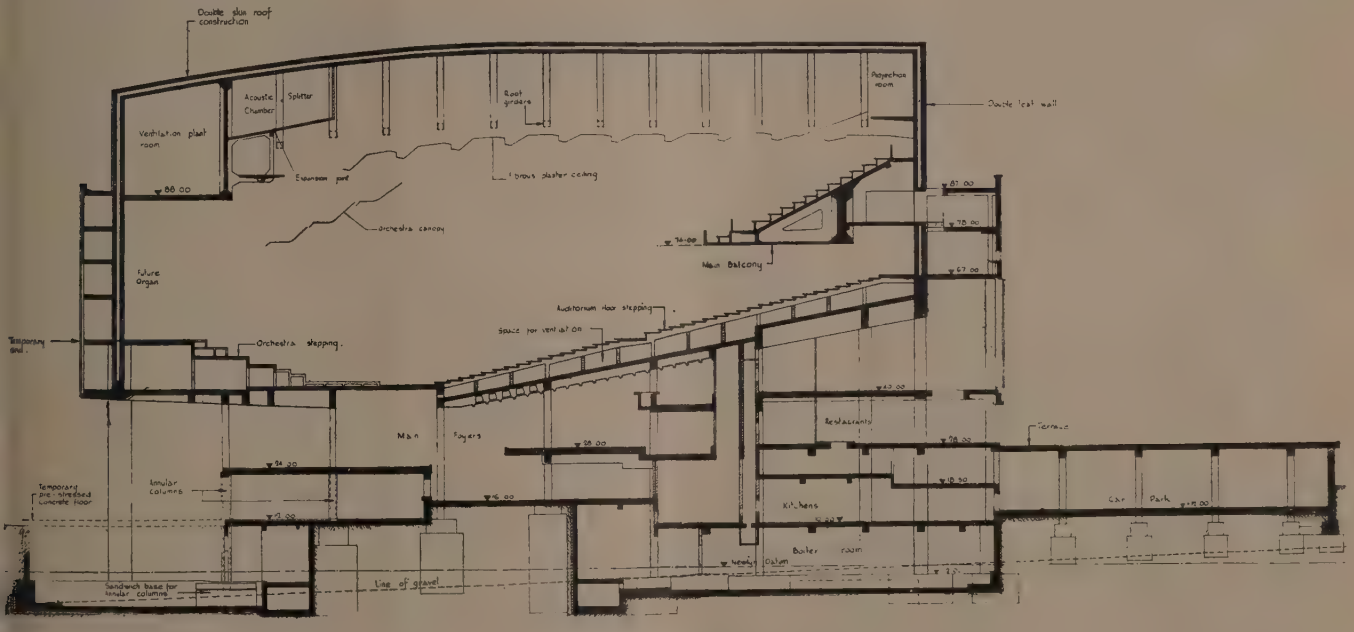
It was later found that there was a considerable seepage of water through the original river wall, and in view of this and the fact that a porous strata existed at — 04.00 to — 05.00 it was decided the risk involved made it necessary to stabilise the bank by continuing the sheet piling, the penetrations varying from 25 ft. to 40 ft. along the river frontage and returning down as far as the access ramp. This portion of piling is situated over the Northern Line Underground Railway and driving over the tunnels was restricted to the hours of 2 a.m. to 4 a.m., during which time engineers were observing the effect (if any) of this work in the railway tunnel.

During the course of the excavation, numerous large obstructions in the form of old foundations were encountered, heavier and far more extensive than had been contemplated, and this caused a certain amount of delay, although the method of excavation greatly facilitated their removal.

Extracts from the record of progress are interesting:

Excavation commenced May 7th, 1949 (completed July 17th, 1949).

Piling commenced June 1st, 1949 (along Hungerford Bridge).



LONGITUDINAL SECTION.

Fig. 3.—Royal Festival Hall, South Bank

Excavation completed June 17th, 1949 (along river frontage).

Piling completed July 10th, 1949.

Dewatering commenced May 19th, 1949 (installation).

Dewatering completed June 19th, 1949 (lowering of water level).

Backfill commenced August 18th, 1949.

Backfill 87½ per cent. complete November 11th, 1949.

Historical items and treasures of great value unearthed during the excavation were not as great as was hoped for, but mainly consisted of clay pipes dating back to 1650, pieces of defective or waste Lambeth Delf Pottery of the early eighteenth century.

The Well Point System

The installation of the well point system commenced on May 18th, 1949. and consisted of 340 well points spaced at 3 ft. intervals around the perimeter of the excavation, each being connected to a 6 in. dia. header pipe around the 1.040 ft. perimeter. Existing ground level was at 15.00 O.D. approximately, the lowest mass excavation being at — 11.00 O.D.

Normal ground water level was at 1.40 O.D., although this varied slightly with the tide, the river Thames being only 16 ft. away at the nearest point. The 6 in. dia. header pipe was installed at 3.00 O.D., i.e., just above water level and rested on a ledge formed in the battered side of the main excavation. The well points were carried down to a depth of — 18.00 O.D., i.e. just above London blue clay at — 20.00 O.D., but sufficiently low to ensure that the highest point of the drainage gradient in the middle of the site was below working level. The well points were jetted down by means of a jetting pump delivering 350 g.p.m. at pressure up to 150 lb. per sq. in. No difficulty was experienced where well sanded ballast occurred for most of the depth, but in the case of some 50 per cent. of the well points, deep layers of clay overlay the water bearing ballast and slowed down the operation considerably. The difficulty met in these cases did not arise from having to penetrate the layers of clay but in having to wash out a hole large enough to allow the coarse particles of sandy ballast to be brought to the surface of the well point to allow the passage of water. The effect of this difficulty was to reduce the maximum rate of jetting from 50 well points a day to a minimum of 10 per day.

The 6 in. dia. header pipe was connected to four diesel well point pumps located in pairs at opposite corners. A fifth pump was installed at another corner as a standby and gate valves were provided in the header pipe in such positions that, should a breakdown occur in any pump, the standby pump could be utilised while repairs were carried out. The installation was completed on June 17th, one month after commencement, during which time excavation had been proceeding down to water level inside the line of well points.

Within 36 hours of the commencement of pumping, the level of the ground water had been lowered at least 10 ft. 6 in. to — 12.00 O.D., i.e. 1 ft. below the lowest points of the main excavation. A few days later it had been lowered a further 2 ft. to — 14.00 O.D., these levels being ascertained by means of an inspection pit formed in the centre of the main excavation.

During the initial stages of pumping, i.e whilst the initial water level was being lowered, it was estimated that the four pumps had a total discharge of approximately 150,000 g.p.h. compared with the maximum pump capacity of 240,000 g.p.h. After several days continuous pumping, the discharge dropped to about 80,000 g.p.h., but in order to maintain the necessary vacuum, the four pumps were kept running continuously, all water being discharged into the Thames.

Three observation well points on each of the four sides of the site were installed to ascertain the effect of tides after pumping had commenced. Soundings recorded over a period of several days indicated that there was a tidal movement of about 12 inches within the excavation, the maximum rise in water level occurring about two hours after high tide. This rise of water level took place below the level of the deepest excavation, and the whole area of the excavation was free of water at all times. Small excavations for isolated foundations to the terracing on the river side of the site, constructed at a later date outside the main area, were excavated by hand, using methods normal for tidal work.

The relative position of the line of well points adjacent to the viaduct varied from 40 ft. to 55 ft. on plan, and the borings indicated that the well points were approximately 24 ft. lower than the viaduct foundations. Borings also indicated that at a depth of approximately 32 ft. below ground level there was an apparently continuous and impervious layer of clay.

It was decided that in order to preserve the water table in the vicinity of the viaduct foundations, the sheet steel pile diaphragm already mentioned should be driven down between the well points and the viaduct, to a sufficient depth into the clay to ensure an adequate seal. This diaphragm was returned eastwards for at least a length of 50 ft. to ensure a gradual water path at the ends. The diaphragm consisted of 30 ft. lengths of No. 3 Larssen sheet steel piling, and was driven from mobile cranes operating in the roadway and by the use of McKierwan Terry Hammers operating on steam. 147 tons of piling was used and was driven completely in 17 days.

This steel piling, including the further piling, already described, along the river front, involving a total of just under 300 tons, which varied in length from 30 ft. to 40 ft., was completed in a total of 32 days. This piling was left in position until after the foundations were constructed, the piling along the river side not being withdrawn until after the new river wall and filling back to the old embankment had been completed along the length of this side.

Foundations and Basements

The construction of the two basements was carried out simultaneously, together with the isolated bases outside. Due to excavation procedure, the basement at the south end was in advance of the remainder. The basement walls generally were 10 in. thick, and although ultimately supported laterally at the top edge by the ground floor construction, they were designed as free cantilevers, giving freedom of construction and allowing back-fill to be carried out without recourse to temporary strutting. The walls varied in height up to a maximum of about 20 ft. and, since no applied finish was to be employed on the internal surfaces, ply faced shuttering was used and the surfaces rubbed down immediately after removing the shuttering.

The basement floor slab was generally positioned flush with the soffit of the column bases occurring within the basement, the bases projecting above floor level an average of 4 ft. This arrangement was decided upon in view of an appreciable part of the south basement having only a temporary roof over at ground level, pending the construction of the smaller Concert Hall to be carried out later, as already explained.

It was, therefore, necessary to load the basement slab to prevent flotation and this could be done within the depth of the upstand bases, the service ducts being

accommodated within this depth. Also, in view of the high cost of keeping the well points system in operation until sufficient superstructure above was constructed over the remainder of the basement to provide dead weight resistance, it was decided to load the whole of the basement slab to obtain this. A further consideration was the simplification and economy in construction of the waterproofing membrane below the floor, which could thus be laid entirely horizontal in one operation instead of having to be returned up the vertical sides of column bases.

Fig. 4.—View of the cleared site before commencement of excavation, 5/5/49.

To minimise the required output of normal mixed concrete, which at this period of the construction was at its peak, due to the quantity required for isolated mass bases under construction at the same time, it was decided that this filling or loading of the basement floor, of which there was a total of approximately 1,300 yds. cube, should be carried out on the grouting principle, weight being the immediate requirement. Stone rejects were deposited in layers of 9–12 inches and a $1\frac{1}{2}$: 1 mix of sand and cement, using a patent add-mixture, was pumped into position by means of a three-inch grouting pump.

Isolated column foundations outside basement areas consisted of reinforced concrete spreader bases immediately below ground floor level, supported by mass concrete piers carried down to the ballast formation. The top of the reinforced concrete base formed the supports to the suspended ground floor slabs.

The mass concrete piers were in many places designed to support a group of reinforced concrete bases, and individual piers varied in volume from 75 yds. cube to as much as 1,800 yds. cube and were in some places as much as 22 ft. deep. Due to the method of excavation previously described, these bases had to be shuttered, patent steel forms being used in conjunction with the tubular steel scaffolding being wired back into the previous lift of concrete. The larger bases were generally cast in layers approximately 2 ft. thick, each layer representing as much as 200 yds. cube for the larger bases.

For the reinforced concrete base construction, 9 in. brick walls were built on top of the mass concrete piers to act as permanent shuttering, the waterproofing membrane being applied to the inner faces before concreting the base, the brick wall ultimately serving as a protective skin to the waterproofing membrane.

The waterproofing of the basement was by means of two layers of a bituminous fabric type of waterproof material, the layers being fused to each other by the application of heat. This system gave the advantage of being able to be carried out under conditions not normally considered practicable when using ordinary asphalt, and it could be protected immediately after fixing. Horizontal waterproofing was laid on a 3 in. thick bed of concrete and was immediately protected by a 2 in. thickness of concrete screed. The steel fixing to slabs and bases was commenced immediately this screed was laid. A 3 in. thick brick skin was provided as protection to the vertical layers.

This waterproof membrane formed a continuous skin over the whole site immediately under the ground floor slab, even where no basement occurred. The total areas covered were 14,000 sq. yds.

The temporary ground floor construction over the basement area at the south end consists of prestressed precast beams and decking units supported on brick piers. This floor will ultimately be removed when the small hall and adjacent areas is constructed. The prestressed beams are 31 ft. span and of I section 30 in. deep. They were manufactured on the Hoyer system in a factory off the site. Each beam weighs 67 cwts. The slabs, also factory produced on the Hoyer system, are $2\frac{1}{2}$ in. thick, 5 ft. 0 in. and 6 ft. 8 in. long.

Superstructure

Construction of the superstructure was carried out in four main stages as follows :—

(a) The whole of the construction brought up uniformly to the auditorium floor level, apart from subsidiary internal staircases.

(b) Concentration on the auditorium area only, to enable roof trusses to be placed.

(c) Completion of work outside auditorium while roof cladding and internal reinforced concrete work proceeded in the auditorium area.

(d) Construction of Car Park and terracing beyond the main building lines.

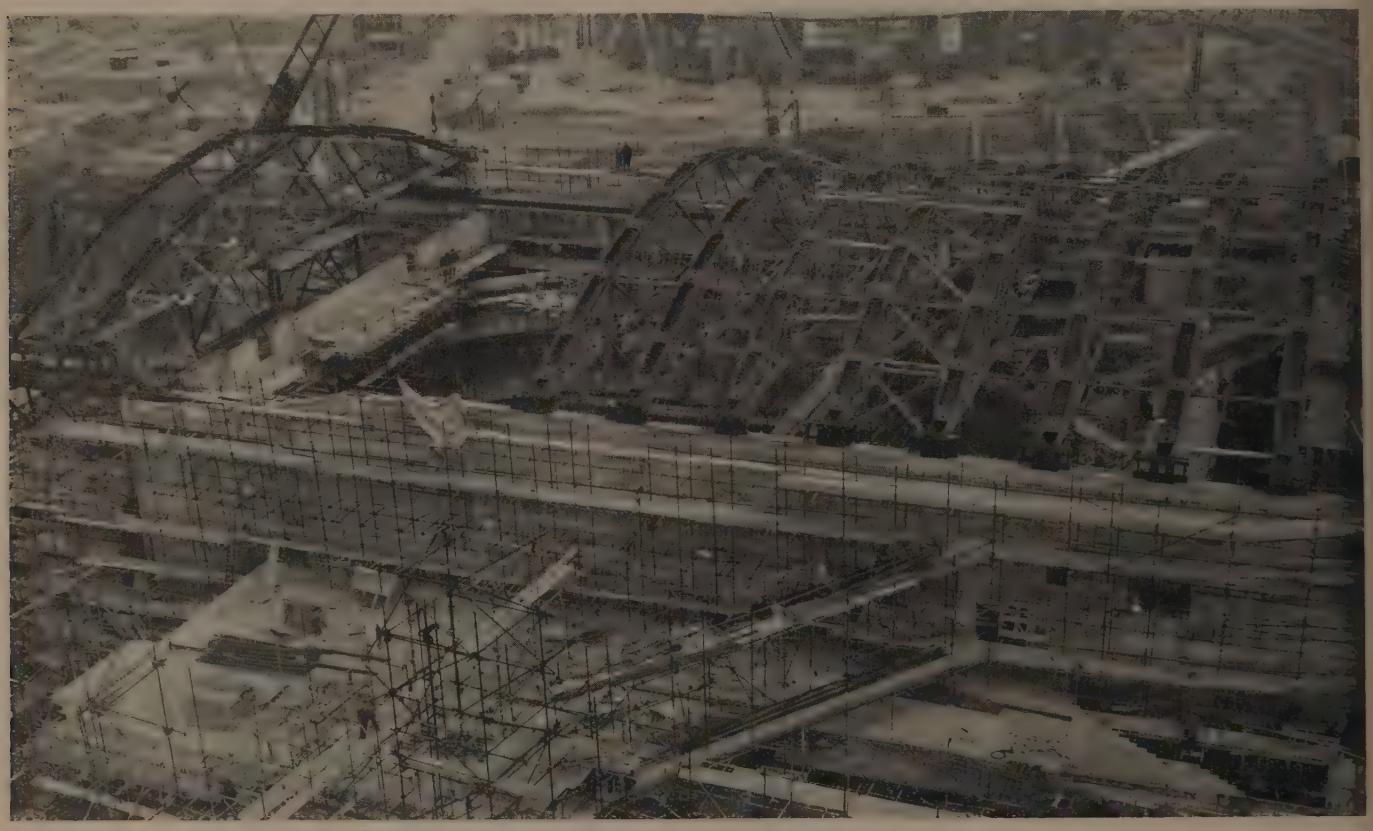
The original plan for construction of the superstructure, prepared before work commenced on site, was

Excavation in progress and commencement of foundation work

to complete all work up to ground floor level and then to concentrate on the auditorium area only. By this arrangement, it was intended to have the auditorium roof temporarily watertight by January, 1950. The construction of the surrounding work would then have proceeded during the winter months, the labour being transferred to all subsidiary work within the auditorium, under cover, during inclement weather. Under this scheme, and in order to make it feasible to get the

auditorium into this condition within the time available, it was the intention to omit all internal work within the auditorium walls, such as balcony construction, staircases and upper floor to auditorium, etc. This portion of work, so constructed, would have been comparable to a large box on stilts, so it was the intention to stiffen temporarily certain pairs of columns between ground level and the sloping auditorium floor, at a height varying from 30 to 50 ft., to act as wind bracing until the staircase blocks at the four corners outside the auditorium area were constructed, these having been designed as already described, as stabilisers for the whole structure.

Due to the additional work encountered below ground level, not foreseeable when this plan was prepared, construction up to ground level was delayed until the end of September, 1949, some eight weeks later than originally intended. It was decided that since the auditorium could not be watertight by January, the main advantages of this scheme would be lost. It was, therefore, abandoned and the whole job replanned to be brought up uniformly to auditorium floor level before concentrating on the auditorium area only, this level not only being most desirable for structural considerations, but having advantages from the point of view of the area made available for the finishing trades to commence operations.



General view showing erection of roof girders in progress

The auditorium walls consist of the two 10 in. thick reinforced concrete walls with a 12 in. cavity between, the two skins being entirely independent above auditorium floor level. Although this construction was designed to meet acoustic requirements, it was of considerable assistance during construction, since it enabled the inner skin only to proceed and support the auditorium roof and thus regain some of the time lost, leaving the outer skin to be constructed with the surrounding work, which it was designed to support.

The construction up to and including the sloping auditorium floor was, as is obviously to be expected in this type of building, very varied and did not lend itself to the adoption of any usual shuttering or labour saving methods, but on the contrary, was complicated, requiring very considerable labour with little repetition. A considerable amount of the propping to beams and slabs was at heights exceeding patent steel prop heights, being up to 41 ft. high, and for these areas use was made of tubular steel supports. It was also necessary to prop through considerable areas where the construction above was heavier than in the designed superimposed load of the work below.

A large proportion of the columns are circular, of 20 in., 26 in. and 32 in. diameter. The shuttering for these was carried out in timber, since the number of uses possible of any one size did not warrant the use of purpose-made steel shutters. The shutters were ply faced and made in two halves, one half 12 ft. long and the other being in two heights of 6 ft. each, to enable the concrete to be placed from a height not greater than 6 ft.

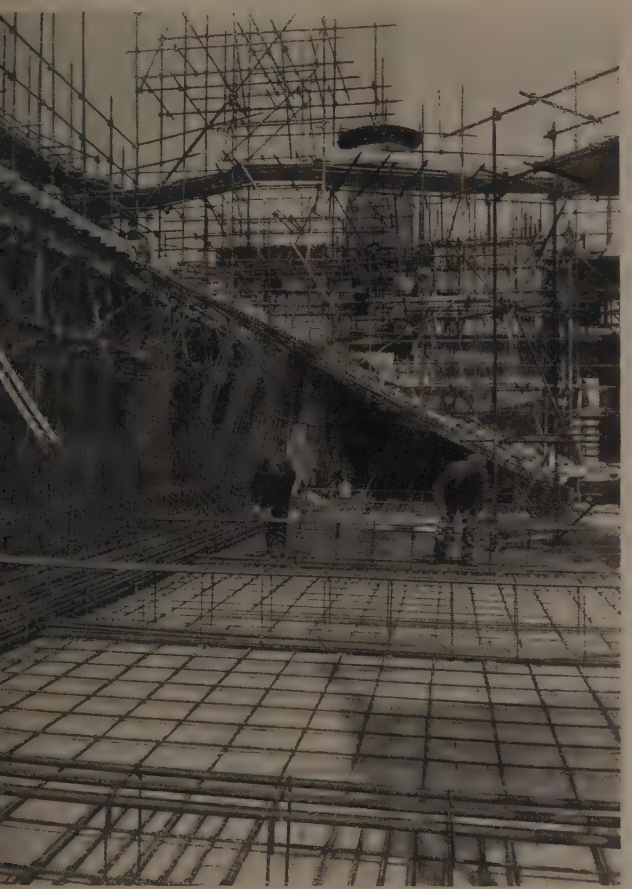
Shuttering to beams and slabs was in wrot boarding, supported on joists. Where normal height propping occurred, 9 in $\times$ 3 in. timber main joists were erected with panels and runners between, to permit earlier release of panels, leaving the main joists and props in position. Due to the variation of sizes of members and the necessity to make the maximum progress possible, an appreciable amount of short ends of timber constantly accumulated. This was made up into standard 2 ft. square palate boards for use elsewhere on the job in conjunction with the normal standard steel panels of similar size.

The concreting of the sloping auditorium floor was completed about the middle of January, and at this stage the arranged change over to concentration on the auditorium area was carried out.

In view of the extremely short period of time allowed for the entire construction, it had been decided to install temporary steam heating for maturing the concrete during periods of frost to enable concreting and the

striking of shuttering to continue more or less at a normal rate. This installation was put into operation in the latter part of January, and continued in use, when necessary, until the auditorium walls were up to roof level at the end of March. This installation is described later.

From the sloping auditorium floor upwards, the auditorium is, as stated, a load carrying box approximately 185 ft. by 117 ft. by an average height of about 49 ft., stiffened vertically by relatively small buttresses projecting from the inner face at 25 ft. centres, and horizontally by slabs 6 ft. 6 in. wide at four levels, i.e., 53.00, 64.00, 78.00 and 86.50, the level of the top of the wall thickening, forming a seating for the roof trusses, being 103.00. The inner edge of these promenade slabs was supported by columns from which cantilever



Reinforcing steel being fixed for the balcony beam and torque box

beams projected inwards at various intermediate levels to support the boxes.

It was also essential, in order to regain some of the time lost, to construct the auditorium walls as rapidly as possible. It was therefore decided to divide the construction into two : (a) the buttresses with their projecting bits of walling, which would be taken up as isolated columns, keeping this construction some 12 ft. 0 in. ahead of (b) the infilling between. This enabled the most skilled craftsmen to be concentrated on the buttress section, thus ensuring accuracy and the desired rate of progress and leaving the greater area of walling as a relatively simple repetitive operation, the alignment of work being controlled by the accuracy of the work on the buttress.

Timber shuttering was used for the buttress construction to enable a joggle joint to be readily formed, and the splice bars left projecting to form up with the remainder of the wall. The method of shuttering used for the rest of the wall consisted of 24 in. square patent steel panels clamped back to tubular scaffolding runners. These runners were only intended to give lineability, the two faces of steel panels being held together during filling by standard metal spacers. The concrete in these walls was filled in 4 ft. lifts and 8 ft. height of shuttering was provided around the whole perimeter of the auditorium. By using an 8 ft. min. length of vertical tube runners, the shuttering to any 4 ft. height was always positioned from the previous lift cast, although in view of the extreme accuracy required, it was essential to be able to make minor adjustments at each lift. The final check after construction was complete to the top of the wall showed a maximum error $\pm \frac{1}{4}$ in.

In order to avoid dismantling the whole of the shuttering to the inner face at each floor level, which would have delayed the wall construction while the promenade slabs were constructed, a horizontal chase was formed in the face of the wall and splice bars cast in, permitting the wall shuttering to continue upwards without interruption. This chase was 2 in. deep and the ends of the U-shaped splice bars were bent longitudinally within the depth of the chase to avoid drilling the shutters and causing difficulties in fixing of steel and removal of shuttering. The splice bars were subsequently bent out, and the face of the chase hacked to enable the promenade slab construction to proceed immediately after the wall shuttering had been raised above slab level. The top of the 10 in. inner skin wall was thickened out to 18 in. for a depth of 21 in., to accommodate the truss bearing plates, and the fixing bolts of which there were No. 10 $1\frac{1}{2}$ in. diameter at each end of each truss. Holes were formed in the top of this thickening to take these bolts by means of a light steel mesh wrapped round tapered timber plugs, the plugs being withdrawn a few hours after concreting. The time taken to carry out this part of the construction from 53.00 level to 103.00 level was 10 weeks, and although low temperatures were experienced for about two weeks, the steam installation proved satisfactory and no time was lost.

The construction at the north or river end of the auditorium lagged slightly, due to the complicated nature of the buttress supports to the main balcony beam which were incorporated with the inner skin wall, the reinforcement at these points being extremely heavy and intricate. To prevent this part of the work lagging too far behind the remainder, it had been decided to allow the walls and buttresses to proceed and to construct the balcony at a later date. The reinforcement was detailed to allow for this, and large haunches and a complete strip of balcony construction was cast monolithic with the adjacent buttresses.

During the critical stage of construction described above, which was essential to allow the auditorium roof construction to commence, a night shift varying between fifteen and twenty men were employed to enable work to be carried on almost continuously at any point that was lagging or that was likely to prove a source of delay. To permit this arrangement, and also to enable the whole labour force, if required, to continue after 4 p.m. during winter months, arc lamps were erected at selected points around the whole perimeter of the building, and two internal towers provided with a group of arc lamps. Mobile arc lamps were also available to give a concentration of light wherever necessary to ensure the work being carried out under good working conditions.

Having completed the work necessary to enable erection and positioning of roof trusses to commence, the main gangs reverted to the construction of the

wings, which had been temporarily stopped at 53.00 level. At the same time, the construction of the balcony was commenced, together with various internal staircases below auditorium level which had been omitted.

Various methods of placing the roof trusses in position were investigated at an early date, but objections were raised against most of the usual methods, mainly due to their preventing or delaying other work proceeding during the same period. It was finally decided to hoist and assemble the trusses at the south end of the job and travel them along the auditorium in pairs by means of bogies at each end running on rails. A 40 ft. high gabbard constructed of ex-military steel trestling was erected close to the building at the south end with a ten-ton 120 ft. jib electric crane mounted on top. The trusses were fabricated in works at Greenwich and delivered by road in half trusses, transport having to be carried out in the early hours of the morning due to traffic considerations.

A temporary steel platform was erected on the 12 in. thick ventilation plant room floor slab, which projected about 25 ft. from the south end wall and ran the full width of the auditorium. This slab was 22 ft. below the general soffit level of the roof trusses, and was used as a working platform for the steel erectors. Each half truss, weighing about ten tons, was hoisted to this assembly platform immediately on delivery, the ends being supported on the travelling bogies and the centre

View from Hungerford Bridge when nearing completion

on the temporary steel platform. Two trusses were assembled at a time, to give lateral stability whilst travelling along the building. The bogies were pulled along by means of hand winches located at the far end of the building, and white marks at 12 in. centres on each bogie track served as guides to enable the movement of the bogies to be controlled to ensure uniform travel and avoid racking. The time taken for moving each pair of trusses varied from six hours for those at the north or far end of the auditorium to half-an-hour for those nearest the assembly platform, the total number of twelve trusses taking four weeks from the date of delivery of the first truss to the positioning of the final pair.

End bearings of the trusses were on rocker plates, the lower plate of the rocker being placed in the finished position and temporarily supported at its correct level by means of steel packing plates, and the top half being attached to the truss itself. Jacks were incorporated in the bogies for lowering the trusses about 4 inches on arrival at the final position. Grouting up of bolt holes was carried out to each pair of trusses after the following pair of trusses had been completely braced and tied, and the 2 in. space between the top of the concrete walls and the underside of the bearing plate filled by means of dry packed concrete, using a 1 : $1\frac{1}{2}$: 3 mix with $\frac{3}{8}$ in. maximum aggregate.

In order to prevent the extension of trusses under their final dead weight load causing cracking of the auditorium walls at the ends of the auditorium, where they were restrained by the end walls, the three trusses at each end were designed to permit adjustments to be made to counteract this extension. This was done by incorporating a pin joint in the top chord at midspan and a sliding joint connection in the tie member, also at midspan, with provision for inserting a 150 ton hydraulic jack to enable the length of the truss to be shortened when part of the dead load was applied. Under partial load, the first truss from the end showed an extension of 0.1 in. and required a force of eighty tons to draw it in 0.28 in. before the remainder of the dead weight was applied. The amount of adjustments made in the trusses varied in accordance with calculations made by the engineer, and was such as to spread out the deformation of the wall uniformly over the end 50 ft. lengths instead of concentrating it all within the end 12 ft. 6 in., which would in all probability have caused serious cracking to occur.

The roof construction consisted of two skins of concrete with a space between varying from 1 ft. 8 in. deep at the crown to 3 ft. 6 in. at the sides, the lower skin spanning on to the steel trusses at 12 ft. 6 in. centres and the upper skin being supported on 4 in. thick reinforced concrete sleeper walls at 6 ft. 3 in. centres parallel to the trusses. A 2 in. thick blanket of fibre glass was draped over the tops of these sleeper walls to form a continuous blanket over the whole of the auditorium, the upper skin slab being thus entirely isolated from the construction below. This arrangement of roof construction was decided upon to meet acoustic requirements, and was the result of exhaustive tests and experiments carried out at the commencement of the contract, both with regard to the roof and wall construction of the auditorium.

To establish the requirements for acoustic purposes, a small test building, 21 ft. by 18 ft. was built adjacent to the public steps leading up to the footpath across Hungerford Bridge, the walls and roof being a replica of the proposed actual construction. It is of interest to note, in passing, that although the proximity of Hungerford Railway Bridge and Viaduct made it appear an unfortunate choice of site for a Concert Hall, it was proved that the problem of noise was no more severe than would have existed on any site for a similar type of building, since the noise of jet aircraft overhead was the governing factor.

The lower skin of the roof construction had been designed to function in conjunction with the top chord of the roof trusses, which were filled with concrete monolithic with the slab. It was undesirable to cast a normal *in situ* slab with the problem of shuttering at a height of about 70 ft. above the auditorium floor below, and a patent pre-centred construction was adopted. This consisted of triangular shaped welded rod lattice ribs with a 2 in. thick concrete sole board cast around the two bottom bars, a single round bar forming the compression member during erection. These ribs were at 1 ft. 9 in. centres, the space between being filled with small precast slabs 2 in. thick, spanning on to the soleboards of the ribs. The ribs and slabs were designed to span between the top chords of the trusses and to support a 4 in. thick *in situ* topping slab and live load during construction. This arrangement, in conjunction with sheet metal tack welded to the lattice bracing on the soffit of the top chord obviated the necessity for any shuttering except in special places, and again saved valuable time in the construction of the roof cladding.

A similar type of construction was used for the upper slab of the double roof, since the space between the two slabs was insufficient for the removing of shuttering and an entirely precast construction did not satisfy the acoustic requirements. In this case, however, the *in situ* topping concrete was only 2 in. thick, and the top member of the steel lattice ribs consisted of a 1 in. by 1 in. steel channel with a continuous hardwood fillet. These fillets served as screeds for the *in situ* slabs, since they were flush with the top surface, but were primarily intended as a means of fixing for the insulating board and copper which completed the roof covering. In both the upper and lower slabs, additional reinforcement was added before the *in situ* concrete was placed. The lower skin of roof construction was completed by July, 1950.

Whilst the roof construction had been proceeding, many finishings, including fibrous plaster ceilings, had commenced below the auditorium floor and, since the latter was by no means watertight due to openings etc., for various services, construction of the balcony had also been completed whilst the roof was in progress. The balcony took the form of a triangular shape torque box, the horizontal slab at the soffit and the sloping slab supporting the precast steppings both acting in conjunction with the main beam itself, the total weight which had to be supported during construction being about 300 tons across the 104 ft. span. Since the floors below were not capable of carrying this very considerable load, propping had to be carried down to the foundations, in some places passing through four floors. In order to allow the installation of Engineering Services to proceed unhindered, temporary lattice steel stanchions in standard 8 ft. lengths were used to divide the span up into four, and thus concentrate the props necessary at intermediate floors. Where necessary, several of these 8 ft. lengths of stanchions were bolted together to give the required height, timbering being used where required to make up the required level. These towers were capable of supporting up to 60 tons, and were supported at each floor on four 20 ton bottle jacks to permit of adjustment. The towers were in pairs at about 25 ft. centres along the main beam, and the bottom 2 ft. 6 in. depth of beam was strong enough to carry the weight of the remainder of the beam above across this 25 ft. span, so that the subsidiary propping between the towers only had to support the weight of this first lift of 2 ft. 6 in. depth of main beam, the auditorium floor immediately below the balcony being capable of taking this weight.

Concreting of the bottom horizontal slab of the balcony construction, which varied in thickness from 5 in. to 11 in., was carried out in four sections with joints running from front to back of the balcony and incorporating the bottom flange of the main beams. The web of the beam was then cast in two lifts across the full width between the brackets left projecting from the buttresses, and finally the sloping top slab, incorporating the top flange of the main beam, cast in four sections similar to the bottom slab. The concrete used in the balcony construction was 1 : 1½ : 3 mix, using rapid hardening cement, and the subsidiary propping was removed ten days after the last section of sloping slab was cast.

In placing the main steel lattice supports, the jacks under each section had been tightened to ensure absolute tightness between floors, deflectometers being positioned adjacent to each prop at every floor to enable this tightening to be controlled and to avoid damage to the structure. Due to the considerable variation on the relative stiffnesses of the floors below, this was of considerable importance. During the whole casting period of the balcony, daily readings were taken at every point and the actual deflection of each intermediate floor tabulated so that the effect of each increment of load was readily seen.

A pair of stanchions at the west end of the beam had to be supported on heavy R.S.J.s spanning about 35 ft. on to adjacent column bases, since at this point no basement existed, and the ground floor slab itself was suspended and totally inadequate to support this pair of temporary stanchions. In this case, the jacks at the base of the stanchion had to be periodically adjusted to take up the deflection in the relatively slender R.S.J.s and to prevent the permanent floor above being seriously overstressed.

After all the subsidiary propping had been removed, leaving only the main props, these were finally lowered by operating the jacks under the props between the balcony and the auditorium floor. This was carried out in pre-determined stages and to a varying amount at each prop, approximately proportional to the anticipated deflection at each point. The maximum deflection was 0.32 in., which compares well with the theoretical calculated deflection of 0.246 in.

The stepped floor above, both the sloping auditorium and balcony slabs, was constructed of precast concrete units. Those to the auditorium were supported on sloping reinforced concrete beams extending through the auditorium at 4 ft. and 7 ft. centres, and supported on brick piers with small reinforced concrete columns for rigidity where beams occurred below. This arrangement permitted all ventilation and other service ducting etc., to be accommodated between the sloping and the stepped floors, to which access is easily obtained.

The precast steppings to the balcony rested directly on the *in situ* construction horizontal stoolings, dowel bars having been incorporated in the sloping top slab.

The construction of the wings was completed to general roof level at 87.00 by the end of July, the work above this level being in the nature of pent houses and not presenting any major problems.

Concreting Methods

The concrete mixes used were 1 : 3 : 6, 1 : 2 : 4, 1 : 1.8 : 3.6 and 1 : 1½ : 3, the approximate quantities of each being as follows :—

8,800—Yds. cube—1 : 3 : 6 mix in blinding and protective screeds.
—Yds. cube—1 : 2 : 4 mix in mass concrete foundations.
27,500—Yds. cube—1 : 1.8 : 3.6 mix in reinforced concrete work generally.
—Yds. cube—1 : 1½ : 3 mix in balcony construction.

The average daily requirements necessary to meet the construction programme were 285 yds. cube during the foundation construction and 104 yds. cube during the construction of the superstructure. The construction of the building, however, not being of a repetitive nature, meant that the productive capacity had to exceed these average figures, and in actual fact, peak outputs of up to 390 yds. cube a day on foundation work, and on superstructure work 190 yds. cube were reached.

Considering the major requirements and conditions, it was decided that the main concrete delivery should be by means of pumps, augmented by independent mixers and hoists stationed at the four corners of the site to cater for staircase construction and small quantities in isolated areas.

Various considerations led to the main mixing station being located on the south side of the Shot Tower, a horizontal distance of some 250 ft. from the centre of the building.

The main mixing station consisted of two 6 in. pumps, each with a maximum output of 20 yds. cube an hour, and each served by two 18/12 enclosed drum mixers, mounted on staging to avoid the pumps having to be sunk far below ground level. Each pair of mixers was served by a weighbatcher with a two-compartment bin of a combined capacity of 13 yds. cube per weighbatcher, the cement being gauged independently. A two-way delivery chute was fitted to the discharge of the weighbatchers.

During the period of the foundation construction, a 5-ton steam crane was employed for loading purposes behind each weighbatcher, but for the superstructure work, one crane was sufficient to keep both batching plants supplied due to the reduced output required. Concrete-lined bins were formed for aggregate storage, with a capacity of 500 yds. cube.

The reinforced concrete was vibrated, various methods being employed to suit the particular requirements and positions of each member. Immersion vibrators were used generally for beams and slabs, electrically driven needle type being employed where the spacing of the reinforcement precluded the use of the normal immersion vibrator. Kango type hammers with special attachments were used on the faces of column and wall shutters. All work was, however, punned in addition to being vibrated, the percentage of steel in columns and at beam junctions being high. Walls, generally, were concreted in four foot lifts and columns in six foot lifts, although in some instances it was necessary to reduce this considerably where the reinforcement was particularly congested at, for instance, splices immediately above floor levels. Compressed air was used for cleaning out the shutters, the compressor also serving to blow out pump pipelines.

In the case of foundations and floor slabs, which were generally 10 in. thick, the concrete was delivered by pipeline direct into position, but walls, etc., were filled by barrowing from a storage hopper supplied by the pump pipeline.

When construction was at a height of about 40 ft., the limit at which the pump would satisfactorily operate was reached, since in addition to this height, a horizontal distance of up to 400 ft. had also to be covered, including a number of bends. One of the two pumps was, at this stage, transferred to the floor of the auditorium and utilised as a booster to the pump remaining at ground level. This reduced output, but at this stage of the work the requirements were also reduced. The ground level pump was used to feed a normal 1½ yds. cube floor hopper positioned above the pump in the auditorium, and this in turn discharged into the hopper of the booster pump. It was found that by using the usual method of electric bell communication, the storage hopper provided a sufficient buffer between signals and this system of relaying concrete proved quite successful and enabled concrete to be pumped a total of 400 ft. horizontally plus 90 ft. vertically from the main mixing station.

All concrete production and placing was under the control of a Concrete Control Engineer, assisted by Supervising Engineers in the various sections of the work. The ratio of coarse to fine aggregate was varied to give the best possible combination, frequent grading tests being carried out and the combination adjusted when warranted by any variation in the grading. The ratio of combined aggregate to cement was not varied. In addition to the cement suppliers' tests, samples of cement were also taken regularly from site for testing in the London County Council Laboratories.

Test cubes were generally crushed at seven days' age with cubes from the same set being periodically tested at 28 days' age. Seven-day crushing strengths obtained with normal Portland cement concrete varied from 2,000 to 3,500 lb. per sq. in., although in the main they were between 2,000 and 3,000 lb. per sq. in for 1 : 2 : 4 mix and from 2,000 to 4,500 lb. per sq. in. for the 1 : 1.8 : 3.6 mix. A similar range of strengths was obtained when using rapid hardening cement during the cold weather period.

As previously mentioned, a steam installation was employed to ensure, as far as possible, the continuity of work during the winter months. This installation consisted of 4 steam boilers capable of producing 600 lb. of steam per hour, and were placed in the basement at the south end. A vertical riser main fed a ring main round the inner face of the auditorium walls at approximately auditorium floor level, the construction proceeding during the winter months being mainly the auditorium walls and promenade slabs, etc. From this ring main, vertical riser pipes were carried up at 25 ft. intervals around the perimeter, these risers being extended with the construction as the work proceeded. From these risers, flexible armoured hose was used to supply steam to any section of the work, sparge pipes being run along the member or area to discharge steam uniformly into a confined space surrounding the member or area. Day and night attendance was maintained, and the system put into operation when the temperature reached about 35°F. A steam supply was also provided to the four staircase blocks, since it was essential to maintain progress at these critical points.

In addition to the steam heating designed to maintain the temperature of the concrete after it had been placed in position, the mixing water was heated to between 120° and 140°F., and rapid hardening cement used. Steam pokers were inserted into the stock piles of aggregates, which were also protected. The top surfaces of slabs were protected by means of timber frames surrounded with building paper and two thicknesses of hessian, the air space in the cavity being about 3 in. deep. In order to avoid loss of temperature along the length of the pump pipeline, this was lagged with hessian, protected by roofing felt. In view of the exposed position of the main mixing station, screens were erected to the full height of the pumps and batching plant to shield the whole of this plant from cold winds.

During the cold weather spells that were experienced, temperatures of the concrete were regularly taken at both the mixing point and at the point of placing, and for 24 hours after placing. With the precautions described, it was found possible to maintain a concrete temperature of about 55°F. with the atmospheric temperature ranging from 38°F. during the day to 28°F. at night. The temperature of the concrete discharged from the mixer was about 67°F.

Reinforcing Steel

Reinforcement was cut and bent on site, a steel-bending yard being established on the south side of Belvedere Road. The structure contains approximately 2,600 tons of reinforcing bars of diameters from ¼ in. to 1½ in.

The plant employed in the steel yard consisted of a 7½ ton, 105 ft. jib electrically driven derrick for unloading and loading purposes, two power bending machines, and various other cutting and hand bending machines. Steel was transported by lorry from the steel yard to various points around the perimeter of the building and hoisted by means of apron hoists.

Steel fixing, generally, was extremely intricate on account of the nature of the job and the size of many members was reduced to meet the architectural requirements.

Reinforcing bars were in some places butt welded by means of a single " V " preparation, in conjunction with a half round steel cover plate. Due to site conditions, it was not possible to employ double " V " welds, the welding having to be done in position, and it being practicable to reverse the bars as would have been necessary if the double "V" system had been employed.

Main bars in the balcony beam proper were welded, the bottom reinforcement in the main flange containing up to 65 1½ in. diameter bars with 12 1¼ in. diameter bars in the top flange at mid-span. This beam contains 86 welded joints. The main beams in the auditorium floor, between the buttresses supporting the main balcony girder act as a tie member to the main portal frame and the longitudinal bars in this line of beams are also welded, involving 72 welds in 1¼ in. and 1½ in. diameter bars. In addition the 44 1½ in. diameter main bars in the bottom of the auditorium wall, spanning 84 ft. clear between the two oval shaped columns were welded.

The steel was delivered in mill lengths. To minimise waste, short lengths of the larger diameters were welded together. This welding was carried out partly in the site steel bending yard and partly in works off the site. Altogether, a total of approximately 1,220 welds were made, this work being supervised and periodically inspected by a representative of Lloyds. Test pieces were cut from each member, and test results obtained before the members were concreted.

General Trades

The portion of the complete building so far described, whilst it forms such an important part of the whole, and governed the speed of the general programme of construction, still represents only 30 per cent. of the total cost. The remainder is, of course, spread over the General Building, Finishings, and Specialist Trades.

As would be expected, practically the whole compass of the building industry is represented in the completed structure. It is considered beyond the scope of this Paper to deal with this in equal detail to the foregoing. The important point to appreciate, however, is the relative volume of work in the two sections and the necessity which existed from the outset to get the other trades fitted in and working on the site, as early as adequate working space could be provided.

The basements were invaded by bricklayers as soon as the shuttering was removed to give them room, and these were fairly rapidly followed by other trades. The Schedule shown in Fig. 6 indicates when the various operations were commenced and the period occupied in completing their work.

Whilst the race to get the roof on was a main concentration, it was also necessary to clothe the building as quickly as possible, and a start was made on the stonework on March 2nd, 1950, on the north east elevation. The stone used is Portland stone, and is generally in plain ashlar, secured to the concrete structure by metal anchors connected to dovetailed slots cast in. As will be seen from the photograph (Fig. 5) whilst the area of stone facing is considerable, a large percentage

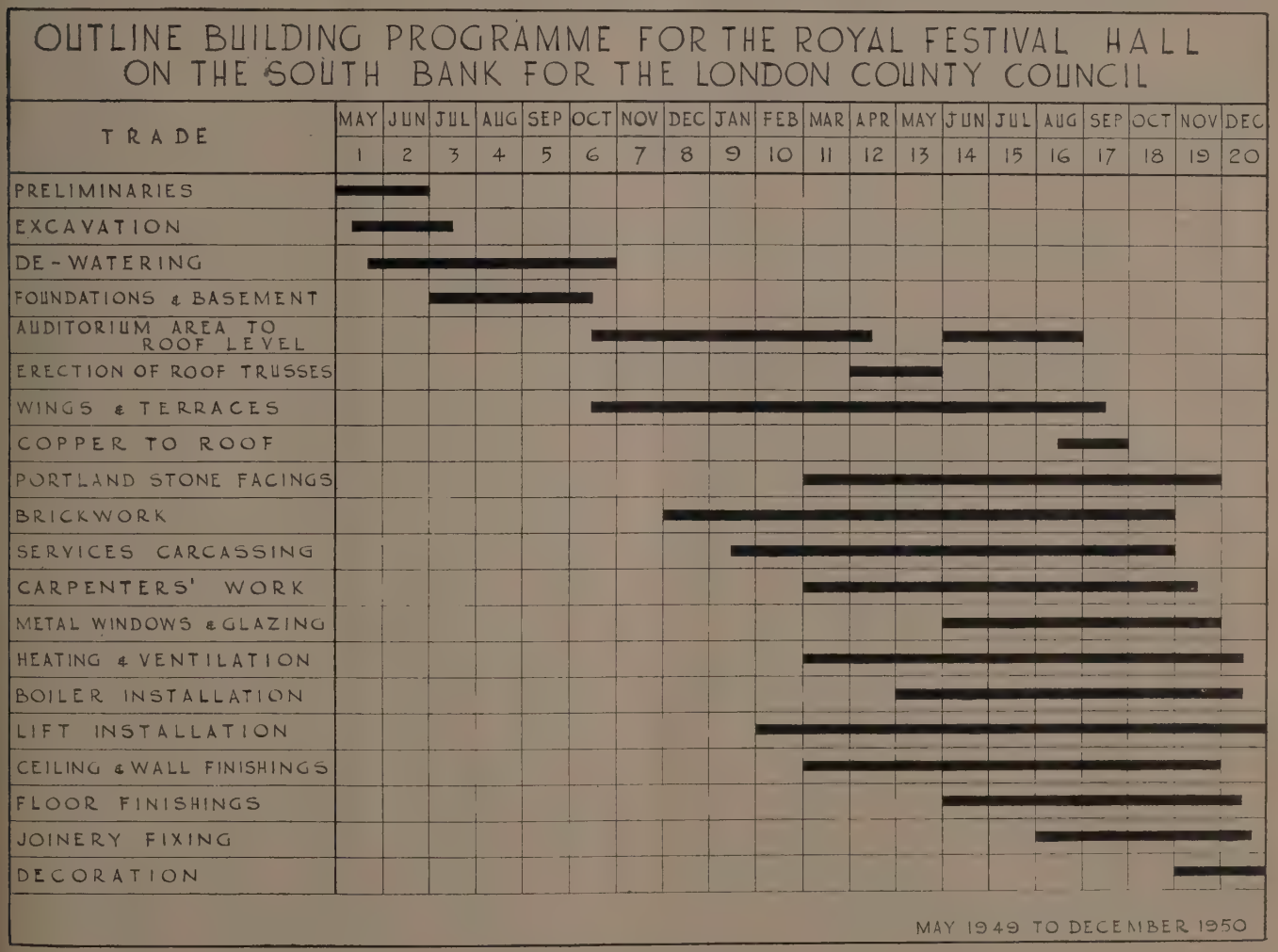

OUTLINE BUILDING PROGRAMME FOR THE ROYAL FESTIVAL HALL ON THE SOUTH BANK FOR THE LONDON COUNTY COUNCIL
TRADE
MAY JUN JUL AUG SEP OCT NOV DEC JAN FEB MAR APR MAY JUN JUL AUG SEP OCT NOV DEC
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
PRELIMINARIES
EXCAVATION
DE-WATERING
FOUNDATIONS & BASEMENT
AUDITORIUM AREA TO ROOF LEVEL
ERECTION OF ROOF TRUSSES
WINGS & TERRACES
COPPER TO ROOF
PORTLAND STONE FACINGS
BRICKWORK
SERVICES CARCASSING
CARPENTERS' WORK
METAL WINDOWS & GLAZING
HEATING & VENTILATION
BOILER INSTALLATION
LIFT INSTALLATION
CEILING & WALL FINISHINGS
FLOOR FINISHINGS
JOINERY FIXING
DECORATION
MAY 1949 TO DECEMBER 1950

Fig. 6.—Building programme

of each elevation is occupied by the large windows already mentioned. These are in aluminium on structural steel framing with teak surrounds.

In addition to the normal windows, are the large glazed entrance doors and screens on the south west and north east elevations. These are constructed of steel frames cladded in aluminium, the glass panels being in armoured plate.

The south elevation, in its temporary finishing, will remain like this until it is possible to construct the small concert hall and permanent elevation when the new Belvedere Road is available.

Continuing with the exterior work, the covering to the main roof over the auditorium, the outline of which forms such a distinctive and intriguing feature of the building is of copper, as earlier stated. The remainder of the roof areas being covered with bitumen sheeting. protected by a cement paving cast *in situ*. The flat areas of roof at 78.00 level are promenade areas, and the gardens there will undoubtedly prove an attractive feature, being pleasantly situated vantage points from which to view the surrounding City.

Internally, and working downwards from the highest point, we have the large space contained within the depth of the Structural Steel Roof. This is occupied largely by the distribution ducts of the ventilating system and the electrical services. The suspended ceiling constituted an important feature in the construction programme, as this is of 2 in. thick shaped fibrous plaster, secured to a steel framework suspended from the Steel Roof Trusses. Advantage was taken of the holes to be provided in the ceiling for lighting, to position steel cables fixed to the steel roof trusses, and thus obtain a suspended scaffold platform which covered the complete area of the auditorium. This scaffold was adjustable in height by means of hand-operated winches, and it was thus possible for the work on the ceiling and in the roof space area to continue without being interrupted, or interrupting, the work below.

The structural work to the boxes is in reinforced concrete the fronts being finished in fibrous plaster slabs.

The side walls of the auditorium are finished in elm veneer panelling, a particularly interesting detail being the knuckle bone design of panelling employed.

The rear or North wall is panelled in leather, fixed to special metal framing supports attached to the wall.

Another particularly interesting feature is the floor finish in front of the platform. This consists mainly of a large area of polished slate, and forms an important part of the acoustical treatment.

The stepped floor has already been mentioned in the first part of this Paper. In the space beneath, that is between the sloping reinforced slab and beam structure, and the steppings which carry the seating accommodation, are the ventilation ducts for extracting the air through openings formed in the recess of the steppings.

The treads of the steppings are finished with timber nosings and cork slabs, also selected to satisfy the acoustic requirements of the hall.

The whole internal surface of the vast hall has, as would be expected, had to be considered from this aspect, and the architects and the acoustical experts have undoubtedly satisfied the requirements by a most intriguing selection of materials which has added considerable interest to the whole project.

Externally, the walls of the auditorium, that is the walls dividing the auditorium from the upper foyers and promenade paces, are covered with slabs of polished Derbyshire marble. These are fixed to the walls by means of copper cramps.

The permanent paving to the adjoining landings and promenade areas consists mainly of hardwood strip flooring and Derbyshire marble.

The restaurant, which is situated at 40.00 level, is delightfully placed, overlooking the Terrace Gardens and the river. The external column finish to this area is again in the attractive Derbyshire marble. No further details are given of this accommodation, as it is felt to be almost certain that members of the Institution and visitors will soon make themselves familiar with the facilities provided.

No attempt will be made to describe the staircases generally, but two interesting and unusual staircases are worthy of some special attention. A circular staircase connecting the two restaurants and the staircase leading from the auditorium to the main foyer ; this latter one being of the dog-legged variety.

In both cases, concrete forms not only the structure but also the finish, with the exception of the treads, which in the former are of black terrazia slabs cantilevered over 2 ft. 4 in. beyond the concrete structure. The latter staircase is similar, except that the treads are of laminated hydulignum, cantilevered over the edge for a distance of 3 ft. 5 in.

The columns in this main foyer at 28.00 area are circular, as described, but are finished in stucco marble, Italian craftsmen being specially brought over to carry out this work. The adjoining columns in the wings are also circular and are covered in strip ash.

The main entrance and foyers at 24.00 level provide an atmosphere of spaciousness, the area being made available by the elevation of the auditorium. This generous area will undoubtedly facilitate the entrance of the patrons and avoid the more customary crush. It is characteristic of the treatment that the term ' Crush Hall ' need not be employed.

A further interesting feature in this area is the huge egg-crate lighting unit in the ceiling, occupying a space of 84 ft. 6 in. by 19 ft. 8 in.

Immediately outside the north entrance is the famous wych elm which, fortunately for the contractors, still appears to be in a mood to take advantage of the coming spring. It has been tended with all the care natural even to builders, who probably at heart are still first class gardeners.

The basements and lower floors, as already described, are extensive. No particular feature of finish has been used, but the ample space provided accommodates the services which will add to the comfort and well being of the visitors to the Concert Hall, housing, as it does, the kitchens, heating, and plant for the mechanical services. An unusual provision is that for connection to the heat pump, which it is intended to form one of the features of the Festival of Britain.

Site Organisation

The construction of the Royal Festival Hall has, so far, been mainly expressed as having involved drawings and charts and so many tons and cubic yards of this and that, but embodied in this interesting and involved structure is the labour of many hundreds of workmen, and at this stage of our review we would like to acknowledge the efforts they made and the good work they have completed.

When planning work of any kind, but particularly on a contract of this type, the importance of the workmen stands out predominantly. To bring together large numbers of workmen of the wide variety of trades and skill required to complete a large and complicated building and to weld these into an efficient team is one which we consider deserves more attention than it normally receives.

This is a problem which concerns all the main parties to the contract, the architects, the engineers, and, of course the contractors. In this particular instance the contractors were fortunate in that they had an appreciable force of key men available ; men between whom a common link already existed, even though they came from different contracts, and who could be relied upon not only for their knowledge and skill but for the loyalty and interest which they would put into the task. The problem of welding together the whole force, as it was gradually built up into a team, that knew what it was building and what it was expected to do, and the importance of their individual work in regard to the whole structure, had to be completed.

We feel it is, therefore, important to give some details of the simple methods by which these considerations were applied and by which the interests of the men in their work was undoubtedly increased, with considerable benefit, not only to the men, but also to the contract.

The contractors set out from the outset to provide good working conditions, although the difficulties and the restricted space on the site made this sometimes more than usually difficult.

In addition to the now usual facilities for welfare of workmen, good canteen accommodation and service was provided, and this provision of space was often used for purposes other than satisfying the ' inner man.'

In the early days of the contract, particularly before the structure began to take on the main shape, which tended to explain itself.

Several means were used to explain to the men the scheme—by simple diagrammatic drawings, charts and models. These and other methods were constantly used to explain the importance of the various sections and the general method of construction. Progress targets were expressed in terms of positions as well as in manhours of work involved and the volume of material to be employed.

These steps were taken to identify the men with their work and to arouse their interest in the building they were constructing. Joint production meetings were held once a fortnight between representatives of management and the men, the latter being represented by a member from each trade. Matters raised were later investigated jointly if it was not possible to reach a decision at the meeting. An incentive scheme was operated for all trades throughout the course of the contract, and the wide and varied nature of the work made it necessary for a small staff to be constantly employed fixing targets for various units of work and agreeing them with the men's representatives.

News letters were issued to the men regularly, giving news of the site and personnel employed, explaining why certain sections were at that time very important ; recording the successes already gained and showing where a lag in progress called for special effort.

This medium was also one of many employed to clear up the differences of opinion which must from time to time occur when large numbers of men are employed on work of this nature. This important task was the responsibility of the Contract Labour Officer, and the success of his work is shown by the fact that the time lost through disputes was infinitesimal. It undoubtedly did much to obtain the good spirit which existed, and it was obvious to the many visitors to the site.

The Contractors' plan of campaign as submitted with the tender visualised establishing on the site a complete staff unit, with a Contract Manager in charge of the contract. This was immediately put into effect, although the Head Office was only over the Bridge. The staff from the outset was a complete unit, but was augmented from time to time as the work developed.

The work on the site was divided into two main divisions — (1) All the reinforced concrete work, (2) Building and General Trades. The reinforced concrete work was in charge of a Senior Engineer responsible to the Contract Manager for carrying out the work in this division in conformity with the general plan, and collaborating with him closely on all matters affecting the Building and Finishing Trades which were the direct responsibility of the Agent—also responsible to the Contract Manager.

Each division had its complement of Supervisory Staff.

The reinforced concrete work above ground level was divided into three sections—the auditorium and the areas on either side, each of which was in charge of a Supervisory Engineer responsible to the Senior Engineer.

The London County Council employed a Resident Engineer and Assistant Resident Engineer on the site during the period of carcass construction, assisted by a Chief Clerk of Works and four Assistant Clerks of Works and Inspectors, also an Engineer responsible for concrete control, who worked in conjunction with the Contractors' Control Engineer.

The Consulting Engineers had an Engineer resident on the site, thus enabling prompt decisions to be given immediately on any matters arising in connection with the structural design and reinforcement.

The Quantity Surveyor had his staff on the site, in charge of the Resident Surveyor. It is very pleasing to be able to state that a most excellent team spirit prevailed throughout the complete organisation, both on the site and their several headquarters, and although the seniors and staff were working at high pressure over long periods, this atmosphere existed throughout the contract, and contributed vastly to the manner in which the work was carried out. It also proved a fine stabiliser on the very few occasions it was tested, as the value of team spirit has been proved before in the heat of the game.

This teamwork, plus the effective work of the remainder of the team—the men engaged on the actual work of construction—really was the factor which produced this imposing structure in the time shown of 20 months. Careful and detailed planning of all operations has been employed throughout. The original master plan prepared and issued to the contract, whilst requiring appreciable adjustment fairly early on in the work for the reasons outlined earlier in the Paper, always served as a valuable guide and stimulus.

A revised master plan was prepared in detail when the construction of the basement and work up to ground level was complete.

Short period detailed progress schedules were prepared regularly, and discussed and attended to at the regular site meetings. These were produced at even shorter intervals during the most critical period of the contract.

For many reasons, and in particular due to the fact that details were not available much ahead of construction, the planning staff had to expend considerable labour and many night hours on the preparation and revisions of these period charts.

There is no doubt, however, but that they proved of very great value, and assisted considerably in ensuring that the various sections were not allowed to lag behind any section of the team.

In view of the short period available by the receipt of drawings and the commencement of work, small key plans were prepared and coloured to indicate areas due to be constructed during the approaching week, and a copy of these was posted in the Architect's office as a solemn warning that it was " now or never " for the details of holes, fixings, etc., required in these areas.

Institution Notices and Proceedings

ORDINARY GENERAL MEETING

An Ordinary General Meeting of the Institution of Structural Engineers was held at 11, Upper Belgrave Street, London, S.W.1, on Thursday, January 25th, 1951, at 5.55 p.m., Mr. Walter C. Andrews, O.B.E., M.I.C.E., M.I.Struct.E., (Vice-President), in the Chair.

The Minutes of the Ordinary General Meeting held on November 23rd, 1950, as published in the Journal in January, 1951, were taken as read, were confirmed and signed.

The following members were elected in accordance with the Bye-Laws. Will members kindly note that the elections, as tabulated below, should be referred to when consulting the Year Book for evidence of membership ?

Students

Bannerjee, Hari Har, B.Sc., Calcutta, c/o Braithwaite Burn & Jessop Construction Co. Ltd., P-13 Mission Row Ext., Calcutta 1, India.

Chate, Hugh Joseph, 31, Cranbrook Road, Redlands, Bristol, 6.

Chilvers, Alan, 13, Ronald Drive, Denton Burn, Newcastle-on-Tyne, 5.

Denson, Francis Ronald, 17, Burnham Avenue, Smithills, Bolton, Lancs.

Duncan, George Alan, 6, Grange Avenue, Billingham, Co. Durham.

Evans, Clifford John, 38, Victoria Road, Penarth, Glams.

Jones, Royston William, 2, Station Terrace, Caerphilly, Glams.

Levell, Peter Arthur, 11, Glenarm Road, Brislington, Bristol, 4.

Malloch, Trevor Stuart, 63, Oban Street, Wellington, New Zealand.

Marshall, John Derek, 78, Thackley Old Road, Windhill, Shipley, Yorks.

Mason, John Francis, 186, Nottingham Road, Borrowash, Derbyshire.

Mobed, Sohrab Framroze, Rustom Building, Room No. 11, Preedy Street, Karachi, 3, Pakistan.

Orpiszak, Boleslaw, 12, Belle Vue Road, Walthamstow, London, E.17.

Russoff, Barry, 20, Bunns Lane, Mill Hill, London, N.W.7.

Stone, Geoffrey Jones, Smallbrook, Staverton, Trowbridge, Wilts.

Van Blankenstein, Louis, 505, Endsleigh Court, Upper Woburn Place, W.C.1.

Wyllie, Derek, 53, Manwood Road, Crofton Park, London, S.E.4.

Graduates

Askew, Harold, 8, Lytton Road, Droylsden, Manchester.

Bent, Geoffrey Malcolm, "The Hawthorns," Belbroughton Road, Blakedown, Nr. Kidderminster, Worcs.

Bewick, John Raymond, 5, Lonsdale Avenue, Houghton Lane, Swinton, Manchester.

Gorecki, Alexander, 26, Farrer Road, Longsight, Manchester.

Hainsworth, Kenneth, 7, Church Walk, Thorntree Estate, Middlesbrough.

Harrison, Richard Hayton, B.Sc. (Civil) Birmingham, Fox Lane Ends, Wrea Green, Preston, Lancs.

King, Gordon Arthur, 395, Bury Road, Bolton, Lancs.

Langdon, John, B.Sc. (Tech.) Manchester, "Wychwood," Barfield, Ryde, Isle of Wight.

Loxton, Peter Pilkington, B.Sc. (Civil) Cape Town, A.M.I.C.E., c/o South Africa House, Trafalgar Square, London.

Pearce, David John, B.Sc.(Tech.), Manchester, 459, Altrincham Road, Wythenshawe, Manchester.

Roberts, Douglas Arthur, B.Sc.(Civil), Leeds, 29, Montagu Crescent, Leeds, 8.

Shaikh, Mohammad Abdul Wahab, 5, Dawood Manzil, Frere Road, Rambaugh, Karachi, Pakistan.

Taylor, Ernest Edward Fred, 32, Roseberry View, Thornaby-on-Tees, Yorks.

Ugona, Samuel Patrick Chukwuma, B.Eng., Sheffield, 191, Mynachdy Road, Cardiff.

Whittaker, John Basil, B.Sc.(Eng.), Manchester, 253, Leeds Road, Ilkley, Yorks.

Wright, Stanley Cempson, B.Sc.(Eng.), London, 35, Preston New Road, Southport, Lancs.

Transfers

Students to Graduates

Bowen, Hugh Derrick, 20, Barnmead Road, Beckenham, Kent.

Grace, John Stephenson, 8, St. Leonards Road, Waddon, Croydon, Surrey.

Hamp, Eric, 11, Cannon Hill Lane, Merton Park, London, S.W.20.

Keats, Gerald, 3, Clifton Gardens, N.W.11.

Staig, Alan Franklin, B.Sc.(Eng.), London, Beenham House, Beenham, Nr. Reading, Berks.

Thompson, Donald, 70, Avon Road, Burnage, Manchester, 19.

Wotton, Thomas Richard, 56, Harlington Road East, Feltham, Middx.

Graduates to Associate Members

Barnes, William Gordon, 82, Thirteenth Avenue, Woodville North, Adelaide, Australia.

Ryss, Jurie, B.Sc.(Eng.), Rand, 801, Africa House, 56/60, Rissik Street, Johannesburg, South Africa.

Associate-Members to Members

Christie, William Dunlop, 27, Ebrington Road, West Bromwich, Staffs.

Members to Retired Members

Bruford, Stanley John, F.R.S.A., A.M.I.Mech.E., Chief Engineer (Construction), Munnar P.O., High Range, Travancore, S. India.

Maltby, George, 19, Waterloo Street, Glasgow, C.2.

White, Arthur Stephen, M.I.Mech.E., Grey Tiles, Lower Road, Bookham, Surrey.

Re-Admission

Associate-Membership

Hanrahan, Edward Thomas, M.Eng., A.M.I.C.E., 90, Bushy Park Road, Rathgar, Dublin.

OBITUARY

The Council regret to announce the deaths of Francis Edson Drury (Past-President), Kenneth Dundas and Edward Meredith (Members).

RESIGNATIONS

Notification was given that the Council had accepted with regret the resignations of George Henry Farleigh, Ernest Henry Macmillen (Members); Oliver Joseph

Jeparatnam Watson, Oswald Wright (Associate Members) ; Norman Frederick Batt (Graduate).

HONOURS AND AWARDS

In offering their sincere congratulations to the following members on the distinctions recently conferred upon them, the Council feel that they are also expressing the good wishes of the Institution.

Order of the British Empire—C.B.E.

Mr. Henry Berry (Honorary Associate).
Dr. Oscar Faber (Past President).
Mr. N. Wynne-Jones (Member).

Order of the British Empire—O.B.E.

Mr. J. E. Swindlehurst (President).
Mr. L. Scott-White (Past President).

Order of the British Empire—M.B.E.

Mr. C. A. Marques (Member).

OVERSEAS REPRESENTATIVE

The Council have appointed Mr. A. J. Francis, M.Sc., A.M.I.C.E., (Associate-Member), to be the Institution's Representative in Victoria and New South Wales, Australia.

REPRESENTATION ON OUTSIDE COMMITTEES

The Council have made the following nominations of members to represent the Institution :—

City of Nottingham Education Committee (Advisory Committee)

Mr. G. Drysdale (Member).
Mr. C. Probert (Associate-Member).

Royal Technical College, Salford (Engineering Advisory Committee)

Mr. J. Kershaw, Junior (Member).

RESEARCH AWARDS

The Council have made the following awards for the Session 1949-50 :—

Research Medal

Professor J. F. Baker, O.B.E. (Member), for a paper entitled " The Design of Steel Frames."

Research Diplomas

Mr. S. Mackey (Associate-Member) and Mr. D. Brotton (Graduate), for a Joint Paper entitled " An Investigation of the Stress Distribution in a Welded Plate Girder for the Margam Plant."

INSTITUTION PUBLICATION

The following publication has been reprinted and it has been necessary to increase the sale price to 2s. 6d. 10(II)1946 " Formulae for Computation of Stresses."

FORTHCOMING MEETINGS

The following meetings will be held at 11, Upper Belgrave Street, London, S.W.1 :—

Thursday, March 1st, 1951

The MacLachlan Lecture. Mr. W. Shearer Smith, A.M.I.C.E. (Associate Member), on " Cold Formed Sections in Structural Practice with a Proposed Design Specification," 6 p.m.

Thursday, March 8th, 1951

Ordinary Meeting at 6 p.m., when Mr. C. L. a'Court, B.Sc., M.I.C.E., will present a paper on " Special Treatment for Portland Cement Concrete."

Thursday, March 29th, 1951

Ordinary General Meeting for the election of members, 5.55 p.m., followed by an Ordinary Meeting at 6 p.m., when Mr. E. Roland Hole (Member), and Mr. F. C. Eales (Associate Member), will present a paper on " The Construction of the Royal Festival Hall."

Thursday, April 5th, 1951

Joint Meeting with the British Section of the Société des Ingénieurs Civils de France at 6 p.m., when a paper on " The Boracic ' Soffioni ' of Tuscany and their Development at Larderello," by Dr. Alfredo Mazzoni will be presented by Mr. Peter Gerard, B.Sc., A.M.I.E.E. (Member). The paper will be illustrated by lantern slides and a film.

Thursday, April 12th, 1951

Ordinary Meeting at 6 p.m., when Lt.-Col. G. W. Kirkland, M.B.E. (Member), and Mr. A. Goldstein, B.Sc.(Eng.), A.C.G.I., D.I.C. (Graduate), will present a paper on " The Design and Construction of a Large Span Prestressed Concrete Shell Roof."

Thursday, April 26th, 1951

Ordinary General Meeting for the election of members, 5.55 p.m., followed by an Ordinary Meeting at 6 p.m., when Dr. K. Hajnal-Konyi (Member), will present a paper on " Comparative Tests on Various Types of Bars as Reinforcement of Concrete Beams."

Thursday, May 24th, 1951

Annual General Meeting.

Members wishing to bring guests to the Ordinary Meetings announced above and to the MacLachlan Lecture are requested to apply to the Secretary for tickets of admission.

LONDON GRADUATES' AND STUDENTS' SECTION

The Annual General Meeting of the Section will be held at 11, Upper Belgrave Street, London, S.W.1, on Tuesday, March 13th, 1951, at 6 p.m., when the film " The Failure of the Tacoma Narrows Bridge " will be shown.

Hon. Secretary : W. S. Twelvetrees, 6, Dell Road, Enfield Wash, Middlesex.

EXAMINATIONS, JULY, 1951

The Examinations of the Institution will be held at centres in the United Kingdom and Overseas on July 17th and 18th (Graduateship), and July 19th and 20th (Associate-Membership).

SESSIONAL PROGRAMME, 1951-52

The Literature Committee are now considering and selecting papers for inclusion in the Sessional Programme for 1951-1952. Members who may wish to offer papers during the coming Session are invited to communicate with the Secretary.

REPRINTS FROM THE JOURNAL

Authors who wish to obtain reprints of their papers published in the Journal, are requested to communicate with the Publishers at least one calendar month before the date of publication of the paper in question.

JOURNAL BINDING, 1950

A binding case can be supplied for the twelve issues of the Journal, January-December, 1950 (Volume 28), price 10/– post free.

The price for binding volumes is 24/– per volume, inclusive. This price is for the half-leather binding which has been in use for some years.

It is requested that all parcels and Journals forwarded for binding should bear the name, address and rank of the member concerned. All volumes for binding must be despatched to the Institution by April 1st, 1951.

An Index will be included in all volumes bound. This index will not be generally distributed, but members and others wishing to have a copy, should apply to the Secretary.

EXAMINATIONS

Preparation for the Examinations of the Institution by Attendance at Technical Colleges

A candidate for Graduateship or Associate-Membership may be able to attend a technical college ; these notes are intended to guide him in choosing the most suitable instruction.

Preparation for the Graduateship Examination

Technical colleges offer :

(a) Full-time courses for degrees or Higher National Diplomas in Building or Engineering.

(b) Part-time day or evening courses for Higher National Certificates in Building or Engineering.

If he obtains a Higher National Certificate or Diploma complying with Appendix II, Section V, of the Regulations Governing Admission to Membership, the candidate will be exempted from the Graduateship Examination.

Alternatively, he may study subjects selected from the available courses and sit the Graduateship Examination. At technical colleges, courses are usually available in Building Science or Engineering Science, Strength of Materials, Theory of Structures and Surveying, but students are not normally allowed to select subjects from National Diploma or Certificate courses unless they can show evidence of sound training in more elementary studies. The advice of the College Authorities should be followed.

Preparation for the Associate-Membership Examination

At some technical colleges there are part-time courses in Structural Engineering which cover the syllabus of the Associate-Membership Examination. At other colleges the candidate must rely on Higher National Certificate courses or on advanced courses in Building, Civil Engineering or Municipal Engineering : these cover only part of the requirements for the Associate-Membership Examination.

Colleges in the first category provide at least two years of instruction in Theory of Structures and in Structural Engineering Design and Drawing, up to Associate-Membership standard. They also give instruction in Structural Specifications, Quantities and Estimates.

The Colleges which have informed the Institution that courses in Structural Engineering are available are :—

Belfast College of Technology ;
Birmingham Central Technical College ;
Bolton Municipal Technical College ;
Bradford Technical College ;
Derby Technical College ;
Dudley and Staffordshire Technical College ;
Glasgow Royal Technical College ;
Liverpool Technical College ;
L.C.C. Brixton School of Building ;
L.C.C. Hammersmith School of Building and Arts and Crafts ;
Manchester College of Technology ;
Middlesbrough Constantine Technical College ;
Salford Royal Technical College.

Colleges in the second category provide instruction in Theory of Structures from which the student may reach Associate-Membership standard, but instruction in Structural Engineering Design and Drawing and in Structural Specifications, Quantities and Estimates is not usually so complete. The colleges which have informed the Institution that such courses are available are :—

Brighton Technical College ;
Cardiff Technical College ;
Huddersfield Technical College ;
Leeds College of Technology ;
London Battersea Polytechnic ;
London Northampton Polytechnic ;
L.C.C. Westminster Technical College ;
Plymouth and Devonport Technical College ;
Preston Harris Institute ;
Wigan Mining and Technical College ;
Woolwich Polytechnic.

Students attending colleges in the first category are advised to take the organised courses in Structural Engineering. Students of Graduate-membership standard will usually be allowed to select subjects from courses provided by colleges in the second category.

THE MACLACHLAN LECTURE

General Conditions

Through the generosity of Mr. John MacLachlan (Retired Member), the Council was able in 1948 to institute an Annual Lecture to be competed for by Associate-Members. The conditions of the presentation are as follows :—

1. The Institution of Structural Engineers shall institute a written lecture to be known as the MacLachlan Lecture, and to be held annually.

2. The subject of the Lecture may be on any aspect of Structural Engineering so long as in every second year the subject shall be confined to steel structures.

3. Entrance into the competition for the Lecture shall be confined to Associate-Members of the Institution.

4. All papers entered for the competition shall be submitted to assessors to be appointed by the Council of the Institution and all such papers (including the prize-winning Lecture) shall be available for publication in the Journal of the Institution at the discretion of the Council.

5. The winner of the competition shall be required to present the Lecture to a meeting of the Institution at which he will be presented with the sum of £17 10s.

6. In the event of there being no winner of the competition in any one or more years, whether because no lecture is submitted or because no lecture submitted is considered to be of sufficient merit to warrant an award, or for any other reason, the Institution shall transfer this sum to the Research Fund of the Institution.

Particulars of the Competition for 1951

1. The MacLachlan Lecture will be given at a meeting of the Institution to be arranged towards the end of 1951.

2. The subject of the Lecture shall be confined to Steel Structures.

3. The work should be submitted as the script of a lecture which the author, if successful in the competition, will deliver before an audience in the course of about one hour. The development of mathematical formulæ and detailed calculations should be avoided as far as possible in the text ; if they are essential they should be embodied in appendices. Photographs, drawings, graphs, etc., which would appear as illustrations to the Lecture in published form, should accompany the script. If additional illustrations would be shown as slides, a list of these should be included.

4. Six copies of each Lecture should be submitted and should be addressed to the Secretary of the Institution.

5. The closing date for the receipt of entries by the Institution is Friday, March 30th, 1951.

DISTRICT SURVEYORS' EXAMINATION

The London County Council has given notice that the next examinations of persons desirous of obtaining a certificate of proficiency to perform the duties of the office of district surveyor will be conducted in London in October, 1951, by the Board established by Section 77 of the London Building Acts (Amendment) Act, 1939. The minimum age limit for candidates is 25.

Possession of this certificate carries eligibility to compete for appointment to vacant positions as District Surveyor at maximum salaries ranging from £1,200 to £1,800 a year (inclusive), or as Assistant District Surveyor (salary scale £840 by £40 to £960 a year).

Particulars of the examination may be obtained from the Architect to the Council, County Hall, Westminster Bridge, S.E.1.

Of the nine candidates who sat for the examination in October, 1950, three were awarded the certificate of proficiency, two of whom were Mr. J. Dufton, A.M.I.Struct.E., and Mr. R. C. Thurlow (Graduate).

PROFESSIONAL POSTS IN GOVERNMENT DEPARTMENTS

The Civil Service Commissioners announce that applications are invited for permanent appointments to the basic (Assistant) grade of Structural Engineer in a number of Departments in England and Scotland. Applications will be accepted at any time up to and including December 31st, 1951. Selected candidates will be interviewed as soon as possible after the receipt of their application forms, and successful candidates may expect early appointments. Candidates are advised to apply as soon as possible.

All candidates must be at least 25 and under 35 years of age on January 1st, 1951, with extension for regular service in H.M. Forces, and up to two years for permanent Civil Servants. All candidates must have the appropriate professional qualifications and experience.

The London salary scale for men aged 30 and over is £600 × £25 — £750. Lower starting salary for younger entrants (from £475 at age 25).

(The next higher grades are :— Main Grade £750 × £25 — £1,000, Senior Grade £1,050 × £35 — £1,270).

Forms of application and full details may be obtained from the Civil Service Commission, Scientific Branch, Trinidad House, Old Burlington Street, London, W.1, quoting No. 3405.

PERSONAL

Mr. A. J. Francis (Associate-Member), has been appointed Professor of Civil Engineering at Melbourne University, Victoria, Australia.

Mr. A. L. Handley (Member), has been appointed District Surveyor to the City of London.

BRANCH NOTICES

Lancashire and Cheshire Branch

The following meetings have been arranged :—

Friday, March 16th, 1951

Fifth Ordinary Meeting. Mr. L. W. Elliott, A.R.I.B.A., A.M.I.C.E., on " Prestressed Concrete in Architecture."

Thursday, April 19th, 1951

Annual Business Meeting.

All meetings will be held in the Reynolds Hall, College of Technology, Manchester, at 7 p.m.

Hon. Secretary : G. Greenlees, A.M.I.Struct.E., 16, Mayfield Avenue, Stretford, Lancs.

Graduates' and Students' Section

The following meeting has been arranged :—

Friday, March 30th, 1951

Annual General Meeting, at the Reynolds Hall, College of Technology, Manchester, 7 p.m. It is requested that as many members as possible should attend the meeting.

Hon. Secretary : F. M. Turner, 11, Colwell Avenue, Stretford, Lancs.

Midland Counties' Branch

The following meetings have been arranged :—

Monday, March 5th, 1951

Films : " The Failure of the Tacoma Narrows Bridge," and " The Construction of the Boulder Dam " will be shown at the Merchant Hall, Albion Street, Derby, at 7 p.m.

Friday, March 30th, 1951

Mr. W. P. Andrews and Mr. P. G. Bowie, A.M.I.C.E., (Member), on " The Solution of the Car Parking Problem Here and in the U.S.A."

Friday, April 27th, 1951

Annual General Meeting, followed by a paper by Mr. H. E. Brooke-Bradley, O.B.E. (Member), on " The Problem of the Supporting Power of the Subsoil."

The meetings will be held at the James Watt Memorial Institute, Birmingham, at 6 p.m.

Hon. Secretary : E. R. Deeley, A.M.I.Struct.E., Arranmoor, Adshead Road, Dudley, Worcester.

Graduates' and Students' Section

The following meetings have been arranged :—

Tuesday, March 20th, 1951

Annual General Meeting. Films : " The Failure of the Tacoma Narrows Bridge," and " The Collapse of the Remagen Bridge."

Saturday, May 5th, 1951

Visit to the Building Research Station, Watford.

Tuesday, May 29th, 1951

Joint Meeting with the Students' Section of the Institution of Civil Engineers. A talk on Code of Practice 114 : 1948—The Structural Use of Reinforced Concrete in Building, will be given by Mr. A. T. Clarke (Associate Member).

The meetings will be held at the James Watt Memorial Institute, Birmingham, at 7 p.m.

Hon. Secretary : M. H. Evans, B.Sc., 107, Metchley Lane, Harborne, Birmingham, 17.

Northern Counties' Branch

The following meetings have been arranged :—

Tuesday, March 6th, 1951

Mr. P. L. Capper, T.D., M.Sc., A.M.I.C.E. (Associate Member), on " Testing Methods in Structural Engineering," at Middlesbrough.

Wednesday, March 7th, 1951

Mr. P. L. Capper will present the same paper at Newcastle.

Tuesday, April 3rd, 1951

Annual General Meeting, at Middlesbrough.

The meetings commence at 6.30 p.m., preceded by tea at 6 p.m., the Middlesbrough meetings being held at the

Cleveland Scientific and Technical Institution, Corporation Road, and that at Newcastle in the Neville Hall, near the Central Station.

Hon. Secretary: J. A. Williams, A.M.I.C.E., A.M.I.Struct.E., 57, Reid Park Road, Jesmond, Newcastle upon Tyne.

NORTHERN IRELAND BRANCH

The following meetings have been arranged:—

Tuesday, March 6th, 1951

Film: "Military Pile Driving."

Tuesday, April 3rd, 1951

Annual General Meeting.

The Meetings will be held in the College of Technology, Belfast, at 7.15 p.m.

Hon. Secretary: S. G. Duckworth, M.I.Struct.E., Lisleen, 13, Finaghy Road North, Belfast.

SCOTTISH BRANCH

The following meeting has been arranged:—

Tuesday, April 17th, 1951

Annual General Meeting at Ca'doro Restaurant, Glasgow, at 6 p.m.

Hon. Secretary: D. G. Drummond, B.Sc., A.M.I.C.E., A.M.I.Struct.E., 11, Woodside Terrace, Glasgow, C.3.

SOUTH WESTERN COUNTIES' BRANCH

Hon. Secretary: E. W. Howells, M.I.Struct.E. 10–12, Market Street, Torquay, Devon.

WALES AND MONMOUTHSHIRE BRANCH

The following meetings have been arranged:—

Friday, March 2nd, 1951

Mr. A. J. Harris, B.Sc., A.M.I.C.E., on "Prestressed Concrete in Civil Engineering Works," at Colwyn Bay.

Tuesday, March 6th, 1951

Mr. G. R. Brueton, A.M.I.C.E. (Associate Member) on "The Fatigue of Materials related to Structural Design," at Swansea.

Thursday, March 22nd, 1951

Mr. G. R. Brueton will present the same paper at Cardiff.

Friday, March 30th, 1951

Mr. A. V. Hooker, A.M.I.C.E. (Associate Member) on "Structural Engineering at Abbey Works," at Colwyn Bay.

Wednesday, April 11th, 1951

Visit to Abbey Works site, Margam, to inspect the buildings for the New Strip Mills for the Steel Company of Wales.

Wednesday, April 11th, 1951

Annual Dinner, at Porthcawl.

Tuesday, May 1st, 1951

Annual General Meeting.

The meetings in Cardiff will be held at the South Wales Institute of Engineers, Park Place, at 6.30 p.m.

The meetings in Swansea will be held at the Mackworth Hotel, at 6.30 p.m.

The meetings at Colwyn Bay will be held at the County Buildings, at 6 p.m.

Hon. Secretary: E. R. Steward, A.M.I.Struct.E., Edrom, Ashleigh Road, Blackpill, Swansea.

WESTERN COUNTIES' BRANCH

The Fourth Meeting of the Session was held on Friday, January 5th, 1951, at the Grand Hotel Bristol, Mr. P. C. Girdlestone, M.A., A.M.I.C.E., M.I.Mech.E., in the Chair. A paper entitled "The Reconstruction of the Colston Hall, Bristol, 1950" was presented by Mr. N. G. T. Ball (Country Associate-Member of Council) to a large audience, and a vote of thanks was proposed by Mr. C. H. Williams (Member).

The following meetings have been arranged:—

Friday, March 2nd, 1951

Mr. F. W. L. Heathcote and Mr. A. H. Chilver, B.Sc. (Student) on "Light Gauge Structural Steel Sections," at College of Technology, Bristol.

Friday, April 6th, 1951

Annual General Meeting, followed by a talk by an Architect, at Grand Hotel, Bristol.

Thursday, April 26th, 1951

Joint Meeting with the Institution of Civil Engineers. Mr. Gilbert Roberts, B.Sc., A.M.I.C.E. on "The Dome of Discovery—Festival of Britain Site," at Reception Room, Bristol University, Queen's Road.

All meetings, except the last will be held at 6.15 p.m. preceded by tea at 5.30 p.m. The last meeting will be held at 5.30 p.m., preceded by tea at 5 p.m.

Hon. Secretary: C. E. Saunders, M.I.Struct.E. Dunkery, Edward Road, Walton St. Mary, Clevedon, Somerset.

YORKSHIRE BRANCH

The Fourth Ordinary Meeting was held on Wednesday, January 17th, 1951, at 6.30 p.m., in the Great Northern Hotel, Leeds, when Mr. H. C. Husband, B.Eng., M.I.C.E., M.I.Mech.E. (Member), was in the Chair and thirty-four members and visitors were present. A lecture entitled "Welded Highway Bridges" was given by Mr. D. J. Davies, B.Sc., A.M.I.C.E., (Associate Member), of the British Constructional Steelwork Association. A discussion followed and Mr. Davies answered questions. A vote of thanks proposed by Mr. C. Wilkinson (Member), and seconded by Mr. S. Mackey, B.Sc., A.M.I.C.E. (Associate Member), was warmly accorded.

The following meetings have been arranged:—

Wednesday, March 21st, 1951

Joint Meeting with the Yorkshire Association of the Institution of Civil Engineers. Mr. J. Guthrie Brown, M.I.C.E., (Member), on "Hydro-Electric Developments, Pitlochry," at Leeds University, at 7 p.m.

Wednesday, April 18th, 1951

Professor R. H. Evans, D.Sc., Ph.D., M.I.C.E., M.I.Mech.E. (Member), on The Institution of Structural Engineers' "Report on Prestressed Concrete."

Wednesday, May 9th, 1951

Annual General Meeting. Mr. J. G. Faber, B.Sc., A.M.I.C.E. (Associate Member), on "Construction of 240 ft. Clear Span Sheds for Aircraft Production."

All meetings to be held at the Great Northern Hotel, Leeds, at 6.30 p.m., except the joint meeting on March 21st.

Hon. Secretary: E. Wrigley, A.M.I.Struct.E., City Engineer's Dept., Civic Hall, Leeds, 1.

UNION OF SOUTH AFRICA BRANCH

Branch Hon. Secretary: A. E. Tait, B.Sc., A.M.I.C.E., A.M.I.Struct.E., P.O. Box No. 3306, Johannesburg. During week-days Mr. Tait can be contacted in the City Engineer's Department, City Hall, Johannesburg 'phone 34–1111 ext. 257.

Natal Hon. Secretary: E. G. Bennett, A.M.I.Struct.E. c/o Reinforcing Steel Co. Ltd., P.O. Box 478, Durban.

Cape Section Hon. Secretary: R. Stubbs, M.I.Struct.E. P.O. Box 1692, Cape Town.

The Tuscan Boracic "Soffioni" and their Development at Larderello*

By Dr. Ing. Alfredo Mazzoni

Introduction

I render cordial thanks to the Institution of Structural Engineers and the British Section of the Societe des Ingenieurs Civils de France for the honour of addressing them to-day on the subject of the strange and fascinating phenomenon: the "Soffioni" of Tuscany, and the industrial undertakings at Larderello.

I also render particular tribute to my friend, Mr. Peter Gerard, who has taken the initiative in this development and translated this paper into English, and who is to read it to you at the Meeting, as my limited knowledge of your language does not allow me to do so.

Synopsis

The paper sets out a general illustration of the industrial application of the vapours emerging from the Tuscan "Soffioni." It shows how these vapours have been captured and utilised throughout the centuries, culminating with a fuller description of their uses in present-day electrical and chemical industries.

Particular reference is made to several power stations in Tuscany, notably that at Larderello. The paper provides an account of the installations at most of the plants mentioned.

Kindred subjects are alluded to, in order to provide an orientation of the existing industry relative to its development. It is shown how these wells were located and finally mastered, how the vapours were utilised in earlier industries, and how these installations were re-equipped to cope with present-day requirements.

The paper is a record of outstanding engineering achievements, in terms of ingenuity, daring, economy, and their resulting benefits to the distributors and consumers of electrical energy.

The implicit conclusion may be drawn, that similar "Soffioni" or "geysers" can be tapped, controlled and employed as cheap sources of power, while the discharge vapours can be guided into the chemical industry.

It is intended to supplement the paper with detailed technical descriptions of the installations and performance at Larderello III, a new generating station which operates on direct vapour intake and eliminates heat exchangers, with the resulting increase in the efficiency of the plant.

1. General Aspect

The phenomenon of the "Soffioni Boraciferi" in Tuscany has always held great interest, as much for its strangeness and scientific importance as for its possibilities of development.

In their natural state, the "Soffioni" manifest themselves as geysers ejecting white clouds of smoke, which consists of water vapour and a mixture of gases, gushing out of the ground in a more or less violent and noisy manner (Fig. 1).

In certain localities the vapour does not gush out of the ground, but instead it bubbles up in pools of muddy water, which it stirs into violent ebullition. These are the "lagoni" or "bulicami," waters, in which the presence of boric acid was detected for the first time in 1777.

These various types of pseudo-volcanic manifestations are encountered in a vast region covering over 80 square miles south of Volterra, bordering on the provinces of Pisa and Grosseto, on the slopes of the mountains which separate the valleys of Cecina and Cornia.

The countryside around these "Soffioni" is almost always hot and scorched, and its colour is of a reddish yellow hue. Sometimes, however, it happens that the vapour emerges amongst sprouting green bushes, which causes visitors some considerable surprise.

Fig. 1.—Natural Soffioni

If we add the loud whistling noise and the smell of sulphur which accompany the phenomenon it gives the passing traveller crossing these solitary regions a strange eerie feeling.

These manifestations are particularly evident in the localities of Larderello, Castelnuovo, Val di Cecina, Sasso Pisano, Monterotondo, Lago, Lustignano and Serrazzano. These localities are situated roughly along an irregular ellipse about nine miles long and about five miles wide. The altitude of these localities varies from between 650 feet in Lago to 1,700 feet in Monterotondo.

These manifestations are also evidenced in more distant places: in Carboli, in Cagnesi and in Travale towards Siena, as well as in various other localities of that region.

Boring operations in Travale produced a powerful "soffione" in December, 1949.

Paper to be read before a Joint Meeting of the Institution of Structural Engineers and the British Section of the Societe des Ingenieurs Civils de France at 11, Upper Belgrave Street, London, S.W.1, on Thursday, April 5th, 1951, at 6 p.m.

The phenomenon of the " soffioni " is very similar to that of the " geysers " of Iceland, the Azores, America or New Zealand, but it differs from these, in that the geysers eject alternate gushes of vapour and boiling water, whereas the " soffioni " eject almost exclusively nothing but water vapour, and any water seeping into the subsoil is entrained within the vapour when it comes to the surface.

The phenomenon of the Tuscan " soffioni " results manifestly from a combination of various sources of heat emanating from the inner regions of the earth.

The following cases of high temperature springs are certainly related to the " soffioni boraciferi " by virtue of their location and of the chemical composition of their water. These are the springs: Bagno Vecchio, near the " soffioni " of Sasso, Perla, Morbo and St. Michele, in the vicinity of Larderello, Bagnolo, near the " soffioni " of Monterotondo, and lastly, Galleraie, near the " soffioni " of Travale.

Much has been written and argued about the origin of the " soffioni," and several theories have been suggested, but so far none has proved entirely satisfactory As mentioned previously, many analogies may be drawn with the geysers and still more with the volcanic "solfataras," particularly with those of Champs Phlegreens, the Aeolian Islands, and the volcanic regions of Iceland, or of the Rocky Mountains in North America.

It can therefore be accepted that " soffioni " are manifestations of underground activities. On the other hand, comparing them with the chemical and physical actions at the level of the earth's crust, one can neither explain the high temperature of 400°F., nor the high pressure which, to our knowledge, may exceed 30 atmospheres, when the gush is stemmed.

It is presumed that the vapours originate from the magmatic mass situated in the volcanic region of Monte-Amiata, Roccastrada, Sassoforte and Campiglia Marittima. There the trachytic magmas have come to the surface with the lava, and this is particularly striking in Amiata. The theory that the boracic vapours of Larderello and its vicinity originate from the above-said trachytic magmas could be well-founded, the more so as the high degree of radio activity of the vapours confirms the direct origin from the deep, eruptive masses.

The deep, subterranean bases of the " soffioni " probably consist of granite miocene rocks which emerge on the Isles of Thyrrene: Elba, Montecristo and Giglio, and on the Continent, quite near the boracic region of Gavorrano and Campiglia. Quite probably the base rests upon an active magmatic reservoir and is not more than 14,000 feet deep.

According to various Scientists, the magmatic reservoir beneath the gases of the " soffioni " would then appear to be 16,000 feet deep, allowing for the upper sedimentary stratum. This is confirmed by various investigations on several active Italian volcanoes where the depth of the magmatic reservoir varies between 6,000 and 16,000 feet.

There is an essential condition for the formation of a zone holding vapour under pressure, a vapour whose characteristics may be determined. This condition is evidently satisfied by the existence of a covering stratum of eocene shale forming a protective anticline which is practically impervious on the whole, although it may allow large infiltrations at the base. Where that covering stratum does not exist the manifestation of vapour may be experienced, but never at significant pressures.

2. Characteristics of the " Soffioni "

With some imagination one can follow the course o the vapour from the bowels of the earth to the light o day. One can visualise the vapour, accumulated i enormous masses, canalised into huge potholes, passin through smaller ones, branching into channels, small an irregular, and finally emerging at the surface.

In the course of its long journey the vapour loses som of its energy due to the irregular obstructions it continu ally encounters. Nevertheless, granting that the pres sure will fall appreciably in the course of the ascent, i seems less likely that the temperature varies the sam way. It is true that there will be a loss of heat cause by the thermal conductivity of the earth, but this los would be offset by a rise due to the friction created i overcoming the continuous resistance and the occasiona obstacles. The state of the fluid is unknown from it origin to the time it gushes out of the borings after i has passed through various strata. Theory and researc have not yet provided an adequate method for th solution of this interesting scientific problem.

The vapour emerges at a temperature which varie between a minimum of 280°F. and a maximum of 420°F according to the boring. This variation depends mostl on the different depths and positions of the borings an partly on the pressures encountered. It is interestin to note, however, that the vapour emerges at groun level at a temperature much higher than that of a full saturated vapour at the same pressure.

Some authorities hold that the vapour is imbue with this elevated temperature at its origin, othe maintain that this temperature increases through frictio en route, and that the vapour is fully saturated at it origin.

Further research should permit a more precise explana tion of this important problem.

Measurements of the pressure of the " soffioni " ca be carried out on open borings for various values o outlet area, or on closed borings—that is to say: b blocking the vapour outlet.

Measurements of the pressure in closed borings hav given results varying as widely as from 5 to 30 atmo spheres and more, according to the locality. It interesting to note that no absolute relation exist between the output, the pressure and the depth of th hole. This is easily understood since the vapour distributed and the pressure depends on the particula channels through which the vapour ascends. Th channels may be directly connected to the origin, o they may cause the vapour to meander considerably

The output is the quantity of vapour ejected per un time and is one of the most important characteristics.

The pressure at open borings varies between 2 to atmospheres and the temperature between 280°F. an 420°F. The mean exit velocity of the vapour is 410 fee per second, with a maximum of about 1,300 feet pe second.

The composition of the fluid varies, though not much from place to place. The ratio: gas to water vapou by weight, is about 6 per cent. at Larderello and 4 pe cent. at Lago. In some localities it is less than 4 pe cent. and in others it is slightly more than 6 per cent.

The gas content of the " soffioni " consists mainly o organic anhydrides in the following order: methan and hydrogen, sulphurated hydrogen, nitrogen, ammoni and the rare gases (helium, argon, neon, kripton, xenon The fluid also contains a fairly constant percentage o boric acid.

The composition of the fluids in the " soffioni " is comparable to that in the " solfataras " as the content of anhydrides is large, and the content of hydrogen and methane is comparatively small. However, whilst the gas content varies considerably in the " solfataras," the composition of the " soffioni " is remarkably constant and regular. A still more surprising result is obtained on comparing certain values such as the ratio : gas to vapour, ammonia content, organic anhydride and hydrogen. It is then found that the " soffioni " of the entire region vary, though little, by a constant amount, according to certain groupings which seem to correspond to three major geological faults, almost parallel to each other. It is supposed that the origin of the vapour lies beyond these faults, the eastern and most important of which corresponds to the localities of Larderello and Castelnuovo which comprises the localities of Sasso and Monterotondo, and the most westerly fault corresponds to the localities of Serrazzano and Lago. Consequently, the plants using the eastern fault receive a high ratio of gas to vapour, while those on the central fault receive a reduced ratio, and those on the western fault receive a still lower ratio of gas to vapour. The content of boric acid, however, is almost constant in all cases.

3. The Old-established Boric Acid Industry

How old are the " soffioni " and how much longer will they last ?

Definite historic indications of their existence appear towards the end of the first century A.D. in the fourth book of the poem " De Rerum Natura," by Lucrece Caro, and later in parchments of the seventh century, and also in documents written about the year 1000 A.D.

At the end of the sixteenth century, descriptions of the phenomenon were given by the physician Ugolino da Montecatini, in his work " De Balneis," and also by Andrea Bacci Elpidiani, Architect of the Pope, in the sixth book of his work " De Thermis." A century later, Paolo Mercula described the " Lagoni " of Leccia, near Monterotondo, in his " Cosmografia Generale," while the " soffioni " of Tuscany were illustrated by Michele Sabanarola. In the eighteenth century, Giorgo Agricola and Giovanni Targioni Tozzetti, describing their travels in 1742, once again drew attention to the phenomenon.

Considering the long history of this phenomenon and its perseverance throughout the ages, one may forecast that it will continue for some period which, although unknown, should certainly be very long and measurable in terms of centuries. We can therefore proceed with industrial developments of these manifestations without fear.

Formerly, these natural manifestations were exploited for the manufacture of alum, vitriol and sulphur, to be used in the woollen industry, which was already flourishing at Florence.

The chemist, Umberto Francesco Hoefer, of Cologne, Director of the Spiceries at the Court of the Grand Duke of Tuscany, discovered, in 1777, that the waters of the " Lagoni " of Monterotondo and Castelnuovo V.C., contained boric acid. This discovery changed that rough and inaccessible region, that region of strange and terrible manifestations, that inferno, into a sphere of production and prosperity.

During that period, Paolo Mascagni investigated a system by means of which he could extract boric acid from the waters of the lagoni by evaporation. Amongst Mascagni's memoirs is the first suggestion that the heat of the volcanic phenomenon might be utilised to economic advantages. He pointed out that this heat might be used to separate the boric acid from the water which contains it, by heating the lead boilers which were proposed for the process of separation. This resulted in an appreciable saving in fuel, which was otherwise provided from the timber neighbouring forests.

Various companies have availed themselves of the discoveries of Hoefer and of the investigations of Mascagni. These companies have set up basic industries in the area. In 1818 the firm of Larderel & Co., of Leghorn, set out to exploit these manifestations. The firm, under the direction of Francois de Larderel, had by that time absorbed all its competitors. They laid out artificial lagoni in the grounds of a small factory near Montecerboli. The works were dubbed " Larderello " in 1846 by the Grand Duke of Tuscany, Leopold II, who selected the name with " motu proprio " in honour of the founder of the firm.

This young industry was soon faced with all the difficulties associated with the extraction of boric acid. De Larderel was the only extractor remaining on the market, and he too would have failed, had he not used all his energy in applying the results of Mascagni's investigations.

De Larderel invented the " lagone couverte " (covered lagone), which was nothing more than a masonry dome, constructed under severe difficulties. The dome was erected over the crevasse that released the natural vapour.

The vapour was conveyed through earthenware pipes so that it flowed underneath the lead boilers in which the water of the lagoni was collected. The concentration obtained in this way was so economical, that the product was able to compete with the borax, obtained directly from the mineral " tinkal," which was imported from abroad.

Further progress was made in 1842, when Adriano de Larderel invented and set up inclined boilers, named after the inventor, " Chaudieres Adriennes " (Adrian boilers). The natural vapour served as a continuous pre-heating system. This boiler system is always employed, because of its simplicity and its low running costs.

In 1884 refineries were opened in England, following the discovery of boron deposits in America. It became therefore necessary to prepare still purer products than hitherto. In the same year, 1884, a refinery for boric acid was established at Larderello, together with a borax factory, while another refinery was set up at Castelnuovo Val de Cecina. The process of collection and treatment of the boric waters, however, remained unchanged until the year 1900, when scientists and renowned technicians were consulted to collaborate with the industry.

4. Extraction of Natural Vapour—Boring Operations—Measurements

(a) Early Research

Direct research on subterranean vapour proved very difficult, as it was not easy to trap a fluid of very high temperature and pressure, which flowed through crevasses and meandered deep down in the earth.

The first tests were carried out by Francois de Larderel in 1832. These tests consisted of soundings by means of rock drills, similar to those adopted for the boring of

artesian wells. The first tangible results, however, were not obtained until the year 1856.

Apart from the ordinary difficulties which one encounters in boring operations, the process of sounding in the areas of the "soffioni" involves problems which are not at all easy to overcome.

In fact, in the region of the "soffioni," one encounters alternate strata of metamorphised rocks and others which are harder and more compact, while there is generally considerable evidence of folding. A further complication arises from the active chemicals within the hot vapour. Various borates will foster the formation of cements, which will make the extraction of the rock drills a difficult, lengthy and uncertain process.

There is yet another complication. If the vapour is travelling in veins, which subject it to intensive pressure, the vapour is liable to explode. This will cause the drills to be propelled out of the earth, together with stones and boiling water. Towers and machines will be damaged and the personnel will be in considerable peril.

In 1924 and the subsequent years soundings were taken with rotating steel drills and central water jets. It was possible to bore holes 17 inches diameter, and reach the remarkable depth of 2,000 feet.

It is interesting to note that the depth is not a criterion of the output of vapour released. Some borings at great depth yielded much less vapour than borings of lesser depth in the same zone. This is not at all surprising, and we now attempt to hit the higher level meanders, which have their origin at a much deeper level, where the flow of vapour exhibits high temperatures and pressures.

It follows, that the more important a vein we strike, the larger still must be the deeper counterpart of that vein, and the larger will be the quantities and the higher the pressures which we shall obtain. It happens, at times, that the vein is small and that its output is consequently not very large. If this vein, however, derives its vapour from another larger vein, not too far removed, then the vapour in the small vein may possess a considerable pressure. Contact between the drill and that vapour may result in the latter overcoming the weight of the drill and the pressure of the water column, and finally there might ensue a violent explosion. After the first eruption, the vapour will no longer exist in bulk, but in most cases it will no longer be possible to use water jets for further sounding. The opportunity to discover larger veins, at greater depths, in this particular sounding, is therefore lost.

One of the most outstanding problems in this field of research is to find a method which will enable us to continue boring for larger veins after we have struck a less important one.

Successful boring operations were carried out, in those days, by means of the rotating drill and its accessories. The vapour was controlled and brought to the surface, and further drilling was carried out, leading into successively lower veins. The output was in the order of 22,000 to 66,000 lb. of vapour per hour.

Knowing that rotary drills could not go to any greater depths, successful application was made of percussion machinery of the Pennsylvania type. The outfit was operated in a boring, where the rotary drill had caused an explosion. By means of this special drilling equipment new wells were reached, and an hourly output of up to 132,000 lb. was obtained.

In 1929 a more powerful boring machinery wa introduced. Simple to work, it could be adapted t any drilling system, whether percussion drilling rotary drilling. The density of the water jet was als augmented by heavy emulsions, obtained by th addition of clay. The increase in the density of th water jet helped to balance the vapour pressure, s that it became possible to advance to greater depths.

At Larderello, a large new sounding had reache the depth of 877 feet. On the morning of the 26th March 1931, the vapour burst forth with great impetus, bu no one was injured and no machinery was damaged The phenomenon presented such an awe-inspirin spectacle, that many people considered it impossib to control this powerful jet. In fact, the output amounte to 485,000 lb. of vapour per hour, at an absolute pressu of 3½ atmospheres and a temperature of 400°F. (Fig. 2

Thanks, however, to the skill of the engineers an spirit of sacrifice displayed by the staff, stimulate by an unparalleled uproar of emerging vapour, it wa possible to control the gusher on the 18th April, 193

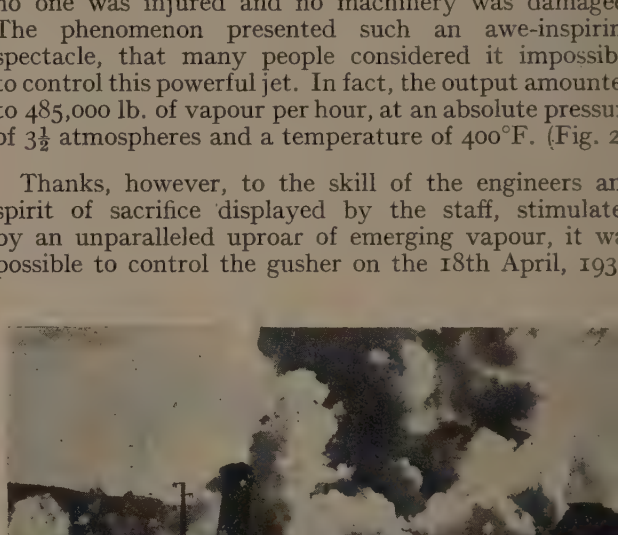


Fig. 2.—Soffionissimo No. 1, at Larderello

when the control valves were fitted and the mouth the "soffione" was connected to the pipes leadin into the factory. The "soffione" was now und control and ready for use.

Another "soffionissimo" was obtained at Larderell The machinery was of the type described above, thoug it was considered improved. On the 22nd April, 193 after nine months of intensive work, an explosio occurred. The boring had reached the depth of 695 fee This "soffionissimo" produced an output of 375,000 l of vapour per hour, at an absolute pressure of 3 a mospheres and a temperature of 356°F.

In the subsequent years the investigation was carrie on methodically, with great vigour, and when th World War broke out in 1939, more than 140 activ wells were in operation after 100,000 feet of borin giving a total output of 4,410,000 lb. of vapour per hou

(*b*) *Present Research*

It should be noted that the technique of borin operations in the Larderello area differs from tho adopted in the boring for oil, minerals, water, etc partly because of the phenomenon of great heat, whic increases the complications and makes the reachin of great depths extremely difficult. In fact, the mudd mixture has at times reached a temperature of 194 203°F., with the consequent formation of hard lum

interfering with the rock drill. The hole had to be by-passed, involving a loss of time and considerable expense. Amongst other difficulties which present themselves during boring operations losses of the muddy mixture, often due to its efflux into crevasses, call for quick remedies.

" Larderello " possess several powerful machines of the rotary type, for borings to a depth of 4,900 feet to 6,600 feet, with an initial diameter of the hole equal to 20 to 28 inches. They also possess various percussion machines, for a depth of 3,300 feet to 4,900 feet, with an initial diameter of the hole equal to 24 inches. Finally they possess numerous smaller machines of both types capable of reaching a depth of some 1,700 feet with an initial diameter of 12 inches to 20 inches.

" Larderello " have recently acquired new machines purchased from the Oil Well Supply Co. of New York, which can reach a depth of 7,400 feet with an initial diameter of 28 inches (Fig. 3).

The following are the boring systems now in use :—

Rotary with tri-conic rock drills.

Percussion, with hollow shaft and a forced circulation of muddy water.

Rotary, with core sample.

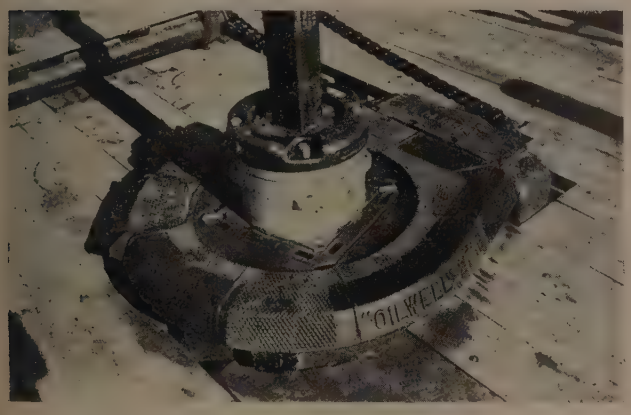


Fig. 3.—Rotary oil well

Rotary System

This is the system most frequently employed. The rotary machines which were put into operation in 1947 and 1949, using the tri-conic rock drills devised by Hughes in America, have shown advances far superior to those of other machinery—considering the type ground through which they have to operate (Fig. 4).

The Percussion System, with hollow shafts and a circulation of muddy water

Until recently, this was the only system in use. It permitted penetration to a great depth, as well as the extraction of a large output of vapour. Because of its slow operation, however, it is at present only employed on borings of medium depth and through fairly easy ground.

Rotary System with core sample

This is used for geological investigations beyond the zone of operations when samples are required to give information on the strata which have been penetrated.

Studies have been undertaken, and successful experiments have been carried out, with a view to obtaining vapour from blocked or sterile wells, and to augment the output from active wells, by bursting a charge at the base of the well using direct detonation. This is a very interesting problem, which may extend the horizon of the field of boring operations, but it is beset by great difficulties and dangers, because the great heat in the wells makes it difficult to use explosives, detonators or even electric fuses.

Geological Studies

Geological studies of the Larderello region were commenced by the late Professor Ferdinando Lotti, and continued by the engineer Emilio Cortese.

A geological department was started in 1926, to plot exact sections for each boring and analyse samples of the rocks encountered during boring. The rock analysis is carried out in a chemical laboratory.

When the boring is completed, a section is drawn, supplemented by notes of accidents which have occurred during the operations, as well as notes showing the output of the vapour and its chemical composition.

The stratification can be determined, by combining all these sections on pre-determined directrices. When all these directrices are assembled, a model is constructed, on which the geological positions of the plants are determined and the various centres of exploitation can be fixed.

The findings of this complex research work confirmed that there is no relation between the output of a vein and the geological period of the stratum in which the vein runs. Some plants have struck very old rocks at a small depth. Further, knowing the geological

Fig. 4.—American rock-bit

formation of the ground, it was possible to foresee the type of rock which would have to be crossed and all the attendant difficulties.

According to observations arising from numerous borings, it is found that the vapour flows best between two strata (particularly between strata of the Eocene

and the Retian periods and strata of the Retian and the Permian periods.)

This is particularly marked at Larderello, Castelnuovo and Serrazzano, which are the centres from which the highest output of vapour is obtained. It is therefore of the greatest importance to know the stratification, in order to be able to determine the position of the boring and to guide it successfully, so that an appreciable output can be assured. To this end, geological models are constructed, showing topographic sections of the subterranean stratification.

Attempts were made to further the knowledge gained from geology by applying geophysics. Unfortunately no success can be claimed up to the present. Recent explorations, both by electrical means and by seismological means, have failed to provide indications of the most suitable positions for borings.

(*c*) *Measurements*

The systematic measurements, which are carried out on each boring, are of the utmost importance. The principal measures deal with vapour output and analysis of boric acid, organic anhydride and ammonia. Output measurements were started in 1908, using nozzles with a known flow area. Measuring the pressure at the upstream end of the nozzle combined with the exit area gives values which can be substituted in an appropriate equation from which the output is determined.

To-day the output is recorded by inserting a stainless steel orifice plate in the pipe, and according to the up and downstream pressures, the output is determined by classical formulæ applied with the calibration constants of the instrument.

This method is most practical since readings can be taken in the pipes leading to the plants, without having to shut down the " soffioni."

The analysis of boric acid, ammonia and organic anhydride is carried out by the usual classical methods.

5. Utilisation of the Vapour

Diligent work, tenacity and courageous endeavour have enabled the vapour to be tapped and guided into pipes at the earth surface. Such efforts have not always met with success and often enough borings

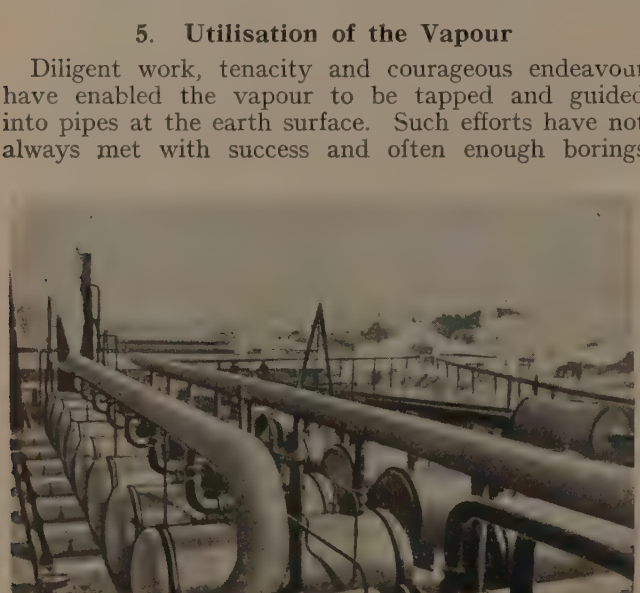


Fig. 5.—Vapour transformers

were carried out through veins which proved of little output although of high pressure with the consequent uncertainty of continuous progress. Long periods of failure were trying to the nerves, but patience and faith eventually lead to successful results.

Once a " soffioni " has blown, and the violent eruption of water and stones has subsided, the vapour is controlled and guided to the place where it is to be used, by means of large diameter steel pipes with adequate thermal insulation. The natural vapour can be employed in two principal actions. It may be introduced directly into velocity and pressure turbines, or it may be utilised in heat transformers (boilers) for use in condenser turbines. In both cases the exhaust vapour is used for the evaporation of boric waters (Fig. 5).

Electrical energy is produced by means of turbo-alternator sets running very economically with great regularity and efficiency, whether the steam is introduced directly into the turbine or used indirectly through vapour transformers.

This electrical energy is distributed by outdoor switching-station (Fig. 6) to the grid and centres of consumption.

Aluminium has been used successfully for electrical connections in control boxes and for the transport of

Fig. 6.—Switching stations

electrical energy. Copper, even when it is covered with protective coating, is subject to corrosion by the atmosphere, rich in hydro-sulphuric acid which, combined with the oxygen and the moisture of the air, produces sulphuric acid. Whatever uses the vapour is put through, the residue of the thermal exploitation will be a condensate, containing almost all the boric acid in solution and parts of the ammonia contents of the vapour. Gaseous residues will consist of the remainder of the ammonia, of carbonic acid, of hydrogen sulphide, of combustible gases, and of the rare gases.

Where pure steam turbines are installed, the above residues will be found in the vapour transformer tubes.

Where, however, natural vapour turbines are installed, the residues are separated in washers at the low pressure side of the turbine. From there, the residues are despatched to the chemical section for further utilisation.

The output from the " soffioni " of 5,500,000 lb. of vapour per hour must be exploited as completely and as rationally as possible. This presents two problems :—

(a) Thermo-electric utilisation.

(b) Chemical utilisation.

(*a*) *Thermo-electric Utilisation*

The first attempts to use the vapour of the " soffioni " for direct mechanical application, were made by the

engineer Raynault in 1894. He used the vapours of the "soffioni" to pre-heat the water of a small multi-tube boiler. The resulting steam drove a small 8 H.P. reciprocating engine.

This result exposed the fallacy of previous theories, propounded by several technicians, who had claimed that the vapour of the "soffioni" would not be applicable for the production of electrical energy, as it would spend all its own energy in overcoming resistance on its attempted journey to the surface of the earth.

Attempts to exploit the vapour had always been hampered by its corrosive action on metallic machinery. The action of the aggressive vapour contents was particularly pronounced in the presence of air.

Similar reasons were responsible for the failure to exploit the kinetic energy of the vapour. This had been attempted by the same Raynault, who had caused the jet from the "soffione" to impound on to a bladed wheel.

A successful experiment, however, was made by Prince Ginori Conti, in April 1904, when he introduced natural vapour at 4.5 atmospheres into a reciprocating engine of 0.75 H.P., after he had withdrawn the accompanying water in an appropriate chamber. In July of that year he coupled his engine on to a small dynamo, which worked regularly for some time, lighting five electric lamps.

When these results became known, a 35 to 40 H.P. Cail piston engine was installed in 1905, also subjected to the natural vapour. This operated a dynamo, which produced electric energy for the lights of the entire plant at Larderello, and for the charging of the accumulators.

Several improvements were introduced in 1913, and it became possible to operate a turbine, which controlled a three-phase alternator of 325 kVA, 4,000 volts and 50 cycles.

The satisfactory results obtained by that machinery and the increase in vapour output led to the installation in 1914 of more powerful machines. They were three reaction turbines with condensers and inlet steam at 1.25 atmospheres, generated by vertical tubular transformers. The turbines operated three 3,500 kVA alternators.

Successive discoveries of greater and more powerful quantities of vapour, at Serrazzano, at increased pressures, led to experiments with an action and back pressure turbine, directly fed with the vapours of the "soffioni." In 1923 a 40 H.P. turbo-generator set was installed at Serrazzano, thus proving the possibility of direct injection.

Other wells with a considerable output were obtained at Larderello, and a 5,000 H.P. action and back pressure turbine was installed at that station (later named Larderello No. 1). Successive additions consisted of three other condenser turbines, so that in 1932 the installed capacity reached 15,000 kVA.

At the same time, two action and back pressure turbines coupled to 1,000 kVA alternators were installed at Castelnuovo V.C. whilst at Serrazzano an action and back pressure group was operating at 750 kVA, and at Lago a condenser turbine was installed, rated at 325 kVA.

As a result of the considerable expansion of the plant which followed, the new Larderello II and Castelnuovo Generating Stations were built before the war, incorporating new 15,000 kVA sets, whilst at Serrazzano, Sasso and Monterotondo new 4,500 kVA sets were installed with direct intake.

In the case of direct intake, the best conditions under which an action and back pressure turbine can function correspond to an inlet pressure of 4.40 atmospheres at a temperature of 365°F. This will result in the most favourable output of vapour, both in quantity and in pressure. The vapour consumption will be about 42 lb/kWh.

On the exhaust side of the turbine, the natural vapour will be at a pressure of about 1.08 atmospheres and a temperature of 220°F. It can then be subjected to an adequate washing process, so that the boric acid and ammonia contents may be extracted.

As I have mentioned before, indirect utilisation of the vapour may be carried out by means of vertical or horizontal tubular heat transformers. The vapours from the "soffioni" are introduced directly into the tubes, where they circulate, in order to give a comprehensive heat exchange with the water in the boilers. The vapour in the tubes will condense and the gases will be released, so that pure steam can be collected in the boilers and directed into the condenser turbines, at a pressure of about 2 atmospheres and with a relative humidity of 2 per cent. In spite of the losses incurred in the best transformers and the energy required to operate the ancillary apparatus, the total consumption of natural vapour only amounts to about 30 lb/kWh, namely 70 per cent. of the consumption in direct intake turbines, thanks to the high vacuum obtained in the condensers.

(*b*) *Chemical Utilisation*

Crude natural boric acid, at various saturations, is obtained from the vapours of the "soffioni" and from the subterranean boric waters. This is treated to attain a commercial saturation of 95 per cent.

Whether the boric solution is obtained by washing the discharge vapour of direct intake plants, or of heat transformers, or whether it be obtained from the subterranean boric waters, the separation of the boric acid is effected by means of evaporation and subsequent cooling of the solution.

Various types of evaporators are used. In the large installations, where the power plant uses pure steam, the process of saturating the boric waters follows upon the process of degassing, all in the vapour transformers working in series. In the small installations, with free exhaust turbines, the process of saturation is carried out by evaporators and also by evaporation surfaces with large exposed areas. This will facilitate the separation and the periodic scooping away of encrustations.

These evaporation plants generally consist of open ponds, in iron, masonry or wood, while the inner surfaces are lined with lead, a metal which is immune from attacks by concentrated boric acid. Steam jackets under the ponds accelerate evaporation. This is similar to the "Adrian" boilers, which you have already seen. Another type of installation at the bottom of the ponds consists of nests of lead tubes, which present the necessary surface for the heat transfer. The efficiency of the process can be increased by mechanical sprinklers of the solution.

When the solution has attained a concentration of 14 to 15 per cent. of boric acid, it is led into an

appropriate chamber, where it is subjected to rapid cooling. A fan blows dry air over the surface of the solution, and a precipitate of boric acid is obtained. The liquid and the boric acid, crystallised in suspension, are led into a filter chamber, where they pass through a filter screen. In this way the solids are separated from the solution, yielding " mother liquor."

The solute goes into rotary driers, in order to eliminate traces of the " mother liquor " which contains boric acid at a concentration of 4 to 5 per cent., all the soluble salts and particularly sulphates of sodium, magnesium and aluminium, which were already present in the original boric water. A concentration of 95 to 97 per cent. should be obtained by using the separation process described above.

A further degree of saturation is obtained from the " mother liquor." This solution is subjected to the same processes as had been applied to the basic solution.

Fig. 7.—Larderello II generating station, June, 1944

The resulting product contains boric acid at a concentration 90 per cent. The various plants refine the 95 per cent. crude boric acid into a solution 99.5 per cent. saturated, or into sodium borate (Borax).

By means of appropriate processes, the ammonia content in the fluids of the " soffioni " is utilised for the production of ammonium bicarbonate and ammonium sulphate.

Carborundum is produced in electric furnaces. This substance is of considerable interest in the field of abrasives, because its hardness is only second to that of the diamond. When crushed into a fine powder, it can be smelted, and consequently it can be used for the manufacture of small objects, such as grinding stones, etc., and these objects find a wide and varied application. The carborunda are also used in the manufacture of atomic piles, in which they serve to decelerate the neutrons.

6. War Damage to the Installations and Repairs which have already been carried out

The first few months of 1944 saw a total power output of the turbo-alternators at the various stations amounting to 170,000 kVA.

The chemical industry had reached a stage where it produced several thousands of tons of crude boric acid per year. This had stimulated industry and provided considerable trade. There was boring equipment for boring operations. A car park had been laid on and a well-furnished office was provided. An extensive chemical laboratory catered for all the research and analysis required for the various activities of the plants.

The bombing during the months of April and May 1944, did not cause extensive damage to the installation, apart from the destruction of half of one of the spectacular reinforced concrete cooling towers, designed by the Consulting Engineers, Messrs. L. G. Mouchel & Partners of London. Grave damage, however, was done by the Germans, when they retreated towards the north between the 13th and 25th of June 1944 (Fig. 7).

The damage, accruing from the retreat of the Germans, may be summarised as follows :—

Wells, vapour transmission pipes, boring machinery, turbo-alternator sets, transformers, cranes, boilers, switchgear, outdoor electric installations and buildings, all this was damaged. All machines in the works were made unserviceable and the principal store was destroyed and burned down. Vehicles were taken away or damaged. As a final act of grace, the Germans destroyed the great bridge which led to the installation at Larderello, and this, in conjunction with the destruction of all the other bridges, left the plants at the end of the war in complete isolation.

Clearing up operations were set in hand as soon as the plant was liberated, and the electric installation was made shipshape once again. The north of Italy, however, was not liberated until April—May 1945, and until then it was impossible to contact the manufacturers of essential machinery. It was therefore only possible to progress at a very slow pace, despite the fine spirit of sacrifice and co-operation displayed by the staff. Nothing could be done until the liberation of Northern Italy. Borings could not be carried out, due to lack of electrical power, and machines could not be re-installed because the manufacturers in Central Italy did not possess the essential designs, which had also been destroyed during the war.

Intensive rebuilding followed the liberation of Italy. At the end of the first year we were able to generate 7,500 kW and a year later, 140,000 kW (Fig. 8).

In the year 1950, the various Generating Stations had a capacity of more than 154,000 kW installed

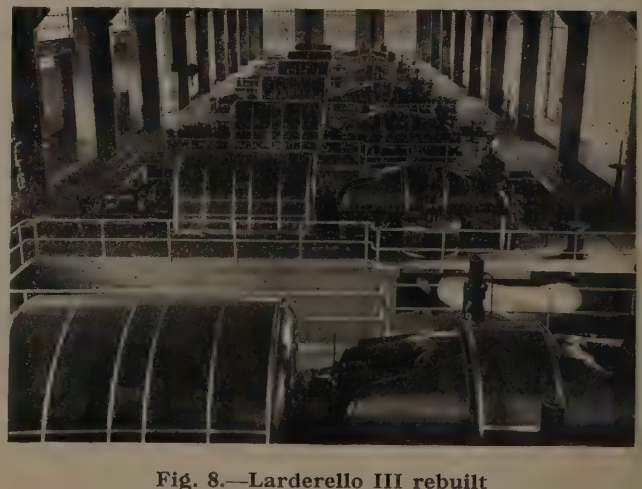


Fig. 8.—Larderello III rebuilt

and the daily output continuously exceeded 3 million kWh. It should be noted that the Larderello plants are not subject to the fluctuations normally associated with winter and summer conditions, but, on the contrary, that the output is absolutely constant.

Repairs on the chemical works and the restoration on the electric plant were carried out simultaneously, and we availed ourselves of the opportunity to improve and simplify the chemical plant.

7. Larderello Generating Station No. III

(*a*) *General Description : Utilisation of natural vapour*

In April 1947 work was started on the new Larderello III Generating Station, which consists of four main 30,000 kVA sets and two 11,250 kVA house sets (one working, the other in reserve). The annual output is estimated at 800 to 900 million kWh.

The turbines are supplied direct with natural vapour with exhaust connected to jet condensers.

Since the fluids from the "soffioni" contain incondensible gases, largely a 4 per cent. content of carbonic anhydride, these gases are extracted from the condensers by means of appropriate turbo extractors so as to maintain the required high vacuum.

By deleting the heat transformers, the turbine inlet pressure is kept at 4.75 atmospheres at about 365°F. This is well in excess of the intake condition in the pure vapour turbine. The vapour consumption is reduced to 21 lb/kWh, which is 50 per cent. less than that of the old direct intake turbines, and 32 per cent. lower than that of the pure vapour intake turbines.

The total installation costs are very much lower than those of a pure vapour intake station, especially in view of the reduced vapour output required, which leads to a considerable saving in boring costs.

On the other hand, it was necessary to install eight turbo-extractors. These machines have been designed for a maximum output of 12,400 lb of organic anhydride per hour. The stainless steel wheels rotate at 4,500 r.p.m., sharing three cooling stages. They are driven by electric motors, rated at 1,000 kW, 4,500 V, and 1,500 r.p.m., equipped with frequency multipliers. Of these machines, only one is operated by a direct vapour intake turbine. In this way an experimental guide is provided for future installations.

The station is provided with four hyperbolic ferro-concrete natural draught cooling towers, built to the designs of Messrs. L. G. Mouchel & Partners, of London. The towers, which are the same as those already installed at Larderello II and Castelnuovo, are 230 feet high, with a 164 feet diameter at the base. They cool 2,640,000 gallons of circulating water per hour, from 106°F. to 88°F. under a temperature drop of 18°F. with an atmospheric dry bulb temperature of 77°F., and a humidity of 60 per cent. (Fig. 9).

The condenser outlet is discharged into a hot water sump. The circulating water pumps consist of four horizontal centrifugal machines delivering 50,000 g/m from the hot water sump to the cooling towers.

In order to avoid flooding, in case of a sudden break in circulation, the hot water sump is provided with an overflow which discharges into a conduit fitted with a self starting Gregoretti syphon, capable of draining the maximum circulation and discharging it into the river Possera, just below.

The oil is air-cooled with two coolers for each 30,000 kVA set and one for each 11,250 kVA set.

(*b*) *The Generating Station*

The tubular steel framed building is of light construction designed for local ground settlements.

The engine room is 300 feet long and 75 feet wide with the 11,250 kVA sets in the centre and the 30,000 kVA sets disposed symmetrically in relation to them.

Alongside, a 50 feet wide room houses the pumps and extractors some 22 feet below engine floor level.

60 ton and 15 ton electric cranes are provided in the engine room and pump house respectively.

Finally, the offices and other ancillary equipment are set out laterally, with the control room well above the engine room floor.

Each of the horizontal 30,000 kVA sets comprises a double inlet 22,000/26,000 kW direct natural vapour intake turbine with jet condensers, and coupled at

Fig. 9.—"Larderello" Mouchel cooling towers

3,000 r.p.m. to the 30,000 kVA alternator, cos $\varphi = 0.8$, 10,500 volts, 50 cycles with a main exciter of 100 kW, 135 volts and auxiliary exciter of one kW, 65 volts.

The two 11,250 kVA sets present the same characteristics, and each one comprises a 9,360 kW turbine coupled at 3,000 r.p.m. to the 11,250 kVA alternator, cos $\varphi = 0.8$, 4,500 volts, 50 cycles with a main exciter of 52 kW, 200 volts and auxiliary exciter of 0.9 kW, 110 volts.

The jet condensers are in two parts working in series, the second part cooling the incondensable gases on their way to the turbo-extractors.

The usual safety apparatus have been incorporated and the inlet valves are automatically controlled so as to cut off the vapour supply if the flow exceeds accidentally 10 per cent. of normal.

The oil pumps are coupled directly to the turbine, but during starting operations lubrication is ensured by a steam pump.

The temperature in the alternators is controlled by three thermo-couples in the stator and three others in the stator windings.

There are four main 30,000 kVA transformers for stepping up 10,500 volts supply to 123,000, 128,000 and 134,000 volts for transmission purposes.

They present no unusual features and are of the outdoor air cooled type with triangle connection on the primary and star connection on the secondary. They are fitted with Buchholz relays and thermo-couples recording the oil temperature on the control panel.

(c) *Wiring diagram*

The main sets are supplied with rigid connections between the alternators and the corresponding transformers with circuit breakers just beyond the transformers, bus bars on the low-voltage side (10.5 kV), a double system of conductors in parallel with circuit breakers between the two systems on the high tension side (130 kV), and five lines at 130 kV at the transmission end.

Both the 10.5 kV and the 130 kV distributions are positioned out of doors near the engine room. The alternators and the 10.5 kV bus bars are connected by means of unipolar lead covered cables, insulated with impregnated paper. The copper conductors are designed for a normal voltage of 15,000 V. In view of the considerable full load current of 1,650 Amps, it was necessary to adopt three parallel cables for each phase. The cables run in a channel, built into the floor, after which they emerge into the open. The channels provide easy access for inspection.

Each generator is equipped with an ultra rapid automatic regulator of the indirect action rheostat type. A stabiliser is not required, since the regulators coupled to the generators and the parallel system, beyond the step-up transformers, will give stabilisation in the presence of the transformer reactance.

The sensitive part of each regulator lies in a separate mechanism, connected by four electric conductors, two of which transmit the feeble correction impulses, while the others cope with changes requiring rapid regulation. In fact, the regulator is characterised by a double action, that is to say, a feeble correction, corresponding to small changes in the voltage and a rapid intervention coming into play with large variations of voltage.

Current transformers are coupled to the neutral conductor of the star connection which, together with the current and voltage transformers at the alternator end, supply the voltage regulator, the measuring instruments and the complex system of protection.

The system of protection consists of the following :— Maximum current relays, maximum voltage relays, differential protection relays, statory and rotary earthing relays. These relays, with the exception of the rotary earthing relays which give only an alarm signal, operate not only on the circuit breakers just beyond the step-up transformers but also on the flux reducing circuit.

The station's auxiliary services, which are of exceptional importance in this case, are supplied by three different sources of energy, namely :—

(1) Supply through 11 120 kVA—4,500 Volt House sets.

(2) Supply by means of two 6,000 kVA transformers stepping down to 4,500 volts the 10,500 transmission voltage.

(3) Supply by means of a 4,750 kVA transformer stepping down to 4,500 volts the 38 kV interconnection grid between generating stations in the district.

The first means of supply is the most commonly used so as to maintain constant voltage and frequency.

The auxiliary services are supplied at 4,500 volts with three different feed possibilities on the operating panel, namely, the supply of the 1,000 kW motors required for the turbo-extractors, the 1,050 kW motors running the cooling water circulating pumps, and of the two 150 kVA transformers stepping down the 4,500 voltage to 220 and 127 volts.

The 700 kVA transformers feed the more important services, such as :— The electric fans used for cooling the lubricating oil, the cooling water pumps attached to the turbo-extractors, and the converter set used for recharging the batteries.

The 150 kVA transformers meet the usual requirements of power (cranes, three phase supply) and lighting.

A 432 Amp./h. storage battery with 86 elements supplies the 160 volt, D.C. circuits for warning lights, as well as the circuit breakers. A 432 Amp./h. battery with 12 elements supplies the 24 volt D.C. circuits of the protective gear and of the signal lights on the control panels.

(d) *Switching Station*

Adjoining the generating station, this outdoor switchgear supplies five separate transmission lines. The transformers can be moved along rails built into the sub-station to an appropriate maintenance building fitted with 90 ton cranes.

The circuit breakers, of the tripolar type and low oil capacity, work on a nominal 165 kV, and 600 Amp. with a standard release power of 1,150 MVA. They are controlled manually by lever and electrically by remote control, using a 160 volt D.C. magnetic coil.

The high voltage switches are of the tripolar rotating spindle type, with expansion contacts fitted directly on the ferro-concrete supports. They are controlled manually by means of small hand wheels, and also electrically by remote control, with 160 volt D.C. motors.

The sub-station conductors are made of Aldrey cord, as protection against the local hydro-sulphuric atmosphere.

It is hoped that this paper has given some idea of the interesting and fascinating phenomenon of the Tuscan "soffioni," and has described how they can be readily utilised both chemically and thermo-electrically.

Valuable boric products of great industrial importance are obtained, and a considerable quantity of energy is at present generated in Central Italy, a region from which transmission to the North and South is very convenient.

In a country like Italy, which is poor in coal and oil, the economic importance of the "Larderello" geothermic generating stations is enhanced when one realises that, altogether, they produced in 1950 a total of 1,300 million kWh. This represented 5 per cent. of the total energy generated in Italy (hydraulically and thermally). It also represented 30 per cent. of the total energy generated by thermal Power Stations, thus corresponding to a saving of about 600,000 tons of coal (based on 32 per cent. thermic efficiency and fuel with a calorific value of 12,000 B.Th.U. per lb).

This energy will be increased by 50 per cent. when Larderello III reaches its full production. It is hoped that many other great installations will follow, and bring their contribution to the solution of an already serious and urgent problem which not only prevails in Italy but also in the whole of Europe : that of meeting the constantly increasing demand for electrical energy both in industry and in the home, for better living conditions leading to social security.

The Design and Construction of a Large Span Prestressed Concrete Shell Roof*

By Lt.-Col. C. W. Kirkland, M.B.E., M.I.Struct.E., M.I.H.E., M.Cons.E.
and A. Goldstein, B.Sc.(Eng.), A.C.G.I., D.I.C.

The structure which is the subject of this paper forms the Garage Unit of a large development scheme for the Bournemouth Corporation Transport Department at Castle Lane, Bournemouth. The garage (Fig. 1) comprises nine barrel vaults each 33 ft. in width and 148 ft. 6 in. span, providing an uninterrupted floor 300 ft. by 150 ft. approx. Parallel to and abutting the western 300 ft. end is a low level Administration Building. A future extension has been planned of identical dimensions to the new garage which will

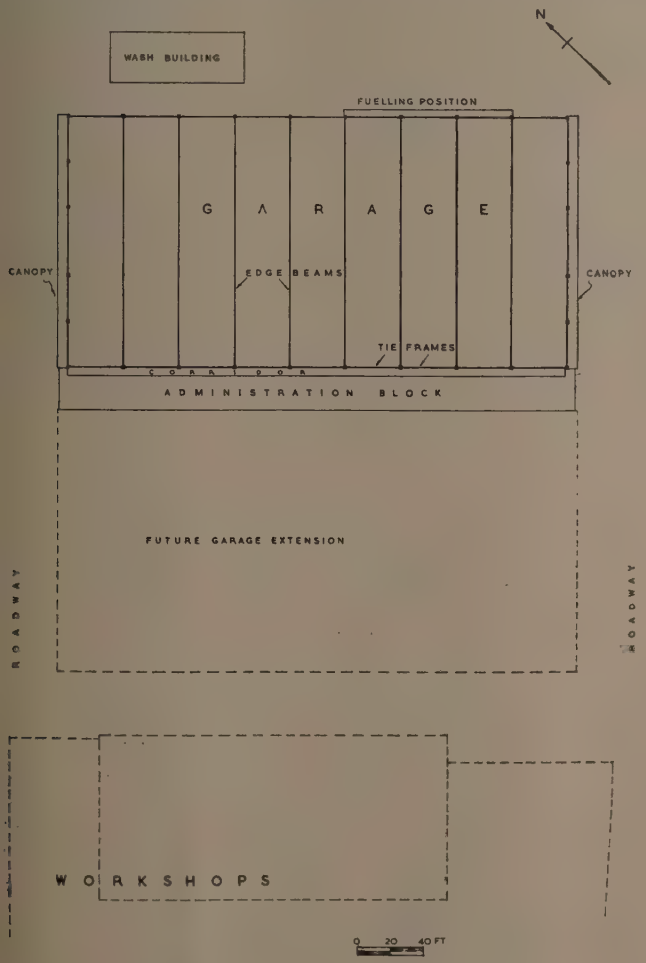


Fig. 1

be constructed at the other side of the Administration Building, the latter eventually serving both the present and future garages.

The structure is to provide accommodation for 99 trolley and double deck motor buses.

Ancillary buildings comprise vacuum cleaning and washing plant, boiler house etc.

A large workshop building with prestressed concrete north light shell roofs, also a concrete framed stores building are now under construction, but these are not dealt with in this paper.

The decision to build the garage in concrete was

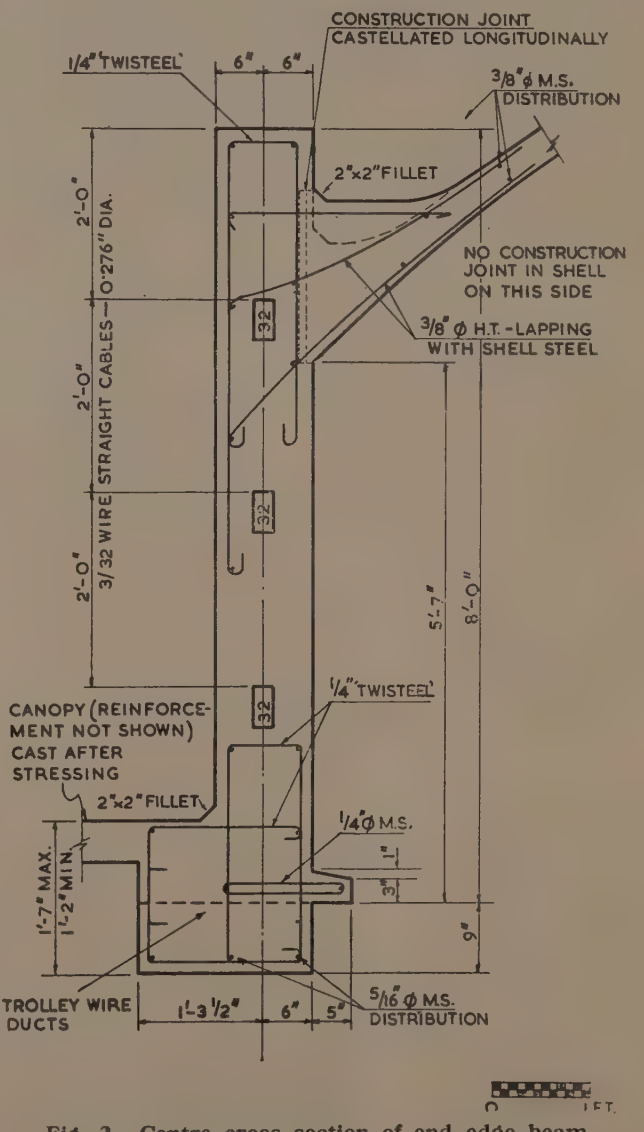


Fig. 2.—Centre cross section of end edge beam

the result of the national economy requirements regarding steel and was to a degree insisted upon by the sponsors, the Ministry of Transport. For the same reason the designers decided to exploit to the full the advantages of prestressing and thereby effected considerable savings in steel and concrete in comparison with normally reinforced concrete shell design.

Literature on the subject of shell design has, with limited exception, been confined solely to either the

* *Paper to be read before the Institution of Structural Engineers at 11, Upper Belgrave Street, London, S.W.1 on Thursday, April 26th, 1951, at 6 p.m.*

construction or the mathematics of shells. The authors have felt the need of literature which would link these two phases of the work, and in presenting this paper trust that by preparing dual text under one cover they will contribute towards closing a gap of which they themselves have been especially conscious.

It is emphasised that in this paper one particular shell only is dealt with, and while many of the concepts presented are of a general nature, the theoretical and practical aspects of different types of shells naturally vary in detail.

At the time of writing, strain and deflection readings are being taken and it is hoped to have some of the comparisons ready for presentation during the discussion.

Design

The development of an efficient and practical design technique has been the aim of engineers ever since the conception of reinforced concrete membranes became reality. Whilst the basic mathematics of shells had been considered much earlier, it was not until the third decade of this century that the greater part of the practical development took place.

The problem of barrel vault design is twofold. A shell is a three dimensional continuous structure and consequently the determination of internal stresses is somewhat complex. A further difficulty is occasioned by the edge conditions which usually add additional indeterminacies.

Sufficient has been written on the various basic stress conditions in a barrel vault roof to obviate the need of a fundamental exposition in this paper. Appendix II gives a bibliography of some of the publications useful for design purposes and Appendix I gives the design methods adopted in this instance in some detail.

The fairly though not unduly complicated design characteristic of shell structures has wider non-technical implications which merit some comment of a general nature.

It is often said that the design of barrel vault roofs is most complicated, and opinion travels swiftly from that belief to the idea that such design would take up excessive time and would be beyond the capacity of the average design office as a business proposition.

The authors would not attempt to prove that shell design is as simple as the normal two dimensional calculations, but do contend that providing reasonable care and forethought is given to the system adopted it is not beyond the technical or financial capacity of any design office employing qualified engineers to undertake work of this nature. In this connection two points should be emphasised.

1. Since the calculations are somewhat lengthy and the various stages interdependent, the computations are best carried out in tabular form. In this way much of the purely arithmetical work can be done by relatively junior staff. The tables used should be designed so as to reduce the work to a minimum, since the majority of the time is taken in writing down the figures, not in obtaining them.

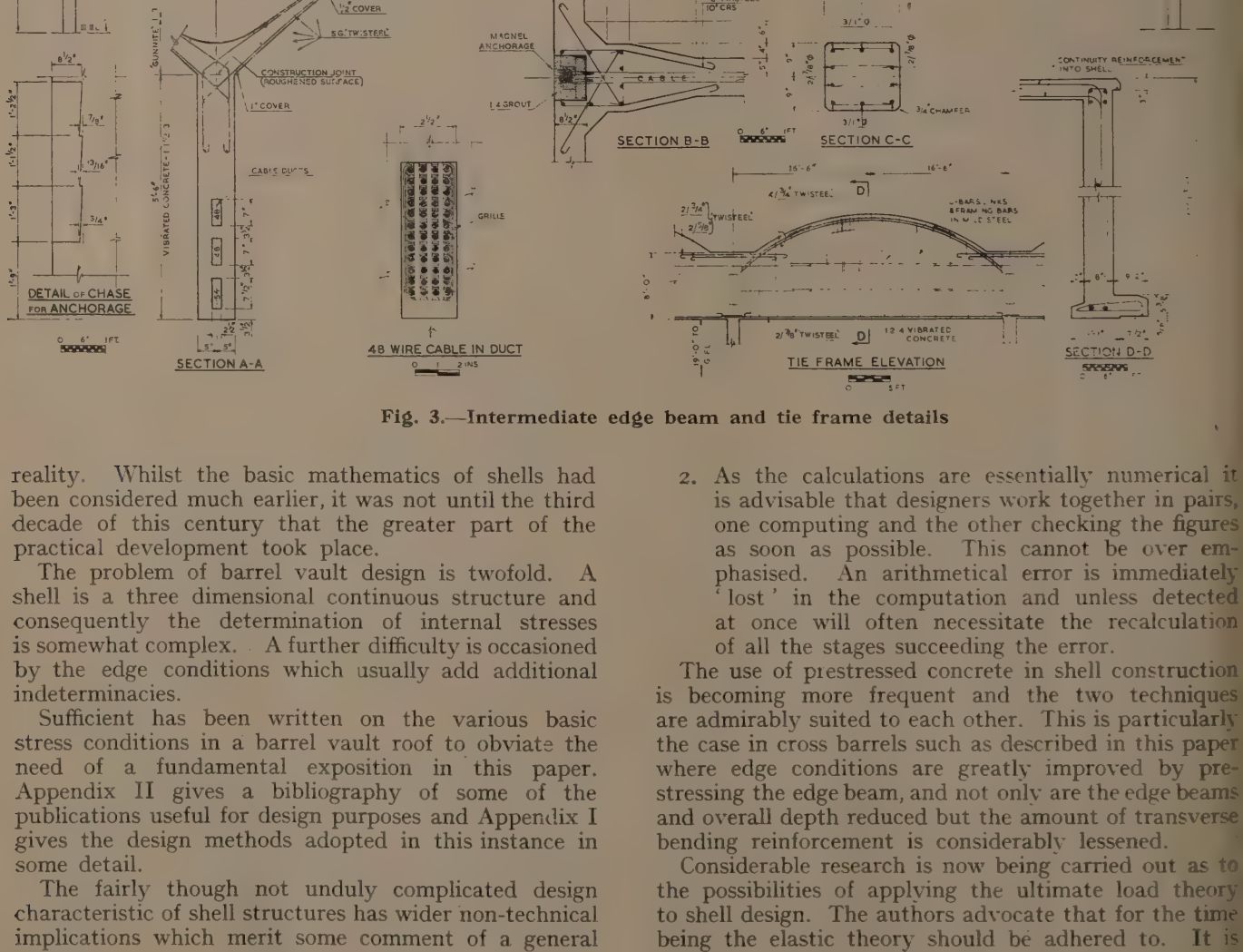


Fig. 3.—Intermediate edge beam and tie frame details

2. As the calculations are essentially numerical it is advisable that designers work together in pairs, one computing and the other checking the figures as soon as possible. This cannot be over emphasised. An arithmetical error is immediately 'lost' in the computation and unless detected at once will often necessitate the recalculation of all the stages succeeding the error.

The use of prestressed concrete in shell construction is becoming more frequent and the two techniques are admirably suited to each other. This is particularly the case in cross barrels such as described in this paper where edge conditions are greatly improved by prestressing the edge beam, and not only are the edge beams and overall depth reduced but the amount of transverse bending reinforcement is considerably lessened.

Considerable research is now being carried out as to the possibilities of applying the ultimate load theory to shell design. The authors advocate that for the time being the elastic theory should be adhered to. It is difficult to see how the ultimate load theory can completely replace the elastic theory in this type of construction, particularly when the last is combined with

Fig. 4.—Assembly of shuttering and corse for intermediate edge beam

prestressing technique, whilst on the other hand it is a fact that where tests have been carried out experimental data confirms the results obtained by the elastic theory over the range of working load.

Finally, the question of time taken over the design of shell structures is one which is of great interest to the practising engineer.

It is difficult to generalise in this respect as conditions will vary from one office to another. It is fairly safe to say, however, that if the full design is carried out from first principles, a cross barrel without edge beams will take approximately four to five days and with edge beams will take approximately a further five days. This does not take into account the prestressing design. The times given are for two men working together, one calculating and one checking.

It will be found that after a certain amount of experience the time taken over the design will be reduced, and if sufficient shells of various shapes have been designed to provide data for plotting curves of shell characteristics the time will be reduced to a much lower minimum.

On the other hand, the first shell to be designed will probably take longer since a good deal of investigation into various stresses, deformations etc., which do not materially influence the amount of reinforcement provided, is likely to be carried out.

In considering the financial implications of the design time, it must be remembered that whilst the time may be a little longer than for normal reinforced concrete framing, the time taken to detail the shells is likely to be less than normal, and it is felt that the difficulties in this connection may have been somewhat exaggerated in the past.

Construction

The necessity for high quality concrete resulted in a separate specification for the prestressed part of the work. In addition, to ensure the best results from the materials used, the superstructure concrete was vibrated, with the exception of the shells. The vibrators used were electrically driven and shutter mounted (Fig. 5). All prestressed concrete work had a specified 28 day cube strength of 6,000 lb per sq. in., while 4,000 lb per sq. in. was specified as a minimum strength at which prestressing could be commenced.

Pressure sprayed concrete was required to attain 4,000 lb. per sq. in. and at 3,000 lb. per sq. in. the shell formwork could be struck.

The foundations are normal 1:2:4 concrete, hand consolidated, and reinforced to cater for the loads and moments induced. Stub bars were left projecting to take the main column reinforcement. The floor slab, again in the interest of steel economy, is unreinforced concrete of 1:2:4 mix, 8 in. in thickness, and consolidated by vibrating screeds. It was cast in sections 33 ft. 0 in. by 13 ft. 6 in. with contraction/expansion joints of a load transference type at the junction of the sections.

The columns are of 1:2:4 concrete, and were cast to the 19 ft. 0 in. height in two lifts and were vibrated.

Following the completion of the first section of columns the prepared shuttering was assembled for the edge beams. The external edge beam, a cross section of which is given on Fig. 2, is simplified from the point of view of construction by the support obtained from

Fig. 5.—Concreting intermediate edge beams

the intermediate columns which frame the door openings. The intermediate edge beams (Fig. 3) are similar throughout, and these are shown in detail.

At an early stage in the design a decision had to be made as to the system of prestressing which would be adopted. The Magnel system was chosen.

The shuttering to the edge beams had therefore to cater for positioning extractable rubber cores (Fig. 4). To provide working access, a soffit and one side only were first assembled. It will be noted that the cable shapes are parabolic in elevation (Fig. 3) and their positions are fixed at specific locations by horizontal "dowels," and vertical "swords." In order to reduce the effect of shrinkage of the concrete, the edge beams were cast in three approximately equal sections.

In the case of the early beams the centre section was cast first and then the two end sections were cast one after the other. These sections of 50 ft. in length are almost twice the general length recommended for shrinkage control, but constructional considerations made it advisable to use this length. Despite the precautions taken, certain small shrinkage cracks did occur but these closed completely on application of the prestressing. The concrete in the edge beams is 1:1½:3 mix, vibrated.

As a result of casting the edge beams in three sections, the rubber cores were of necessity left in place in the first beams for three to seven days after the first concrete was poured. Considerable difficulty was experienced in extracting these cores after so long a time, and it was further found that after extraction of the cores the cleaning tool which is inserted to clean the ducts was ineffective. As a consequence the standard grills which separate the wires in the cables were found to

Fig. 6.—Intermediate edge beam complete.—Note tie frame return at remote end

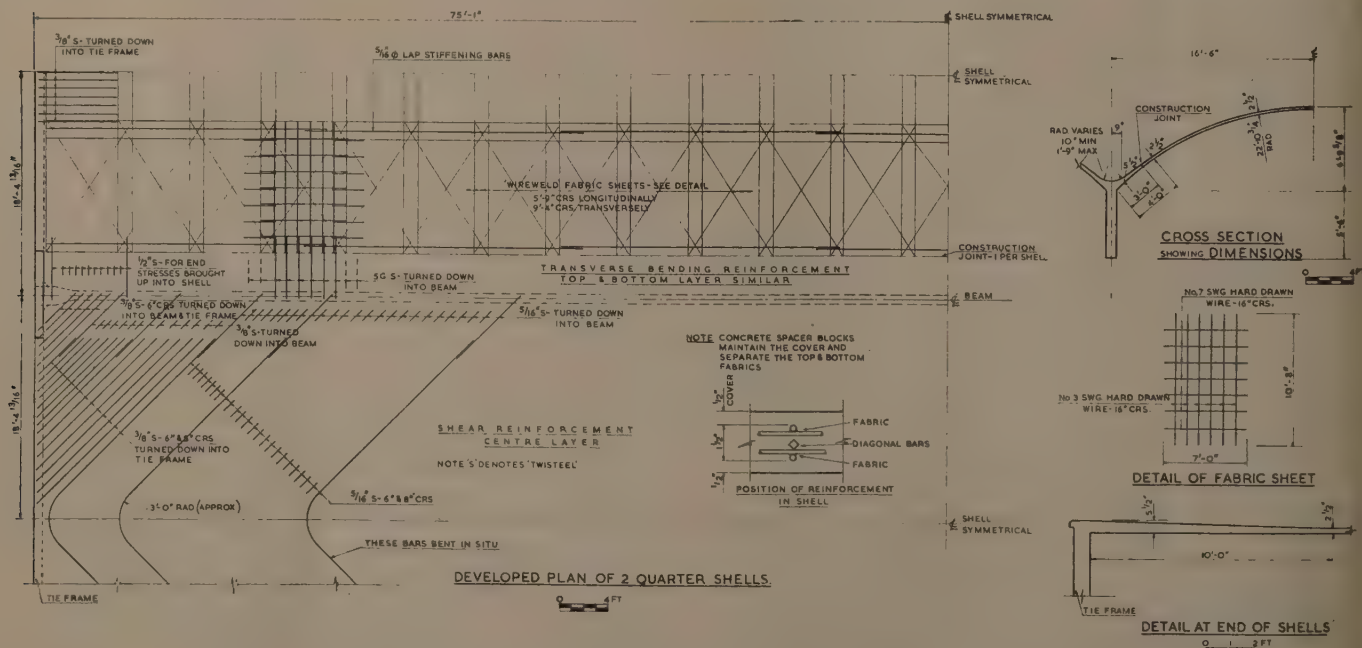


Fig. 7.—Intermediate shell details

be too large to pass through the ducts. Special grills had to be supplied in these cases.

In the case of the subsequent edge beams, cores were positioned in the concrete for each of the beam sections consecutively, working from one end. The day following casting each section of the beam, the composite rubber cores were withdrawn and the cable ducts cleaned of fins of concrete by means of the cleaning tool. This procedure proved to be satisfactory and the standard grills now pass through the ducts with ease.

The progress planning of the work had been based on extraction of the cores from the full length of the beam, and the amended method might have caused

Fig. 8.—Inserting cable in end edge beam.—Cable is supported on revolving spinner and is winched through the ducts from the remote end

delays had not the contractors provided additional shuttering and worked on two edge beams simultaneously.

Strain hardened, square twisted bars were provided to project from the top of the edge beams to act as " starter " bars to the shell reinforcement and to ensure that the membrane and beam would act as a monolithic structure (Fig. 6).

Twenty-four hours after completion of concreting, the formwork to the sides of the edge beams was removed for re-use, but the soffits were kept in position until completion of the prestressing.

The architectural design required a canopy to be constructed projecting from the external edge beam. Some consideration was given to the possibility of casting this unit while the shuttering to the edge beam was in position. It was found, however, that the effect of prestressing would be reduced in the beam, and in consequence the canopy was not cast until prestressing had been completed.

Shuttering to the tie frames (Fig. 3) proceeded simultaneously with the edge beams. The concrete used was 1:2:4 vibrated mix. A short return of the tie frames was cast with the end section of the edge beam (Fig. 6) and castellated construction joints were used between the returns and the centre portions of the tie frames. No special economies could be obtained by prestressing the tie frames, and normal strain-hardened reinforcement was used. As with the edge beams, starter bars were left projecting from the tie frames to connect with the end of the shell membrane. A point of interest is that due to the arching action of the shell the tie frames were designed to take the vertical loads and the shell thrust. Reinforcement to meet these conditions was provided at the top and bottom of the tie frames.

It was decided to form the shell (Fig. 7) of pressure-sprayed concrete as it was felt that by so doing the difficulties of manual compaction could best be overcome. It was anticipated that a high grade concrete could be placed by these means which would contain a minimum of voids and also provide a dense soffit. An original proposal to line the inside of the shells with soft building board was overruled by the sponsors on grounds of economy, and while this measure is regretted, it has permitted a detailed examination of the soffit of the shell to be made. This has proved gratifying, although in the first shell, at points where temporary stoppages in the work had occurred, small deposits of entrapped rebound sand were found which could be scraped away with ease. Thorough investigation has shown these patches to be local only, and not to penetrate more than $\frac{1}{4}$ in. into the membrane. They have been removed and the affected areas made good. The alternative of hand placed vibrated concrete was considered, but rejected on the grounds that top shuttering would prove a difficulty and also that vibration might have an adverse effect on recently placed uncured concrete, particularly as the height of the struts to the shell formwork facilitates the transmission of vibration despite considerable bracing. Fig. 14 shows the application of the shell concrete.

The formwork to the shell was of steel sheets sprung to the required radius and tied to purlins of tubular scaffolding which spanned between tubular spandrels (Fig. 13) bent to radius. The whole was supported on a tubular scaffolding " birdcage " which incorporated raking struts. These raking struts were provided to resist the unbalanced thrusts of the shells which occurred when the soffit of one shell was struck prior to the adjacent shell being completed.

The problem of treatment of the shell formwork to prevent adhesion of the concrete caused some concern. Shutter grease was used on the end shell, but by the time

Fig. 9.—East side of garage under construction. ' Corrugated ' chase in foreground forms the construction joint for a short canopy over the fuelling position

the reinforcement had been fixed the elements and the workmen had combined to remove most of it. Some had contaminated the reinforcement where concrete spacer blocks had become broken or misplaced. It was found, however, that the minute quantity remaining by the time concreting commenced was completely eliminated by the first blast from the cement gun and removed with the rebound particles which were disposed of through apertures left in the formwork. The use of limewash was considered but rejected on the grounds that its life would be shorter than the shutter grease. The second shell was cast against

Fig. 10.—West side of garage under construction

Fig. 11.—Underside of first two vaults completed—without soffit finish

untreated formes, and no difficulties were experienced in striking.

On completion of the first two shells and after the pressure sprayed concrete had reached the specified strength, the operations preceding prestressing the external edge beam were begun. The cables which had been bench assembled on the site previously were inserted and winched into the ducts (Fig. 8) and the distribution plates buttered with a thin coat of cement mortar were offered to the parent concrete in the appropriate chases. After the mortar had hardened (normally three days) tensioning of the wires was commenced. As the anchorages were 150 ft. apart, normal methods of communication were superseded by use of

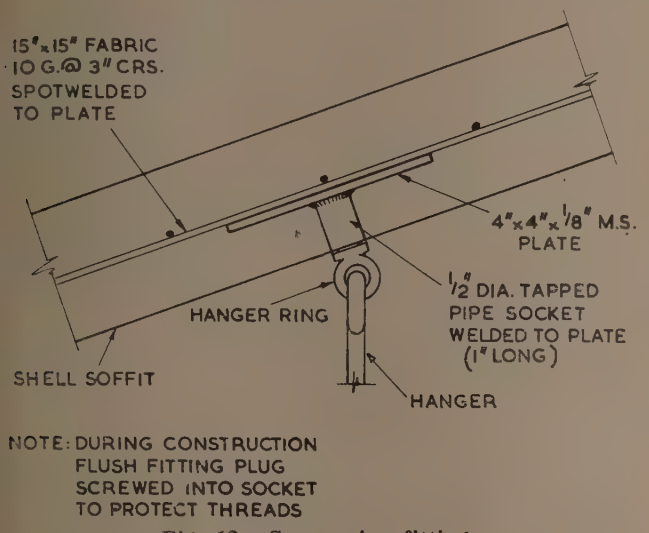


Fig. 12.—Suspension fitting

a balanced armature self-energising field telephone set. The supervisor working on the jack indicated to his assistant at the remote end the pairs of wires to be tensioned and instructed them as to the driving and wedging of the wires.

The operation proceeded smoothly. The time taken to prestress one pair of 0.276 in. dia. wires varied from seven minutes to ten minutes. As the operatives gained experience in the use of the equipment the time taken in prestressing was reduced. The extension on the wires obtained was $7\frac{1}{2}$ in. to $7\frac{3}{4}$ in. on 150 ft. length of cable, representing a compressive force in the concrete of 508 tons approx. With the 15 per cent. allowance for shrinkage, creep, etc., the final prestress is not less than 430 tons.

Prior to the prestressing operation, a number of strain gauge stations were established and gauge plugs were fixed to the concrete, using a jig with location points at 10-in. centres. During the prestressing operation readings were taken using a specially designed direct reading strain gauge incorporating a Mercer high sensitivity shock proof dial gauge reading to 0.0001 in. For purposes of recording gauge readings the engineers also used a field telephone installation. The senior engineer supervised the readings from the Resident Engineer's office and the figures were read direct to him by the engineer using the instrument at the various stations. A further telephone was positioned at the stressing end of the beam and continuous 3-way communication was thus established. A direct check was kept by the senior engineer as to the behaviour of all parts of the structure during prestressing. This procedure, which eliminated field errors by easy reference to master data sheets in the office during the stressing operations, is to be recommended. At the time of writing, sets of readings have been taken for the external edge beam, the first and second shells and the first intermediate edge beam only. Analyses of the complete figures will be made in due course, and correlated to the "E" value of the concrete should provide an interesting comparison between design and actual stresses. It is hoped to publish these figures at a later date.

During prestressing, the first intermediate edge beam "hogged" approx. $\frac{1}{4}$ in. from the soffit support at its centre and accurate deflection readings will be obtained from future beams.

The finish to the shell is to be bituminised felt, but one complete barrel at the centre is to be left uncovered for a period to determine the behaviour of the pressure sprayed concrete and for observation generally.

An attempt at rigid quality control of concrete work has been made. Test cube results from the concrete in various parts of the work have been used as keys to the time and strength control of various operations. This has been a necessary precaution for the prestressed concrete work and its application has not produced delays.

Figs. 9 and 10 are general progress photographs which show progressive stages of work on the tie frames.

Fig. 11 is an interior view of the first two shells.

Fig. 15 shows the top of the finished shells and in the background can be seen the reinforcement and shuttering to further shells prior to concreting.

The following is a table of reinforcement (of the shell structure only) expressed in terms of pounds per sq. ft. covered :—

	Mild Steel	H.T. Steel	Prestressing wire (7 m/m)	Total
Shells	0.037	1.05		1.087
Edge Beams ...	0.087	0.38	0.92	1.387
Tie Frames ...	0.045	0.25		0.295
Columns ...	0.34			0.34
Foundations ...	0.26			0.26
Totals	0.769	1.68	0.92	3.369

The Fire Committee have insisted upon a sprinkler installation being incorporated and the mains for this service, together with the electrical conduits to the parabolic reflector lamps which are to illuminate the lanes between the garaged vehicles have had to

Fig. 13—Formwork and scaffolding to shell

be catered for. The obvious objections to the cutting of holes for fixing services to prestressed concrete work could only be overcome by early planning of these services.

The sprinkler mains and electric conduits are positioned on the side of the edge beams and will be carried by steel brackets secured by bolts which have

been cast in the edge beam concrete. These bolts can be seen in Fig. 11, which shows the first intermediate edge beam with the soffit support removed, and it will be appreciated that considerable care was necessary in locating these fixings in the formwork to the degree of accuracy required, and in such a way that on striking, the formwork remained undamaged and fit for re-use.

The branch pipes to the sprinkler nozzles which are situated at approximately third points on the curve of the shell, and at 9 ft. centres had also to be supported, and a special fitting was devised comprising a female

Fig. 14.—Pressure spraying the shell

tapped body to take the future screwed rod sling, spot welded to a piece of wire fabric 15 in. square (Fig. 12). This unit was placed in position accurately on the shell formwork and securely wired to the shell reinforcement. The tapped socket was closed with a flush fitting screw plug to prevent entry of concrete into the threaded body. This plug was subsequently removed from the soffit of the shell.

Top lighting of the shell by glass dome lights had to be abandoned for reasons of economy and as the degree of illumination provided by glazing in the walls etc. amounts to 9.5 lumens per sq. ft. (average) at a working plane 5 ft. above floor level, and with the doors open this figure is increased to 15 lumens, this was justifiable. The general use of the garage will be at night time, when artificial illumination becomes essential, while during the daylight hours only a small amount of

Fig. 15.—Top of first two vaults completed.—Dark patches in gutter consist of the rebound sand prior to removal by brushing

general servicing is likely. A line of inspection pits in which artificial lighting has been provided will cater for general servicing, wheel changing etc. Major repairs will be carried out in the workshop building now under construction.

The ancillary buildings, mentioned earlier, were constructed in conventional reinforced concrete, and the work was carried out concurrently with that in the garage building.

The Architects for the work are Messrs. Jackson and Greenen, F./A.R.I.B.A., A.M.T.P.I., of Bournemouth, to whom the authors would like to acknowledge the ideal degree of collaboration which resulted in both engineer and architect factually working in complete harmony as members of a team, captained by them, and as well, to thank them and the Bournemouth Corporation Transport Committee for permission to present this paper.

The Contractors are as follows :—

General Contractor	James Drewett & Son, Bournemouth.
Specialist Sub-Contractor for vibrated concrete work in the superstructure and for prestressed work.	The Vibrated Concrete Construction Co., Ltd.
Specialist Sub-Contractor for pressure sprayed concrete work.	The Gunite Co., Ltd.
Suppliers of prestressing equipment. (Magnel System).	Stressed Concrete Designs Ltd.

Acknowledgement is also made to all those who have kindly lent photographs for inclusion in this paper.

Appendix I

Design and Calculations for Barrel Vault

The calculations are carried out using Finsterwalder's approach. For the full explanation of the theory reference should be made to the works given in Appendix II. The symbols in this paper are similar to those employed by the above author to facilitate cross reference.

The calculations can be divided into the general headings of :

(1) Intermediate Shell ;
(2) End Shell ;
(3) Correction for junction effects ;
(4) Prestressing Calculations—Intermediate Shell ;
(5) Prestressing Calculations—End Shell ;
(6) Effects of additional columns under end edge beam.

Due to shortage of space it is impossible to reproduce the entire calculations here, but items (1) and (2), which contain the bulk of the work and the basic principles will be dealt with in detail.

It must be emphasised that the amount of detailed calculation given in the following is more than what would be necessary for a normal design, e.g. the numbers are given to 6—8 digits. This is not necessary throughout the calculation. The simultaneous equations will generally have to be solved with a calculating machine, but for the remainder of the work experience soon shows that not all the evaluations are particularly sensitive. As in this instance all the original calculations had been carried out with a machine they are reproduced here in a similar manner. Again, not all the deflections obtained in this instance would be necessary for a normal design.

Design Loading

SHELL (lb. per sq. ft) :

O.W. (3″)		=	36
Covering	... ...	=	1
Super (total)	... ...	=	15
Total 'g_o'	... ...	=	52 lb./sq. ft.

INTERMEDIATE EDGE BEAM (lb. per ft. run):

One half edge beam considered on each side of the shell.

O.W. (72 × 5)	...	= 360
Thickening	... ...	= 100
Rain	... ...	= 10
Plant	... ...	= 25
Total 'q_1'	... ...	= 495 lb./ft. run

END EDGE BEAM (lb. per ft. run)

O.W. (96 × 12)	...	= 1,152
Thickening	... ...	= 100
Plant	... ...	= 25
Rain	... ...	= 10
Canopy (6′ × 12″ × 4″)		= 288
Brickwork (8 × 45)	...	= 360
Total 'q_0'	... ...	= 1,935 lb./ft.

Subsequent to the design, a number of minor architectural modifications were made to the end edge beam, resulting in a small increase in the load (Fig. 2). The effects of these modifications, particularly with intermediate columns provided, is negligible.

Schedule of Symbols used

Reference should be made to Fig. A.1.

L = total length of shell.
R = radius of shell
d_s = thickness of shell.
$a_s = \dfrac{cR^2}{L^2}$, $b_s = \sqrt{3}\cdot\dfrac{R}{d_s}$ } shell factors.
g = total design load on shell (lb./sq. ft.).
d_{B1} = depth of intermediate edge beams.
d_{Bo} = depth of end edge beams.
$2b_{B1}$ = width of intermediate edge beams.
b_{Bo} = width of end edge beams.
q_1 = total design load on intermediate edge beams (lb./ft. run).
q_o = total design load on end edge beams (lb./ft. run).
ω = angular measure from origin.
ω_K = total angle subtended by shell.
Φ_K = semi angle subtended by shell.
x = longitudinal measure along shell from origin at centre.

Acting on a unit element of the shell. For sign convention see Fig. A.1:

M_s = transverse bending moment in shell
T_s = transverse thrust in shell.
T_1 = longitudinal force in shell.
N_s = radial shear force in shell.
S = plane shear force in shell positive when producing tension along a diagonal of increasing values of ω and x.

ζ = radial deflection of shell—positive when towards centre.
η = tangential deflection of shell—positive when in direction of increasing values of ω.
v = tangent rotation in shell.

At springings, i.e. $\omega=0$ (see equations 13):

$\triangle A$ = vertical deflection of shell.
$\triangle B$ = horizontal deflection of shell.
V = vertical force in shell.
H = horizontal force in shell.

p = longitudinal load distribution expressed as series $p=4/c \cos cx$ where $c=\dfrac{\pi}{L}$, -3π, $+5\pi$ etc.
c = loading term, π, -3π, $+5\pi$ etc.
M = longitudinal B.M. derived from $p\left(\dfrac{pL^2}{8}\right)$ in series.
δ = deflection derived from p, $\left(\dfrac{5}{384}\cdot\dfrac{pL^4}{EI}\right)$ in series.

Y_A, Y_B, Y'_A, Y'_B = integration constants determined from edge conditions of shell.
J_1, K_1, J'_1, K'_1, = roots to characteristic equation of shell.
Jn, Kn, $J'n$, $K'n$ = $(n-1)^{th}$ derivatives of roots.
α, β, α', β' = oscillation factors (see Table A).
a, b, a', b' = fundamental constants (equation 5).
X_A = vertical symmetrical force at shell-beam junction.
X_E = vertical antisymmetrical force at shell-beam junction.
X_B = horizontal symmetrical force at shell-beam junction.

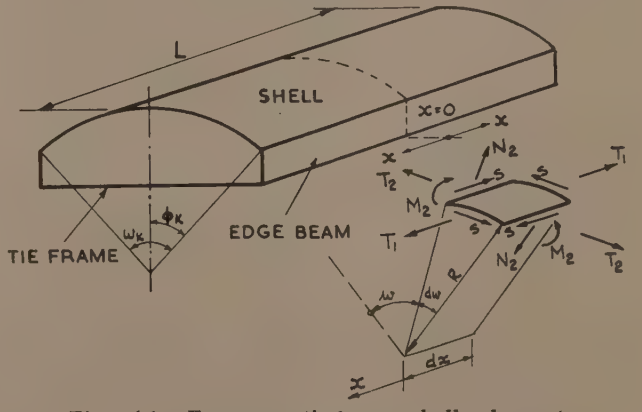


Fig. A1.—Forces acting on shell element (All shown positive)

X_F = horizontal antisymmetrical force at shell-beam junction.
X_D = longitudinal symmetrical force derivative $\left(\dfrac{\delta S}{\delta x}\right)$ at shell-beam junction.
X_H = longitudinal antisymmetrical force derivative $\left(\dfrac{\delta S}{\delta x}\right)$ at shell-beam junction.
A, B, A', B' = integration constants for various unit loads acting above (symmetrically and antisymmetrically) at shell edge.
'm' = denotes membrane condition.
δmA = vertical deflection of shell edge in membrane condition.

δmB = horizontal deflection of shell edge in membrane condition.

$\triangle AA$, $\triangle AB$, $\triangle DB$, etc. denote hypothetical gaps between shell and edge beam for unit edge loads. First suffix indicates the type of gap, e.g. 'A' for vertical symmetrical deflection, 'F' for horizontal antisymmetrical deflections, 'D' for longitudinal symmetrical stress etc. Second suffix indicates type of edge load causing gap, e.g. 'A' for vertical symmetrical etc.

$\triangle mA$, $\triangle mB$, etc. denote hypothetical gaps under membrane conditions where 'A,' 'B' etc. indicate type of gap.

Design Dimensions

L = 150 ft. 0 in.
d_s = $2\frac{1}{2}$ in.
R = 23 ft. 0 in.
d_{B1} = 6 ft. 0 in.
$2.b_{B1}$ = 10 in.
d_{BO} = 8 ft. 0 in. (upstand).
b_{BO} = 12 in.

$$a_s = \frac{c^2R^2}{L^2} = \frac{\pi^2 \times 23^2}{150^2} = 0.232045$$

$$b_s = \frac{3.R}{d_s} = \frac{1.732053 \cdot 23 \cdot 12}{2.5} = 191.2187$$

ω_K = 91° — 40′ — 44″ = 1.6000984 RADs
Φ_K = 45° — 50′ — 22″ = 0.8000492 RADs
Sin Φ_K = 0.7173913
Cos Φ_K = 0.6966714

Design

The loading, taken as uniformly distributed on the shell is expressed longitudinally in a Fourier's series thus :—

$$p = \frac{4}{\pi}\left\{\frac{\cos\pi x}{L} - \frac{1}{3}\frac{\cos 3\pi x}{L} + \frac{1}{5}\frac{\cos 5\pi x}{L} - \ldots.\right\} = 1 \quad (1a)$$

which on successive integration gives equivalent bending moments and deflection :—

$$M = \frac{4}{\pi}\cdot\frac{L^2}{\pi^2}\left\{\frac{\cos\pi x}{L} - \left(\frac{1}{3}\right)^3\frac{\cos 3\pi x}{L} + \left(\frac{1}{5}\right)^3\frac{\cos 5\pi x}{L} - \ldots.\right\}$$

$$= \frac{L^2 - 4x^2}{8} \quad \ldots \quad \ldots \quad \ldots \quad (1b)$$

$$E.I.\delta = \frac{4}{\pi}\cdot\frac{L^4}{\pi^4}\left\{\frac{\cos\pi x}{L} - \left(\frac{1}{3}\right)^5\frac{\cos 3\pi x}{L} + \left(\frac{1}{5}\right)^5\frac{\cos 5\pi x}{L} - \ldots\right\} = \frac{5}{384}L^4 - \frac{L^2x^2}{16} + \frac{x^4}{24} \quad \ldots \quad \ldots \quad (1c)$$

The entire calculations should, in theory, be carried out for each load term of the series, i.e. $p = \frac{+}{c}$ where $c = \pi, -3\pi, +5\pi$, etc., but in practice it is nearly always permissible to use only the first load term, i.e. $c = \pi$ and $p = \frac{+}{\pi}$, as has been done in this paper.

The membrane unit forces and deflections are given by the following :—

$$T_2 = -gR\cos(\omega - \Phi_K) \cdot p \quad \ldots \quad \ldots \quad \ldots \quad (2a)$$

$$\frac{\delta S}{\delta x} = -2g[\sin(\omega - \Phi_K)] \cdot p \quad \ldots \quad \ldots \quad \ldots \quad (2b)$$

$$T_1 = -\frac{2g}{R}[\cos(\omega - \Phi_K)] \cdot M \quad \ldots \quad \ldots \quad (2c)$$

$$E.d_s.\eta = 4g[\sin(\omega - \Phi_K)].M + \frac{2g}{R^2}[\sin(\omega - \Phi_K)].\delta \quad (2d)$$

$$E.d_s.\zeta = gR^2[\cos(\omega - \Phi_K)].p + 4g[\cos(\omega - \Phi_K)].M + \frac{2g}{R^2}[\cos(\omega - \Phi_K)].\delta \quad \ldots \quad \ldots \quad (2e)$$

Since for the calculations the centre of the shell ($x = 0$) is considered it is more convenient to work in $\delta S/\delta x$ for shear force. $\delta S/\delta x$ will be maximum at $x = 0$ (as are all other stresses, deformations etc. considered) and a simple integration at the end of the calculation will give the shear force S.

As can be seen, the membrane stresses for a shell of this nature are statically determinate. However, in order that the membrane stresses only be maintained it is necessary to provide edge restraints of a rigidity which is impractical, undesirable and frequently, as in this instance, impossible.

Calculations must therefore be made for the secondary stress condition using the general elastic theory. The term 'secondary' may be misleading; in this shell the secondary stresses are much greater than the membrane stresses.

In the calculations the various forces are considered as unit forces, i.e. forces (in lb.) acting on an element of the shell 1 ft. square. To obtain stresses, the forces are eventually divided by the cross-sectional area of the element (i.e. 12 in. × $2\frac{1}{2}$ in. = 30 sq. in.).

All unit forces, deformations etc., can be expressed as a sum of damped oscillations travelling across the shell from the edge, considered and varying according to a cosine relationship (1*a* to 1*c*) along the length of the shell.

Thus at $x = 0$:

Unit Force Deformation etc. =

$$Y_A(\alpha.e^{J1\omega}.\sin K_1\omega + \beta e^{J1\omega}.\cos K_1\omega) + Y_B(\alpha.e^{J1\omega}.\cos K_1\omega - \beta.e^{J1\omega}.\sin K_1\omega) + Y'_A(\alpha'.e^{J'1\omega}.\sin K'_1\omega + \beta'.e^{J'1\omega}.\cos K'_1\omega) + Y'_B(\alpha'.e^{J'1\omega}.\cos K'_1\omega - \beta'.e^{J'1\omega}.\sin K'_1\omega) \quad \ldots \quad (3)$$

i.e. $= Y_A.a + Y_B.b + Y'_A.a' + Y'_B.b' \quad \ldots \quad \ldots \quad (4)$

where
$a = (\alpha.e^{J1\omega}.\sin K_1\omega + \beta.e^{J1\omega}.\cos K_1\omega)$
$b = (\alpha.e^{J1\omega}.\cos K_1\omega - \beta.e^{J1\omega}.\sin K_1\omega)$
$a' = (\alpha'.e^{J'1\omega}.\sin K'_1\omega + \beta'.e^{J'1\omega}.\cos K'_1\omega)$ (5
$b' = (\alpha'.e^{J'1\omega}.\cos K'_1\omega - \beta'.e^{J'1\omega}.\sin K'_1\omega)$

The values a, b, a' and b' are called the fundamental constants.

In equation (3), J_1, K_1, J'_1, and K'_1 are four roots of the characteristic shell equation obtained from the elastic stress strain equilibrium. J_1 and J'_1 determine the damping of the oscillations, K_1 and K'_1 the wave lengths. α, β, α' and β' are constants (oscillation factors) which are different for each force or deformation but which can be obtained from successive derivatives of J_1, K_1, J'_1 and K'_1 and the geometrical constants of the shell include Y_A, Y_B, Y'_A and Y'_B are integration constants which can be obtained from the edge conditions.

Differences of opinion exist as to the accuracy of the various characteristic equations proposed by different workers in this field. It is not the purpose of this paper to discuss these. It is generally accepted, however, that for a long shell such as is considered here, Finsterwalder's equation will give very accurate results and the forces in the shell not considered here can be neglected. This equation is :

$$\frac{d^8f}{d\omega^8} + \frac{2d^6f}{d\omega^6} + \frac{d^4f}{d\omega^4} = -2as\left\{\frac{d^6f}{d\omega^6} - \frac{2d^4f}{d\omega^4} + \frac{d^2f}{d\omega^2}\right\}$$

$$+ a_s^2 \left\{ \frac{d^4f}{d\omega^4} + \frac{d^2f}{d\omega^2} \right\} + 4a_s^2 b_s^2 f = 0 \quad \ldots \quad (6)$$

where $a_s = \frac{cR^2}{L^2}$; $b_s^2 = \frac{3R^2}{L^2}$; f = function for M_2

The general solution to the above equation is :

$$f = A_1 e^{J_1\omega} \sin K_1\omega + B_1 e^{J_1\omega} \cos K_1\omega + A'_1 e^{J'_1\omega} \sin K'_1\omega + B'_1 e^{J'_1\omega} \cos K'_1\omega \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

where A_1, B_1, A'_1 and B'_1 are constants.

There are eight roots to equation (6), four of which produce damped oscillations. These four roots (J_1, K_1, J'_1 and K'_1) must first be obtained.

Now considering J_1 and K_1 (the procedure is similar for J'_1 and K'_1), if

$$\frac{d^n J_1}{d\omega^n} = J_{1+n} \text{ and } \frac{d^n K_1}{d\omega^n} = K_{1+n}$$

the various derivatives of the roots required can be obtained successively from

$$\left.\begin{aligned} J_n &= J_1 . J_{n-1} - K_1 . K_{n-1} \\ K_n &= J_1 . K_{n-1} + K_1 . J_{n-1} \end{aligned}\right\} \quad \ldots \quad \ldots \quad (8)$$

Substituting equation (7) in equation (6) and transforming with equation (8) the following simultaneous equations are obtained.

$$(J_2^4 - 6J_2^2 K_2^2 + K_2^4) + (2 - 2a_s)(J_2^3 - 3J_2K_2^2) + (1 - 4a_s + a_s^2)(J_2^2 - K_2^2) + J_2 . (a_s^2 - 2a_s) + 4a_s^2 b_s^2 = 0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (9a)$$

$$(4J_2^3 - 4J_2K_2^2) + (2 - 2a_s)(3J_2^2 - K_2^2) + 2J_2(1 - 4a_s + a_s^2) + (a_s^2 - 2a_s) = 0 \quad \ldots \quad \ldots \quad \ldots \quad (9b)$$

There is no direct solution to the above two equations, but if the damping is strong (as it generally is) a close approximation is given by

$$\left.\begin{aligned} J_2 &= \frac{a_s}{2} + Z \; ; \; J'_2 = \frac{a_s}{2} + Z' \\ K_2 &= -K'_2 = \sqrt{\left[-\frac{a_s^2}{8} + \sqrt{\left(\frac{a_s^4}{64} + a_s^2 b_s^2 \right)} \right]} \\ \text{where } Z &= -Z' = \sqrt{\left[\frac{a_s^2}{8} + \sqrt{\left(\frac{a_s^4}{64} + a_s^2 b_s^2 \right)} \right]} \end{aligned}\right\} \quad (10)$$

The values obtained from equations (10) can be corrected using the Newtonian correction. If they be substituted in equations (9), an error $\triangle_1$ and $\triangle_2$ will be obtained instead of zero. The correction for J_2 and K_2, *viz.* $\triangle J_2$ and $\triangle K_2$ is obtained from the following equations which are derived from equations (9) through partial differentiation with some simplification :—

$$\triangle J_2(4J_2^3 - 12J_2K_2^2 - 6a_sJ_2^2 + 6a_sK_2^2 + 2a_s^2J_2) + \triangle K_2(4K_2^3 - 12J_2^2K_2 + 12a_sJ_2K_2 - 2a_s^2K_2) = -\triangle_1 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (11a)$$

$$\triangle J_2(12J_2^2 - 12a_sJ_2 + 2a_s^2 - 4K_2^2) + \triangle K_2(4a_sK_2 - 8J_2K_2) = -\triangle_2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (11b)$$

One correction is usually sufficient and finally the roots are obtained from

$$\left.\begin{aligned} J_1 &= -\sqrt{\left[\sqrt{\left(\frac{J_2Z}{2} \right)} + \frac{J_2}{2} \right]} \; ; \\ K_1 &= \sqrt{\left[\sqrt{\left(\frac{J_2Z}{2} \right)} - \frac{J_2}{2} \right]} \\ J'_1 &= -\sqrt{\left[\sqrt{\left(\frac{J_2{}^1Z^1}{2} \right)} + \frac{J_2{}^1}{2} \right]} \; ; \\ K'_1 &= \sqrt{\left[\sqrt{\left(\frac{J_2{}^1Z^1}{2} \right)} - \frac{J_2{}^1}{2} \right]} \end{aligned}\right\} \quad (12)$$

The various derivatives of the roots are obtained from equation (8).

The roots and derivatives for the shell described are given in Table I.

The oscillation factors are obtained next from Table A and are shown in Table II.

It is now necessary to decide at what intervals the forces in the shell are to be evaluated. It is generally sufficient to calculate the forces at the springings, crown and quarter points of the shell.

The values of the various oscillations $e^{J_1\omega} \sin K_1\omega$ etc., are next calculated for the points chosen and are shown in Table III. In this connection two points are to be borne in mind :—

(1) Since the shell subtends a fairly small angle, disturbances at one edge will influence the remote edge : consequently oscillations originating from both edges are taken into account.

(2) As will be seen later, it greatly facilitates the calculation to divide the system of loads, edge conditions etc., into a symmetrical system and an 'antisymmetrical' system. The oscillations from the two edges are therefore added for the symmetrical case and subtracted for the antisymmetrical case.

The fundamental constants for the various points in the shell can now be calculated and are shown in Table IV.

Care must be given to the signs as, in accordance with the convention adopted, symmetrically arranged values of T_1, M_2, T_2 and ζ have equal signs and of N_2, $\delta S/\delta x$, η and v have unequal signs and *vice versa* (Fig. A2). In the symmetrical condition, therefore, the symmetrical oscillation values are used for T_1, M_2, T_2 and ζ and the antisymmetrical oscillation values for N_2, $\delta S/\delta x$, η and v. The reverse applies in the antisymmetrical case.

In the same way the fundamental coefficients at $\omega = \frac{1}{4}\omega_K$ and $\omega = \frac{3}{4}\omega_K$ for the symmetrical condition have equal signs in the case of T_1, M_2, T_2 and ζ and unequal signs for N_2, $\delta S/\delta x$, η and v and again the reverse is true in the antisymmetrical condition. It is advantageous to mark the values of N_2, $\delta S/\delta x$, η and v with an asterisk to show that care is needed.

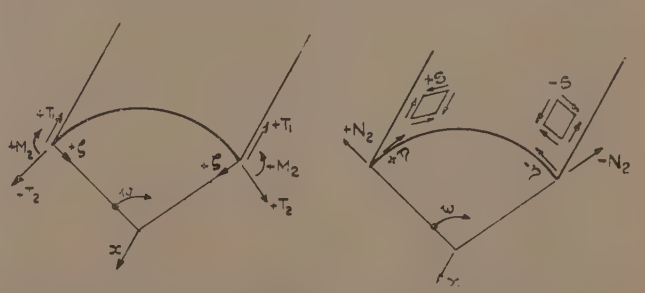


Fig. A2.—Symmetrical arrangement of forces etc. showing sign inequalities

The fundamental coefficients for the vertical and horizontal loads and deflections at the springings are obtained from

$$\left.\begin{aligned} V &= +T_2 \sin \Phi_K - N_2 \cos \Phi_K \\ H &= -T_2 \cos \Phi_K - N_2 \sin \Phi_K \\ \triangle A &= -\eta \sin \Phi_K + \zeta \cos \Phi_K \\ \triangle B &= +\eta \cos \Phi_K + \zeta \sin \Phi_K \end{aligned}\right\} \quad (13)$$

Note :—The origin springing is considered here.

The remaining step in the calculation prior to the direct evaluation of the various forces is the determination of the integration constants Y_A, Y_B, Y'_A, Y'_B.

In the case of a shell without edge beams, i.e. a shell with free edges, the above determination is relatively simple. At a free edge the following conditions exist :—

$M_2 = 0$; $N_2 = 0$; $T_2 = 0$; $S = 0$.

TABLE 'A'

Oscillation Factors

	α	β	α′	β′
M_2	-1	0	-1	0
N_2	$-\frac{1}{R}.J_1$	$-\frac{1}{R}.K_1$	$-\frac{1}{R}.J'_1$	$-\frac{1}{R}.K'_1$
T_2	$+\frac{1}{R}.J_2$	$+\frac{1}{R}.K_2$	$+\frac{1}{R}.J'_2$	$+\frac{1}{R}.K'_2$
$\frac{\delta S}{\delta x}$	$-\frac{1}{R^2}(J_1+J_3)$	$-\frac{1}{R^2}(K_1+K_3)$	$-\frac{1}{R^2}(J'_1+J'_3)$	$-\frac{1}{R^2}(K'_1+K'_3)$
T_1	$-\frac{1}{a_s R}(J_2+J_4)$	$-\frac{1}{a_s R}(K_2+K_4)$	$-\frac{1}{a_s R}(J'_2+J'_4)$	$-\frac{1}{a_s R}(K'_2+K'_4)$
$E.d_s.\eta$	$+\frac{2}{a_s}(J_1+J_3)-\frac{1}{a_s^2}(J_3+J_5)$	$+\frac{2}{a_s}(K_1+K_3)-\frac{1}{a_s^2}(K_3+K_5)$	$+\frac{2}{a_s}(J'_1+J'_3)-\frac{1}{a_s^2}(J'_3+J'_5)$	$+\frac{2}{a_s}(K'_1+K'_3)-\frac{1}{a_s^2}(K'_3+K'_5)$
$E.d_s.\zeta$	$-J_2+\frac{2}{a_s}(J_2+J_4)$ $-\frac{1}{a^2_s}(J_4+J_6)$	$-K_2+\frac{2}{a_s}(K_2+K_4)$ $-\frac{1}{a^2_s}(K_4+K_6)$	$-J'_2+\frac{2}{a_s}(J'_2+J'_4)$ $-\frac{1}{a^2_s}(J'_4+J'_6)$	$-K'_2+\frac{2}{a_s}(K'_2+K'_4)$ $-\frac{1}{a^2_s}(K'_4+K'_6)$
$E.d_s.R.v$	$-J_3+\frac{2}{a_s}(J_1+2J_3+J_5)$ $-\frac{1}{a_s^2}(J_3+2J_5+J_7)$	$-K_3+\frac{2}{a_s}(K_1+2K_3+K_5)$ $-\frac{1}{a_s^2}(K_3+2K_5+K_7)$	$-J'_3+\frac{2}{a_s}(J'_1+2J'_3+J'_5)$ $-\frac{1}{a_s^2}(J'_3+2J'_5+J'_7)$	$-K'_3+\frac{2}{a_s}(K'_1+2K'_3+K'_5)$ $-\frac{1}{a_s^2}(K'_3+2K'_5+K'_7)$

Knowing the values of the fundamental coefficients at the edge for the above forces, equations (3) can be written down for the four forces and equated to zero. Solving the four simultaneous equations thus obtained gives the integration constants.

Where a shell acts together with edge beams as is the case under consideration the solution is more difficult. It is this condition which seems to have received less attention by workers in this field and it is thought that a somewhat detailed exposition of the solution is in order.

The approach is as follows :—

(1) All the statically determinate forces on the shell and edge beam are assumed to be acting (Fig. A3).

(2) A hypothetical cut is assumed to exist between the beam and the shell.

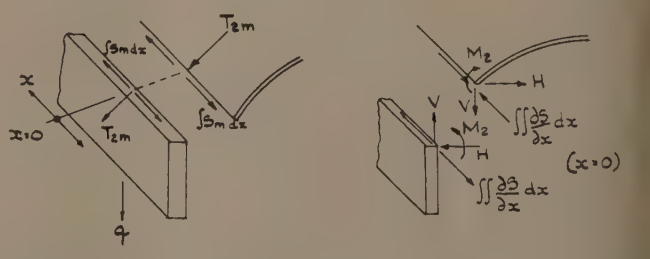
(3) Notwithstanding the assumed cut all forces between beam and shell are assumed to be transmitted.

(4) Due to the above the beam and edge of the shell will deflect certain amounts which will not be equal but which can be calculated.

(5) Due to the inequality of the deflections in (4) a hypothetical gap between the beam and the shell is introduced.

(6) (a) This gap (or difference in deflection and stresses) is closed by the application of certain forces (equal and opposite) to the beam and shell at the junction.
(b) Alternately it can be said that the gap does not occur due to the existence of certain unknown forces at the junction of beam and shell.

(7) The edge forces in (6) cause the secondary stresses in the shell which can be calculated once the edge forces are known, since the integration constants can then be determined.



FORCES PRODUCING 'GAP' IN MEMBRANE CONDITION

EQUILIBRIATING EDGE FORCES TO CLOSE GAP

Fig. A3

It will be clear that the edge gaps and therefore the edge forces, which are functions of the gaps, are dependent on the relative stiffnesses of the beam and shell and are statically indeterminate.

TABLE I

Roots and Derivatives of Shell Equation

Derivative Order	1	2	3	4	5	6	7
J	−2.77354	+6.25562	−9.37958	−5.08096	+113.8147	−584.9529	+2205.717
K	+1.19871	−6.64934	+25.94088	−83.19147	+224.6443	−486.6290	+648.496
J'	−1.15351	−6.98767	+27.25067	+4.55513	−273.4459	+586.8925	+1284.464
K'	+2.88414	−6.653752	−12.47825	+92.9885	−94.1253	−680.0815	+2477.158

TABLE II

Oscillation Factors

	α	β	α'	β'
M_2	— 1	0	— 1	0
N_2	+ 0.12059	— 0.052118	+ 0.050153	— 0.12539
T_2	+ 0.27198	— 0.28910	— 0.30381	— 0.28929
$\delta S/\delta x$	+ 0.022974	— 0.051304	— 0.049333	+ 0.018136
T_1	— 0.22010	+ 16.83344	+ 0.45578	— 16.17650
$E.d_s.\eta$	— 2044.296	— 4419.901	+ 4797.221	+ 1897.129
$E.d_s.\zeta$	+ 10961.861	+ 9814.904	— 10998.226	+ 11654.149
$E.d_s.v.R$	— 44212.682	— 18501.838	— 16128.411	— 43266.429

The gaps are expressed as differences in vertical and horizontal deflection, longitudinal stress and edge rotation. The forces required to close the gap consist of a vertical force V, a horizontal force H, a longitudinal force S (or $\delta S/\delta x$ for computation) and a transverse moment M_2 (Fig. A3). It can be seen that four indeterminacies exist for each edge beam, i.e. a total of eight statically indeterminate forces. This can be reduced to three and six respectively by neglecting the torsional deformation of the edge as is permissible where the edge beam is fairly stiff compared to the shell.

In the case of the intermediate shells which are symmetrical in properties and loading only three indeterminate forces need be solved, since the three forces at the remote edge will be identical. The forces to be determined are accordingly called V, H and $\delta S/\delta x$ (or X_A, X_B and X_D).

In the end shell, non-symmetrical conditions exist; the edge forces are therefore divided into symmetrical forces X_A, X_B and X_D (V, H, $\delta S/\delta x$) and antisymmetrical forces X_E, X_F and X_H (Fig. A4). In the same way two sets of integration constants will be obtained and the final secondary forces in the shell will consist of the symmetrical plus the antisymmetrical forces.

Flexibility Coefficient Method to determine the Statically Indeterminate Edge Forces

The effect of unit loads acting on the shell edge must first be determined. This is done for each load acting alone in turn. The loads are distributed along the x-direction in the usual cosine distribution with an intensity of $4/\pi$ for the first loading term, in the centre ($x = 0$).

TABLE III

Oscillation Values

Location	$e^{J1\omega} \sin K_1\omega$				$e^{J1\omega} \cos K_1\omega$			
	Vibration from origin springing	Vibration from remote springing	Total Value		Vibration from origin springing	Vibration from remote springing	Total Value	
			symmetrical	anti-symmetrical			symmetrical	anti-symmetrical
Origin springing	0	+ 0.0111147	+ 0.0111147	— 0.0111147	+ 1.0	— 0.0040227	+ 0.9959773	+ 1.0040227
$\frac{1}{4}\omega_k$	+ 0.1521201	+ 0.0355354	+ 0.1876555	+ 0.1165847	+ 0.2925419	+ 0.0047274	+ 0.2972693	+ 0.2878145
$\frac{1}{2}\omega_k$	+ 0.0890029	+ 0.0890029	+ 0.1780058	0	+ 0.0624402	+ 0.0624402	+ 0.1248804	0
$\frac{3}{4}\omega_k$	+ 0.0355354	+ 0.1521201	+ 0.1876555	— 0.1165847	+ 0.0047274	+ 0.2925419	+ 0.2972693	— 0.2878145
Remote springing	+ 0.0111147	0	+ 0.0111147	+ 0.0111147	— 0.0040227	+ 1.0	+ 0.9959773	— 1.0040227

Location	$e^{J'1\omega} \sin K'_1\omega$				$e^{J'1\omega} \cos K'_1\omega$			
	Vibration from origin springing	Vibration from remote springing	Total Value		Vibration from origin springing	Vibration from remote springing	Total Value	
			symmetrical	anti-symmetrical			symmetrical	anti-symmetrical
Origin springing	0	— 0.1571608	— 0.1571608	+ 0.1571608	+ 1.0	— 0.0153697	+ 0.9846303	+ 1.0153697
$\frac{1}{4}\omega_k$	+ 0.5763438	— 0.0534105	+ 0.5229333	+ 0.6297543	+ 0.2553570	— 0.2447402	+ 0.0106168	+ 0.5000972
$\frac{1}{2}\omega_k$	+ 0.2943471	+ 0.2943471	+ 0.5886942	0	— 0.2669652	— 0.2669652	— 0.5339304	0
$\frac{3}{4}\omega_k$	— 0.0534105	+ 0.5763438	+ 0.5229333	— 0.6297543	— 0.2447402	+ 0.2553570	+ 0.0106168	— 0.5000972
Remote springing	— 0.1571608	0	— 0.1571608	— 0.1571608	— 0.0153697	+ 1.0	+ 0.9846303	— 1.0153697

TABLE IV

Fundamental Constants

	Symmetrical Loading Condition				Antisymmetrical Loading Condition				
	a	*b*	*a'*	*b'*	*a*	*b*	*a'*	*b'*	
	$\alpha e^{1\omega}\sin K_1\omega + \beta e^{J1\omega}\cos K_1\omega$	$\alpha e^{J1\omega}\cos K_1\omega - \beta e^{J1\omega}\sin K_1\omega$	$\alpha' e^{J'1\omega}\sin K'_1\omega + \beta e^{J'1\omega}\cos K'_1\omega$	$\alpha' e^{J'1\omega}\cos K'_1\omega - \beta e^{J1\omega}\sin K'_1\omega$	$\alpha e^{J1\omega}\sin K_1\omega + \beta e^{J1\omega}\cos K_1\omega$	$\alpha e^{J1\omega}\cos K_1\omega - \beta e^{J1\omega}\sin K_1\omega$	$\alpha e^{J'1\omega}\sin K'_1\omega + \beta' e^{J'1\omega}\cos K'_1\omega$	$\alpha' e^{J'1\omega}\cos K'_1\omega - \beta' e^{J'1\omega}\sin K'_1\omega$	
At Springing $\omega = 0 = \omega_k$	M_2	0.0111147	− 0.9959773	+ 0.1571608	− 0.9846303	+ 0.0111147	− 1.0040227	− 0.1571608	1.0153697	M_2
	$*N_2$	− 0.0536678	− 0.1205952	0.1194423	+ 0.076308	− 0.0505678	+ 0.1206832	− 0.1313517	+ 0.0296742	N_2 *
	T_2	0.2849160	+ 0.2741022	− 0.2371003	− 0.3447995	0.2932880	+ 0.2698638	0.3414877	− 0.2628245	T_2
	$*\delta S/\delta x$	− 0.0517653	+ 0.224960	+ 0.0106619	− 0.0529415	0.0508419	− 0.0234516	− 0.0256108	0.0457245	$\delta S/\delta x$ *
	T_1	+ 16.76327	− 0.4063082	− 15.99959	− 2.093533	+ 16.903598	0.0338818	16.3534966	3.0051010	T_1
	$*Ed_s\eta$	− 4414.9590	− 2101.6456	+ 2680.2221	+ 4512.7990	− 4424.8432	1986.9466	+ 1114.0353	− 5021.6438	$Ed_s\eta$ *
	$Ed_s\zeta$	+ 9897.2599	+ 10808.6755	+ 13203.5182	− 8997.0112	+ 9732.5491	+ 11115.0470	− 10104.7798	12998.8407	$Ed_s\zeta$
	$*Ed_sRv$	− 18084.855	− 44596.170	− 46466.174	− 9576.513	− 18918.821	− 43829.185	− 40066.682	− 22680.309	Ed_sRv *
	V	− 0.1670075	+ 0.1126233	− 0.0868817	− 0.2965627	− 0.1751732	+ 0.1095214	− 0.1534715	− 0.2092212	V
	H	+ 0.2369936	− 0.277473I	+ 0.2508679	+ 0.1895421	+ 0.2406023	− 0.2745835	+ 0.3321355	+ 0.1618143	H
	$Ed_s\angle_A$	+ 10062.3916	+ 9037.7974	+ 2775.7455	− 9548.8646	+ 9954.7326	+ 9168.9530	+ 6240.5119	− 12058.4042	$Ed_s\angle_A$
	$Ed_s\angle_B$	+ 4024.4321	+ 6289.8934	+ 11339.3232	3269.0697	+ 3899.3844	+ 6589.5891	+ 8025.1976	5826.8197	$Ed_s\angle_B$
At Quarter Point $\omega = \frac{1}{4}\omega_k = \frac{1}{3}\omega_k$	M_2	0.1876555	− 0.2972693	0.5229333	− 0.0106168	− 0.1165847	0.2878145	− 0.6297543	0.5000972	M_2
	$*N_2$	− 0.0009415	+ 0.0407834	− 0.0311269	+ 0.1040504	+ 0.0071362	+ 0.0456276	+ 0.0248951	+ 0.0661068	N_2 *
	T_2	− 0.0349020	+ 0.1351038	− 0.1619448	+ 0.1480560	− 0.0514986	+ 0.1119856	− 0.3360020	+ 0.0302486	T_2
	$*\delta S/\delta x$	− 0.0120875	+ 0.0125934	− 0.0219978	− 0.0360927	− 0.0109398	+ 0.0164568	− 0.0256054	− 0.0100079	$\delta S/\delta x$ *
	T_1	+ 4.962762	− 3.224314	+ 0.0666019	+ 8.464070	+ 4.819247	− 2.025868	− 7.802791	+ 10.415157	T_1
	$*Ed_s\eta$	− 1510.4454	− 73.08519	+ 3969.8196	+ 1204.3520	− 1697.5244	221.7123	+ 2528.7683	− 941.1406	$Ed_s\eta$ *
	$Ed_s\zeta$	+ 4974.7234	+ 1416.8041	5627.6089	− 6211.1085	+ 4102.8572	+ 2010.7150	− 1097.9730	12839.4324	$Ed_s\zeta$
	$*Ed_sRv$	− 10479.620	− 10568.020	31794.355	+ 19181.446	13796.781	− 9671.101	− 8893.434	+ 22454.223	Ed_sRv *
At Crown $-\omega = \frac{1}{2}\omega_k$ $\delta x, \eta$ and $\nu = 0$	M_2	− 0.1780058	− 0.1248804	− 0.5886942	+ 0.5339304	+ 0.0149570	+ 0.0243365	+ 0.0964778	+ 0.0470426	N_2
	T_2	+ 0.0123114	+ 0.0854271	− 0.0243895	− 0.3325202	− 0.0023173	+ 0.0120013	0.0387256	+ 0.0156637	$\delta S/\delta x$
	T_1	+ 2.062988	− 3.023935	+ 8.905443	+ 9.279655	− 915.8556	+ 531.4756	+ 1811.1617	− 3678.2110	$Ed_s\eta$
	$Ed_s\zeta$	+ 3176.9641	− 378.1883	− 12697.0963	− 988.4425	− 10180.631	− 2227.863	+ 13606.559	+ 34082.144	Ed_sRv

At Springing $\omega = 0$ Signs change at ω_k.

At Quarter Point $\omega = \frac{1}{4}\omega_k$ Signs change at $\frac{1}{3}\omega_k$

At Crown $-\omega = \frac{1}{2}\omega_k$ M_2, T_2, T_1 and $\zeta = 0$

To obtain the effects of these unit loads on the shell, e integration constants A, B, A' and B' (for unit ads) are determined from the following equations :—

A i.e. in direction of V :

$$A.a+B.b+A'.a'+B'.b' = 4\pi;\ 0;\ 0;$$

B i.e. in direction of H :

$$A.a+B.b+A'.a'+B'.b' = 0;\ 4/\pi;\ 0;$$

D i.e. in direction of $\delta S/\delta x$:

$$A.a+B.b+A'.a'+B'.b' = 0;\ 0;\ 4/\pi;$$
$$A.a+B.b+A'.a'+B'.b' = 0;\ 0;\ 0;$$

symmetrical forces. (14a)

$$\text{E}: A.a+B.b+A'.a'+B'.b' = 4/\pi;\ 0;\ 0;$$
$$\text{F}: A.a+B.b+A'.a'+B'.b' = 0;\ 4/\pi;\ 0;$$
$$\text{H}: A.a+B.b+A'.a'+B'.b' = 0;\ 0;\ 4/\pi;$$
$$\text{J}: A.a+B.b+A'.a'+B'.b' = 0;\ 0;\ 0;$$

antisymmetrical forces (14b)

assumed to be cut down the centre, and it will be seen that the horizontal deflection in all cases is zero, due to the opposing forces from the neighbouring shell.

Having determined the unit load effects, the edge deflections and stresses for the membrane condition are next evaluated.

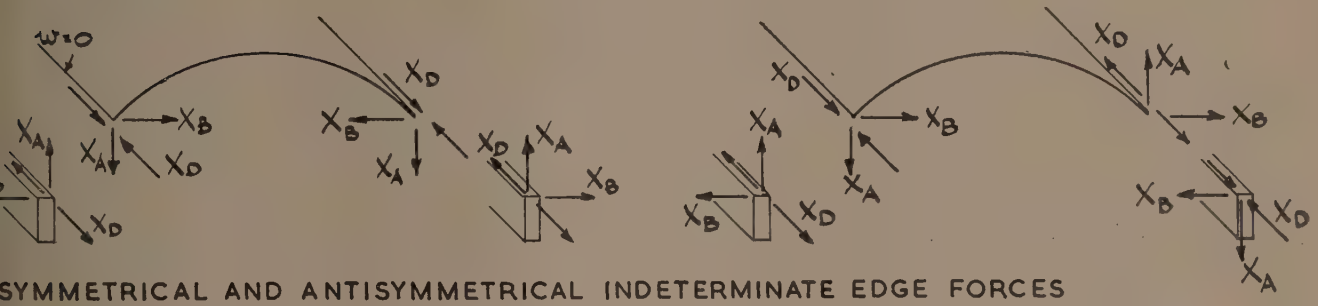
For the shell the edge deflections and longitudinal stresses are determined from the following equations derived from equations (2) :

$$E.d_s.\delta_{mA} = g R^2\cos^2\Phi_K.p+4gM+\frac{2g.\delta}{R^2} \quad (15a)$$

$$E.d_s.\delta_{mB} = g R^2\sin\Phi_K\cos\Phi_K.p \quad \ldots \quad (15b)$$

$$\left(\frac{T_1}{d_s}\right)_m = \frac{2g}{Rd_s}\cos\Phi_K.M \quad \ldots \quad \ldots \quad \ldots \quad (15c)$$

For the beam, the deflections and stresses are calculated from statical formulæ considering all the load on the beam, *viz.* its own weight and loading and the forces transmitted (across the hypothetical gap) from the shell in membrane condition (T_2 and S — Fig. A3).



SYMMETRICAL AND ANTISYMMETRICAL INDETERMINATE EDGE FORCES

POSITIVE SYMMETRICAL AND ANTISYMMETRICAL DEFLECTIONS AND STRESSES

Fig. A4

It is seen that for each unit load acting four simul-neous equations can be solved for A, B, A' and B'. e edge rotation is neglected and therefore only three s of solutions are necessary.

Table V shows the equations (14) and Table VI gives e values of A, B, A' and B' for each edge load acting ne.

From this, the vertical and horizontal deflections and ngitudinal stresses occurring at the shell edge ($\omega=0$) n be evaluated for each load acting alone according the formula

$$A.a+B.b+A'a'+B.'b'$$

The intermediate and end shells must now be con-lered separately.

Intermediate Shell

The vertical and horizontal deflections and longi-dinal stresses of the beam at the junction with the ell for unit loads acting at that point can be determined m normal statical formulæ. The edge beam is

Table VIII shows all the above deflections and stresses and by subtraction the hypothetical edge gaps (or stress differences) are found.

The actual gap is zero. Therefore, the sum of the hypothetical gaps due to unit edge loads multiplied by the actual edge forces together with the hypothetical gap for the membrane condition must be zero. In the symmetrical case (intermediate shell) this is expressed by

$$\left.\begin{aligned} X_A.\triangle_{AA}+X_B.\triangle_{BA}+X_D.\triangle_{DA}+\triangle_{mA} &= 0 \\ X_A.\triangle_{AB}+X_B.\triangle_{BB}+X_D.\triangle_{DB}+\triangle_{mB} &= 0 \\ X_A.\triangle_{AD}+X_B.\triangle_{BD}+X_D.\triangle_{DD}+\triangle_{mD} &= 0 \end{aligned}\right\} \quad (16)$$

Table IX gives the equation and Table X the solution for X_A, X_B and X_D.

From these values the integration constants are evaluated thus :—

$$\left.\begin{aligned} Y_A &= A_A.X_A+A_B.X_B+A_D.X_D \\ Y_B &= B_A.X_A+B_B.X_B+B_D.X_D \\ Y'_A &= A'_A.X_A+A'_B.X_B+A'_D.X_D \\ Y'_B &= B'_A.X_A+B'_B.X_B+B'_D.X_D \end{aligned}\right\} \quad (17)$$

All unit forces in the shell can now be calculated according to equation (4).

TABLE V
Equation Coefficients for A, B, A′ and B′

	A	B	A'	B'	Edge Load Condition $V = +\frac{4}{\pi}$	$H = +\frac{4}{\pi}$	$\frac{\delta S}{\delta x} = +\frac{4}{\pi}$
Symmetrical Edge Load	−0.1670075	+0.1126233	−0.0868817	−0.2965627	+1.273240	0	0
	+0.2369936	−0.2774731	+0.2508679	+0.1895421	0	+1.273240	0
	−0.0517653	+0.0224960	+0.0106619	−0.0529415	0	0	+1.273240
	−18084.855	−44596.179	−46466.174	−9576.513	0	0	0
Antisymmetrical Edge Load	−0.1751732	−0.1095214	−0.1534715	−0.2092212	+1.273240	0	0
	+0.2406023	−0.2745835	+0.3321355	+0.1618143	0	+1.273240	0
	−0.0508419	+0.0234516	+0.0256108	−0.0457245	0	0	+1.273240
	−18918.821	−43829.185	−40066.682	−22680.309	0	0	0

TABLE VI
Integration Constants for Unit Edge Loads

	Symmetrical Edge Load — Edge Load Condition $V = +\frac{4}{\pi}(A)$	$H = +\frac{4}{\pi}(B)$	$\frac{\delta S}{\delta x} = +\frac{4}{\pi}(D)$	Antisymmetrical Edge Load — Edge Load Condition $V = +\frac{4}{\pi}(E)$	$H = +\frac{4}{\pi}(F)$	$\frac{\delta S}{\delta x} = +\frac{4}{\pi}(H)$
A	+8.529806	+2.344767	−37.245720	+16.918251	+7.626834	−37.688623
B	−0.0074318	−2.613341	−6.495420	+2.957040	−.669143	−5.716444
A'	+1.529665	+2.205412	+18.002786	−1.088125	+1.613779	+13.476666
B'	−8.651522	−2.958999	+13.233851	−17.904536	−7.919711	+18.677261

TABLE VII
Edge Deflections and Longitudinal Stresses ($\omega = 0$)

	Symmetrical Edge Load — Edge Load Condition $V = +\frac{4}{\pi}(A)$	$H = +\frac{4}{\pi}(B)$	$\frac{\delta S}{\delta x} = +\frac{4}{\pi}(D)$	Antisymmetrical Edge Load — Edge Load Condition $V = +\frac{4}{\pi}(E)$	$H = +\frac{4}{\pi}(F)$	$\frac{\delta S}{\delta x} = +\frac{4}{\pi}(H)$
T_1	+890.7678	+54.12988	−4499.8185	+1199.3704	+378.0151	−3845.4874
$\triangle_A$	+0.00192467	+0.000541937	−0.00524933	+0.00508424	+0.00220452	−0.0070981
$\triangle_B$	+0.000553467	+0.000338798	−0.000365622	+0.00221604	+0.00103339	−0.0022681

TABLE VIII
Edge Deflection and Stress Differences (Intermediate Shell)
'Hypothetical Gaps' ($\omega = 0$)

		Load Condition $V\ (A)$	$H\ (B)$	$\frac{\delta S}{\delta x}\ (D)$	Membrane m
Vertical Deflection	Shell	+0.00192467	+0.000541937	−0.00524933	+0.0198262
	Edge Beam ...	−0.00245082	0	−0.00735247	+2.1289376
	Difference ...	$\triangle_{AA}$ = +0.00437549	$\triangle_{BA}$ = +0.000541937	$\triangle_{DA}$ = +0.00210314	$\triangle m_A$ = −2.109111
Horizontal Deflection	Shell	+0.000553467	+0.000338798	−0.000365622	+0.000214255
	Edge Beam ...	0	0	0	0
	Difference ...	$\triangle_{AB}$ = +0.000553467	$\triangle_{BB}$ = +0.000338798	$\triangle_{DB}$ = −0.000365622	$\triangle m_B$ = +0.000214
Longitudinal Stress	Shell	+890.7678	+54.12988	−4499.8185	−43890.0
	Edge Beam ...	+1161.0552	0	+4644.2207	−921940.0
	Difference ...	$\triangle_{AD}$ = −279.2874	$\triangle_{BD}$ = +54.12988	$\triangle_{DD}$ = −9144.0392	$\triangle m_D$ = +878050.0

Note :—Only symmetrical L.C. necessary for Inner Shell.

INTERMEDIATE SHELL

TABLE IX
Equation Coefficients for Edge Forces

X_A	X_B	X_D	$-\triangle m$
+ 0.00437549	+ 0.000541937	+ 0.00210314	+ 2.109114
+ 0.000553467	+ 0.000338798	— 0.000365622	— 0.000214255
— 270.2874	+ 54.129878	— 9144.0392	— 878,050.0

TABLE X
Edge Forces

X_A	+ 546.5908 lb/ft.
X_B	— 812.5531 lb/ft.
X_D	+ 75.05764 lb/ft.^{2}

TABLE XI
Integration Constants

Y_A	— 38.5102
Y_B	+ 1631.885
Y'_A	— 1276.868
Y'_B	— 1331.197

End Shell

The procedure is similar to that for the intermediate shell except that, since the edge conditions are now unsymmetrical all deflections, forces etc., are divided into symmetrical groups (e.g. $\frac{\delta_1 + \delta_0}{2}$) and antisymmetrical groups ($\frac{\delta_1 - \delta_0}{2}$). The edge considered as origin ($\omega = 0$) is the edge adjacent to the first intermediate shell (or 'junction edge.')

To maintain clarity the four intermediate columns supporting the end edge beam are not considered in the calculation reproduced here. A method for determining their effect is outlined later.

Table XII gives the deflections and longitudinal stresses of the intermediate (δ_1) and end (δ_0) edge beams from which the symmetrical values ($\triangle A$, $\triangle B$, $\triangle D$) and the antisymmetrical values ($\triangle E$, $\triangle F$, $\triangle H$) are obtained for each edge load condition (symmetrical and antisymmetrical).

Table XIII gives the shell and beam deflections and the hypothetical gaps.

Where symmetrical and antisymmetrical edge forces are considered equation (16) becomes

$$X_A.\triangle_{AA}+X_B.\triangle_{BA}+X_D.\triangle_{DA}+X_E.\triangle_{EA}+X_F.\triangle_{FA}+X_H.\triangle_{HA}+\triangle_{mA}=0$$
$$X_A.\triangle_{AB}+X_B.\triangle_{BB}+X_D.\triangle_{DB}+X_E.\triangle_{EB}+X_F.\triangle_{FB}+X_H.\triangle_{HB}+\triangle_{mB}=0$$
$$X_A.\triangle_{AD}+X_B.\triangle_{BD}+X_D.\triangle_{DD}+X_E.\triangle_{ED}+X_F.\triangle_{FD}+X_H.\triangle_{HD}+\triangle_{mD}=0$$
$$X_A.\triangle_{AE}+X_B.\triangle_{BE}+X_D.\triangle_{DE}+X_E.\triangle_{EE}+X_F.\triangle_{FE}+X_H.\triangle_{HE}+\triangle_{mE}=0$$
$$X_A.\triangle_{AF}+X_B.\triangle_{BF}+X_D.\triangle_{DF}+X_E.\triangle_{EF}+X_F.\triangle_{FF}+X_H.\triangle_{HF}+\triangle_{mF}=0$$
$$X_A.\triangle_{AH}+X_B.\triangle_{BH}+X_D.\triangle_{DH}+X_E.\triangle_{EH}+X_F.\triangle_{FH}+X_H.\triangle_{HH}+\triangle_{mH}=0 \qquad (18)$$

These equations are shown in Table XIV.

The solution of this set of six simultaneous equations yields the edge forces, *viz.* the symmetrical ones X_A, X_B, X_D and the antisymmetrical ones X_E, X_F and X_H, all of which are shown in Table XV.

TABLE XII

Edge Beam Deflections and Stresses (End Shell)

		Edge Load Condition						
		A	*B*	*D*	*E*	*F*	*H*	*m*
Vertical Deflection	δ_1	— 0.00245082	0	— 0.00735247	— 0.00245082	0	— 0.00735247	+ 2.128938
	δ_0	— 0.00043081	0	— 0.00086162	+ 0.00043081	0	+ 0.00086162	+ 1.026915
	$\delta_1 + \delta_0$	— 0.00288163	0	— 0.00821409	— 0.00202001	0	— 0.00649085	+ 3.155853
	$\delta_1 - \delta_0$	— 0.00202001	0	— 0.00649085	— 0.00288163	0	— 0.00821409	+ 1.102023
	$\triangle A$	— 0.00144082	0	— 0.00410704	— 0.00101001	0	— 0.00324543	+ 1.577926
	$\triangle E$	— 0.00101001	0	— 0.00324543	— 0.00144082	0	— 0.00410704	+ 0.551012
Horizontal Deflection	δ_1	0	0	0	0	0	0	0
	δ_0	0	— 0.0275718	+ 0.0137859	0	+ 0.0275718	— 0.0137859	— 14.97629
	$\delta_1 + \delta_0$	0	— 0.0275718	+ 0.0137859	0	+ 0.0275718	— 0.0137859	— 14.97629
	$\delta_1 - \delta_0$	0	+ 0.0275718	— 0.0137859	0	— 0.0275718	+ 0.0137859	+ 14.97629
	$\triangle B$	0	— 0.0137859	+ 0.0068929	0	+ 0.0137859	— 0.00689294	— 7.48815
	$\triangle F$	0	+ 0.0137859	— 0.0068929	0	— 0.0137859	+ 0.00689294	+ 7.48815
Longitudinal Stress	δ_1	+ 1161.0552	0	+ 4644.2207	+ 1161.0552	0	+ 4644.2207	— 921940.0
	δ_0	+ 136.0569	— 2176.9784	+ 1723.4200	— 136.0569	+ 2176.9784	— 1723.4200	— 1479730.0
	$\delta_1 + \delta_0$	+ 1297.1121	— 2176.9784	+ 6367.6407	+ 1024.9983	+ 2176.9784	+ 2920.8007	— 2401670.0
	$\delta_1 - \delta_0$	+ 1024.9983	+ 2176.9784	+ 2920.8007	+ 1297.1121	— 2176.9784	+ 6367.6407	+ 557790.0
	$\triangle D$	+ 648.5561	— 1088.4892	+ 3183.8204	+ 512.4992	+ 1088.4892	+ 1460.4004	— 1200835.0
	$\triangle H$	+ 512.4992	+ 1088.4892	— 1460.4004	— 648.5561	— 1088.4892	— 3183.8204	— 278895.0

TABLE XIII

Edge Deflection and Stress Differences (End Shell) ($\omega = 0$)
'Hypothetical Gaps'

			Edge Load Condition						
			A	*B*	*D*	*E*	*F*	*H*	*m*
Symmetrical Deflections	Horizontal Deflection	Shell	+ 0.00192467	+ 0.000541937	— 0.00524933	0	0	0	+ 0.019826
		Edge Beam	— 0.00144082	0	— 0.00410704	— 0.00101001	0	— 0.00324543	+ 1.577926
		△ A	+ 0.00336549	+ 0.000541937	— 0.00114228	+ 0.00101001	0	+ 0.00324543	— 1.558100
	Vertical Deflection	Shell	+ 0.000553467	+ 0.000338798	— 0.000365622	0	0	0	+ 0.0002143
		Edge Beam	0	— 0.01378588	+ 0.006892939	0	+ 0.0137859	— 0.00689294	— 7.488146
		△ B	+ 0.000553467	+ 0.0141247	— 0.00725856	0	— 0.0137859	+ 0.00689294	+ 7.488360
	Longitudinal Stress	Shell	+ 890.7678	+ 54.1299	— 4499.8185	0	0	0	— 43890.0
		Edge Beam	+ 648.5561	— 1088.4892	+ 3183.8204	+ 512.4992	+ 1088.4892	+ 1460.4004	— 1200835.0
		△ D	+ 242.2117	+ 1142.6191	— 7683.6389	— 512.4992	— 1088.4892	— 1460.4004	+ 1156945.0
Anti-symmetrical Deflections	Horizontal Deflection	Shell	0	0	0	+ 0.00508424	+ 0.00220452	— 0.00709813	0
		Edge Beam	— 0.00101001	0	— 0.00324543	— 0.00144082	0	— 0.00410704	+ 0.551012
		△ E	+ 0.00101001	0	+ 0.00324543	+ 0.00652505	+ 0.00220452	— 0.00299108	— 0.551012
	Vertical Deflection	Shell	0	0	0	+ 0.00221604	+ 0.00103339	— 0.00226815	0
		Edge Beam	0	+ 0.0137859	— 0.00689294	0	— 0.01378588	+ 0.00689294	+ 7.488146
		△ F	0	— 0.0137859	+ 0.00689294	+ 0.00221604	+ 0.0148193	— 0.00916109	— 7.488146
	Longitudinal Stress	Shell	0	0	0	+ 1199.3704	+ 378.0151	— 3845.4874	0
		Edge Beam	+ 512.4992	+ 1088.4892	+ 1460.4004	+ 648.5561	— 1088.4892	+ 3783.8204	+ 278895.0
		H	— 512.4992	— 1088.4892	— 1460.4004	+ 550.8143	+ 1466.5043	— 7029.3078	— 278895.0

TABLE XIV

Equation Coefficients for Edge Forces (End Shell)

X_A	X_B	X_D	X_E	X_F	X_H	$-\triangle m$
+ 0.00336549	+ 0.000541937	— 0.00114228	+ 0.00101001	0	+ 0.00324543	+ 1.558100
+ 0.000553467	+ 0.0141247	— 0.00725856	0	— 0.0137859	+ 0.00689294	— 7.488360
+ 242.2117	+ 1142.6191	— 7638.6389	— 512.4992	— 1088.4892	— 1460.4004	— 1156945.0
+ 0.00101001	0	+ 0.00324543	+ 0.00652505	+ 0.00220452	— 0.00299108	+ 0.551012
0	— 0.0137859	+ 0.00689294	+ 0.00221604	+ 0.0148193	— 0.00916109	+ 7.488146
— 512.4992	— 1088.4892	— 1460.4004	+ 550.8143	+ 1466.5043	— 7029.3078	+ 278895.0

TABLE XV

Edge Forces (End Shell)

X_A	+ 659.2629 lb/ft.
X_B	— 567.2708 lb/ft.
X_D	+ 114.1298 lb/ft²
X_E	— 64.24995 lb/ft.
X_F	— 95.98079 lb/ft.
X_H	— 48.67047 lb/ft.²

TABLE XVI

Integration Constants (End Shell)

Symmetrical	Y_A	+ 42.419
	Y_B	+ 736.251
	Y'_A	— 204.862
	Y'_B	— 2514.696
Anti-symmetrical	Y_A	+ 15.297
	Y_B	+ 152.457
	Y'_A	— 740.896
	Y'_B	+ 1001.475

Finally, the symmetrical integration constants are given by:

$$\left.\begin{aligned} Y_A &= A_A.X_A+A_B.X_B+A_D.X_D \\ Y_B &= B_A.X_A+B_B.X_B+B_D.X_D \\ Y'_A &= A'_A.X_A+A'_B.X_B+A'_D.X_D \\ Y'_B &= B'_A.X_A+B'_B.X_B+B'_D.X_D \end{aligned}\right\} \quad (19a)$$

And the antisymmetrical integration constants are given by:

$$\left.\begin{aligned} Y_A &= A_E.X_E+A_F.X_F+A_H.X_H \\ Y_B &= B_E.X_E+B_F.X_F+B_H.X_H \\ Y'_A &= A'_E.X_E+A'_F.X_F+A'_H.X_H \\ Y'_B &= B'_E.X_E+B'_F.X_F+B'_H.X_H \end{aligned}\right\} \quad (19b)$$

These values are shown in Table XVI.

The unit forces are then calculated from equation (4) using both symmetrical and antisymmetrical integration constants and fundamental constants.

The values of the unit forces etc., for both intermediate and end shells are shown in Table B and Fig. A5. The forces in the end shell are those obtained without considering the intermediate columns supporting the end edge beam.

Correction for Junction Effect

As stated previously, the intermediate edge beams are considered as bisected and immovable horizontally. This is not strictly correct for the edge beam between the end and first intermediate shell ('junction edge beam') and can be seen by the discrepancies between the values of H, V, T_1, $\triangle A$, M_2 and $\frac{\delta S}{\delta x}$ at the springing on each side of the junction edge beam. V, M_2 and $\frac{\delta S}{\delta x}$ may differ without causing trouble, but the differences in H, $\triangle A$ and T_1 cause a tendency for the shells to separate. These differences can be balanced out and the effects of this correction can be distributed from the junction edge beam over each adjacent shell.

These effects are generally negligible but if required can be calculated in the following manner.

Denoting the differences in H, T_1 and $\triangle A$ at the junction edge beam by δH, δT_1 and $\delta \triangle A$ it will be clear that the forces and deflections to be imposed at the springing for correction are $\frac{\delta H}{2}$, $\frac{\delta T_1}{2}$ and $\frac{\delta \triangle A}{2}$. The following edge conditions then exist:

$$T_1 = \frac{\delta T_1}{2}; H = \frac{\delta H}{2}; \triangle A = \frac{\delta \triangle A}{2}; \delta = 0 \quad \ldots \quad (20)$$

If the fundamental constants are now evaluated with oscillations from one springing only (since the edge disturbance takes place at one edge only), then using equations (4) and (20) four equations are obtained for four integration constants which will give the corrections throughout the adjacent shells. Care must be taken over the signs as an origin change takes place in the first intermediate shell.

The corrections have been evaluated and are shown in Table B. It can be seen that they are quite small. With prestressing, the deflection and stress differences at the junction edge beam are smaller; the corrections therefore are generally less than those shown.

Effects of Additional Columns under End Edge Beam

The stresses in the end shell and end edge beam are very large. Architectural requirements, however, necessitated the use of four intermediate columns along the 150 ft. sides of the building and these columns were used to support the end edge beam. The effects on the shell and beam stresses is greatly to reduce them. The precise determination of the effects is very difficult, but a close approximation can be made considering the effects of the columns as an edge condition at the outer edge of the end shell producing an upward deflection of such an order so as to make the final downward deflection very small, depending on the spacing and size of the columns. Having determined the appropriate edge condition, new integration constants can be calculated in the manner explained previously.

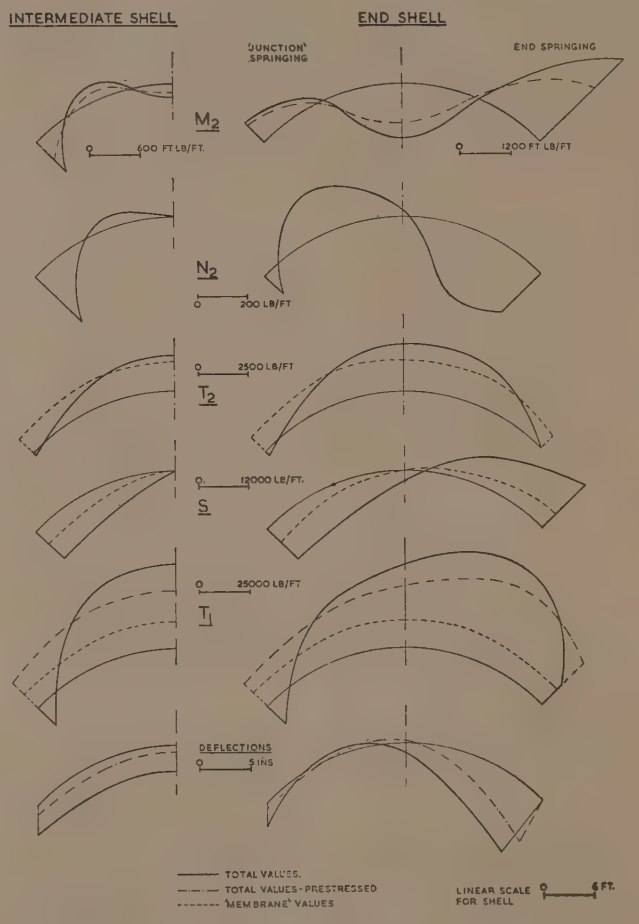


Fig. A5

This was done in the shell being described prior to the calculation of reinforcement and prestressing forces.

Effects of Prestressing Forces

As can be seen from the text and illustrations, the edge beams were prestressed after the casting of the shell. To determine the effects of the prestressing actions on the shell, the prestressing effects at the shell-beam junction are calculated and inserted in the calculation when considering the hypothetical gaps in the membrane condition. Further calculations must also be made to determine the effects at the end of the beam and shell ($x = L/2$). Shortage of space prohibits a detailed

TABLE B

Values of Unit Forces and Deflections (Non-Prestressed)

Intermediate Shell.

ω	—	M_2	N_2	T_2	$\frac{\delta S\,\delta r}{S}$	T_1	η	ζ	v
0	Membrane	0	0	1091	− 95	−9144	−0.100	−0.108	—
Springing	Total	−535	−237	−150	$\frac{+192}{+9167}$	15764	−2.04	−1.07	0
	Membrane	0	0	−1405	−51.0	−12105	−0.091	−0.210	—
$\frac{1}{8}\omega_R$	Total	+202	−33	−1173	$\frac{-140}{+7114}$	−28950	−1.08	+2.80	+0.001
	Membrane	0	0	−1523	0	−13125	0	−0.24	0
$\frac{1}{4}\omega_R$	Total	−156	0	−1796	$\frac{0}{0}$	−41874	0	+2.70	0

Deflection at Springing :— Vertical 2.85 in. ; Horizontal 0 in.

End Shell

0	Membrane	0	0	−1091	−95	−9144	−0.100	−0.108	—
Inner Springing	Total	−936	−69	−97	$\frac{+188}{-8070}$	−15211	−1.62	−1.56	0
$\frac{1}{8}\omega_R$	Membrane	0	0	−1405	−51.0	−12105	−0.091	−0.210	—
	Total	−173	−170	−1350	$\frac{+169}{+8069}$	−19860	−0.109	−0.713	−0.012
$\frac{1}{4}\omega_R$	Membrane	0	0	−1523	0	−13125	0	−0.24	0
	Total	−1319	−21	−2291	$\frac{+47}{+2244}$	−40440	−0.727	+0.966	+0.012
$\frac{1}{2}\omega_R$	Membrane	0	0	−1405	−51.0	−12105	−0.091	−0.210	—
	Total	−10	+280	−1942	$\frac{-151}{-7200}$	−51790	+0.26	+4.58	+0.041
Edge Springing	Membrane	0	0	−1091	−93	−9144	−0.100	−0.108	—
	Total	−2576	−212	−21	$\frac{-297}{-14180}$	−13090	−0.118	−7.10	0

Deflection at End Springing :— Vertical 5.03 in. ; Horizontal 4.27 in. (inwards).

Corrections

0		+10	+32	+191	$\frac{+23}{+1098}$	−2608	−0.158	+0.382	0
$\frac{1}{4}\omega_R$		+61	−10	+32	$\frac{+12}{+573}$	−3117	−0.007	+0.32	−0.002
$\frac{1}{2}\omega_R$		−38	−8	−28	$\frac{-1.2}{-57}$	−927	−0.060	−0.051	−0.002
$\frac{3}{4}\omega_R$		−56	−2	−19	$\frac{-2.5}{-120}$	−225	−0.051	−0.103	−0.0005
ω_R		−13	−5	+5	$\frac{-1.4}{-67}$	+535	+0.003	−0.001	+0.0007
Units		*LB FT*	*LB FT*	*LB FT*	$\frac{LB\ FT^2}{LB\ FT}$	*LB FT*	Ins.	Ins.	Rad's.

explanation of these methods, but it is hoped to make them the subject of a future paper.

The effect of the prestressing actions on M_2, T_1, ζ and η, the main design, criteria is indicated graphically in Fig. A5. It will be noted that considerable stress and deflection reductions take place.

Appendix II

Bibliography

[1]U. Finsterwalder. 'Die querversteiften zylindrischen Schalengewolbe mit kreissegmentformingem Querschnitt.' (Segmental cylindrical shells with cross stiffeners). Ingenieur - Archiv 1933.

[2]U. Finsterwalder. 'Die Theorie der zylindrischen Schalengewolbe, System Zeiss-Dywidag und ihre Anwendung auf die Grossmarkthalle in Budapest.' (The Theory of Cylindrical Shells, Zeiss-Dywidag System and its application to the Market Hall in Budapest). International Association for Bridge and Structural Engineering, 1932.

[3]Fr. Dischinger. 'Die strenge Theorie der Kreiszylinderschale in ihrer Anwendung auf die Zeiss-Dywidag Schalen.' (The exact theory of the circular cylindrical shell in its application to the Zeiss-Dywidag Shells). Beton und Eisen, 1935.

[4]A. Aas Jakobsen. 'Uber das Randstörungs Problem an Kreiszylinderschalen.' (The Problem of Edge Disturbances on Cylindrical Shells). Der Bauingenieur. 1939.

[5]A. Aas Jakobsen. 'Sur le calcul de la voute cylindrique circulaire.' (The calculation of Cylindrical Shells). Travaux. 1937.

[6]H. Schorer. 'Line Load Action on Thin Cylindrical Shells.' Proceedings—American Society of Civil Engineers. 1935.

[7]C. E. Molke and J. E. Kalrinka. 'Principles of Concrete Shell Dome Design.' JOURNAL OF THE AMERICAN CONCRETE INSTITUTE. 1938.

[8]R. S. Jenkins. 'Theory and Design of Cylindrical Shell Structures.' Modern Building Techniques—Bulletin No. 1. London 1947.

For the convenience of readers, the name of one library where each of the above publications may be obtained or referred to is given below.

*[1]Patents Office Library.
[2] and [7]Institution of Structural Engineers.
[3]Cement and Concrete Association.
*[4] and [5]Science Library.
[6] and [8]Institution of Civil Engineers.
*Photostat Service.

NOTE :—When referring to publication [2] care should be taken as there are several printers' errors. Cross reference to [1] will indicate these.

The Encasing Structure for the Hot Water Accumulator for the Pimlico District Heating Scheme

Discussion on Mr. Guthlac Wilson's Paper★

The CHAIRMAN (Mr. W. C. Andrews, O.B.E., M.I.C.E., Vice President) apologised for the absence of the President, Mr. Leslie Turner, and the Secretary, Major R. F. Maitland, both of whom were visiting the South African Branch.

Proposing the thanks of the meeting to Mr. Wilson for his paper, Mr. Andrews said that although the structure with which he was concerned was relatively small, it dwarfed the buildings around it; so that if any mistakes had been made in design or in respect of its appearance, they would remain as a monument to whoever was responsible. The paper gave very useful information and was an antidote to what might be called purely theoretical papers. It was obvious that a good deal of ingenuity had had to be exercised, not only from the point of view of cost, but also from that of appearance. The Institution very much appreciated the trouble Mr. Wilson had taken to write the paper and to present it to the meeting.

(The vote of thanks was warmly accorded and Mr. Wilson briefly responded).

Mr. S. B. DONKIN, M.I.C.E., M.I.Mech.E., M.I.E.E. (Consulting Engineer for the Pimlico District Heating Scheme) paid tribute to Mr. Wilson in connection with the structure and thanked him for his excellent and interesting paper, which described the fundamentals of the design; the fact that the structure was erected with very considerable speed was an indication of the merit of the design. It was important to note, for example, that on the completion of erection, the accuracy was to within a maximum of $\frac{3}{8}$ in. vertically, and within $\pm \frac{1}{8}$ in. in diameter, so conforming to a true circle. That was particularly interesting to Mr. Donkin, because he had been responsible for the accumulator inside the structure. The accumulator shell was 126 ft. high, and had a diameter of about 29 ft. and it was entirely welded on site; it was built of prefabricated plates, all machined at the edges, and they fitted together extraordinarily well. To build a cylindrical shell of prefabricated plates to be as true as Mr. Wilson's structure was found to be quite impossible. As a matter of interest, he said the maximum out-of-vertical was of the order of $1\frac{1}{2}$ in., but not continuously all round; it varied from place to place. The maximum deviation from a true circle in plan was of the order of $\pm 4\frac{1}{4}$ in. That would probably be regarded as a very considerable deviation, but when one saw the structure and did not measure it, it appeared to be perfect. When it was remembered that the structure had to be built of plates, 6 ft. high, clipped together and then welded in all seams, with nothing to guide the erectors other than some weighted wire vertical lines, it would be appreciated that the result was excellent. The figures quoted were obtained before the water was put into the accumulator; the water had since been put in, and perhaps the pressure of that water would have reduced the deviation from the true circle in plan. That matter would have to be checked later.

Mr. Donkin also pointed out that the glazing bars were placed in the reverse way to the usual placing

*Read before the Institution of Structural Engineers at 11, Upper Belgrave Street, London, S.W.1, on Thursday, April 13th, 1950, published in the "Structural Engineer," Vol. XXVIII, No. 4, page 85.

of them for windows or for sloping roofs. That was done for two reasons, i.e., it was required to erect the glazing panes from the galleries on the inside, and it was desired to take them out and to replace them from the inside. The problem had puzzled the manufacturers for some time, but Mr. Wilson had helped them to solve it, and they had produced a very excellent design of glazing which enabled the panes to be taken out and replaced as required. The glass used was ¼ in. thick, and, with the approval of the architects, it was not plain glass, but was mottled or rough cast on one side and smooth on the other. The smooth side was on the outside of the tower, of course, so that it could be cleaned. There was an arrangement at the top to which a cradle could be attached, so that men could clean the glass outside when necessary.

A point which had yet to be decided was the colour, for one was concerned to please the inhabitants of the estate as well as the architects and the housing committee of the Westminster City Council; the architects had told him that evening that they had not yet settled it.

One regretted that Mr. Wilson had not yet been able to obtain a photograph of the completed structure with all the glazing fitted, but the glazing had been held back, because some pipes which had to be fitted around the accumulator were arriving very late, as was not unusual in these days.

Discussing the operation of the accumulator, Mr. Donkin said it would contain about 500,000 gallons of hot water. It was to be heated at Battersea Station by back pressure sets and pumped to the top of the tower, the water being carried back to Battersea from the bottom of the tower.

The Battersea Power Station of the British Electricity Authority could generate electricity and heat the circulating water at times when they wanted to do so, and hot water from the accumulator could be used by the Council as and when it was required by them.

Mr. PHILIP POWELL, discussing the architectural aspects of the Pimlico accumulator, said that originally it was not intended that the present system of district heating should be applied to the housing scheme; therefore, the idea of a scheme, involving the siting of an enormous water vessel in its midst, had come rather as a bombshell when it was put forward, and many people had commiserated with him and his partner. The hope was expressed that it would not ruin their scheme. Right from the beginning, however, he and his partner had welcomed it.

Nevertheless, the siting of the structure had presented rather a difficult problem, because there was a very high density of development on the estate—about 60 flats per acre. But with the help of Mr. Donkin and Mr. Wilson, they had arrived at what they felt was a happy solution; it helped the scheme, it did not block the view from the flats and they felt that the tower and accumulator represented something which was really worth having, from all points of view.

There had been some difficulty, because many outside authorities had tried to persuade them to adopt the most extraordinary designs for the structure. It was suggested that it might be constructed as a buttress to one of the buildings, or that it might be a mock shot tower—anything else except an accumulator. But he and his partner had rather horrified some people when they had said they hoped, that in the end, it would look something like a hot water accumulator; and they had tried to work on those lines.

At the same time, they did not just want something which looked like a plain elongated gasholder; such a structure, with paint on it, might in time become rather shabby and rather out of keeping with the scheme. However, the various suggestions made did not deflect from their desire to make it look like an honest industrial structure.

Then Mr. Donkin had come to their help, for he had decided that it was necessary to provide access galleries, to ensure easy maintenance of the cylinder. That had rather dictated that the vessel should have an outer casing, and that the casing should be in the form of waterproof sheeting. Glass had been chosen for practical and æsthetic reasons. Very strong glass could be obtained at a very cheap price, it would provide a waterproof job, it was easy to maintain and it would withstand the London atmosphere. Above all, the glass covering had made it possible to show the beautiful structure which Mr. Guthlac Wilson had provided.

The architects had had to work very closely with the district heating engineers and the structural engineers. And had, no doubt, been an abominable nuisance; to them, it had, however, been a great pleasure to be associated with a structure which, in their opinion greatly enhanced the appearance of the scheme as a whole.

Mr. A. S. BROOKS, discussing the problems in connection with the glazing, said the glazing bars were of extruded aluminium, with continuous spring clips. When his Company were approached with regard to the glazing of the structure, they had felt rather anxious about the bars being fixed inside-out, which meant that the wind pressure would come on to the back of the spring clips. Therefore, they had carried out tests at the works on aluminium glazing under pressures corresponding to heavy wind loads applied both normally and reversed.

The tests were conducted in the standard pressure testing chamber. It consisted of a rectangular steel cabinet, approximately 7 ft. 6 in. high by 4 ft. 6 in. wide and 4 ft. long, divided into two compartments, 3 ft. and 1 ft. long respectively, by a diaphragm, having an aperture 6 ft. 9 in. by 4 ft. The larger compartment was fitted with an access door having a small window and a suitably baffled air inlet, which was connected to the compressed air supply mains through a finely adjustable valve. A safety valve, a large relief valve and a pressure gauge were also fitted. The smaller compartment was connected to an orifice type air flow meter and fitted with a pressure gauge. (These instruments were not required for this particular series of trials). Suitable gauging rods were applied to measure the deflection on the frame or window under test at various points; they passed through glands, so that their movement could be observed from outside the cabinet.

The glazing assembly or other sample undergoing tests was clamped in the aperture of the diaphragm, a sub-diaphragm being used as necessary if the tests were carried out on a smaller article or assembly.

In the tests, with the load applied from the cover strip side, the pressure was increased steadily until the glass fractured. Air pressures were applied, starting from 2.6 lb./sq. ft. and being increased by small increments to 65.0 lb./sq. ft. At the latter pressure, with a bar deflection of 1 5/32 in., the glass reached a deflection of 1 5/16 in. and fractured. It was calculated that under a wind pressure of 30 lb./sq. ft. the 2½ in.

bars with unsupported lengths of 5 ft. 9 in. and spaced at 1 ft. 9¾ in. would deflect $\frac{7}{16}$ in. It was concluded that when a load of 30 lb./sq. ft. was applied from what was normally the under side of the glazing bar, there was no danger of collapse of the cover strip or of the glass lifting.

Mr. DONKIN said that when the accumulator was first designed as a vertical structure, he was asked constantly why the cylinder was not designed horizontally, for thereby he would avoid a high structure, which was subject to adverse criticism. The architects, of course, had saved the situation; the answer was quite simple. If the accumulator vessel were horizontal, it would be necessary to use a standpipe, to retain a head of water at a level above the top of the flats; therefore, practically the whole of the cylinder would be subjected to the highest pressure, which was about 55 lb./sq. in., and the cost of the horizontal cylinder would have been very much greater.

Mr. S. M. REISSER (Member), said the tank was all-welded, which was the normal and natural way in which to build a tank in these modern times. But he wondered whether alternative designs were put forward for a riveted structure, and which of the two types of construction Mr. Donkin would choose.

Mr. DONKIN replied that no alternative designs were prepared for a riveted accumulator, because they had known that it would be more expensive than a welded vessel; further, the noise caused by the riveting during construction would have been a great nuisance to the district. The noise due to gouging the back side of the welds was very considerable, but it was not so continuous as the noise due to riveting.

Mr. O. BONDY (Member), who felt sure Mr. Reisser did not mean to suggest that he would prefer that the seams of the accumulator were riveted rather than welded, said he hoped that in the future, another paper might be presented to the Institution on the design of heavy plate structures such as accumulators, as distinct from the encasing structure.

He wondered whether the accumulator, which he supposed was erected after the casing was completed, would stand on its own feet and would withstand the wind load, if any, or whether the designers had relied on the glazing remaining intact. Were there interconnections between the vertical accumulator and the vertical legs of the encasing structure?

The part of the paper in which wind pressure was discussed was of particular interest, because there were not many structures of such height and of such diameter in existence on which wind loads could be measured. He supposed that wind tunnel tests had been considered, and perhaps, figures derived from them; or was he right in assuming that the basis for the wind load distribution shown in one or two of the diagrams was that referred to in the footnotes?

Mr. Bondy added that he must be one of many who were extremely glad to see that structures of such importance, and which were so much in the public eye, were being erected by welding; he felt there could be no better proof of the confidence placed in welded structures than that both the inner accumulator and the outer casing had been designed for welding and completed by welding. Because of that fact, and because of the wind pressure figures and diagrams given in the paper, if for no other reasons, the paper should be extremely useful for future reference. Therefore, he was glad to offer his tribute to the author.

The CHAIRMAN said Mr. Wilson had presented a very interesting practical example of the results of collaboration between engineers and architects in an endeavour to arrive at the best possible solution, and he felt it would be agreed that success had been achieved.

Commenting on the point that the total out-of-vertical of the tower was ⅜ in., he asked how the measurements were taken, because it would seem that they would not be easy in the case of so high a structure erected in such an exposed position. He did not, of course, question the figures.

Mr. R. C. BUCHANAN, M.B.E. (Resident Engineer on the site) said the measurements were made by wires with weights at the ends, and on a calm day.

The CHAIRMAN asked if there were an oil bath or anything of that kind.

Mr. BUCHANAN replied that there was not.

Mr. WILSON, in a brief reply to the discussion, thanked the speakers for their interesting contributions.

Dealing with Mr. Donkin's reference to the relative accuracies in the construction of the tank and the tower, he said he had been amazed by the way in which the tank was erected; he had never seen one erected before, and the ingenious clips and temporary connections that were made were such as to call for his admiration. He had not expected that a thin shell tank, which had internal angle stiffening rings only 5 in. wide to be as nearly circular as a tower having 2 ft. 6 in. circular girders to form walk-ways at every 18 ft. of height, as the stiffness of the ring members was a measure of the accuracy attainable in diametrical dimension. He would think there must certainly have been a difference in the circular dimension as the result of the water pressure.

The co-operation of all concerned had been very much valued, and it was felt that it had resulted in the erection of a nice looking tower of which all would be proud.

Commenting on the remarks of Mr. Reisser, he said that an alternative design for a riveted tower had not been considered. As Mr. Donkin had pointed out in respect of the tank, riveting would be very noisy and offensive to everybody around. Also, they had not felt very confident in bolting such a high structure subjected to wind pressures.

Replying to Mr. Bondy, he said the accumulator was erected after the casing. It did not stand on its own feet and had had to be designed for wind pressure, because it was erected before the glazing was put on the tower, and no one could guarantee that there would not be a storm before the glazing was completed. There were no connections between the accumulator and the casing.

Wind pressure was estimated on the bases given by the references in the footnotes. Those references were to reports of tests on similar structures, although of not so great a ratio of height to diameter, but it was felt that they were adequate for the purpose.

The CHAIRMAN having expressed the thanks of the meeting to Mr. Wilson for his replies to the discussion, the meeting closed.

Institution Notices and Proceedings

ORDINARY GENERAL MEETING

An Ordinary General Meeting of the Institution of Structural Engineers was held at 11, Upper Belgrave Street, London, S.W.1, on Thursday, February 22nd, 1951, at 5.55 p.m., Mr. Walter C. Andrews, O.B.E., M.I.C.E., M.I.Struct.E., (Vice-President), in the Chair.

The Minutes of the Ordinary General Meeting held on December 14th, 1950, as published in the Journal in February, 1951, were taken as read, were confirmed and signed.

The following members were elected in accordance with the Bye-Laws. Will members kindly note that the elections, as tabulated below, should be referred to when consulting the Year Book for evidence of membership ?

STUDENTS

BARRITT, Norman, 64, Malcolm Street, Heaton, Newcastle-on-Tyne 6.

BOTING, Henk, P.O. Box 820, Bloemfontein, O.F.S., South Africa.

BRAITHWAITE, Lawrence, 31, Stafford Street, Belgrave, Leicester.

CHODZKO-ZAJKO, Witold Jozef, 43, Bromley Road, Walthamstow, E.17.

COX, Anthony James, 18, Fontarabia Road, Battersea, S.W.11.

CROSSLAND, Roy, 45, Park Road, Coldean, Brighton, Sussex.

GABRIELCZYK, Ryszard January, 276, Amhurst Road, Hackney, London, N.16.

GRAFF, Samuel, B.Sc. (Eng.) Rand., c/o City & Guilds College, Exhibition Road, S.W.7.

HARGREAVES, Paul Ian, 32, Grange Road North, Hyde, Cheshire.

MASON, Peter Michael, 54, St. George's Road, Norton, Stourbridge, Worcs.

MOORE, Bernard George, Brookhill Lodge, Cowfold, nr. Horsham, Sussex.

ROSE, Douglas Frederick, 54, Broad Lawn, New Eltham, London, S.E.9.

SELBIE, Robert Ian Eddleston, York Cottage, Pinner Hill, Pinner, Middlesex.

VILES, David Vivian, 23, Warwick Road, Bishop's Stortford, Herts.

WEAVER, Charles Louis Harry, 125, Sumner Road, Peckham, London, S.E.15.

WILLIAMS, Lionel Woolf, 62a, Alandale Drive, Pinner, Middlesex.

GRADUATES

ARMSTRONG, John Bryan, 71, Granville Avenue, West Hartlepool, Co. Durham.

BEDFORD, Thomas Edwin, B.Sc. (Tech.) Manchester, 28A, Tatton Road South, Heaton Moor, Stockport, Cheshire.

BHANDARKAR, Prafullachandra Ganesh, B.Sc., B.E.(Civil), Bombay, 6B, Harjivandas Estate, nr. Broadway Cinema, Vincent Road, Dadar, Bombay.

CARTWRIGHT, John Desmond, 12, Pine Grove, Kings Heath, Birmingham 14.

CLARK, Frank, B.Sc. (Eng.) Glasgow, 3, Park Lane, Wallington, Surrey.

EVANS, Phillip George, 14, Dale Drive, Whitby, Ellesmere Port, Cheshire.

FARRINGTON, Denis James, 13, West Central Drive, Swinton, Manchester.

GEDGE, David Gregory, B.Sc. (Eng.) London, A.C.G.I., 31, Collingwood Road, Great Yarmouth, Norfolk.

LETHEREN, Dennis, A.M.I.Mun.E., 21, Abbotts Drive, N. Wembley, Middlesex.

PITCHFORD, Brian Edward, 17, Leafield Road, Darlington, Co. Durham.

ROBERTS, Albert James, St. Margaret's Cottage, Oxford Passage, Cheltenham, Glos.

ROBINSON, William Crathern, 34, Eastvale, The Vale, Acton, W.3.

WEST, Robert Ernest, 65, Abbotts Drive, Wembley, Middlesex.

WHIPP, Alan, 163,Harcourt Street, Reddish, Stockport, Cheshire.

WILKIE, David Gordon, 149A, Laburnum Road, Redcar, Yorks.

WILLIAMS, David Whitlock Peers, 42, Claremont Road, Sparkbrook, Birmingham 11.

WRIGHT, Donald Hilton, B.Sc. (Eng.) London, 28, Porter Road, Derby.

ASSOCIATES

NICHOLLS, Ralph George, A.R.I.B.A., Waterloo House, 41, St. Mary Street, Bridgwater, Somerset.

SHEPHARD, Major, Arthur, Office of the Chief Engineer, Headquarters, Western Command, Queen's Park, Chester.

MEMBERS

BROWNSTON, Alfred John, "Shanklin," 39, Wellington Hill West, Henleaze, Bristol.

DOBELL, Curzon, 211, East 37th Street, New York 16, N.Y., U.S.A.

TRANSFERS

Students to Graduates

BALASUBRAMANIAM, Kumarasamy, B.Sc.(Eng.), London, Junior Assistant Engineer, P.W.D. University Scheme, Peradeniya, Ceylon.

BOWERING, Brian Arthur, c/o 10, Belper Street, Garston, Liverpool, 19.

BRYANT, Michael Ernest, 30, Groveland Road, Beckenham, Kent.

COLE, Eric Maxwell, 38, Western Road, Newhaven, Sussex.

GRAHAM, Leslie, 18, Deal Street, Bury, Lancs.

HARDY, Donald, 33, Oak Street, Elland, Yorks.

KERR, Gordon Cecil, 81, Elmers End Road, Anerley, S.E.20.

MEACOCK, Peter Henry, Deepdene, London Road, Daventry, Northants.

NORMAN, John Leonard, 106, Warner Place, Hackney Road, E.2.

WHEELER, John Bingham, 261, Grove Lane, Handsworth, Birmingham, 21.

Graduates to Associate Members

AINSCOW, Kenneth Edwin, B.Sc.(Tech.), Manchester, 12, St. Helen's Road, Dorchester, Dorset.

ANG BOO SOKE, 127, Joo Chiat Place, Singapore, 15.

RUFFLE, Nigel John, B.Sc.(Eng.), London, Lowburn Cottage, Ireshopeburn, Bishop Auckland, Co. Durham.

Associate Members to Members

GENTRY, William Gibson, A.M.I.C.E., 26, Cranford Gardens, Acklam, Middlesbrough, Yorks.

LALLEMAND, Joseph Francois, P.O. Box 105, Kitwe, Northern Rhodesia.

Members to Retired Members

INMAN, Frederick Philip, F.I.A.A., M.Inst.R.A., Santa Monica, Westhill, Paignton, S. Devon.

JONES, John Arthur, F.R.I.C.S., 33, Birch Grove, Acton, W.3.

LEATHERDALE, Francis Harold, Tasli, Hawks Hill, Leatherhead, Surrey.
MCINTYRE, Stewart, Cruachan, Longniddry, East Lothian.
NORTHCOTT, James William, 14, Holmcote Gardens, Highbury New Park, N.5.
SWINGLEHURST, James Gordon, 9, Ashburnham Gardens, Upminster, Essex.
WELLS, William, A.M.I.Mech.E., Antofagasta (Chile) & Bolivia Rly.Co.Ltd., Casillas "S–T," Antofagasta, Chile, S. America.

OBITUARY

The Council regret to announce the deaths of Edward Eric BILLINGTON, Alfred George CURTIS, Sydney Hubert WHITE (Members); George FINCKEN (Retired Member); Harold Walter FYSON, and Harry Turner STAGG (Associate Members).

RESIGNATIONS

Notification was given that the Council had accepted with regret the resignations of Frank BAXTER, Edwin Forbes BOTHWELL, F.R.I.B.A., Richard GREENHALGH (Associates); Samuel Ernest Lovell BENDALL, Charles William CLARKE, Joseph Conan HAMER, Reginald George IMBER, Aye MAUNG, O.B.E., Anant Hiralal PANDYA, George RICHWOOD (Associate Members); Major Bruce Theodore BOSWELL, Noel Leslie COSTAIN, Michael Alexander HASTILOW (Graduates); Alan Graeme GIBBS, Peter Hugo GUENSBERGER, Peter William HANSEN, Donald Alec MARCHANT, Ronald Philip PERRY, and John Havelock THORNTON (Students).

SPECIAL NOTICE

It has been necessary to make the following alterations to the Sessional Programme as previously announced :—

The Paper by Lt.-Colonel G. W. Kirkland and Mr. A. Goldstein, which was to have been given on April 12th, has been transferred to April 26th, 1951.

The Paper by Dr. K. Hajnal-Konyi, which was to have been given on April 26th, has been transferred to May 17th, 1951.

FORTHCOMING MEETINGS

The following meetings will be held at 11, Upper Belgrave Street, London, S.W.1. :—

Thursday, April 5th, 1951

Joint Meeting with the British Section of the Societe des Ingenieurs Civils de France at 6 p.m. when a paper on "The Tuscan Boracic 'Soffioni' and their Development at Larderello," by Dr. Alfredo Mazzoni, will be presented by Mr. Peter Gerard, B.Sc., A.M.I.E.E. (Member). The paper will be illustrated by lantern slides and a film.

Thursday, April 26th, 1951

Ordinary General Meeting for the election of members, 5.55 p.m., followed by an Ordinary Meeting at 6 p.m. Lt.-Col. G. W. Kirkland, M.B.E. (Member), and Mr. A. Goldstein, B.Sc.(Eng.), A.C.G.I., D.I.C. (Graduate), will present a paper on "The Design and Construction of a Large Span Prestressed Concrete Shell Roof."

Thursday, May 17th, 1951

Ordinary Meeting at 6 p.m., when Dr. K. Hajnal-Konyi (Member), will present a paper on "Comparative Tests on Various Types of Bars as Reinforcement of Concrete Beams."

Members wishing to bring guests to the Ordinary Meetings announced above are requested to apply to the Secretary for tickets of admission.

Thursday, May 24th, 1951

Annual General Meeting.

Thursday, June 28th, 1951

Ordinary General Meeting for the election of members only.

LONDON GRADUATES' AND STUDENTS' SECTION

The next meeting of the Section will be held at 11, Upper Belgrave Street, S.W.1, on Tuesday, April 17th, at 6 p.m., when Mr. S. M. Reisser, B.Sc., A.M.I.C.E. (Member), will give a lecture on "The Arc Welding Process and its Application to Structural Engineering."

On Saturday, May 5th, there will be a visit to Murex Welding Processes Ltd., Waltham Cross, Herts, conducted by Mr. S. M. Reisser, when there will also be a demonstration of arc welding.

Hon. Secretary : W. S. Twelvetrees, 6, Dell Road, Enfield Wash, Middlesex.

EXAMINATIONS, JULY, 1951

The Examinations of the Institution will be held at centres in the United Kingdom and Overseas on July 17th and 18th (Graduateship), and July 19th and 20th (Associate Membership).

DRURY MEDAL AWARD

The third competition for the above award will take place in 1951, and the subject will be the design of a reinforced concrete garage with provision for erection above it of future superstructure, the garage being subjected to unbalanced earth pressure.

Graduates and Students of the Institution who wish to compete are invited to apply for full details to the Secretary; envelopes to be marked in the top left-hand corner "Drury Medal Award."

The closing date for the competition is October 1st, 1951.

The general conditions of the competition are as follows :—

1. The competition shall be for Graduates and Students of the Institution of not more than 25 years of age.
2. The subject of the competition shall be a design of a structural character, that is to say, primarily structural design, not planning.
3. The subject of design and conditions shall be prepared and issued biennially by a group of five members appointed by the Council.
4. The Literature Committee shall appoint a jury of not less than five to examine the works submitted and to interview candidates, if found necessary.
5. In order to show that the work submitted is solely the work of the competitor, the documents submitted shall be countersigned by a corporate member of the Institution, or, failing this, shall be accompanied by a declaration on a prescribed form signed by the candidate in the presence of a Justice of the Peace or a Commissioner for Oaths.

REPRINTS FROM THE JOURNAL

Authors who wish to obtain reprints of their papers published in the Journal, are requested to communicate with the publishers at least one calendar month before the date of publication of the paper in question.

DESPATCH OF JOURNAL

As the result of an official order which came into force on February 1st, it is no longer permitted to use the wrapping paper in which the Journal has hitherto been despatched. For this reason, the Journal is now being wrapped for postage in a substitute paper.

BRANCH NOTICES

LANCASHIRE AND CHESHIRE BRANCH

The following meeting has been arranged :—

Thursday, April 19th, 1951

Annual Business Meeting to be held in the Reynolds Hall, College of Technology, Manchester, at 7 p.m.

Hon. Secretary : G. Greenlees, A.M.I.Struct.E., 16, Mayfield Avenue, Stretford, Lancs.

GRADUATES' AND STUDENTS' SECTION

Hon. Secretary : F. M. Turner, 11, Colwell Avenue, Stretford, Lancs.

MIDLAND COUNTIES BRANCH

The following meeting has been arranged :—

Friday, April 27th, 1951

Annual General Meeting, followed by a paper by Mr. H. E. Brooke-Bradley, O.B.E. (Member), on " The Problem of the Supporting Power of the Subsoil," to be held at the James Watt Memorial Institute, Birmingham, at 6 p.m.

Hon. Secretary : E. R. Deeley, A.M.I.Struct.E., Arranmoor, Adshead Road, Dudley, Worcs.

GRADUATES' AND STUDENTS' SECTION

The following meetings have been arranged :—

Saturday, May 5th, 1951

Visit to the Building Research Station, Watford.

Tuesday, May 29th, 1951

Joint Meeting with the Students' Section of the Institution of Civil Engineers. A talk on Code of Practice 114 : 1948 — The Structural Use of Reinforced Concrete in Building, will be given by Mr. A. T. Clarke (Associate Member), to be held at the James Watt Memorial Institute, Birmingham, at 7 p.m.

Hon. Secretary : M. H. Evans, B.Sc., 107, Metchley Lane, Harborne, Birmingham, 17.

NORTHERN COUNTIES' BRANCH

The following meeting has been arranged :—

Tuesday, April 3rd, 1951

Annual General Meeting to be held at the Cleveland Scientific and Technical Institution, Corporation Road, Middlesbrough, at 6.30 p.m., preceded by tea at 6 p.m.

Hon. Secretary : J. A. Williams, A.M.I.C.E., A.M.I.Struct.E., 57, Reid Park Road, Jesmond, Newcastle-upon-Tyne.

NORTHERN IRELAND BRANCH

The following meeting has been arranged :—

Tuesday, April 3rd, 1951

Annual General Meeting to be held in the College of Technology, Belfast, at 7.15 p.m.

Hon. Secretary : S. G. Duckworth, M.I.Struct.E., Lisleen, 13, Finaghy Road North, Belfast.

SCOTTISH BRANCH

The following meeting has been arranged :—

Tuesday, April 17th, 1951

Annual General Meeting at Ca'doro Restaurant, Glasgow, at 6 p.m.

Hon. Secretary : D. G. Drummond, B.Sc., A.M.I.C.E., A.M.I.Struct.E., 11, Woodside Terrace, Glasgow, C.3.

SOUTH-WESTERN COUNTIES BRANCH

An Ordinary General Meeting of the South Western Counties Branch was held at the Globe Hotel, Newton Abbot, at 7 p.m. on Friday, February 16th, 1951.

A very interesting and instructive lecture, illustrated by lantern slides, was delivered by Mr. F. S. Snow, M.I.C.E., M.I.Mech.E., (Past President), entitled " Recent Examples and further application of Shell Concrete Construction."

The new Branch Chairman, Mr. L. F. Vanstone, L.R.I.B.A., M.I.Struct.E., presided, and members attended from Plymouth and Barnstaple districts.

The lecture was followed by a discussion in which all members took part, and Mr. Snow expressed the hope that he would be able to have certain tables printed which would considerably simplify the design of shell concrete roofs.

Hon. Secretary : E. W. Howells, M.I.Struct.E., 10-12, Market Street, Torquay, Devon.

WALES AND MONMOUTHSHIRE BRANCH

The following meetings have been arranged :—

Wednesday, April 11th, 1951

Visit to Abbey Works site, Margam, to inspect the buildings for the New Strip Mills for the Steel Company of Wales.

Wednesday, April 11th, 1951

Annual Dinner, at Porthcawl.

Tuesday, May 1st, 1951

Annual General Meeting, to be held at the Mackworth Hotel, Swansea, at 6.30 p.m.

Hon. Secretary : E. R. Steward, A.M.I.Struct.E., Edrom, Ashleigh Road, Blackpill, Swansea.

WESTERN COUNTIES BRANCH

The Fifth Meeting of the Session was held at the College of Technology, Bristol, on Friday, February 2nd, 1951, and was a joint meeting with the South-Western Association of the Institution of Civil Engineers.

Mr. P. C. Girdlestone, M.A., M.I.Struct.E., M.I.Mech.E., A.M.I.C.E. (Branch Chairman) was in the Chair, and a lecture on " A Chapter of Engineering a Century Ago " was given by Professor Andrew Robertson, D.Sc., F.R.S., M.I.C.E.

The lecture was a most interesting account of the experimental work and construction of the Britannia Tubular Bridge over the Menai Strait and referred briefly to Brunel's ship *The Great Eastern.*

A vote of thanks was proposed by Mr. G. E. Scott, O.B.E., M.I.C.E. (Branch Chairman of the Institution of Civil Engineers).

The following meetings have been arranged :—

Friday, April 6th, 1951

Annual General Meeting, followed by a talk by Mr. Evelyn Freeth, A.R.I.B.A., A.A.Dip., A.R.W.A., at Grand Hotel, Bristol, at 6.15 p.m., preceded by tea at 5.30 p.m.

Thursday, April 26th, 1951

Joint Meeting with the Institution of Civil Engineers. Mr. Gilbert Roberts, B.Sc., A.M.I.C.E., on " The Dome of Discovery—Festival of Britain Site," at Reception Room, Bristol University, Queen's Road, at 5.30 p.m., preceded by tea at 5 p.m.

Hon. Secretary : C. E. Saunders, M.I.Struct.E., Dunkery, Edward Road, Walton St. Mary, Clevedon, Somerset.

YORKSHIRE BRANCH

The following meetings have been arranged :—

Wednesday, April 18th, 1951

Professor R. H. Evans, D.Sc., Ph.D., M.I.C.E., M.I.Mech.E., (Member), on The Institution of Structural Engineers' " Report on Prestressed Concrete."

Wednesday, May 9th, 1951

Annual General Meeting. Mr. J. G. Faber, B.Sc., A.M.I.C.E., (Associate Member), on " Construction of 240 ft. Clear Span Sheds for Aircraft Production."

The meetings will be held at the Great Northern Hotel, Leeds, at 6.30 p.m.

Hon. Secretary : E. Wrigley, A.M.I.Struct.E., City Engineer's Dept., Civic Hall, Leeds, 1.

UNION OF SOUTH AFRICA BRANCH

Branch Hon. Secretary : A. E. Tait, B.Sc., A.M.I.C.E., A.M.I.Struct.E., P.O. Box No. 3306, Johannesburg. During week-days Mr. Tait can be contacted in the City Engineer's Department, City Hall, Johannesburg, 'phone 34–1111 ext. 257.

Natal Hon. Secretary : E. G. Bennett, A.M.I.Struct.E., c/o Reinforcing Steel Co. Ltd., P.O. Box 478, Durban.

Cape Section Hon. Secretary : R. Stubbs, M.I.Struct.E., P.O. Box 1692, Cape Town.

Comparative Tests on Various Types of Bars as Reinforcement of Concrete Beams*

By Dr. K. Hajnal-Konyi, M.I.Struct.E., A.M.I.C.E.

Synopsis

Comparative tests on beams reinforced with various types of large size bars, both in ordinary grade and high grade concrete, have proved the superiority of cold worked steel over steel having a natural yield point regarding factor of safety and warning before failure. They have also proved the advantage of a ribbed pattern as against plain bars from the point of view of bond and crack formation and the necessity of increasing the strength of reinforcement with improved bond in order to avoid failure without warning. Suggestions are made for certain revisions of the Code of Practice C.P. 114 (1948) and B.S. 1144 so as to allow the economical use of high quality steel as reinforcement of concrete.

1. Principles of Comparing Various Types of Bars

The purpose of the tests to be described and discussed in this paper was to compare various types of bars from the point of view of their suitability as reinforcement of concrete.

Before details of the bars are given, it is necessary to clarify the principles which were established as a basis of comparison.

The Americans assess the relative value of various types of bars mainly from bond tests (1) and (2)†. However, the usual bond test on pull-out specimens has a very limited scope and gives a one-sided picture. Even if the test is modified as suggested by the A.C.I. Committee 208 (3), it only covers one aspect of a very complex subject and does not deal with many others which are of equal importance.

In the author's opinion, *the value of a reinforcing bar can only be assessed on the basis of beam tests.* The test beams must be so designed that they fulfil a number of conditions, i.e.

(1) They must fail either in tension or by bond slip but not in compression or by shear.

(2) The diameter of the bars used must be as large as possible.

(3) The distance between the end of the beam and the section at which the maximum bending moment occurs must be the minimum compatible with the practical use of the bar.

The main considerations in assessing the bars are :

(a) Type of failure (i.e. whether in tension or by bond slip).

(b) Maximum stress reached at failure both in itself and in relation to the yield point and the tensile strength of the bar.

(c) Formation of cracks, i.e., their number and maximum width at various stages of loading.

(d) Warning before failure.

There are, of course, also other factors to be taken into account, as will be discussed later.

Condition (1) can be satisfied by limiting the percentage of the reinforcement and by providing sufficient links.

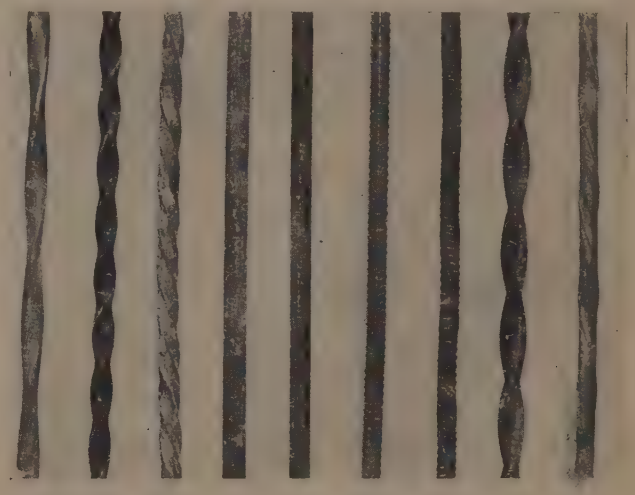


Fig. 1

(1) 1 in. Diameter Oval Bar twisted to 6 in. pitch.
(2) 1 in. ,, ,, ,, 12 in. pitch.
(3) $\frac{7}{8}$ in. Square Twisted Bar.
(4) $1\frac{1}{4}$ in. Diameter M.S. Bar.
(5) $1\frac{1}{8}$ in. ,, H.T. Bar.
(6),(8) 1 in. ,, American Ribbed Bar.
(7),(9) 1 in. ,, ,, ,, ,, twisted.
(10) $\frac{3}{4}$ in. ,, Twin Twisted Bar.
(11) 1 in. ,, Twisted Bar.

Condition (2) is particularly important since the width of the cracks increases approximately with the diameter of the bars (4). Tests made with small size bars do not allow conclusions regarding stresses at which larger size bars can be used without the danger of corrosion. Since it is difficult to obtain very large sizes of various types, a cold worked bar of 1 in. diameter complying with B.S. 1144 seems to be a fair compromise.

Condition (3) must be fulfilled in order to allow the development of the maximum average bond stress which may occur in a practical case.

2. Description of the Bars used in the Tests

The tests comprised 17 different bars and were carried out in two series. In Series 1, 11 bars were investigated; see Fig. 1. The selection of bars for Series 2 was based on the results of Series 1. The six bars tested in Series 2 are shown in Fig. 2.

*Paper to be read before the Institution of Structural Engineers at 11, Upper Belgrave Street, London, S.W.1, on Thursday, May 17th, 1951, at 6 p.m.

†For references see p. 148

The bars have been numbered 1 to 17. Their geometrical properties and the principles of their selection are described in the following notes, their strength properties are given in Tables 1 and 2.

No. 1 was twisted from a M.S. bar of elliptical cross section. The ratio of the major axis of the ellipse to the minor axis was 3 : 2. The dimensions of the bar were so chosen that its cross sectional area was equal to that of a 1 in. diameter round bar. This bar is referred to as " oval twisted." The pitch was 6 in., i.e., the bar was purposely " overtwisted " as against B.S. 1144, according to which the pitch must be between 8D and 14D. For a bar of elliptical cross section D may be interpreted as the diameter of a circle of equivalent area.

No. 2 was similar to Bar No. 1, but the intended pitch was 12 in. Actually, only one of the two specimens used in the tests was twisted to the required degree, the pitch of the other was 17 in., i.e., the bar was " undertwisted."

No. 3 was a 7/8 in. square twisted bar, as available on the British market.

No. 4 was a $1\frac{1}{4}$ in. diameter plain round bar of M.S. quality. On the basis of a ratio of permissible stresses of 18,000 lb/sq. in. to 27,000 lb/sq. in., as specified

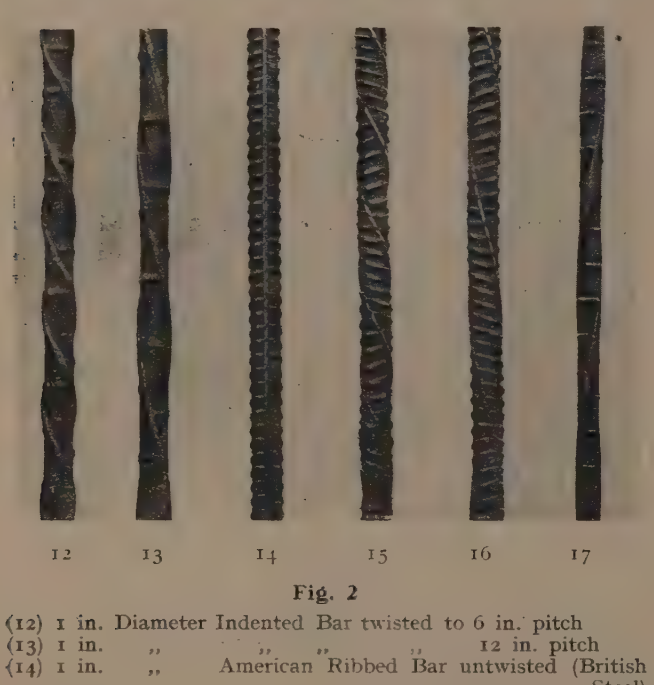


Fig. 2

(12) 1 in. Diameter Indented Bar twisted to 6 in. pitch
(13) 1 in. " " " " 12 in. pitch
(14) 1 in. " American Ribbed Bar untwisted (British Steel)
(15) 1 in. " " " twisted to 6 in. "
(16) 1 in. " " " " 12 in. "
(17) 0.71 in. Diameter Tentor Bar.

in the Code of Practice C.P. 114, 1948 (hereafter referred to as the " Code ") for M.S. bars and cold worked bars respectively, this is the nearest size M.S. bar equivalent to a 1 in. diameter cold worked bar.

No. 5 was a $1\frac{1}{8}$ in. diameter plain round bar of high tensile quality to B.S. 785.

No. 6 was a 1 in. diameter " deformed " bar of M.S. quality complying with the latest U.S.A. standards (A.S.T.M. Designation A305–47T), obtained from U.S.A. Its pattern can clearly be seen in Fig. 1. It is referred to as " American ribbed " bar.

No. 7 was twisted from bar 6.

No. 8, also supplied from U.S.A., was similar in shape to No. 6, but specified as of " high tensile " quality. (Actually, as may be seen from Table 1, its ultimate strength was less than the lowest limit specified for high tensile steel in B.S. 785, but its yield point reached the minimum requirement for high tensile steel).

No. 9 was twisted from Bar No. 8, as No. 7 from No. 6.

No. 10 was a twin-twisted bar of $\frac{3}{4}$ in. diameter (of the single bars) as available on the British market.

No. 11 was a 1 in. diameter Torsteel bar, rolled and twisted, but not marketed, in this country. Torsteel is rolled as a round bar with two small longitudinal ribs diametrically opposed which form two spirals after twisting.

The result of twisting bars 6 and 8 was not satisfactory. The stress-strain diagram of both the twisted and untwisted bars was rather different from that of bars to B.S. 785 produced in this country. It was therefore decided to roll similar bars here (bar 14, Fig. 2) and twist them both to 6 in. and 12 in. pitch (bars 15 and 16). It was further decided to investigate also Tentor bars imported from Denmark (bar 17). A Tentor bar is first tensioned to or beyond its yield point and then twisted (Tentor = tension + torsion). The raw material used may be M.S. or a special alloy steel [5]. These Tentor bars were not obtainable in larger sizes than .71 in. diameter. In order to have a bar of similar shape comparable in size with the others, 1 in. Indented bars were bought in this country and twisted to both 6 in. and 12 in. pitch (bars 12 and 13). It may be seen from Fig. 2 that bars 12, 13 and 17 are very similar in shape, they differ only in the degree of twist. In twisting bars 12 and 13, the ordinary twisting process was adopted. The twist of Tentor bars is less than specified in B.S. 1144. Judged by their shape, they appear to be " undertwisted " although they are, in fact, fully developed, as proved by their strength properties and stress-strain diagram.

An explanation is necessary regarding the use of the terms " undertwisted " and " overtwisted." As stated in the author's paper [6] " The degree of twist can be characterised by the figure : Size of bar in inches × number of twists per foot." This figure is called work hardening unit (W.H.U.). The minimum pitch specified in B.S. 1144, i.e. 8D, corresponds to 1.5 W.H.U., the maximum, i.e. 14D, to .858 W.H.U. Bars with less than .858 W.H.U. are called " undertwisted," bars with more than 1.5 W.H.U. are called " overtwisted." The use of W.H.U.s in describing a bar has the advantage that a higher degree of twist is characterised by a higher figure, whereas the usual notation based on the pitch, is an inverse measure of cold work. For untwisted bars the pitch $=\infty$ whilst W.H.U. $= 0$.

The preceding definition of the terms " undertwisted " and " overtwisted," based on B.S. 1144, is only applicable to bars developed by twisting alone and not to bars developed by the Tentor process.

3. Design of the Test Beams

Details of the test beams are shown in Fig. 3.

The cross section was so determined that with the maximum permissible steel stress according to the Code acting in the reinforcement, the calculated maximum compressive stress in the concrete is slightly below the permissible limit for concrete of 1 : 2 : 4 mix and a cube strength of 3,000 lb/sq. in. In order to avoid an excessive cover to the reinforcement, the section was tapered to 4 in. at the bottom.

The length of the beams was governed by the following considerations :

According to the Code, Clause 307 : " a bar in tension should extend from any section on the side further

from the point of maximum moment, for a distance such that the average bond stress does not exceed B_1, the appropriate permissible bond stress given in Tables 4, 5 and 6, in Clause 303. This condition will be satisfied if the length measured from such section is at least equal to n times the diameter of the bar where

$$n = \frac{\text{the tensile stress in the bar}}{\text{four times the permissible bond stress } B_1}$$

The maximum permissible stress is 27,000 lb/sq. in. For concrete of 1 : 2 : 4 mix B_1 = 120 lb/sq. in.

Hence $n = \frac{27{,}000}{4 \times 120} = 56.2$

Since it may be expected that the bond strength of cold worked bars exceeds that of a plain round bar by at least 10 per cent., advantage can be taken of Clause 307 (h) relating to "deformed" bars and n reduced by 10 per cent., i.e. to 50.6. The beam was so designed that the distance from the end to the section of the maximum bending moment is 50 in. With a length of 6 in. beyond the theoretical point of support, the distance between the support and the maximum bending moment is 44 in. To obtain reliable results, it is desirable to extend the maximum bending moment over one-third of the span. In this way, a theoretical span of 3 × 44 in. = 11 ft. is arrived at, resulting in an overall length of 12 ft.

Thus the beams represented the most unfavourable conditions for cold worked bars of 1 in. equivalent diameter in 1 : 2 : 4 concrete with regard to bond stresses under the Code.

All cold worked bars were left protruding at the ends so as to allow the observation of any movement of the bar ends relative to the concrete. Plain bars with a natural yield point were provided with a standard hook.

The tapered cross section shown in Fig. 3 has been maintained in all beams reinforced with a single bar. With Tentor bars, having a smaller cross section than all the others, beams reinforced with 2 and 3 bars respectively were also tested (beams 18 and 19). These were of rectangular cross section of 8 in. width and 18 in. overall depth with the number of links increased from 12 to 17 on both sides.

4. Test Programme and Method of Testing

Although it is always desirable to have at least one repetition for every test, it appeared to be preferable to vary the concrete strength for the second test instead of repeating the first with the same mix, since in view of the great number of bars to be compared it was not possible to have four tests for each bar. For each bar one beam was made in concrete of 1 : 2 : 4 mix with a target cube strength of 3,000 lb/sq. in. on the day of testing (beams marked "L") and one in concrete of 1 : 1½ : 3 mix, with a target cube strength of 6,000 lb/sq. in. on the day of testing (beams marked "H"). The only exception was made in the case of Tentor bars, with which 3 "L" beams were tested (reinforced with 1, 2 and 3 bars respectively) and only 1 "H" beam (reinforced with 3 bars). Altogether, 11 + 8 = 19 beams were tested in "L" concrete and 11 + 6 = 17 beams in "H" concrete (see Tables 1 and 2).

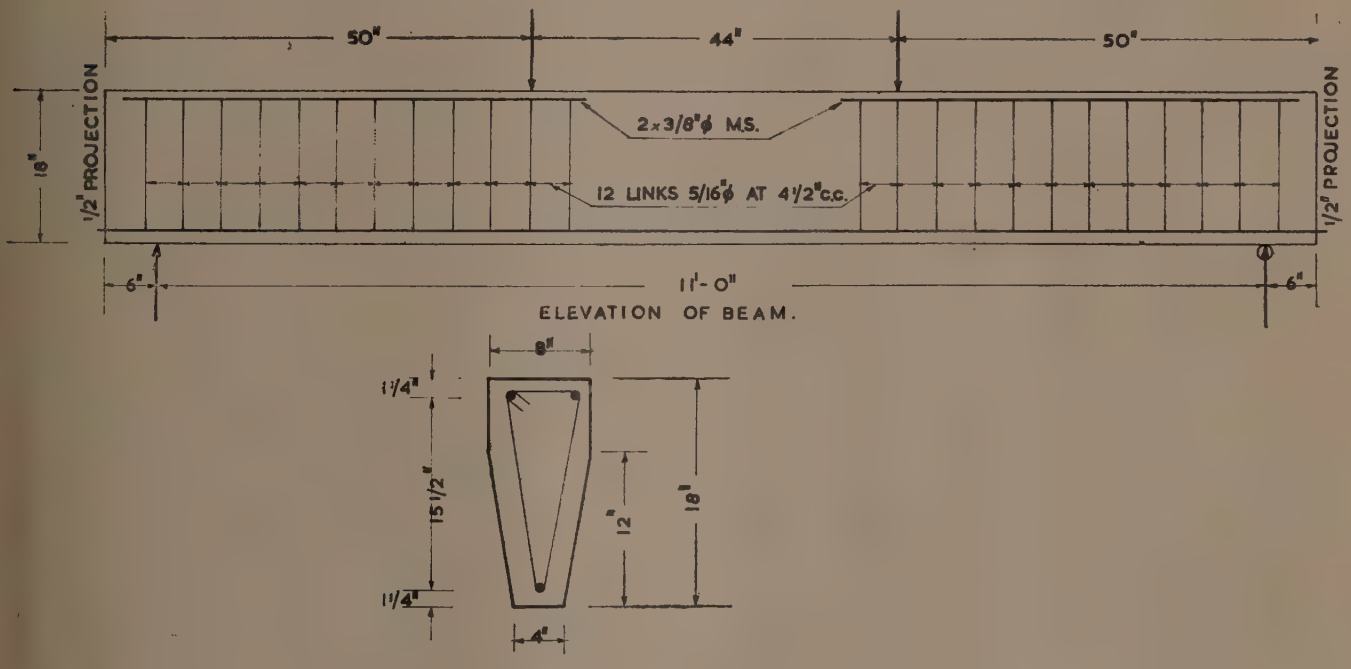


Fig. 3.—Test beam

All tests were carried out at Mr. R. H. Harry Stanger's laboratories and all figures in this paper are based on information contained in Mr. Stanger's original reports.

The beams were supported on steel rollers at a span of 132 in. and the load was applied at the third points. At the points of support and loading, steel distribution plates 6 in. wide were inserted between the rollers and the concrete.

Load was applied in increments of 2,000 lb total load and the beam examined for cracks. At every 4,000 lb the load was maintained for at least 5 minutes, the deflection being recorded at the beginning and

end of this time interval. The cracks were also measured to record the maximum width.

After applying a load of 20,000 lb, corresponding to an excess of slightly more than 50 per cent. over the permissible working load according to the Code for beams with cold worked bars of 1 in. diameter, the load was released and the permanent deflection and the crack widths observed. The only exception to this procedure was adopted in the case of beams 19 where, in view of the much higher ultimate load to be expected, the load was released at 40,000 lb.

In Series 2, on 10 beams measurements were taken of the change of strain at the centre section.

The cement used was normal Portland cement, the aggregate re-graded Ham river. The water : cement ratio was as follows :

	Series 1	Series 2
"L" concrete... ...	.55	.55
"H" concrete... ...	.42	.44

At least 2 cubes were made for each beam. In addition, three prisms 6 in. × 6 in. × 18 in. were cast for beams 9L and 7H in Series 1, and for beams 13L and 15H in Series 2.

5. Analysis of Test Results

(A) Calculation of Stresses

Tables 1 and 2 contain the analysis of test results for Series 1 and 2 respectively. The greater part of these tables is self-explanatory, but some explanations are nevertheless necessary.

The values for the maximum bending moment (M) include the effect of dead weight of the beam (Line 14).

The steel stresses at failure have been worked out both on the assumption of a rectangular stress distribution in the compressive zone (Whitney's theory) and according to the standard method of design with $m = 15$. Values with an index "p" refer to a rectangular stress distribution, values with an index "m" to the standard method. For further details, see Fig. 7 in the author's paper [8].

When Whitney's theory is applied, it is necessary to make an assumption regarding the value of the concrete stress in the rectangular stress block (c_p). The ratio prism strength : cube strength was found to be as follows :

	Series 1	Series 2
"L" concrete... ...	.76	.87
"H" concrete... ...	.70	.67

Whitney's theory is based on the cylinder strength, for which he suggests a reduction factor of .85. The cylinder strength is slightly higher than the prism strength. In Table 1, the value of c_p has been assumed uniformly at 2/3 of the average cube strength. This corresponds to a reduction factor for the prism strength of .88 for "L" concrete and .95 for "H" concrete. In order to keep the two series consistent, the reduction factors applied to Series 2 were :

$$\text{For "L" concrete} \quad \frac{.87}{.76} \times .667 = .76$$

$$\text{For "H" concrete} \quad \frac{.67}{.70} \times .667 = .64$$

It is realised that the ratio prism strength : cube strength was probably different in each beam and that the method adopted in assessing c_p from the average cube strength is an approximation. On the other hand, the effect of a possible variation of c_p on the tensile stress at failure is very small within the range of percentages included in the two series which were all far below the limiting value of "balanced" design.

TABLE 1.—Analysis of Test Results of Series 1

1	No. of Beam		1L*	1H	2L	2H*	3L	3H	4L*	4H	5L*	5H*	6L	6H	7L*	7H	8L	8H	9L*	9H	10L*	10H	11L*	11H	1
2	Reinforcement		1" Oval Twisted				⅞ Sq. Twisted		1¼" D Mild Steel		1⅛" D High Tensile		1" Dia. Amerdian Ribbed M.S.		1" Dia. Amerdian Ribbed M.S. Twisted		1" Dia. Amerdian Ribbed		1" Dia. Amerdian Ribbed M.T. Twisted		¾" D Twin Twisted		1" Dia. Torsteel		2
3	Pitch	in.	6·4	6·5	12·0	17·0	8·0	9·5	—	—	—	—	—	—	11·5	7·5	—	—	12·0	21·5	9·0	9·0	12·5	11·5	3
4	Work Hardening Unit (W.H.U.)	in.	1·875	1·846	1·000	706	1·313	1·105	—	—	—	—	—	—	1·043	1·60	—	—	1·0	596	1·000	1·000	860	1·043	4
5	Sectional Area [A_s]	sq. in.	·785	·786	·785	·782	·782	·777	1·267	1·257	·959	1·012	·777	·773	·783	·756	·769	·779	·775	·772	·825	·860	·768	·772	5
6	Yield Point [y]	lb/sq. in.	73,300	73,920	68,530	61,150	68,770	62,940	38,530	37,860	58,100	53,960	40,100	43,080	53,310	53,810	51,970	51,520	65,410	63,670	59,580	56,900	67,200	70,780	6
7	Ultimate Strength [u]	lb/sq. in.	84,670	85,790	77,500	75,450	79,520	72,130	65,180	62,750	51,640	51,620	64,740	64,960	70,780	78,620	81,880	79,740	80,840	83,550	72,580	68,220	81,310	84,270	7
8	Ratio $\frac{y}{u}$		·865	·861	·855	·810	·865	·873	·591	·567	·930	·955	·619	·672	·753	·781	·627	·646	·719	·761	·821	·822	·838	·840	8
9	Cover	in.	1·35	1·25	1·45	1·40	1·45	1·40	1·40	1·38	1·38	1·30	1·25	1·50	1·25	1·80	1·80	1·38	1·50	1·55	1·40	1·25	1·46	1·25	9
10	Effective Depth (d)	in.	16·05	16·15	15·95	16·00	15·83	15·88	16·97	16·07	16·19	16·14	16·25	16·20	16·25	15·70	16·20	16·15	16·00	15·86	15·86	16·00	16·05	16·25	10
11	Percentage $\left[\frac{100\,A_s}{bd}\right]$	%	·608	·606	·615	·610	·614	·608	·991	·980	·771	·784	·598	·596	·602	·602	·593	·603	·606	·606	·577	·672	·598	·594	11
12	Average Cube Strength	lb/sq. in.	3,170	6,265	3,080	6,348	2,860	6,200	2,890	6,430	3,140	6,395	2,980	6,370	2,890	6,100	2,870	6,060	2,720	6,280	3,040	6,180	3,050	6,250	12
13	Maximum Load	lb.	36,000	45,000	36,000	36,000	34,000	36,000	34,000	36,000	36,000	38,000	27,000	34,000	37,000	36,000	28,000	28,000	36,000	38,000	36,000	37,750	34,200	42,000	13
14	Maximum Bending Moment (M)	lb. in.	814,700	1,012,700	814,700	814,700	770,700	814,700	770,700	814,700	814,700	858,700	608,700	550,700	726,700	814,700	594,700	638,700	814,700	858,700	814,700	853,200	773,100	948,700	14
15	$\frac{M}{bd^2c_p}$		·187	·116	·194	·084	·159	·096	·196	·092	·183	·098	·121	·082	·179	·102	·151	·078	·219	·101	·179	·101	·186	·111	15
16	$\frac{p}{d}$		·896	·936	·851	·981	·988	·949	·890	·982	·887	·848	·936	·965	·901	·849	·918	·960	·874	·947	·901	·947	·897	·941	16
17	Depth of Neutral Axis (n_p)	in.	3·34	2·00	3·48	1·97	3·97	1·63	3·51	1·34	3·33	1·68	2·08	1·04	3·22	1·70	2·93	1·78	4·03	1·88	3·14	1·70	3·31	1·92	17
18	Lever Arm [a_p] (Plastic Theory)	in.	14·38	15·15	14·31	15·21	14·15	15·17	14·72	15·25	14·52	15·30	15·21	15·68	14·64	14·85	14·67	15·60	13·98	15·10	14·38	15·15	14·40	15·29	18
19	Steel Stress at Failure (Plastic Theory) [c_p]	lb/sq. in.	72,170	84,830	73,030	68,470	69,660	69,130	42,780	42,480	56,160	55,470	42,870	45,430	63,400	72,560	57,000	62,880	73,170	73,640	66,410	63,480	70,150	80,700	19
20	Lever Arm [a_m] (Standard Method)	in.	14·20	14·29	14·10	14·15	14·09	14·14	13·76	13·81	14·14	14·08	14·39	14·35	14·38	13·90	14·35	14·30	14·16	14·12	13·95	14·08	14·21	14·39	20
21	Steel Stress at Failure (Standard Method) [c_m]	lb/sq. in.	73,080	88,340	73,580	73,600	68,540	74,140	44,720	46,540	57,660	60,240	45,310	49,650	64,510	77,530	53,860	57,350	74,230	78,750	68,010	70,450	71,000	85,160	21
22	Ratio $\frac{c_p}{c_m}$		·985	1·148	·986	1·130	1·043	1·036	1·110	1·172	1·045	1·078	1·065	1·040	1·183	1·213	1·001	1·096	1·145	1·157	1·115	1·151	1·044	1·133	22
23	Ratio $\frac{y}{c_m}$		·837	·829	·847	·807	·876	·858	·858	·836	·911	·856	·892	·899	·896	·923	·827	·823	·877	·881	·915	·946	·863	·867	23
24	Maximum Shear [S]	lb.	18,700	23,200	18,700	18,700	17,700	18,700	17,700	18,700	18,700	19,700	11,700	12,700	16,700	18,700	13,700	14,700	18,700	19,700	18,700	19,580	17,800	21,700	24
25	Average Bond Stress [B_1]	lb/sq. in.	365·2	415·0	367·7	368·4	353·1	377·0	219·7	231·4	258·6	271·5	224·1	244·3	321·6	373·1	263·8	284·4	366·2	387·2	350·7	363·6	347	418·8	25
26	Permissible Steel Stress to C.P. 114	lb/sq. in.	27,000	27,000	27,000	27,000	27,000	27,000	18,000	18,000	25,750	25,750	20,000	20,000	27,000	27,000	25,750	25,750	27,000	27,000	27,000	27,000	27,000	27,000	26
27	Permissible Bending Moment	lb. in.	300,950	304,100	298,840	298,870	297,870	298,690	313,780	312,440	383,870	387,070	223,630	221,870	304,130	283,710	284,210	286,770	298,340	294,250	323,450	317,010	284,740	300,070	27
28	Factor of Safety $\left[\frac{(14)}{(27)}\right]$		2·707	3·330	2·725	2·726	2·580	2·748	2·466	2·608	2·239	2·338	2·706	2·483	2·380	2·872	2·092	2·227	2·749	2·918	2·819	2·608	2·630	3·155	28

NOTE — In Beams marked thus × bar slipped at one end
In Beams marked thus ×× bar slipped at two ends

The average bond stresses (Line 25) have been calculated from the formula

$$B_1 = \frac{\text{stress at failure} \times \text{sectional area}}{\text{perimeter} \times \text{length}}$$

For "stress at failure," the stress calculated by the standard method has been assumed so as to make the values comparable with those specified in the Code. For bars of other than round cross section, i.e., Nos. 1, 2, 3, 10, the "perimeter" has been assumed as that of a round bar of equal sectional area, in accordance with a Note to Clause 307 of the Code. The "length" is the distance between the end of the beam and the point of loading, i.e. 50 in. for all bars without hooks.

In the case of hooked bars (Nos. 4 and 5) allowance has been made for the hooks in accordance with Clause 307 (d) of the Code. This Clause implies that a standard hook may be assumed as equivalent to a length of bar equal to 16 times its diameter. By adding 16 times the diameter to the straight length, the following lengths are obtained.

For a $1\frac{1}{4}$ in. bar, 65.0 in.
For a $1\frac{1}{8}$ in. bar, 63.5 in.

The permissible steel stress to the Code (Line 26) has been taken as 27,000 lb/sq. in. for all cold worked bars. For bars having a natural yield point, the following assumptions have been made :

Bar No. 4 has no guaranteed minimum yield point. The actual yield point is 37,850 lb/sq. in. and 38,580 lb/sq. in. respectively. It is therefore consistent with the Code to assume a permissible stress of 18,000 lb/sq. in. On the other hand, the yield point of the American ribbed type bars of M.S. quality (No. 6 supplied from U.S.A. and No. 14 rolled in this country) is in no case less than 40,000 lb/sq. in. It seems, therefore, to be justified to classify them as " mild steel with guaranteed minimum yield point " and to allow 20,000 lb/sq. in. For bars 5 and 8, both having a yield point of not less than 51,500 lb/sq. in., a stress of 25,750 lb/sq. in. has been allowed.

The permissible bending moments (Line 27) have been worked out from the actual cross sectional area (Line 5), the lever arm obtained by the standard method (Line 20), and the permissible stress in Line 26.

The factor of safety (Line 28) is the ratio $\frac{\text{bending moment at failure}}{\text{permissible bending moment}}$ which is the same as the ratio $\frac{\text{steel stress at failure to the standard method}}{\text{permissible steel stress}}$

(B) Discussion of Test Results

(a) Work Hardening

The degree of work hardening may be judged by the figures in Lines 4 and 8. The figures in Line 4 are a purely geometrical measure, not applicable to bars developed by the Tentor process. The figures in Line 8 are, on the other hand, of equal significance for any type of cold worked bar. It may be seen from Tables 1 and 2 that for all single twisted bars, excluding only the American ribbed type bars and the undertwisted oval bar in Beam 2H, but including the Tentor bars, the ratio $\frac{\text{yield point*}}{\text{ultimate strength}}$ varies between the narrow limits of .826 and .873.

TABLE 2.—Analysis of Test Results of Series 2

1	N° OF BEAM		12L	12H	13L	13H	14L	14H	15L	15H	16L	16H	17L	18L	19L	19H	1
2	REINFORCEMENT		1" INDENTED TWISTED				1" AMERICAN RIBBED TYPE		1" AMERICAN RIBBED TYPE TWISTED				·71" TENTOR	N°2 ·71" TENTOR	N°3 ·71" TENTOR		2
3	PITCH	in	6·0	6·0	12·0	12·0			6·0	6·0	12·0	12·0	12·0	12·0	12·0	12·0	3
4	WORK HARDENING UNIT (W.H.U)	in	2·0	2·0	1·0	1·0			2·0	2·0	1·0	1·0	0·71	0·71	0·71	0·71	4
5	SECTIONAL AREA [A_s]	sq. in	·7298	·7298	·7353	·7353	·7795	·7795	·7706	·7706	·7724	·7724	·401	·802	1·203	1·203	5
6	YIELD POINT [y]	lb/sq. in	73,200	73,700	61,400	60,500	41,400	40,100	69,400	70,300	63,400	64,100	84,900	86,800	85,900	86,000	6
7	ULTIMATE STRENGTH [u]	lb/sq. in	85,300	86,000	73,000	70,100	67,400	66,100	82,900	83,600	79,300	81,500	98,600	101,800	100,600	102,300	7
8	RATIO $\frac{y}{u}$		·856	·857	·841	·863	·614	·607	·837	·841	·799	·787	·861	·853	·854	·842	8
9	COVER	in	1·35	1·50	1·35	1·20	1·45	1·50	1·45	0·90	1·30	1·00	1·45	1·20	1·20	1·30	9
10	EFFECTIVE DEPTH (d)	in	16·15	16·00	16·15	16·30	16·05	16·00	16·05	16·60	16·20	16·50	16·19	16·44	16·44	16·34	10
11	PERCENTAGE $\left[\frac{100\,A_s}{bd}\right]$	%	·565	·570	·569	·564	·607	·609	·600	·580	·596	·585	·310	·610	·914	·936	11
12	AVERAGE CUBE STRENGTH	lb/sq. in	3,230	5,760	3,080	6,230	3,150	5,530	2,990	5,890	2,690	5,700	2,950	3,120	3,220	5,630	12
13	MAXIMUM LOAD	lb.	38,600	41,500	33,000	35,000	23,000	23,000	37,000	44,500	36,000	41,000	24,500	48,000	67,500	72,000	13
14	MAXIMUM BENDING MOMENT (M)	lb. in	871,900	935,700	748,700	792,700	528,700	528,700	836,700	1,001,700	814,700	924,700	561,700	1,083,300	1,512,300	1,611,300	14
15	$\frac{M}{bd^2\,C_p}$		·170	·125	·153	·095	·107	·073	·178	·121	·190	·117	·119	·211	·285	·219	15
16	$\frac{a_p}{d}$		·906	·933	·917	·950	·943	·962	·901	·935	·894	·937	·936	·880	·828	·875	16
17	DEPTH OF NEUTRAL AXIS (n_p)	in	3·04	2·14	2·68	1·63	1·83	1·22	3·18	2·16	3·43	2·08	2·07	3·95	5·66	4·01	17
18	LEVER ARM [a_p] (PLASTIC THEORY)	in	14·63	14·93	14·81	15·48	15·13	15·39	14·46	15·52	14·48	15·46	15·16	14·47	13·62	14·04	18
19	STEEL STRESS AT FAILURE (PLASTIC THEORY) [f_p]	lb/sq. in	81,700	85,900	68,800	69,600	44,800	44,100	75,100	83,800	72,800	77,400	92,400	93,300	92,300	95,400	19
20	LEVER ARM [a_m] (STANDARD METHOD)	in	14·34	14·20	14·34	14·48	14·20	14·16	14·21	14·72	14·35	14·63	14·78	14·55	14·23	13·86	20
21	STEEL STRESS AT FAILURE (STANDARD METHOD) [f_m]	lb/sq. in	83,300	90,300	71,000	74,500	47,800	47,900	76,400	88,300	73,500	81,800	94,800	92,800	88,300	96,600	21
22	RATIO $\frac{(18)}{(6)}$		1·116	1·166	1·121	1·150	1·082	1·100	1·082	1·192	1·148	1·207	1·088	1·075	1·075	1·109	22
23	RATIO $\frac{(19)}{(7)}$		·958	·999	·942	·993	·695	·667	·906	1·002	·918	·950	·937	·917	·917	·934	23
24	MAXIMUM SHEAR [s]	lb	20,000	21,450	17,200	18,200	12,200	12,200	19,200	22,950	18,700	21,200	12,950	24,850	34,600	36,850	24
25	AVERAGE BOND STRESS [B_1]	lb/sq. in	387·0	419·5	332·4	348·7	237·2	237·7	374·8	433·2	361·4	402·2	340·9	333·7	317·5	347·3	25
26	PERMISSIBLE STEEL STRESS TO C.P. 114.	lb/sq. in	27,000	27,000	27,000	27,000	20,000	20,000	27,000	27,000	27,000	27,000	27,000	27,000	27,000	27,000	26
27	PERMISSIBLE BENDING MOMENT	lb in	282,620	279,860	284,650	287,430	221,420	220,710	295,680	306,350	299,220	305,080	160,030	315,040	462,200 (1)	450,320	27
28	FACTOR OF SAFETY $\left[\frac{(14)}{(27)}\right]$		3·085	3·343	2·630	2·758	2·388	2·395	2·830	3·270	2·723	3·031	3·510	3·439	3·272 (1)	3·578	28

NOTE (1) ACCORDING TO THE STANDARD METHOD, THE CONCRETE STRESS CORRESPONDING TO THIS MOMENT IS 1,222 lb/sq in. THE PERMISSIBLE MOMENT IS THEREFORE GOVERNED BY THE PERMISSIBLE CONCRETE STRESS OF 1,000 lb/sq in, AND HAS TO BE REDUCED TO 378,100 lb.in, INCREASING THE FACTOR OF SAFETY TO 4·0

The low figures in Line 8 for bars 7 and 9 indicate that the twisting of the American bars has not been successful. The yield point of bars 7 remained below the minimum requirements of B.S. 1144, although bar 7H was overtwisted. y and u for bars 15 and 16, twisted from British steel, comply with the requirements of B.S. 1144 but the ratios y/u for bars 16 are lower than .800.

In bars 13, the minimum yield point and ultimate strength required in B.S. 1144 have been reached, although the raw material for these bars had an average ultimate strength of only 27·05 t/sq. in.

(b) Type of failure and stresses at failure

A small slip was observed in several bars in Series 1, but the significance of a slip can only be assessed in relation to the stress at failure, particularly in relation to

*The term " yield point " when applied to cold worked bars is misleading. According to B.S. 1144 it is the stress at a strain of .005. As will be seen from Fig. 6, in comparison with Fig. 5, cold worked bars do not " yield."

the ratio $\frac{\text{stress at failure}}{\text{yield point}}$ (Line 22).

In all cold worked bars, with the exception of 1L the yield point has been exceeded. The calculated values of t exceed the yield point by 1.3 to 18.9 per cent. in "L" concrete and by 9.8 to 21.3 per cent. in "H" concrete. In bars with a natural yield point, the yield point has at least been reached in bar 8L and exceeded in all others, the maximum excess being 11.0 per cent. in "L" concrete and 12.2 per cent. in "H" concrete.

In bar 1L the ratio t_p/y is only .985. Although in view of the uncertainty of c_p (see p. 136) and the variation of the yield point along the bar it is possible that the yield point has been reached also in this bar, the simultaneous slip at both ends indicates failure by bond. With all other cold worked bars where slip occurred, this happened at a stress well above the yield point and was therefore a secondary effect.

In beam 4L the first slip was observed at a load of 32,000 lb, corresponding to a stress of 5 per cent. above the yield point. At this stage the hooks came into action and the load could be further increased to 34,000 lb., when slip occurred also at the other end and the beam failed.

In beam 5L the first slip was observed at a load of 32,000 lb, at a stress of only 91 per cent. of the yield point. The load could be increased to 36,000 lb when vertical cracks occurred at the end of the beam where the bar had slipped and the hook split the concrete. At this stage, the calculated stress exceeded the yield point by 1.9 per cent.

It may be concluded that all beams reinforced with cold worked bars have failed in tension with the exception of beam 1L which has failed in bond, and all beams reinforced with bars having a natural yield point have also failed in tension, although in beam 5L slip at one end of the bar has already occurred at a stress well below the yield point. Consequently, the only beams for which significant values for the bond strength can be calculated from the test results are 1L and 5L. For all other bars the average bond stress in Line 25 represents only a limit which is at least equal to but may be substantially exceeded by the average bond strength.

The average bond strength in beam 1L was 365 lb/sq. in. Related to a permissible bond stress of $120 \times 1.1 = 132$ lb/sq. in., it gives a factor of safety of 2.76. The fact that in spite of this high factor of safety failure occurred by bond slip before the yield point of the steel was exceeded is due to the exceptionally high yield point of bar 1L (22 per cent. higher than specified in B.S. 1144). It was the highest of all bars in Series 1 used in "L" concrete and, apart from the Tentor bars (which are not comparable with regard to bond stresses because of their smaller diameter), was only equalled by bar 12L.

The calculated stress at failure in bar 2L is slightly higher than in 1L, but the difference is practically negligible. The yield point of bar 2L was substantially lower than that of bar 1L, so that in bar 1L a higher tensile stress at failure should have been reached had the bond been better (as in bar 12L).

For beam 5L a closer investigation is necessary. The required minimum bond length according to the Code is obtained as follows:

$$n = \frac{25{,}750}{4 \times 120} = 53.6 \quad \text{Min L} = 53.6 \times 1.125 = 60.2 \text{ in.}$$

The available length, with allowance for a hook according to the Code, was 63.5 in. (see p. 137). The calculated average bond stress at failure was 256.6 lb/sq. in., but at first slip it was only 229 lb/sq. in. Related to the permissible bond stress of 120 lb/sq. in., the factor of safety against first slip is only 1.91.

Actually, the average bond stresses as calculated in Line 25 with an arbitrary allowance for a hook are misleading, since the hook does not come into action before slip occurs. The average bond strength is better assessed without allowance for hooks, i.e., by assuming a bonded length of say 49 in. With this assumption, the average bond stress at first slip was 297 lb/sq. in. After slipping, the force was transferred to the hook and the concrete was split. This was caused by the excessive pressure of the hook on the concrete (see Fig. 4 showing the end of beam 5H which failed in a similar way). To avoid this type of failure it is

Fig. 4.—Beam No. 5. 1¼ in. diameter high tensile steel. High-grade concrete

necessary to increase either the bonded length (without allowance for the hook) or the diameter of the hook, or both. The result of both beams 5L and 5H shows that the requirements of the Code in this respect do not go far enough in the case of plain bars of high tensile quality. The arbitrary allowance of 16D for the hook does not seem justified.

The figures of 365 and 297 lb/sq. in., or say 300 lb/sq. in., may be considered as truly representative of the average bond strength of oval twisted and plain round bars respectively in "L" concrete, related

to the minimum embedded length according to the Code. In short lengths, as may be used in pull-out specimens, much higher values are obtained, but such values are of little significance for practical design.

The calculated average bond stress at first slip in beam 4L, without allowance for a hook, was $\frac{65}{48} \times \frac{1.05}{1.11} \times 219.7 = 275$ lb/sq. in. In this beam, the length of the bar without allowance for the hook, was adequate according to the Code formula (48 in. as against 47 in.) The slip was due to yielding and not to exceeding the average bond strength.

Both beams 1L and 5L have demonstrated the importance of higher bond strength for bars having a higher yield point. In all other beams the bond strength was sufficient in relation to the yield strength of the bars.

Table 3 contains a comparison of the calculated stresses at failure. The beams have been divided into two groups: (A) reinforced with bars having a natural yield point; (B) reinforced with cold worked bars. The ratios t_p/y, t_p/u, the values of t_p and the factors of safety, are reproduced from Tables 1 and 2. The ratios of t_p/y in "H" concrete to those in "L" concrete have been added. The factors of safety will be discussed in Section (d).

In each group the beams have been listed according to the ratio $\frac{\text{stress at failure}}{\text{yield point}}$ in "L" concrete. The two groups are distinctly separated by the value of the stress at failure. In Group (A) this stress varies between 42,480 and 56,160 lb/sq. in., in Group (B) between 63,400 and 95,400 lb/sq. in.

An important feature of Table 3 is the fact that all ratios H/L for t_p/y are greater than 1 with the exception of beam 6 where the ratio is only .973. This exception is most probably due to an over-estimation of the load at failure in beam 6L. In this beam failure occurred immediately on reaching the maximum load, whilst in beam 6H the maximum load was maintained for 11 minutes. It is probable that the failure of beam 6L would have been caused by a smaller load if it had been sustained for a longer period. The ratio $t_p/y = 1.069$ is probably too high in view of the sudden and unexpected failure of this beam. In the other 4 beams of Group (A) the ratio H/L for t_p/y varies between 1.009 and 1.025, which shows that the quality of the concrete has only a very small influence on the excess of the yield point at failure if the steel has a natural yield point.

In Group (B) we have to exclude from the comparison of H/L bars 2, 7 and 9, which were not sufficiently developed.

For the remaining 9 pairs of beams the ratio H/L varies between 1.026 and 1.165. This indicates that the improvement of the quality of concrete has a greater effect on the stress at failure in cold worked bars than in bars belonging to Group (A).

It should be noted that the variation of the ratio t_p/y for "H" concrete is comparatively small. In Group (A) (5 values) the limits are 1.026 and 1.122, in Group (B) (12 values), 1.098 and 1.213. Although there is a small over-lapping, the superiority of Group (B) is obvious.

The ratio $\frac{\text{stress at failure}}{\text{ultimate strength}}$ is not characteristic of bars having a natural yield point, since failure depends

TABLE 3.—Comparison of stresses at failure

No. OF BEAM	DESCRIPTION OF BAR	$\frac{t_p}{y}$ "L"	$\frac{t_p}{y}$ "H"	$\frac{H}{L}$	$\frac{t_p}{u}$ "L"	$\frac{t_p}{u}$ "H"	$\frac{H}{L}$	t_p (lb/sq. in.) "L"	t_p (lb/sq. in.) "H"	FACTOR OF SAFETY TO CODE "L"	FACTOR OF SAFETY TO CODE "H"	$\frac{L}{2 \cdot 456}$	$\frac{H}{2 \cdot 456}$	$\frac{H}{2 \cdot 608}$	No. OF BEAM
	(A) BARS HAVING A NATURAL YIELD POINT.														
8	AMERICAN RIBBED H.T	1·001	1·026	1·025	·627	·665	1·057	52,000	52,880	2·092	2·227	·852	·907	·854	8
5	1⅛" DIA. H.T.	1·019 (1)	1·028	1·009	·611	·605	·990	56,160	55,470	2·239	2·339	·912	·952	·897	5
6	AMERICAN RIBBED M.S.	1·069 (7)	1·040	·973 (7)	·662	·699	1·056	42,870	45,430	2·266	2·483	·928	1·011	·952	6
14	AMERICAN RIBBED BRITISH M.S.	1·082	1·100	1·017	·665	·667	1·003	44,800	44,100	2·388	2·395	·972	·975	·918	14
4	1¼" DIA. M.S.	1·110	1·122	1·011	·656	·636	·970	42,780	42,480	2·456	2·608	1·000	1·062	1·000	4
	(B) COLD WORKED BARS.														
1	OVAL TWISTED 6" PITCH	·985 (2)	1·148	1·165 (2)	·852 (2)	·989	1·161 (2)	72,170 (2)	84,830	2·707	3·320	1·102	1·356	1·277	1
3	SQUARE TWISTED 7/8"	1·013	1·098	1·084	·876	·959	1·094	69,560	69,130	2·590	2·746	1·055	1·118	1·058	3
11	TORSTEEL	1·044	1·133	1·085	·863	·952	1·103	70,150 (3)	80,200	2·630	3·165	1·071	1·285	1·210	11
18	TENTOR ·71" DIA.	1·075	—	—	·917	—	—	93,300	—	3·439	—	1·400	—	—	18
19	TENTOR ·71" DIA.	1·075	1·109	1·032	·917	·934	1·019	92,300	95,400	4·000 (3)	3·579	1·629	1·487	1·372	19
15	AMERICAN RIBBED BRITISH M.S. 6" PITCH	1·082	1·192	1·102	·906	1·002	1·106	75,100	83,800	2·830	3·270	1·152	1·331	1·254	16
17	TENTOR ·71" DIA.	1·088	—	—	·937	—	—	92,400	—	3·510	—	1·429	—	—	17
2	OVAL TWISTED	1·098	1·120 (3)	1·020 (3)	·942	·907 (3)	·963 (3)	73,080	68,470 (3)	2·725	2·726 (3)	1·110	1·110	1·045	2
10	TWIN TWISTED	1·115	1·151	1·032	·915	·946	1·034	66,410	66,480	2·519	2·609	1·026	1·062	1·000	10
12	INDENTED TWISTED 6" PITCH	1·116	1·166	1·045	·958	·999	1·043	81,700	85,900	3·085	3·343	1·256	1·361	1·282	12
13	INDENTED TWISTED 12" PITCH	1·121	1·150	1·026	·942	·993	1·054	71,000	74,500	2·680	2·758	1·071	1·123	1·058	13
16	AMERICAN RIBBED BRITISH M.S. 12" PITCH	1·148	1·207	1·051	·918	·980	1·035	72,800	77,400	2·725	3·081	1·109	1·234	1·162	16
9	AMERICAN RIBBED H.T. TWISTED	1·149	1·157 (4)	1·007	·827	·881 (4)	1·066 (4)	78,170	73,540 (4)	2·749	2·918	1·119	1·188	1·119	9
7	AMERICAN RIBBED M.S. TWISTED	1·189	1·213	1·020 (6)	·896	·923	1·030	63,400	72,560	2·369	2·672	·973 (6)	1·169	1·101	7

NOTES:

(1) SLIPPED AT A STRESS EQUAL TO 91% OF THE YIELD POINT.
(2) FAILED IN BOND IN "L" CONCRETE.
(3) UNDERTWISTED, W.H.U = ·706 IN "H" CONCRETE.
(4) UNDERTWISTED, W.H.U = ·558 IN "H" CONCRETE.
(5) PERMISSIBLE MOMENT LIMITED BY CONCRETE STRESS.
(6) UNSATISFACTORILY DEVELOPED.
(7) SEE DISCUSSION IN TEXT.

entirely on the yield point. It has only been included in Table 3 to illustrate the fact that the stress at failure is generally within 60 to 70 per cent. of the ultimate strength. The ratio H/L for t_p/u is rather erratic, depending on the variation of the ratio $\frac{\text{yield point}}{\text{ultimate strength}}$ of the bars used in " L " and " H " concrete respectively.

Conditions are different for Group (B). We have to exclude from the comparison beam 1L which failed in bond, both beams 7, the reinforcement of which did not comply with B.S. 1144, and beams 2L and 9H with undertwisted bars. After omitting these beams, we find that the ratio t_p/u varies between .863 and .958 in " L " concrete and between .934 and 1.002 in " H " concrete*.

If we exclude Tentor bars for reasons to be explained in Section (c) the variation of t_p/u in " H " concrete is reduced to the interval .946 to 1.002. This means in other words that with all cold worked bars of M.S. quality complying with B.S. 1144, the ultimate tensile strength could be approached in " H " concrete within 5.4 per cent., in contrast to bars having a natural yield point where the deficiency was between 30 and 40 per cent. of the ultimate strength.

The ratio H/L for t_p/u varies for comparable beams in Group (B) between 1.018 and 1.095. Thus, the influence of the concrete quality is distinctly reflected in the figures.

(c) Stress-strain diagrams

In order to appreciate the reasons for the variations of the stress ratios in Table 3, it is necessary to consider the stress-strain diagrams of the bars. For lack of space, it is impossible to reproduce here all stress-strain diagrams, and only the following have been selected, which are characteristic of the remainder :

Group (A)

Bar 4, 1¼ in. dia. M.S.
,, 6, 1 in. ,, American ribbed M.S. } superimposed Fig. 5

Group (B)

Bar 1, Oval twisted 6¼ in. pitch
,, 2, ,, ,, 12 in. ,,
,, 9, American ribbed high tensile twisted } superimposed Fig. 6

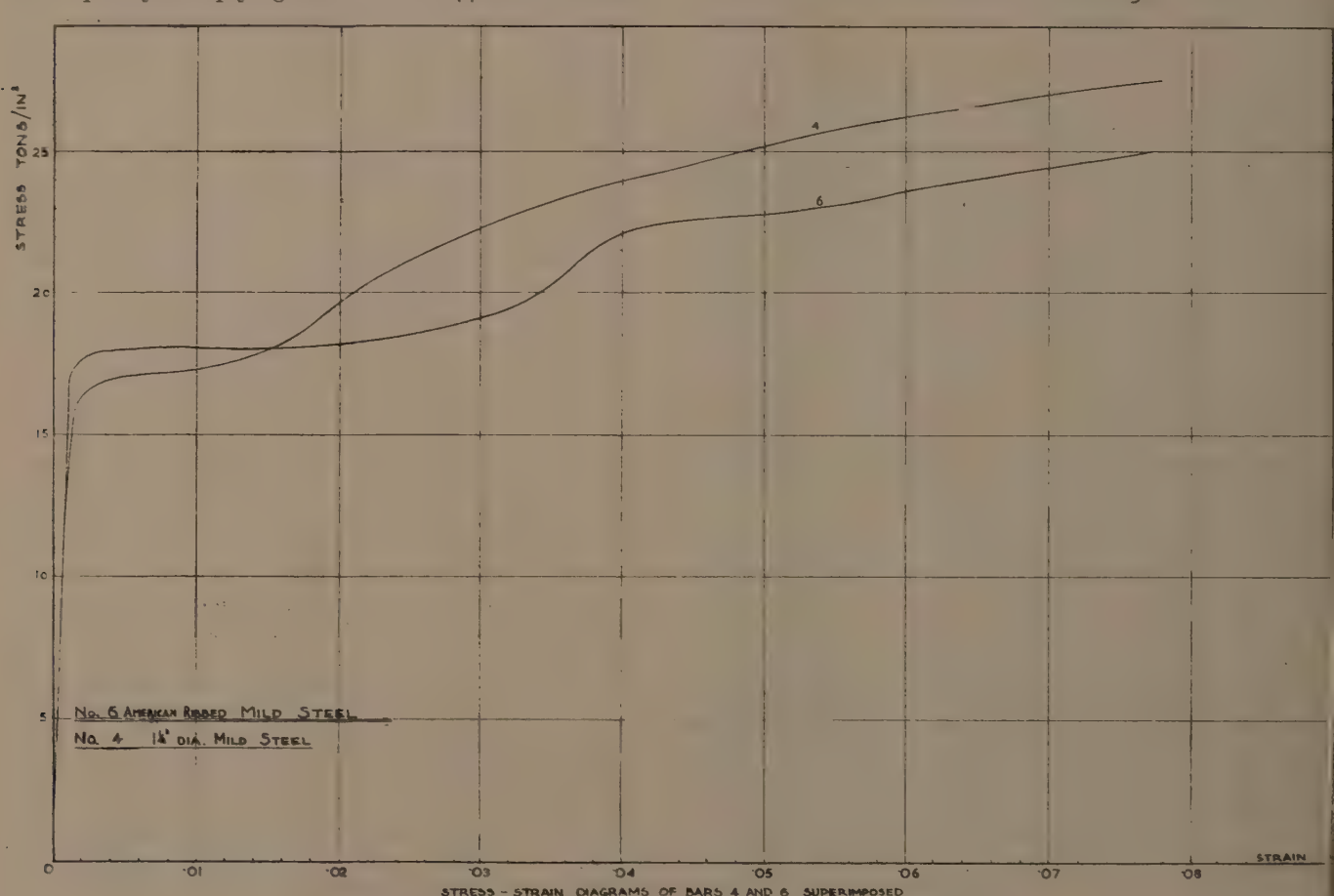


Fig. 5.—Stress-strain diagrams of bars 4 and 6 superimposed

The stress-strain diagrams are confined to the firs part, since stresses corresponding to a strain of mor than about .04 cannot be utilised in reinforced concrete

Fig. 5 shows the difference in behaviour of the Britis M.S. used in beams 4 and the American M.S. used i beams 6. Although the yield point of the British ste is lower than that of the American steel, the two lin intersect each other at a strain of about .015. Th stress in the British bar at a strain of .02 is high than that of the American bar at a strain of .03. Th stress-strain diagrams for bars 8 and 5 are similar t that for bar 6, whilst the stress-strain diagram f bar 14 is similar to that of bar 4 in the critical rang

*On the basis of stresses worked out by the standard method, the ratio $\frac{\text{stress at failure}}{\text{ultimate strength}}$ exceeds 1 in all 8 cases to be considered in " H " concrete. In 15H it amounts to 1.056. In none of the beams could the steel be broken. This indicates that the standard method does not give a reliable measure of the steel stress at failure. The calculated concrete stress at failure in beam 19L is 4,000 lb./sq. in., although the cube strength was only 3,220 lb/sq. in. On the other hand, the identity of the ratios t_p/y and t_p/u in 18L and 19L should be noted.

This appears to be the explanation for the comparatively low figures t_p/y for bars 8, 5 and 6*.

Fig. 6 shows the difference between an overtwisted bar (1), a bar in the lower range of work-hardening according to B.S. 1144 (2L), and a bar which could not be cold worked satisfactorily (9L). This diagram explains the low ratio between yield point and ultimate strength in bar 9.

The high value of t_p/y for the American ribbed type bars 16, 9, 7, can only be explained by their very high bond strength. The fact that for 16H, t_p/u = .950, although y/u = .788, whilst for 2H, t_p/u = .907 although y/u = .810, is an indication of the superior bond of the American ribbed type bar in comparison with the oval twisted bar. This high bond value has, however, practically no influence on the stress at failure in bars having a natural yield point, as was clearly demonstrated by bars 8, 6, 14.

Another important characteristic of a cold worked bar is the strain at which the maximum stress is reached. The following values are typical :

Bar No.	Description	Strain on reaching max. stress per cent.
1	1 in. oval $6\frac{1}{4}$ in. pitch (overtwisted) ...	3.6
2	1 in. oval 12 in. pitch	4.4
17	Tentor	5.2
9	American ribbed H.T. unsatisfactorily developed	7.2

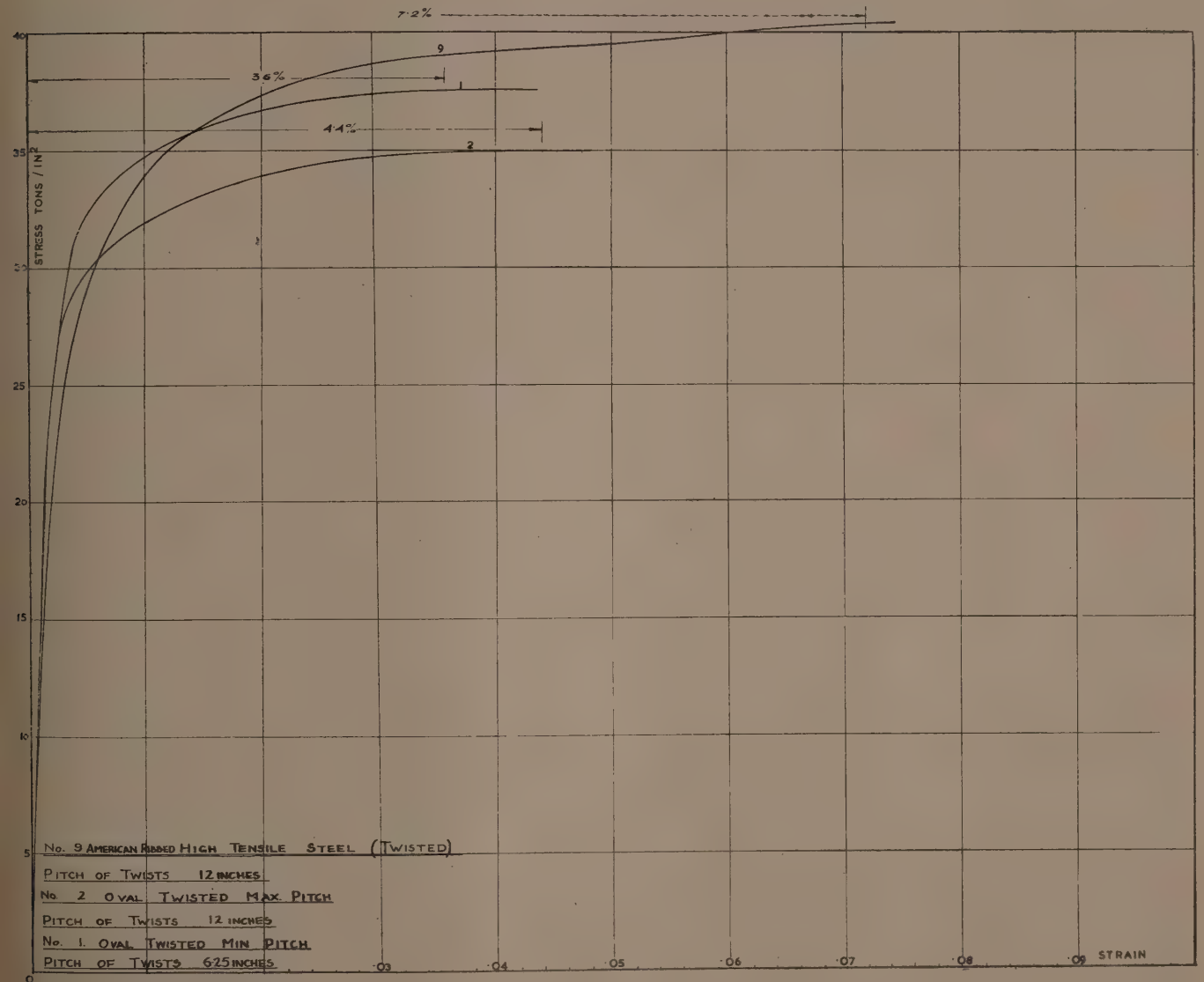


Fig. 6.—Stress-strain diagrams of bars 1, 2 and 9 superimposed

For all bars with a ratio .80 < y/u < .88 other than Tentor bars the strain at which the maximum stress was reached was between 3 and 4.4 per cent. For bars with a natural yield point this strain is more than 10 per cent.

A too low value of this strain is undesirable. On the other hand, this strain must not be too high, as exemplified by bar 9L.

The strain of 5.2 per cent. obtained for Tentor bars explains the reason for the comparatively low values of t_p/y and t_p/u for this type of reinforcement in relation to Indented twisted bars (12 and 13). It should be borne in mind, however, that owing to the higher strength

*Regarding the ratio t_p/y = 1.069 for 6L, see the preceding remarks.

of the Tentor bars used in the tests, the stresses at failure are much higher than in any other bar. The variation of the stress at failure in the three beams in "L" concrete (17L, 18L, 19L) is remarkably small (92,300 to 93,300 lb./sq. in.), particularly in view of the variation of the percentage of the reinforcement.

It would be desirable to include in B.S. 1144 an upper and a lower limit for the strain at which the maximum stress is reached. The minimum elongation after failure, as specified at present, is of doubtful practical value, since it is the result of the combination of two different phenomena: uniform elongation and necking, and depends also on the gauge length.

For the determination of numerical values of the limits, further investigations are necessary.

(d) Factors of safety

Table 3 also contains the factors of safety repeated from Line 28 in Tables 1 and 2. They vary between 2.092 (8L) and 4.0 (19L). A better comparison is obtained if we take as a basis the factors of safety for the commonly used M.S. bar 4. This may be done in two ways. The factor of safety for 4L (2.456) may be adopted as unity either for both "L" and "H" concrete, or for "L" concrete only, and the factor of safety for 4H (2.608) as unity for "H" concrete. The latter way seems to be more logical and has been adopted in the following.

It should be noted that in the two relevant columns all figures for Group (A) are smaller than 1 (except, of course, for bar 4 which has been chosen as unity), whilst all figures for Group (B) are greater than 1, with two exceptions: 7L and 10H. As has already been mentioned, the twisting of the American ribbed type bars was unsatisfactory and the yield point of bar 7 was far below the minimum specified in B.S. 1144. Thus, bar 7L has to be excluded from any comparison. The other exception, 10H, shows that the factor of safety of the twin-twisted bar in "H" concrete was exactly the same as that of bar 4. This is due to the lower yield point of twin-twisted bars (minimum specified in B.S. 1144: 54,000 lb./sq. in.).

These figures lead to the conclusion that in all cases where steel with a higher natural yield point has been used in the tests, the factor of safety based on the Code has been reduced, whilst in all cases with single twisted bars (excluding 7L) the factor of safety has been increased in comparison with mild steel bars.

The disqualification of cold worked bars of very high strength in comparison with "high tensile" steel to B.S. 785 may be further illustrated by taking beam 5L as a basis of comparison. This beam failed under a bending moment of 814,700 lb. in. and its permissible bending moment is 363,820 lb. in. Beam 18L failed under a bending moment of 1,083,300 lb. in., i.e. 33 per cent. more than beam 5L, but it is only allowed to carry 315,040 lb. in., i.e. 13.4 per cent. less. The discrepancy is even greater with beam 19L, which failed under a bending moment of 1,512,300 lb. in., i.e. 85.5 per cent. more than 5L, but is allowed to carry only 378,100 lb. in., i.e. 4 per cent. more than 5L.

6. Development of Cracks

The load at which first cracks occur depends mainly on the quality of the concrete, the shape of the beam, the amount of steel, and is hardly affected by the quality or shape of the reinforcement. It is therefore not necessary to deal with the question of first cracking.

Tables 4 and 5 contain a summary of the observations of crack development in the two series. The reading of

TABLE 4.—Crack widths and number of cracks. Series 1

1	NO. OF BEAM.		1L	1H	2L	2H	3L	3H	4L	4H	5L	5H	6L	6H	7L	7H	8L	8H	9L	9H	10L	10H	11L	11H	1
2	REINFORCEMENT.		MIN. PITCH	OVAL TWISTED "I"	MAX. PITCH		⅜" SQUARE TWISTED		¼"DIA. MILD STEEL		⅝"DIA. HIGH TENSILE		1"DIA. AMERICAN RIBBED N.S.		1"DIA. AMERICAN RIBBED N.S. TWISTED		1"DIA. AMERICAN RIBBED H.T.		1"DIA. AMERICAN RIBBED N.S. TWISTED		¾"DIA. TWIN TWISTED		1"DIA. TORSTEEL		2
3	PERMISSIBLE LOAD.	lb.	12650	12790	12560	12550	12490	12450	12330	13170	15510	15650	9130	9050	12790	11860	11880	12000	12440	12340	13610	13830	12370	12610	3
4	LOAD AT B.M. 50% HIGHER THAN PERMISSIBLE.	lb.	19450	19700	19350	19340	19250	19180	18960	20270	23780	23990	14210	14000	19700	19310	19350	18520	19180	19030	21010	21260	19070	19430	4
5	MAX. CRACK WIDTH AT PERMISSIBLE LOAD.	m m	.16	.10	?	.07	.12	.02	.12	?	.15	.10	?	0	.16	.10	.10	.10	.02	.10	.14	.16	.11	.06	5
6	MAX. CRACK WIDTH AT B.M. 25% HIGHER THAN PERMISSIBLE	m m	.20	.11	.10	.20	.20	.10	.18	.05	.20	.15	?	.09	.20	.14	.14	.14	.15	.15	.22	.24	.19	.10	6
7	MAX. CRACK WIDTH AT B.M. 50% HIGHER THAN PERMISSIBLE	m m	.28	.20	.20	.20	.28	.10	.21	.05	.30	.20	.06	.13	.20	.23	.18	.18	.27	.19	.28	.35	.24	.19	7
8	MAX. CRACK WIDTH AT B.M. 75% HIGHER THAN PERMISSIBLE	m m	.38	.24	.24	.31	.37	.17	.35	.10	.40	.26	.11	.17	.28	.26	.24	.23	.33	.23	.37	.49	.31	.24	8
9	MAX. CRACK WIDTH AT B.M. 100% HIGHER THAN PERMISSIBLE	m m	.46	.39	.31	.49	.45	.25	.44	.14	.55	.31	.19	—	.60	.33	.35	.40	.57	.34	.47	.64	.55	.31	9
10	STRESS AT WHICH MAX. CRACK WIDTH = 0.25 m m	lb/sq.in.	36850	48880	49740	44440	38000	54320	28220	46940	35660	44800	—	45090	44980	44030	43910	43450	39490	50530	38620	34370	47390	49550	10
11	Nº OF CRACKS AT PERMISSIBLE LOAD.		8	3	?	4	10	6	8	?	13	10	?	0	14	5	6	4	8	7	11	10	8	5	11
12	Nº OF CRACKS AT B.M. 25% HIGHER THAN PERMISSIBLE		11	7	8	6	14	8	9	6	14	12	?	8	16	11	10	8	14	12	12	12	12	8	12
13	Nº OF CRACKS AT B.M. 50% HIGHER THAN PERMISSIBLE		11	10	8	11	17	12	10	8	14	15	5	11	18	14	11	12	19	15	14	15	13	12	13
14	Nº OF CRACKS AT B.M. 75% HIGHER THAN PERMISSIBLE		12	15	12	16	21	17	11	10	14	16	7	15	19	16	16	16	24	20	17	16	15	13	14
15	Nº OF CRACKS AT B.M. 100% HIGHER THAN PERMISSIBLE		12	17	14	17	21	20	12	14	14	16	10	17	23	22	19	20	28	22	18	17	16	16	15
16	Nº OF CRACKS AT FAILURE		13	21	14	18	21	24	12	17	14	17	11	20	29	25	19	21	34	24	21	19	16	19	16
17	Nº OF CRACKS WHEN MAX. WIDTH = 0.25 m m		11	16	13	15	16	20	11	17	14	16	—	20	19	16	17	19	18	22	13	13	13	14	17
18	STRESS AT 12 TONS/SQ. IN. OF MAX. CRACK WIDTH PERMISSIBLE STRESS.		1·440	1·810	1·843	1·646	1·408	2·044	1·570	2·608	1·393	1·740	—	2·284	1·665	1·632	1·765	1·921	1·463	1·870	1·430	1·280	1·570	1·847	18

the crack widths was taken in multiples of .05 mm., the probable error of the readings is therefore not more than ± .03 mm. The maximum crack widths in the Tables were obtained by interpolation between two readings.

It may be seen from these Tables that in the great majority of beams cracks occurred far below the permissible bending moment. The maximum crack width at the permissible bending moment was .18 mm. (10H). In beam 4L, which again may be considered as a basis of comparison, the maximum crack width under the permissible bending moment was .12 mm. and in 5L, .15 mm. The effect of the quality of concrete was rather ambiguous, as already established by Wästlund and Jonsson(4). In several cases, larger maximum widths and less cracks were observed in "H" concrete than in "L" concrete.

Although a "permissible width of cracks" is not yet generally agreed upon, a width of .01 in. = .25 mm. may be considered as a convenient limit(6). Line 10 contains the stresses, calculated by interpolation, at which a maximum crack width of .25 mm. was reached in the various beams. They vary within wide limits. The lowest figure, 28,200 lb./sq. in., occurs in 4L, the highest, 63,280 lb./sq. in., in 19L. This latter is not directly comparable with the others since the reinforcement of this beam consisted of 3 bars of .71 in. dia. In the corresponding beam 19H, the stress was only 49,120 lb./sq. in. The lowest figure with a cold worked bar is 34,570 lb./sq. in. (10H), and for a single twisted cold worked bar, 38,000 lb./sq. in. (3L), closely followed by 1L (38,850 lb./sq. in.). In the majority of beams the stress in Line 10 is between 40,000 lb./sq. in. and 50,000 lb./sq. in.

It should be remembered, of course, that the cracks become wider with repeated loadings, but this can only be determined by fatigue tests. This was beyond the scope of the present investigations.

Line 18 in Tables 4 and 5 contains the ratio $\frac{\text{Stress at .25 mm. max. crack width}}{\text{permissible stress}}$. This may be considered as a kind of "factor of safety" against reaching a crack width of .25 mm. at first loading. The lowest figure (1.280) is obtained for 10H, the second lowest (1.393) for 5L. The figures for all single twisted bars are higher than for 5L.

The maximum crack width observed in a beam is, however, more or less incidental, depending on local conditions, and does not appear to be suitable for comparing various bars, unless a very great number of repetitions is available so that averages can be taken. It seems to be more appropriate to compare the bars on the basis of the number of cracks before failure. Such a comparison must include the stress itself, since with a given shape of bar more cracks may be developed at higher stresses. Table 6 contains such a comparison based on the average number of cracks in "L" and "H" concrete, and includes the average stress at failure. It should be pointed out that the spacing of the cracks is not uniform.

The bars are listed in the sequence of diminishing number of cracks. Where two or more bars have the same number of cracks, they are listed in the sequence of increasing stresses at failure.

TABLE 5.—Crack widths and number of cracks. Series 2

1.	Nº OF BEAM		12L	12H	13L	13H	14L	14H	15L	15H	16L	16H	17L	18L	19L	19H	1.
2	REINFORCEMENT		1" INDENTED TWISTED TO 6' PITCH		1" INDENTED TWISTED TO 12' PITCH		1" AMERICAN RIBBED TYPE		1" AMERICAN RIBBED TYPE TWISTED TO 6' PITCH		1" AMERICAN RIBBED TYPE TWISTED TO 12' PITCH		.71" TENTOR	Nº 2 .71" TENTOR	Nº 3 .71" TENTOR		2
3	PERMISSIBLE LOAD	lb.	11,810	11,690	11,910	12,030	9,030	9,000	12,410	12,880	12,570	12,840	6 240	13,080	19,770(1)	19,230	3
4	LOAD AT B.M 50% HIGHER THAN PERMISSIBLE	lb	18,240	18,050	18,380	18,570	14 070	14,020	9 ,130	19 860	19 370	19,700	9 880	20 240	30 270	29 460	4
5	MAX.CRACK WIDTH AT PERMISSIBLE LOAD	m m	·15	·10	·15	·10	·05?	–	·06	·11	·11	·11	·10	·10	·10	·10	5
6	MAX. CRACK WIDTH AT B.M 25% HIGHER THAN PERMISSIBLE	m m	·20	·17	·19	·18	·10	·15?	·10	·16	·15	·15	·15	·11	·11	·10	6
7	MAX.CRACK WIDTH AT B.M 50% HIGHER THAN PERMISSIBLE	m m	·21	·23	·23	·23	·13	·15	·18	·20	·19	·20	·17	·15	·15	·17	7
8	MAX. CRACK WIDTH AT B.M 75% HIGHER THAN PERMISSIBLE	m m	·30	·28	·29	·27	·15	·20	·21	·24	·22	·28	·22	·20	·15	·23	8
9	MAX. CRACK WIDTH AT B.M 100% HIGHER THAN PERMISSIBLE	m m	32	·37	·44	·61	·15	?	·27	29	·27	·33	·29	·20	·20	·25	9
10	STRESS AT WHICH MAX. CRACK WIDTH = 0.25 m m	lb/sq.in	42,100	44, 640	43, 890	43, 460	>41, 790	>41, 930	50, 290	48, 540	49, 690	44, 840	48, 370	58, 900	63, 280	49, 120	10
11	Nº OF CRACKS AT PERMISSIBLE LOAD		16	7	8	10	6	-	11	10	12	8	4	10	11	8	11
12	Nº OF CRACKS AT B.M 25% HIGHER THAN PERMISSIBLE		18	11	11	13	10	8	14	14	16	13	11	12	14	11	12
13	Nº OF CRACKS AT B.M 50% HIGHER THAN PERMISSIBLE		19	13	15	17	12	11	16	17	18	16	12	17	15	13	13
14	Nº OF CRACKS AT B.M 75% HIGHER THAN PERMISSIBLE		21	17	19	21	14	12	17	20	21	18	14	19	20	15	14
15	Nº OF CRACKS AT B.M 100% HIGHER THAN PERMISSIBLE		23	18	20	23	18	12	22	21	22	21	16	21	21	19	15
16	Nº OF CRACKS AT FAILURE		26	19	20	24	19	12	25	27	24	25	19	22	29	27	16
17	Nº OF CRACKS WHEN MAX WIDTH = 0.25 m m		18	16	18	19	-	-	19	20	22	18	14	21	26	15	17
18	STRESS AT .25 m m MAX. CRACK WIDTH / PERMISSIBLE STRESS		1 559	1·653	1·626	1 610	>2 090	>2 097	1·863	1 798	1 840	1 661	1 792	2 182	2 344	1 819	18

NOTE (1) THE ASSUMED PERMISSIBLE LOAD CORRESPONDS TO A PERMISSIBLE STEEL STRESS OF 27000 lb/sq in., DISREGARDING THE LIMITATION IMPOSED UPON THE PERMISSIBLE LOAD BY THE CONCRETE STRESS

It is interesting to compare the various American ribbed type bars. With too low stresses at failure (beams 6 and 14), only averages of 15.5 cracks were developed. With H.T. quality (beams 8), the average was 20, and with increased stress at failure (beams 16, 15, 7, 9, achieved by cold work), the average number was increased to 24.5 to 29.

On the other hand, the plain round bars do not show a significant increase (4 and 5) and even with oval twisted bars (1 and 2) which failed at stresses of the same order as the twisted American ribbed type bars, no substantial increase of the number of cracks took place. Torsteel was only slightly better.

The comparatively favourable placing of twin-twisted bars is probably due to their larger perimeter in comparison with single twisted bars.

With regard to Tentor bars, the difference in the percentage of steel in beams 17, 18, 19, has also to

be taken into account. Beam 17, with a very low percentage, gives a too unfavourable picture, whilst the opposite applies to beam 19. Beam 18, immediately following beams 12 and 13, seems to relegate Tentor bars to their appropriate place.

The number of each type tested was far too small to obtain a reliable classification of the bars from Table 6 but the superiority of the American ribbed type bar, *if produced with satisfactory strength properties*, is quite definite. On the other hand, the poor crack control of plain round and oval twisted bars is also conspicuously shown.

Regarding the relative position of Torsteel, twin-twisted bars, and square twisted bars, many further tests would be necessary. The comparative performance of square twisted bars according to Table 3 is much less favourable than according to Table 6.

7. Warning Before Failure

Whilst cracking under working loads is an undesirable though inevitable feature of normal reinforced concrete structures, and a limitation of crack widths is of vital importance, a widening of the cracks before the maximum load is reached is an essential requirement of safety in order to give visible warning of the approaching danger. A nominal factor of safety, however high, is not good enough, since the failure of a structure depends on many circumstances which cannot be taken into account. An exaggeratedly high level of the factor of safety is undesirable from the point of view of national economy and *it is far more reasonable to design structures with a comparatively lower factor of safety in such a manner that they look dangerous at a stage when their load-bearing capacity is, in fact, not yet exhausted.* This requirement is on the whole satisfactorily fulfilled with slabs and beams in ordinary grade concrete reinforced with M.S., although not with columns(7).

The present tests have demonstrated that *with bars with improved bond which have a natural yield point, or even with plain M.S. bars if used in concrete of high strength, only inadequate warning may be expected.* This will be discussed with reference to Fig. 7 which shows 12 typical load-deflection diagrams (7 in "L" and 5 in "H" concrete) and has to be interpreted in conjunction

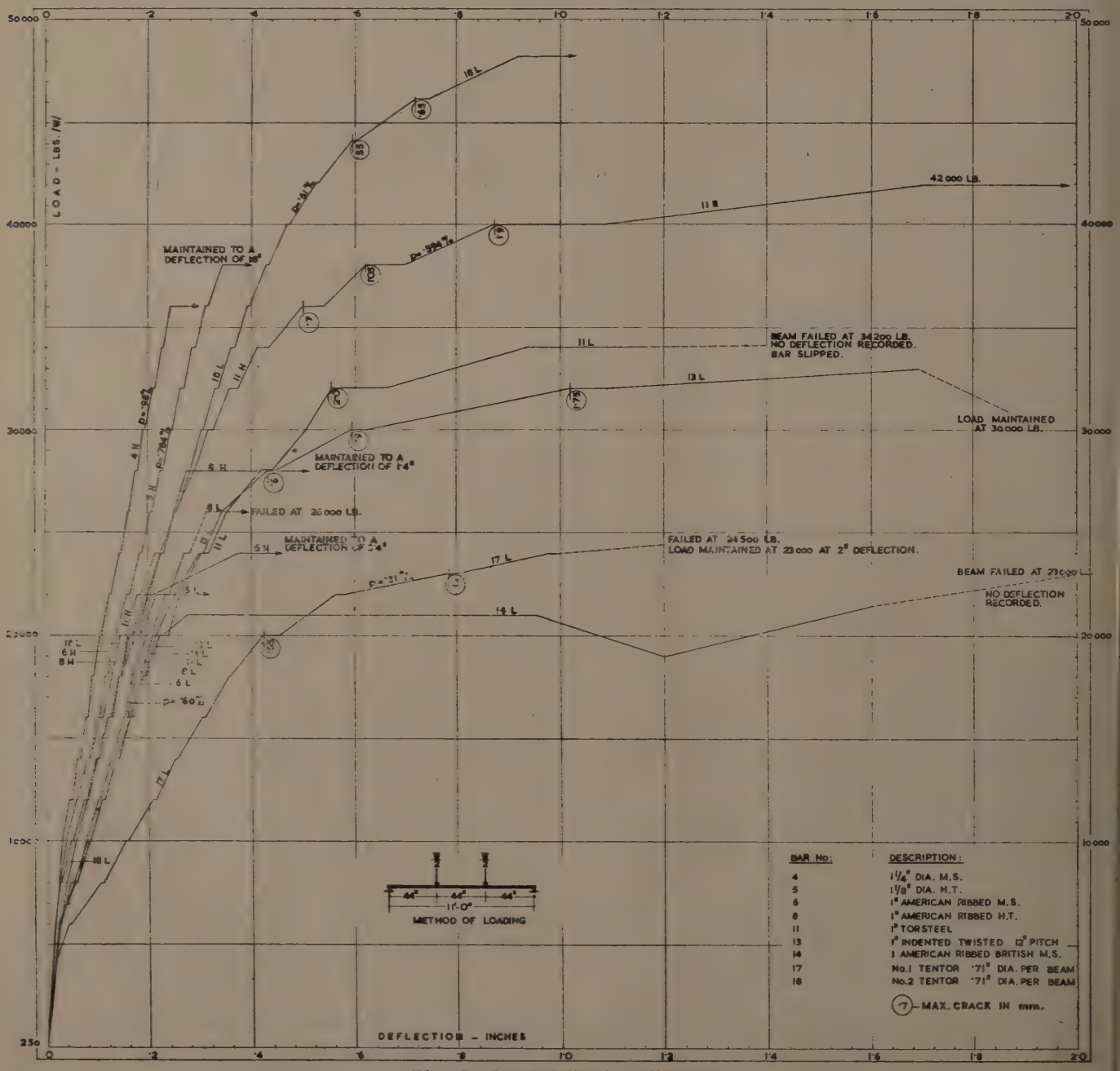

LOAD - LBS. /W
DEFLECTION - INCHES
42 000 LB.
MAINTAINED TO A DEFLECTION OF 2"
BEAM FAILED AT 34200 LB. NO DEFLECTION RECORDED. BAR SLIPPED.
LOAD MAINTAINED AT 30000 LB.
MAINTAINED TO A DEFLECTION OF 1·4"
FAILED AT 25000 LB.
MAINTAINED TO A DEFLECTION OF 1·4"
FAILED AT 24500 LB. LOAD MAINTAINED AT 23000 AT 2" DEFLECTION.
BEAM FAILED AT 23000
NO DEFLECTION RECORDED.
METHOD OF LOADING
BAR No: DESCRIPTION:
4 1¼" DIA. M.S.
5 1⅛" DIA. H.T.
6 1" AMERICAN RIBBED M.S.
8 1" AMERICAN RIBBED H.T.
11 1" TORSTEEL
13 1" INDENTED TWISTED 12" PITCH
14 1 AMERICAN RIBBED BRITISH M.S.
17 No.1 TENTOR ·71" DIA. PER BEAM
18 No.2 TENTOR ·71" DIA. PER BEAM
(7) - MAX. CRACK IN mm.

Fig. 7.—Load-deflection diagrams

with the formation of cracks. Both the quality of the concrete and the percentage of the reinforcement are clearly reflected in the diagram.

The essential point about the load-deflection lines is their shape. All beams reinforced with steel having a natural yield point (with the exception of 6H) show

TABLE 6

Beam No.	Description of Reinforcement	Average No. of cracks at failure	Average Stress at failure lb/sq. in.
9	American ribbed H.T. twisted ...	29	74,405
19	Tentor (3 bars of .71 in. dia. per beam)	28	93,850
7	American ribbed M.S. twisted ...	27	67,980
15	,, ,, British M.S. twisted 6 in. pitch	26	79,450
16	,, ,, British M.S. twisted 12 in. pitch ...	24.5	77,900
3	Square twisted	22.5	69,395
12	Indented twisted 6 in. pitch ...	22.5	83,800
13	,, ,, 12 in. ,, ...	22	69,200
[1]18	Tentor (2 bars of .71 in. dia. per beam)	22	93,300
8	American ribbed H.T. (untwisted) ...	20	52,440
10	Twin-twisted	20	65,945
[1]17	Tentor (1 bar of .71 in. dia.) ...	19	92,400
11	Torsteel	17.5	75,175
1	Oval twisted 6 in. pitch	17	78,500
2	,, ,, 12 in. and 17 in. pitch ...	16	70,750
[2] 6	American ribbed M.S. (untwisted) ...	15.5	44,150
14	,, ,, British ,,	15.5	44,450
5	1¼ in. dia. H.T.	15.5	55,815
4	1¼ in. ,, M.S.	14.5	42,630

NOTES

[1]One beam only.

[2]The number of cracks counted for 6L was only 11 ; that for 6H, 20 ; it is probable that the actual number for 6L was greater, since the 11 cracks were counted at a load of 20,000 lb. No observations could be made under the maximum load of 22,000 lb. since the beam failed suddenly.

a sudden break where the slope of the line changes into horizontal, whereas all beams reinforced with cold worked steel show a gradual change of slope long before failure. Most of the beams, reinforced with steel having a natural yield point, failed at a maximum crack width of .15 to .25 mm.

The behaviour of beam 14L is of particular interest. At a load of 20,000 lb the maximum width of the cracks

Fig. 8.—Beam No. 4. 1¼ in. diameter mild steel. High-grade concrete

was .15 mm. On reaching a load of 22,000 lb. momentarily, the steel began to yield, the deflection increased rapidly, and the load fell off to 21,000 lb. The load of 22,000 lb. is not shown in the diagram since no reading of the deflection could be taken. With increasing deflection the load dropped to 19,000 lb. then increased gradually to 21,500 lb., when a deflection of 1.604 in. was measured. The load could be further increased to 23,000 lb. when the concrete was crushed.

Beam 14H (not shown on the diagram) behaved in the same way. The only difference was that owing to the lower yield point of the reinforcement, yielding had already begun when the load of 20,000 lb. was reached for the first time. After a second loading to 20,000 lb. the load was increased momentarily to 21,500 lb. and dropped again to 20,000 lb. After ex-

Fig. 9.—Beam No. 14. 1 in. diameter American ribbed bar untwisted. High-grade concrete

ceeding a deflection of .9 in. the load began to rise again until the maximum of 23,000 lb. was reached at a deflection of 2.744 in. when the concrete was crushed.

The load-deflection line of beam 14L indicates the existence of an upper and a lower yield point which could not be shown by the method adopted by Mr. Stanger in determining the stress-strain diagram. His figures relate throughout to the upper yield point.

There are two reasons why the oscillation of the stress at yielding could be demonstrated in beams 14.

(1) The comparatively short range of yielding.

(2) The excellent bond of the American ribbed type bar.

The combination of these two factors made it possible that in beam 14L at a deflection of 1.2 in., in beam 14H at a deflection of .9 in., the load, after having fallen below a maximum previously reached, began to rise again. With the long range of yielding of bars 6 and 8, the beams failed before the steel stress could rise again, in spite of the good bond. With the plain bars 4 and 5, the bond was not good enough to prevent the concentration of yielding at one crack (Fig. 8 showing beam 4H after failure should be compared with Fig. 9 showing beam 14H after failure). The tensile zone of the central portion of beam 14H completely disintegrated and fell down when the beam was removed from the testing machine. The essential feature of this picture are six wide cracks, three on either side of the crack where failure occurred. (The extension of the central crack to the top occurred when the beam was

taken down from the machine). This formation of cracks after yielding was characteristic of all beams reinforced with American ribbed type bars, in contrast to beams reinforced with plain round bars

Apart from beams 14 which represent a very special case, *the only beams reinforced with steel having a natural yield point which gave reasonable warning before failure were beams* 4L *and* 5L (not indicated in Fig. 7) i.e. the beams with plain bars in ordinary quality concrete. *If, however, plain steel of M.S. quality is used in concrete of a cube strength of the order of* 6,000 *lb./sq. in.* (*beam* 4H) *or if steel of very high bond value like the American ribbed type bar is used, even in concrete of ordinary quality, no warning can be expected even if the yield point is as high as* 52,000 *lb./sq. in.* (*beam* 8L).

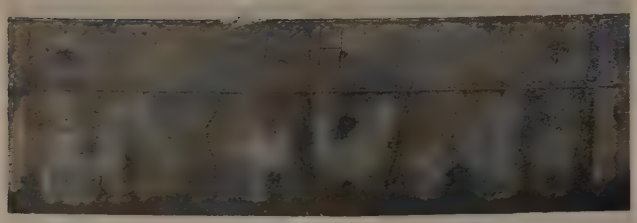


Fig. 10.—Beam No. 13. 1 in. diameter indented bar twisted to 12 in. pitch. High-grade concrete

Thus, while the improved bond is of great advantage under working loads, since it increases the number of cracks and reduces their average width, it is disadvantageous from the point of view of failure risk unless the yield point of the steel exceeds 60,000 lb/sq. in. At stresses of 50,000 lb/sq. in. no cracks wider than .25 mm. can be expected; with small size bars even of smaller width. The number of tests with American ribbed type bars (six) is sufficient to arrive at the conclusion *that the use of this type of bar as reinforcement can only be recommended if it is produced with a yield point substantially higher than the guaranteed minimum for high tensile steel to B.S.* 785. Beams 14 which might be quoted as contradicting this conclusion do, in fact, confirm it since the yielding began in the same way as in the four other beams and the short range of yielding of the bars used in these beams cannot be relied upon. Expressed in another way, *the bond of the American ribbed type bar is too good for steel of B.S.* 785 *quality.*

In his paper read before this Institution on September 30th 1943(6), the Author made the following statement:

> "It is therefore obvious that if an advantage is to be obtained from the use of high tensile steel as reinforcement of concrete, the higher strength must be accompanied by higher bond."

The present tests have proved that the inversion of this statement is also true. *If an advantage is to be obtained from improved bond, the higher bond must be accompanied by higher strength.*

The danger of unexpected failure occurs, however, even with plain round bars if the quality of the concrete is too good (4H). The second conclusion to be drawn from these considerations is therefore that *high quality concrete requires high strength steel as reinforcement,* preferably higher than high tensile steel to B.S. 785.

High tensile steel to B.S. 785 has the disadvantage that its bond strength in ordinary grade concrete is not good enough for the higher working stresses which are permitted in the Code, as demonstrated by beam 5L. A further disadvantage is that it cannot be distinguished from ordinary M.S. on the site. This is a source of danger which could, of course, be overcome by special rolling marks as specified, for instance, in Germany.

In contrast to the beams with bars having a natural yield point, *the load-deflection lines for beams with cold worked bars in Fig.* 7 (11L, 11H, 17L, 18H) *show that all these beams have fully satisfied the requirements of warning before failure. They all became unserviceable at loads which could still be safely carried and had to be increased to cause failure.* Beams 19L and 10H which are not shown in Fig. 7 because they failed at much higher loads behaved in a similar way. The maximum width of cracks in beam 19L at 66,000 lb was .6 mm. The beam failed at a load of 67,500 lb but could still maintain 66,000 lb. Beam 19H showed cracks of .7 mm. width at 68,000 lb. of .8 mm. at 70,000 lb. failed at 72,000 lb. and maintained a load of 69,000 lb. The warning is obviously better with lower percentages but even with the high percentage used in 19L which far exceeded the "economic" percentage according to the Code, it was far better than in any of the beams reinforced with steel having a natural yield point.

There is a very great difference between "warning" as in beams 14 and warning with reinforcement where the yield point has been completely eliminated.

Fig. 10 showing beam 13L after failure is characteristic of the formation of cracks in beams with cold worked bars.

Fig. 11 shows the central part of beam 17L in the testing machine before failure under a load of 24,000 lb. (max. load 24,500 lb.) (The marking of this beam as 19L should read 17L, since beams 17, 18, 19 have been re-numbered in this paper).

8. Workability of Steel

It is essential that it should be possible to bend bars to any required shape. For this reason, the cold bend test specified in B.S. 1144 is of the utmost importance.

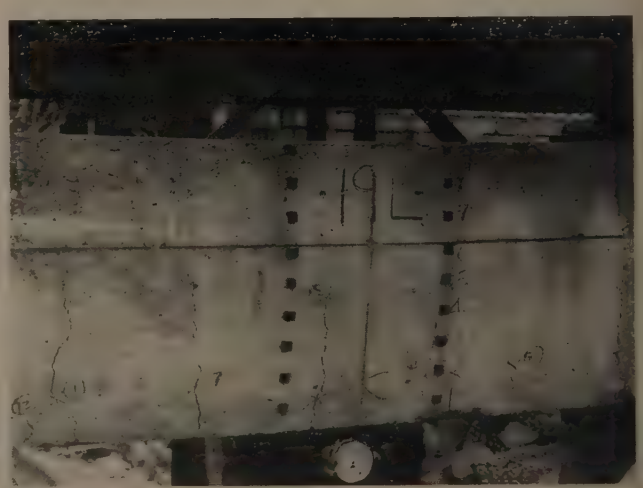


Fig. 11.—Beam No. 17L. 0.71 in. diameter Tentor bar. Low-grade concrete. Under load of 24,000 lb. prior to failure

From the preceding considerations one might arrive at the conclusion that the twisted American ribbed type bar would be the ideal reinforcement, combining the advantages of high bond, good crack control, and adequate warning. Such a conclusion would, however, be misleading, since these bars did not lend themselves to cold working. This has already been pointed out in connection with bars 7 and 9 (American material) and also applies to bars 15 and 16 (British M.S.). The development of bars 7 and 9 by cold work was quite inadequate, yet bar 9 could not pass the cold bend test in spite of the long range of the stress-strain diagram before reaching the maximum stress and an elongation exceeding the requirements of B.S. 1144. Bar 15, admittedly overtwisted, was so brittle that it broke at an angle of 45° and even bar 16, with a pitch of only 12 in., i.e. in the lower range of B.S. 1144 and with a ratio of $y/u < .8$, i.e. below the lowest limit of the normal range, was fissured when bent to 180°*.

It would require further investigations to find out the reason for the unsuitability of bars of this shape for cold working. This is,beyond the scope of this paper. If American ribbed type bars were produced with a natural minimum yield point of 60,000 lb/sq. in., it would be a different matter, although even a minimum yield point of 65,000 lb/sq. in. would not render them equivalent to cold worked single twisted bars to B.S. 1144 from the point of view of warning.

All other bars passed the cold bend test satisfactorily, but the bending of the oval twisted bars (1 and 2) was very difficult because of their shape. The bars had the tendency of " springing " at right angles to their plane. Special provisions would be necessary on bending machines for bending oval twisted bars in a satisfactory manner and without danger to the workmen.

The cold bend test as specified in B.S. 1144, Clause 12, is rather ambiguous since two alternative methods (" by pressure or by blows from a hammer ") are permitted. In the author's opinion, only one method, i.e. bending in an ordinary bar bending machine, should be adopted. This is the nearest to practical conditions. It would also be essential to specify a definite temperature range.

A standard test for cold bending and re-bending (i.e. straightening again) would also be very useful.

9. Conclusions and Suggestions

(A) Assessment of the various bars

The tests have proved the superiority of cold worked bars over bars with a natural yield point with regard to factor of safety, bond, crack formation, and warning before failure.

The *American ribbed type bar* (6, 7, 8, 9, 14, 15, 16) is superior to all others regarding bond and crack control, but for this very reason cannot be recommended for use as reinforcement unless its yield point is at least 60,000 lb./sq. in. Otherwise it does not give enough warning before failure. All attempts at cold working of this type of reinforcement have given unsatisfactory results.

Among the cold worked bars, the *oval twisted* (1 and 2) proved to be the least efficient in bond and crack control. The difficulty of bending does not commend it for practical application.

The strength properties of *twin-twisted bars* (10) are inferior to those of single twisted bars. As long as the ceiling of 27,000 lb./sq. in. for permissible stresses is adhered to, the lower strength does not affect the economy of design in comparison with single twisted bars but the factor of safety is lower.

Of the remaining cold worked bars no marked difference could be found between *square twisted steel* (3) and *Torsteel* (11). On the other hand, the *Indented twisted bars* (12 and 13) and *Tentor bars* (17, 18, 19) showed certain advantages. Although results for Tentor bars may be affected in their favour, because of their smaller diameter, it is reasonable to assume that the 1 in. diameter Indented twisted bars are representative of the shape of Tentor bars. The stress at failure of bar 12L (81,700 lb/sq. in.) was much higher than in any other bar of 1 in. diameter in " L " concrete, yet no slip occurred at failure. The greater extension of Tentor bars may be affected in their favour, because of comparison with single twisted bars to B.S. 1144 (which nevertheless allowed the utilisation of at least 91 per cent. of the ultimate strength even in " L " concrete) is bound to be an advantage in the case of impact loading, although no such tests could be included in the present series.

Regarding *plain round bars* (4 and 5) the tests have demonstrated that beams in high strength concrete reinforced with M.S. may fail without warning, and even with M.S. steel to B.S. 785 the warning may be insufficient.

(B) Suggestions for the amendment of the Code and B.S. 1144

(a) Code

The Code greatly favours bars having a natural yield point and does not allow the economical use of single twisted bars.

(1) The efficiency of hooks is over-estimated by the arbitrary allowance of 16D as equivalent bond length.

(2) The standard hook is too small in the case of high tensile steel.

(3) The ceiling stress of 27,000 lb/sq. in., irrespective of the strength properties, shape, and size of the reinforcement, is too low. Much higher stresses could be allowed for suitable types of reinforcement without danger of corrosion or reduction of the factor of safety.

(4) The standard method of design, based on the modular ratio, reduces the calculated moment of resistance of a given concrete section with increasing steel stress. Thus, at the " economic " percentage, mild steel cannot be replaced by an equivalent area of high tensile steel without the addition of steel in compression. This is not justified by experimental evidence. The maximum resistance of a concrete section in bending is independent of the quality of the steel and should be so in the design. This could be achieved in different ways and is essential for the economical use of cold worked steel.

(b) B.S. 1144

(1) The term " yield point " is misleading, since cold worked bars do not yield. It should be replaced by " proof stress."

*In fairness to the American ribbed type bar it should be added that the bars rolled in this country had a carbon content of .30 per cent. (i.e., the upper limit permissible for British M.S.) and that the corners of the ribs were rather sharp. The corners of the ribs of the original bars supplied from U.S.A. were rounded off.

(2) An upper and lower limit should be specified for the ratio $\frac{\text{proof stress}}{\text{ultimate strength}}$.

(3) An upper and lower limit should be specified for the strain at which the maximum stress is reached.

(4) The specification of a "minimum elongation" included in Clause 9 should be abolished.

(5) Only one method should be specified for cold bend tests, based on the use of an ordinary bar bending machine.

(6) Limits of temperature should be included in the specification of the cold bend test.

(7) Provisions regarding the pitch should be extended so as to include bars cold worked by the Tentor process.

Acknowledgement

The author is indebted to Mr. C. Harry Stanger and members of his staff for their assistance in carrying out the tests.

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Book Reviews

Photoelasticity: Principles and Methods, by H. T. Jessop and F. C. Harris. (London, Cleaver-Hume Press, 1949.) 184 pages, 164 figures, 10 in. by 6¼ in. 28s.

The photoelastic method of stress analysis owes its development very largely to the classic work of Coker and Filon at University College, London. The authors of the present book were closely associated with the late Professor Filon in his pioneer work, and are therefore especially fitted to produce a text-book on this subject.

The book is intended primarily for those who wish to undertake practical work with photoelastic equipment. Since the method is now in general use in most research laboratories as well as in Universities and Technical Colleges, a book of this character will meet a real need.

The early chapters deal in a clear manner with the fundamental principles of compound stresses and strains. Although the notation may be strange to some readers, and not always in accordance with British Standards, it is clearly explained and consistently adhered to in the text.

The relevant principles of optics are next explained, leading up to polarisation and double refraction. Chapter V deals with the theory of photoelasticity.

The next chapter describes the ways in which the directions and magnitudes of the principal stresses in a loaded plate are deduced from photo-elastic observations. Chapter VII gives an account of the "frozen stress" method of investigating three-dimensional stresses which is now being developed and which promises to extend considerably the practical applications of photo-elasticity.

Chapters VIII and IX are devoted to practical matters—the apparatus required and the materials available.

Finally, in Chapter X, a number of well-illustrated examples are given with the calculation of the stresses from actual observations.

In general, the book is well set out and clearly printed, and the illustrations are good. The authors have wisely reduced the mathematics to an essential minimum.

It is to be regretted that no list of references is included. While a comprehensive bibliography would be out of place in a book of this type, a short and carefully selected set of references would be invaluable to readers who desire further information on specific aspects of the subject.

P. L. C.

Welding Design and Processes, by B. R. Hilton. (London, Chapman & Hall, 1950.) 342 pages, 8½ in. by 5½ in. 36s.

This is one of the most useful books yet published for the designer of welded fabrications and the author has managed to cover an ambitiously wide field in a book of convenient size.

With such a wide scope it is only natural that some of the statements are generalised and rather sweeping, and a bibliography to which reference could be made for amplification of particular points would be a valuable addition to the volume.

In Chapter II the statement that intermittent fillet welds should not be used in stressed positions is far too sweeping without amplification. In Table VIII the maximum electrode sizes given appear unnecessarily restrictive especially for structural work with modern techniques.

It is a pity that some reference could not have been made to the increased stresses now permitted under B.S.S. 449 and other advantages now allowed under this specification.

The subject-matter in Chapter III is well treated but some of the metallurgical explanations are a little misleading.

The examples of design given in Chapter VIII and the commentary on their chief points forms one of the most useful sections of the work. It is a pity no examples are given of structural work, for which the allowable weld stresses and some of the joint details shown are unnecessarily restrictive.

Even as it stands, however, there is so much of general application expressed in such a common-sense way that a study of this book will well repay the designer of welded fabrications whether in the structural or mechanical field.

P. C. G. H.

Breaking Tests on Three Full Size Prestressed Concrete Bridge Beams

By Dr. P. W. Abeles, M.I.Struct.E.

Synopsis

This paper describes laboratory tests carried out to destruction on three prestressed concrete bridge beams, two of which were composite, partially prestressed beams, containing tensioned and untensioned wires. Load deflection curves and strain measurements have proved the full co-operation of tensioned and untensioned wires up to failure, the resilience being similar to that of ordinary prestressed concrete beams without untensioned wires.

Introduction

In the course of electrification between Manchester, Sheffield and Wath, seventeen arch bridges had to be replaced because of their insufficient headroom for the overhead contact wire and catenary. Since a minimum construction depth was required, a slab construction was adopted composed of prestressed concrete beams of inverted T-shape with *in situ* concrete filling and decking, the prestressed concrete beams being capable of carrying alone without intermediate support the additional *in situ* concrete when placed, and later, in combination with the latter, the additional dead weight and the live load. In the design 1 of Fig. 1, employed for the first bridge contract let by the railway authorities in 1948 (i.e., before these tests were carried out), and built in 1949, ordinary mild steel bars were placed in the *in situ* concrete. In the second design of Fig. 1, as employed with the bridges of three further contracts, untensioned high-strength steel wires 0.2 inches diameter, and placed within the T-beams, have been used, being of the same properties as the tensioned wires (tensile strength 100 tons per sq. in.).

It is a special feature of these bridge designs that partial prestressing is employed, concrete tensile stresses of 500 to 600-lb. per sq. in. being permitted for roads at which no higher loadings are expected than the Ministry of Transport loading. It has been shown in publication (1) on the basis of numerous tests that cracking at first loading does not commence, as long as the bending tensile strength of the concrete (i.e., the modulus of rupture which can be assumed for high strength concrete as approx. 1,000 lb. per sq. in.) is not exceeded. Complete freedom from cracks under repeated application of the working load can, therefore, be obtained, if the concrete tensile stress is not higher than 50 to 60 per cent. of the bending tensile strength.

More particulars about design and erection of these bridges will be given in a paper, " Some Developments in Prestressed Concrete," to be presented to the Institution of Structural Engineers. In the present article, three full-size beam tests are discussed which were carried out in connection with the first bridge contract in 1949. Fig. 2 shows the cross sections of test beams P1, T1, and T2. The purpose of these tests with regard to P1 was to examine the properties of a prestressed concrete beam, as employed in the bridges design 1, and with regard to T1 and T2, to investigate the behaviour of a combined section with untensioned wires embodied in the precast prestressed beam, the main aim being to study the behaviour of such a novel construction before final decisions were made with regard to design 2 of Fig. 1. Designs 1 and 2 of Fig. 1 were first proposed in 1945 in the author's closure to paper (2), such composite construction having been suggested by Dr. K. Hajnal-Konyi in 1944 (see also discussion to (2)).

The three beams were tested at the Testing Works of Mr. R. H. Harry Stanger, at Elstree, and the prestressed beams were manufactured by Dow-Mac, Ltd., at Tallington, Lincs. The concrete was composed of 2.8 parts, by weight, of coarse aggregates (3/16 to ½ in.), 1.41 parts of fine aggregates (B.S. sieve 100 to 3/16 in.), and 1 part of ordinary Portland cement, a water cement ratio of approx. 0.42 being used. The aggregates were obtained from pits near the factory and washed before use. A horizontal mixer with weigh batching equipment was

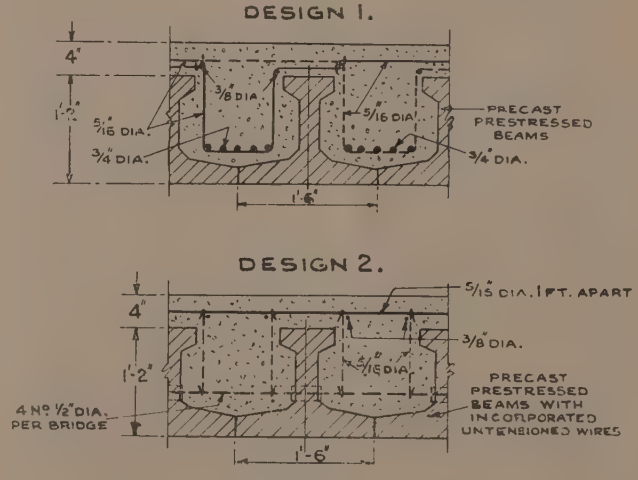

Fig. 1.—Two bridge deck designs

employed. The beams were cast in the open, in inverted position, using steel moulds and employing external vibrators. Beams P1 and T1 were manufactured in December, 1948, and T2 in March, 1949. The 28 days' cube and prism strength amounted to approx. 8,500 and 6,770 lb. per sq. in. respectively (cubes of 6 in. side and prisms 4 in. by 4 in. by 16 in.).

Breaking test on prestressed concrete beam P1

The purpose of this test was, as already stated, to examine generally the properties of prestressed concrete in a full-size beam and, in particular, to determine the modulus of rupture and the deflection at a load, corresponding to a concrete tensile bending stress of 750 to 800 lb. per sq. in. This loading was chosen for subsequent checking tests carried out during manufacture and beams were only accepted if cracks did not develop at this loading and the deflection did not exceed that

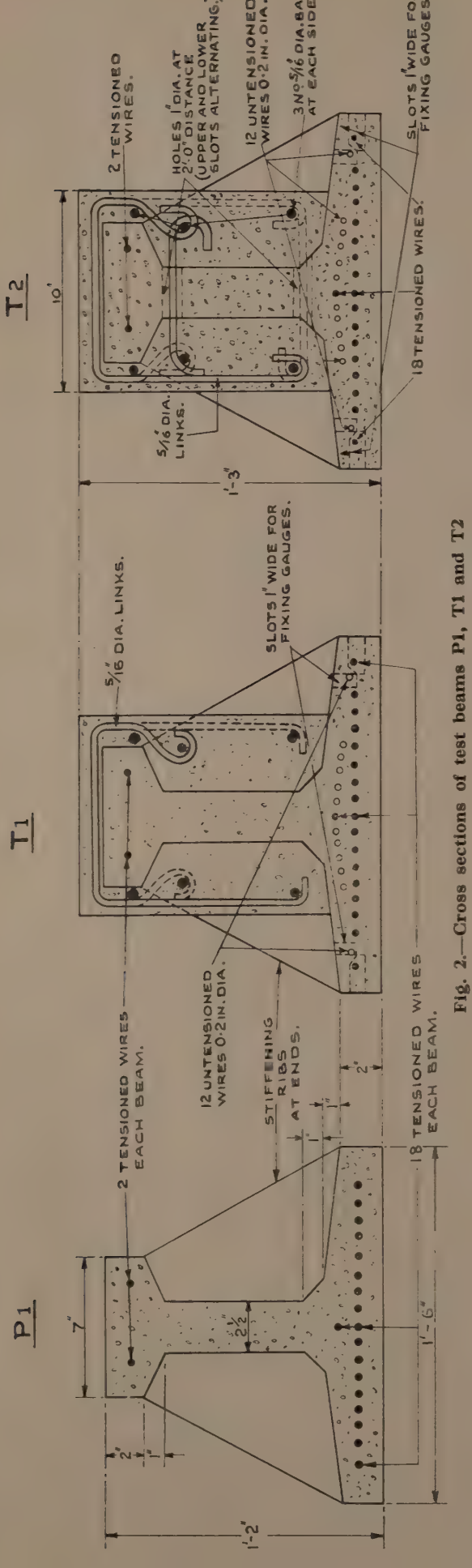

Fig. 2.—Cross sections of test beams P1, T1 and T2

obtained in the original test by more than 30 per cent. All the checking test deflections were well within this limit.

Fig. 3 shows the loading arrangement for testing beam P1 of 28 ft. 6 in. length at Elstree in January, 1949. Particulars about the loading arrangement and test results are seen in Fig. 4. Beam P1 was 30 days old when first tested and was selected at random from a number o

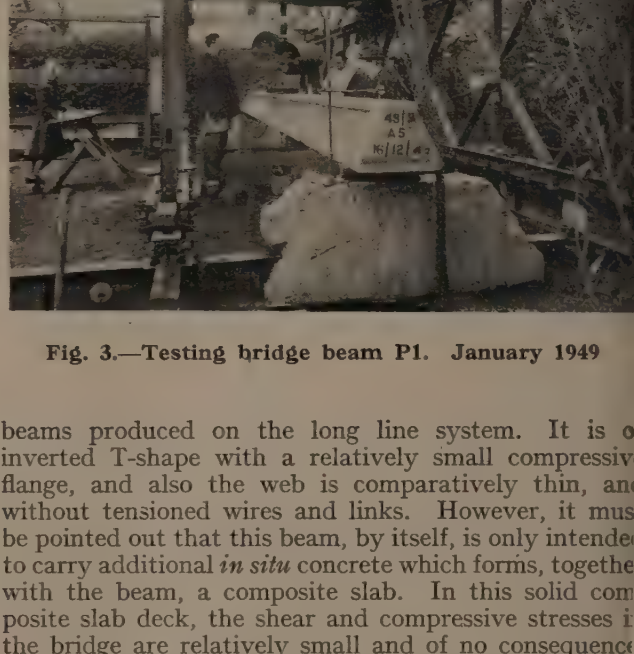

Fig. 3.—Testing bridge beam P1. January 1949

beams produced on the long line system. It is o inverted T-shape with a relatively small compressiv flange, and also the web is comparatively thin, an without tensioned wires and links. However, it mus be pointed out that this beam, by itself, is only intende to carry additional *in situ* concrete which forms, togethe with the beam, a composite slab. In this solid com posite slab deck, the shear and compressive stresses i the bridge are relatively small and of no consequence However, when tested by itself, beam P1 is relativel weak in the compressive zone and must be considere as an over-reinforced beam which fails, through sudde compression failure, as seen in Fig. 5.

There are eighteen tensioned wires 0.2 in. dia. of strength of 100 tons per sq. in. in the tension flange, an two tensioned wires in the compression flange. Th arrangement of wires allows for handling the beam i any manner, without the possibility of cracks developin in the top flange. At the ends of the beam stiffener rib are provided to avoid horizontal cracks between tensi flange and web at release of the tensioned wires. Th initial tensioning force was 2.1 tons per wire (stre 150,000 lb. per sq. in.), but firstly, this force was i creased by 5 per cent. for a few minutes to offset cre of steel.

On the basis of measurements upon special specime of shortening due to elastic deformation of the concret shrinkage and creep, between casting and testing, t actual losses were obtained, which agreed well with t assumed losses (effective prestress at testing 123,500).

In the first place (first and second loading), a lo $2W = 5.3$ tons, representing the basis of the checki test, was applied, as shown in Fig. 4. The permane deflections were measured upon release to "zero loading at $2W = 250$ lb. Later (at the third loadi the load was increased to $2W = 6$ tons, correspondi to a concrete tensile stress of 1010 lb. per sq. in., at whi condition cracks did not develop and the measur deflection remained between the two limits calculat

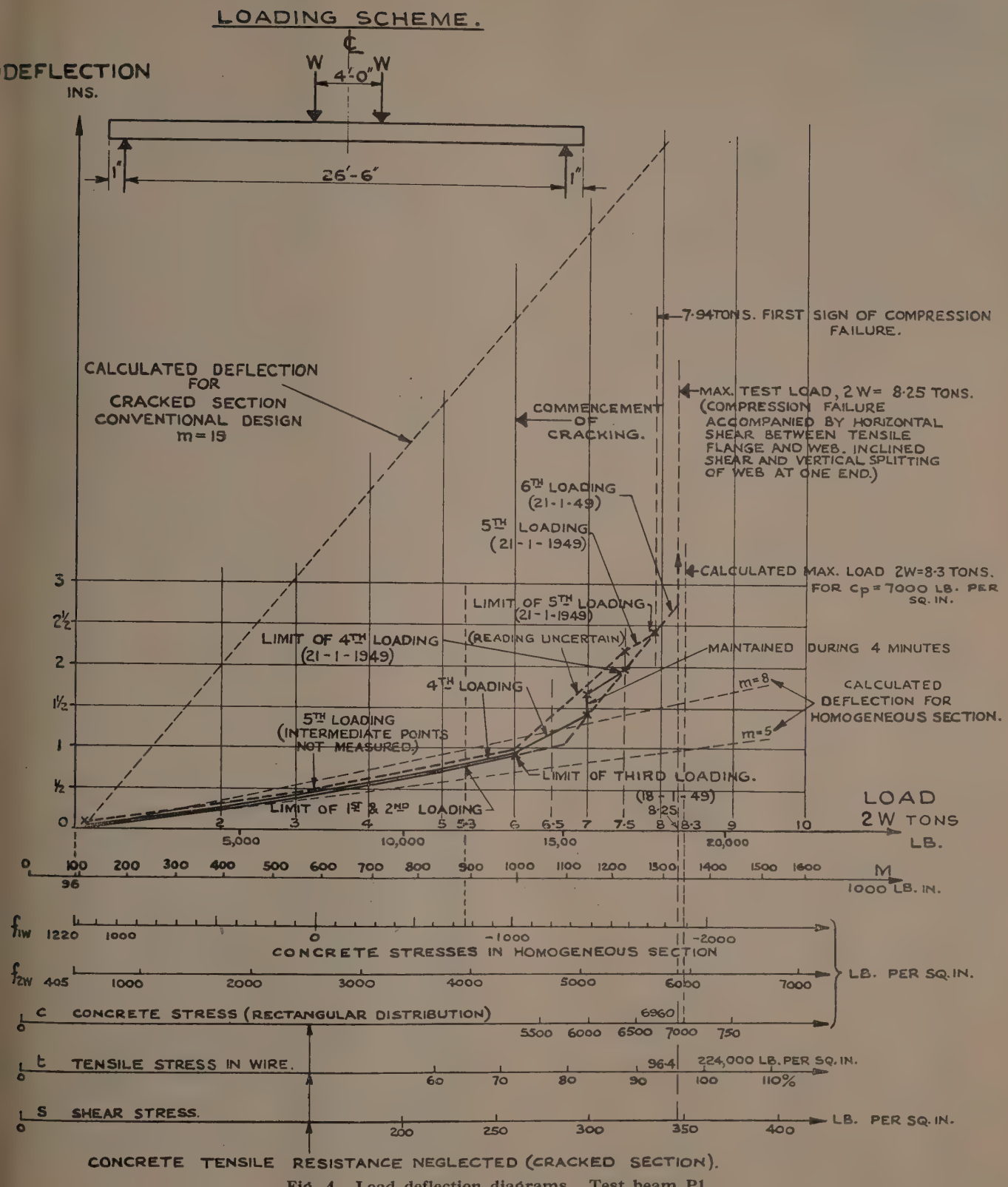
LOADING SCHEME.
W
4'-0"
W
1"
26'-6"
1"
DEFLECTION INS.
7·94 TONS. FIRST SIGN OF COMPRESSION FAILURE.
CALCULATED DEFLECTION FOR CRACKED SECTION CONVENTIONAL DESIGN m=15
COMMENCEMENT OF CRACKING.
MAX. TEST LOAD, 2W= 8·25 TONS. (COMPRESSION FAILURE ACCOMPANIED BY HORIZONTAL SHEAR BETWEEN TENSILE FLANGE AND WEB. INCLINED SHEAR AND VERTICAL SPLITTING OF WEB AT ONE END.)
6TH LOADING (21-1-49)
5TH LOADING (21-1-1949)
CALCULATED MAX. LOAD 2W=8·3 TONS. FOR Cp = 7000 LB. PER SQ. IN.
LIMIT OF 5TH LOADING (21-1-1949)
LIMIT OF 4TH LOADING (21-1-1949)
(READING UNCERTAIN)
MAINTAINED DURING 4 MINUTES
4TH LOADING
m=8
CALCULATED DEFLECTION FOR HOMOGENEOUS SECTION.
5TH LOADING (INTERMEDIATE POINTS NOT MEASURED.)
m=5
LIMIT OF THIRD LOADING. (18-1-49) 8·25
LIMIT OF 1ST & 2ND LOADING
LOAD 2W TONS
LB.
M
1000 LB. IN.
CONCRETE STRESSES IN HOMOGENEOUS SECTION
LB. PER SQ. IN.
C CONCRETE STRESS (RECTANGULAR DISTRIBUTION)
6960
t TENSILE STRESS IN WIRE.
96·4
224,000 LB. PER SQ. IN.
S SHEAR STRESS.
LB. PER SQ. IN.
CONCRETE TENSILE RESISTANCE NEGLECTED (CRACKED SECTION).

Fig. 4.—Load deflection diagrams. Test beam P1

for $m = 5$ and $m = 8$ (based on $E_s = 30 \times 10^6$ lb. per sq. in.), which are plotted in the diagram.

Two days later this loading was repeated, but this time (the fourth loading), cracks occurred at $2W = 6$ tons. When the loading was increased to $2W = 7.5$ tons, corresponding to a nominal tensile stress of 1,570 lb. per sq. in., cracks of a maximum width of 0.3 mm. developed. On removal of this load, the cracks closed up entirely, and the permanent deflection was insignificant.

The fifth loading was carried up to a load of $2W = 7.94$ tons, and when the first signs of compression failure were noticed, the load was reduced. On renewed loading, compression failure took place quite abruptly at a load $2W = 8.25$ tons, followed by heavy shear cracks (particularly horizontal shear between tensile flange and web), as seen from Fig. 6. The stresses at failure correspond to 96.3 per cent. of the ultimate tensile strength of the steel and to a concrete prism strength of 6,970 lb. per sq. in. based on an

approximately uniform stress distribution, as indicated in Fig. 5. The prism strength obtained from specimens (4 in. by 4 in. by 16 in.) at 28 days amounted to 6,770 lb. per sq. in., and the beam was broken when the concrete was 35 days old, which gives good agreement between calculation and test.

The very good agreement between the failure load of $2W = 8.3$ tons, according to Fig. 5, calculated previous to the test, and the actual failure load $2W = 8.25$ tons is only accidental, since it is, as a rule, difficult to predict the ultimate resistance of over-reinforced beams as shown in (3). In the present case,

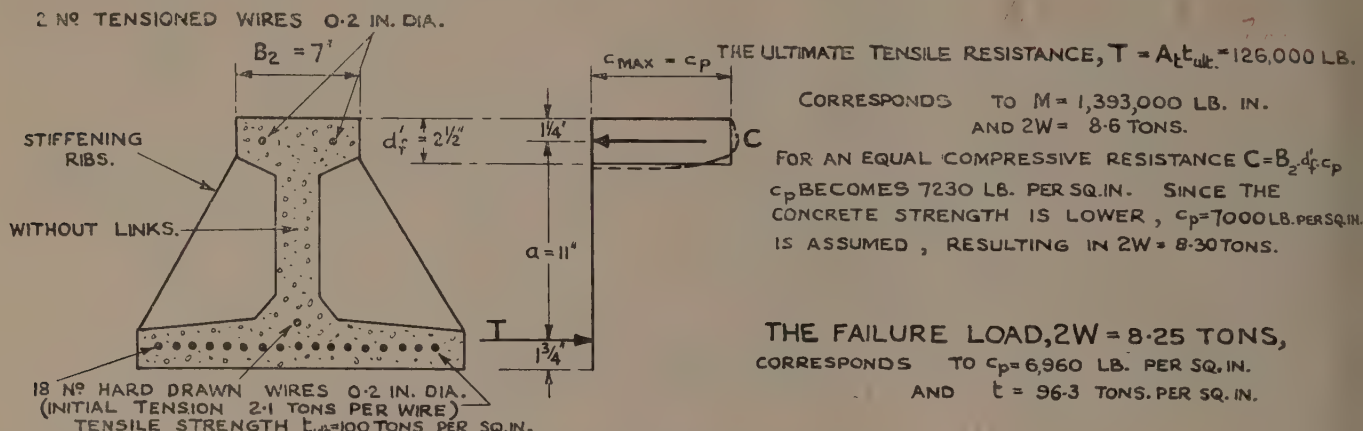
2 Nº TENSIONED WIRES 0·2 IN. DIA.
$B_2 = 7"$
$c_{MAX} = c_p$
THE ULTIMATE TENSILE RESISTANCE, $T = A_t t_{ult.} = 126,000$ LB.
CORRESPONDS TO M = 1,393,000 LB. IN. AND 2W = 8·6 TONS.
FOR AN EQUAL COMPRESSIVE RESISTANCE $C = B_2 \cdot d_f' \cdot c_p$
c_p BECOMES 7230 LB. PER SQ.IN. SINCE THE CONCRETE STRENGTH IS LOWER, c_p = 7000 LB. PER SQ.IN. IS ASSUMED, RESULTING IN 2W = 8·30 TONS.
STIFFENING RIBS.
WITHOUT LINKS.
$d_f' = 2\frac{1}{2}"$
$1\frac{1}{4}"$
C
$a = 11"$
T
$1\frac{3}{4}"$
THE FAILURE LOAD, 2W = 8·25 TONS,
CORRESPONDS TO c_p = 6,960 LB. PER SQ. IN.
AND t = 96·3 TONS. PER SQ. IN.
18 Nº HARD DRAWN WIRES 0·2 IN. DIA.
(INITIAL TENSION 2·1 TONS PER WIRE)
TENSILE STRENGTH $t_{ult.}$ = 100 TONS PER SQ.IN.

Fig. 5.—Stresses at failure. Test beam P1

the compressive flange is relatively shallow, and the full prism strength was developed in this flange. In over-reinforced rectangular beams it would be more difficult to determine the actual ultimate resistance so closely, as in the present case.

In Fig. 4, in addition to the loading $2W$ and bending moment, also the stresses f_{1w} and f_{2w} (the concrete stresses calculated for a homogeneous section in the bottom and top fibre respectively) are indicated. Obviously, the concrete tensile stresses f_{1w} (tension negative sign) after cracking are only nominal, since no concrete tensile stress exceeding the modulus of rupture can be developed. However, in view of the well-established resilience of prestressed concrete, such nominal tensile stresses are of interest, giving an indication of the extent of loading. Further, also the concrete and steel stresses c and t respectively, computed according to Fig. 5 for a cracked section with rectangular concrete stress distribution, are shown in Fig. 4, together with the shear stress s, calculated in conventional manner for the web of P1.

Reference may be made to the deflection line plotted in Fig. 4, computed for a cracked section according to conventional design ($m = 15$). It is seen that the deflection curve of P1 after cracking is nearly parallel to this theoretical line. This is in good agreement with observations made on the occasions of previous tests carried out by the L.N.E.R. (1) which were, in fact, the basis for the design 1 of Fig. 1 employing partial prestressing, since these tests had established without doubt the resilience of prestressed concrete.

Breaking test on partially prestressed composite beam T1

In order to produce a condition somewhat similar to a composite bridge structure design 2 of Fig. 1, a section according to T1, Fig. 2 was chosen, and a concrete casing of the same concrete properties was added after seven days to a prestressed beam also from the first cast, as P1, but containing for the first time additional untensioned wires (i.e. twelve in this case). It may be mentioned that previous to this test, to the author's knowledge, no investigations were made with regard to a prestressed concrete beam with pre-tensioned and untensioned wires, except for a preliminary test on a small size beam, as described in (2), the whole question of partial prestressing being discussed in (4).

The untensioned wires in beam T1 of the same strength properties as the tensioned wires were slightly tightened to keep them in position. The concrete casing was necessary to ensure that the tensile resistance of the tensioned and untensioned wires could be fully utilised, which required increase of the compressive and shear resistance, the section of the prestressed beam alone being too weak to carry a load corresponding to the resistance of the tensioned wires, as the test on P1 has shown, not to mention the increased resistance

Fig. 6.—Test beam P1. After failure—January 1949

due to the untensioned wires. There was no special connection between the prestressed concrete component and the casing, excepting adhesion between the two concretes, the bond being somewhat improved by roughening of the smooth surface of the prestressed beam.

Fig. 7 shows the results of these tests. Similar to Fig. 4, the various stresses for a homogeneous and cracked section are plotted in this diagram. In addition to the stresses shown in Fig. 4, those of fibres 3 and 4

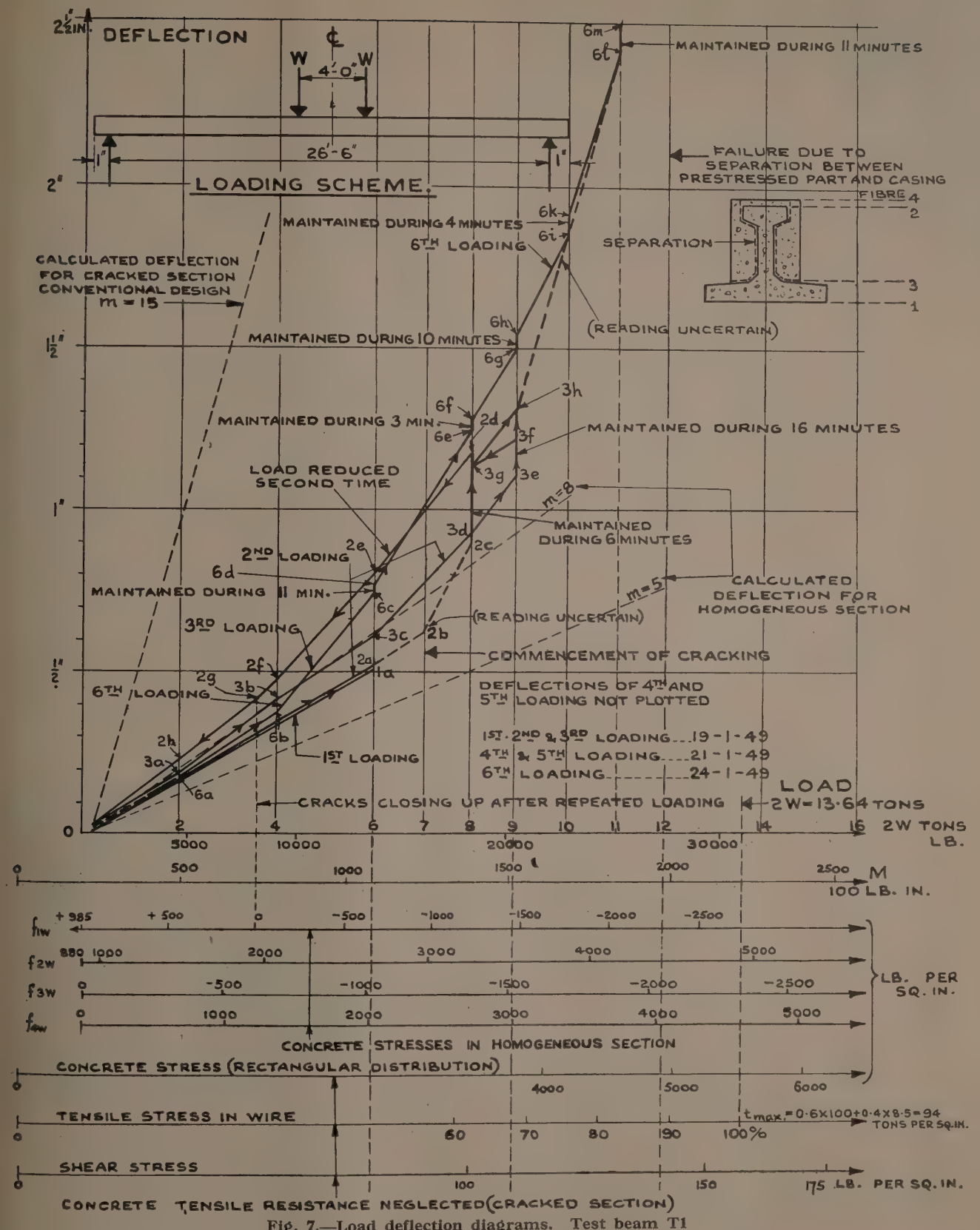

Fig. 7.—Load deflection diagrams. Test beam T1

of the composite section are also seen. The maximum resistance, based on the stress t_{max} has been computed on the assumption that at failure the ultimate tensile strength is reached only in the tensioned wires, but not in untensioned, for which only 85 per cent. of the strength was taken into account. The average maximum stress thus amounts to $0.6 \times 100 + 0.4 \times 85 = 94$ tons per sq. in., resulting in a maximum load $2W = 13.64$ tons, as indicated in Fig. 8.

This beam was tested in January 1949, and showed clearly that both tensioned and untensioned wires, as well as precast concrete with additional casing, co-operated very well together. In the diagram Fig. 7, the deflection curves of four loadings: 1, 2, 3 and 6 of the six loadings carried out are plotted, and references are given for the principal points of the deflection curves. The first loading was carried up to $2W = 6$ tons, corresponding to the checking load (concrete tensile

stress at 775 lb. per sq. in.) when no cracks occurred and an insignificant permanent deflection remained. Also, in this case, the deflection line remained well within the two limits of $m = 5$ and $m = 8$, plotted in the diagram. First visible cracks were detected at the second loading at $2W = 7$ tons (point 2b, deflection reading uncertain), corresponding to a concrete tensile stress of approx. 950 lb. per sq. in. At $2W = 8$ tons (point 2c), five cracks were detected, the maximum width being 0.25 mm. The load was maintained for approximately six minutes, when one crack extended up to 7 in. from the top fibre. As soon as the load was reduced to $2W = 4$ tons total (point 2f), corresponding to a concrete tensile stress of approx. 200 lb. per sq. in., the cracks were still visible, but not measurable. However, at a further

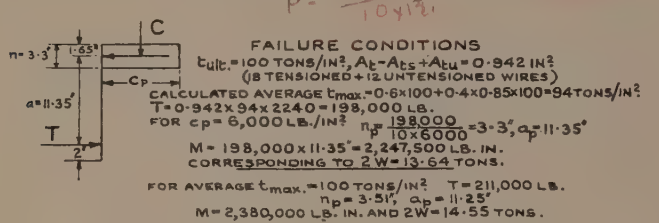

Fig. 8.—Stresses at failure. Test beam T1

reduction to approx. $2W = 3.6$ tons (point 2g), corresponding to a zero stress in the concrete bottom fibre, the deflection suddenly diminished, and all cracks closed up entirely. At the third loading, the load was gradually increased to $2W = 9$ tons (point 3e) and this was maintained for sixteen minutes, when nine cracks were detected with a maximum width of 0.3 mm. The load was reduced to $2W = 8$ tons (point 3g) and again increased to $2W = 9$ tons (point 3h). On reducing the load, the cracks closed up entirely as before. It must be pointed out that the nominal concrete tensile stress at this loading of $2W = 9$ tons amounted to 1,580 lb. per sq. in. and the bending moment at this loading was approx. 68 per cent. of the test maximum expected ($2W = 13.64$ tons). The fourth and fifth loadings, not plotted in Fig. 7, were also carried out to a loading of $2W = 9$ tons. While nine cracks were detected at the third loading at $2W = 9$ tons, at further loadings, the number increased up to twenty-one at the fourth loading and to forty-one at the fifth loading, but the maximum width of 0.3 mm. was not exceeded in any case.

It is interesting to investigate the various load deflection curves in Fig. 7, particularly loading 2, showing a hysteresis curve for a complete cycle of loading (2a to 2e). The deflections at $2W = 2$, 4 and 6 tons on the decrease of this loading (points 2h, 2f and 2i, respectively) are much larger than those at corresponding increases of later loadings. Not only at the third loading (points 3a, 3b and 3c) but even at the sixth loading (points 6a, 6b and 6c) these deflections are considerably less than the corresponding values of the descending second loading. These differences are remarkable, and indicate the importance of rest for recovery. Even deflections 3a and 3b of the third loading, which immediately followed the second loading, are larger than those of the sixth loading (points 6a and 6b respectively) carried out three days later. In the case of beam T1 the general course of the deflection curves after cracking corresponds particularly well with the slope of the theoretical deflection line computed for a cracked section and $m = 15$, plotted in Fig. 7. Also the influence of creep can be clearly seen from this figure, at the points where the loading was maintained for a certain time (points 2c—2d and 3e—3f).

As stated, one of the cracks already extended up to 7 in. from the compression fibre when the load $2W = 8$ tons was reached the first time. If a triangular stress distribution in the compression zone is assumed, the mean tensile stress of the tensioned and untensioned wires at their combined centre of gravity in a cracked section amounted to 153,000 lb. per sq. in., which would be 69 per cent. of the maximum strength. It is remarkable that on removal of loading the cracks closed up completely in spite of existence of non-tensioned wires, for which permanent extension might have been expected.

The sixth loading was carried on up to $2W = 12$ tons, when sudden failure occurred, the concrete casing being separated from the prestressed beam, and the latter failed similarly to test beam P1, due to compression followed by shear. At this loading 89 per cent. of the calculated maximum moment was reached. At failure, which occasioned separation between casing and prestressed beam, the latter now alone became obviously too weak, since compression failure had occurred with P1 already at a loading $2W = 8.3$ tons. It was noticed that the roughening of the very smooth surface of the prestressed component was insufficient, and it was, therefore, decided to repeat the test.

Test on partially prestressed composite beam T2

The test on T1 had already proved without doubt the suitability of a partially prestressed beam with tensioned and untensioned wires, since it was seen that after repeated loading up to $2W = 9$ tons (when the calculated mean stress in the tensioned and untensioned wires in a crack amounted to 153,000 lb. per

Fig. 9.—Testing composite partially prestressed test beam T2. December 1949 (measurement of width of cracks)

sq. in. and already forty-one cracks had developed) the cracks closed up entirely and became invisible. However, it was desirable to prove that the full tensile resistance can be reached at failure, and it was thus necessary to repeat the test. As seen in Fig. 2 for T2 a prestressed beam was chosen which had two holes in the web, to enable links to be threaded through these holes. Moreover, more attention was paid to a thorough roughening of the surface. This was particularly important, since not sufficient assistance was to be expected by the two links threaded through the web. It must be emphasised that the whole shape

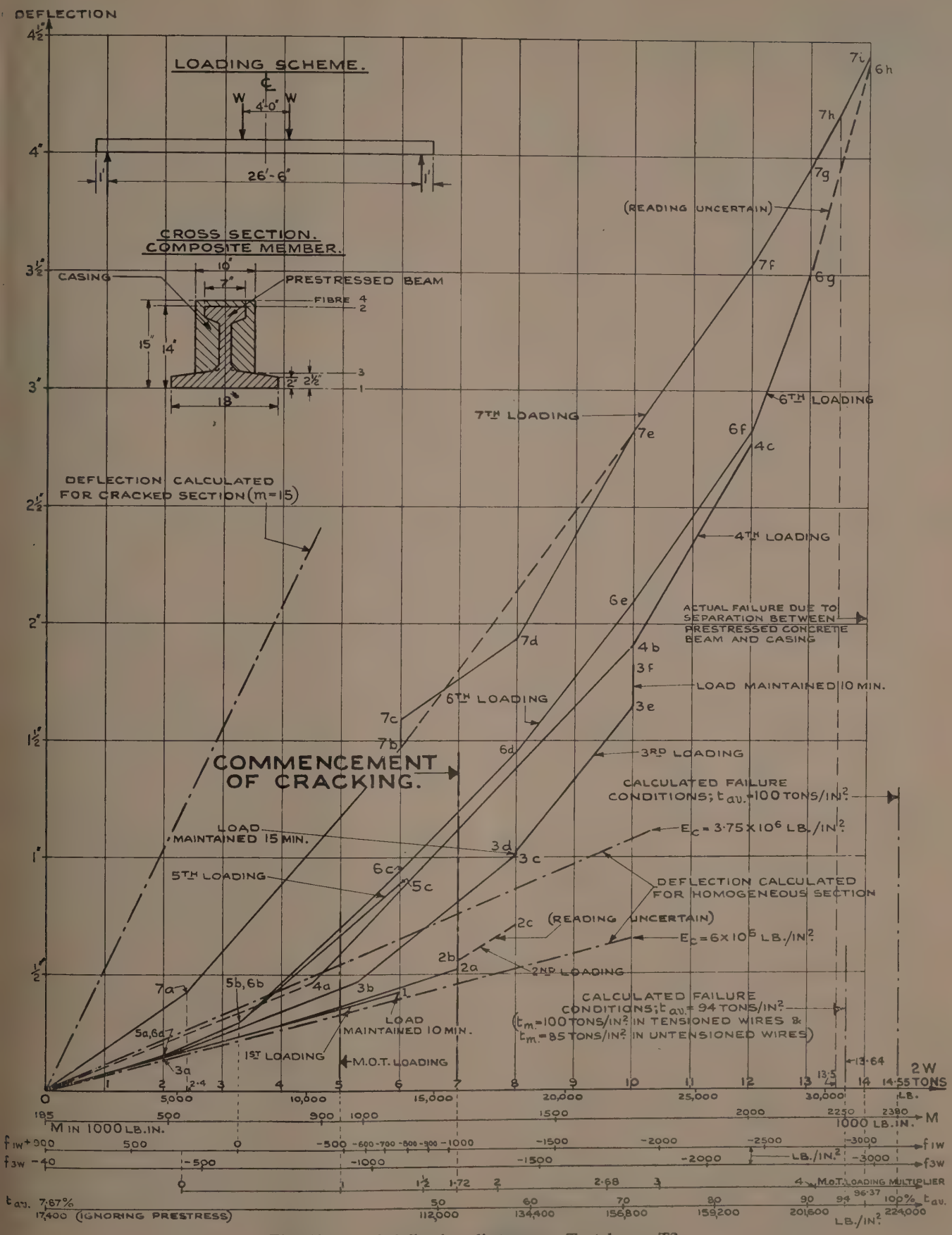

Fig. 10.— ad deflection diagrams. Test beam T2

of the test beam, T, is not suitable for producing a member corresponding to a composite slab in which the *in situ* concrete is securely wedged between adjacent pre-cast beams. However, the shape selected seems to be the best solution if only one pre-cast beam is used.

The second test beam, T2, was cast in March 1949, and tested in December 1949. Fig. 9 is a photograph of the test, showing measurements of the width of cracks. Fig. 10 gives the load-deflection curves and the calculated stresses in a homogeneous and cracked

section similar to Figs. 4 and 7. These stresses are plotted, together with measured maximum widths of cracks, in Fig. 11, whereas Fig. 12 shows the stress distribution in the section for various loadings (actual and nominal concrete stresses in a homogeneous section). The type of support during hardening influences the magnitude of the concrete stresses in the beam and consequently the creep losses, certain stresses due to creep and differential shrinkage are transmitted to the casing concrete, as shown in Fig. 12 (a) at load $2W = 0$. The quality of the concrete casing was, as with beam T1, the same as that of the prestressed beam. With regard to the stress diagrams of Fig. 12 (b and c), it is interesting to note that the increase of the E modulus with age, the beam T2 being nine months old at testing, whereas beam T1 was but one month in age. In Fig. 10 scales are plotted for the concrete tensile stresses only (in the fibres 1 and 3) and for the average steel stress in the tensioned and untensioned wires, based on failure conditions. A scale has been added showing the multiplier for the M.o.T. loading, corresponding to unity at a concrete tensile stress of somewhat above 500 lb. per sq. in. This is based on the assumption that higher loads than M.o.T. loading will not occur, ensuring freedom from cracks even under fatigue. However, with public roads, a further occasional increase of live load is possible. The Railway Executive has, therefore,

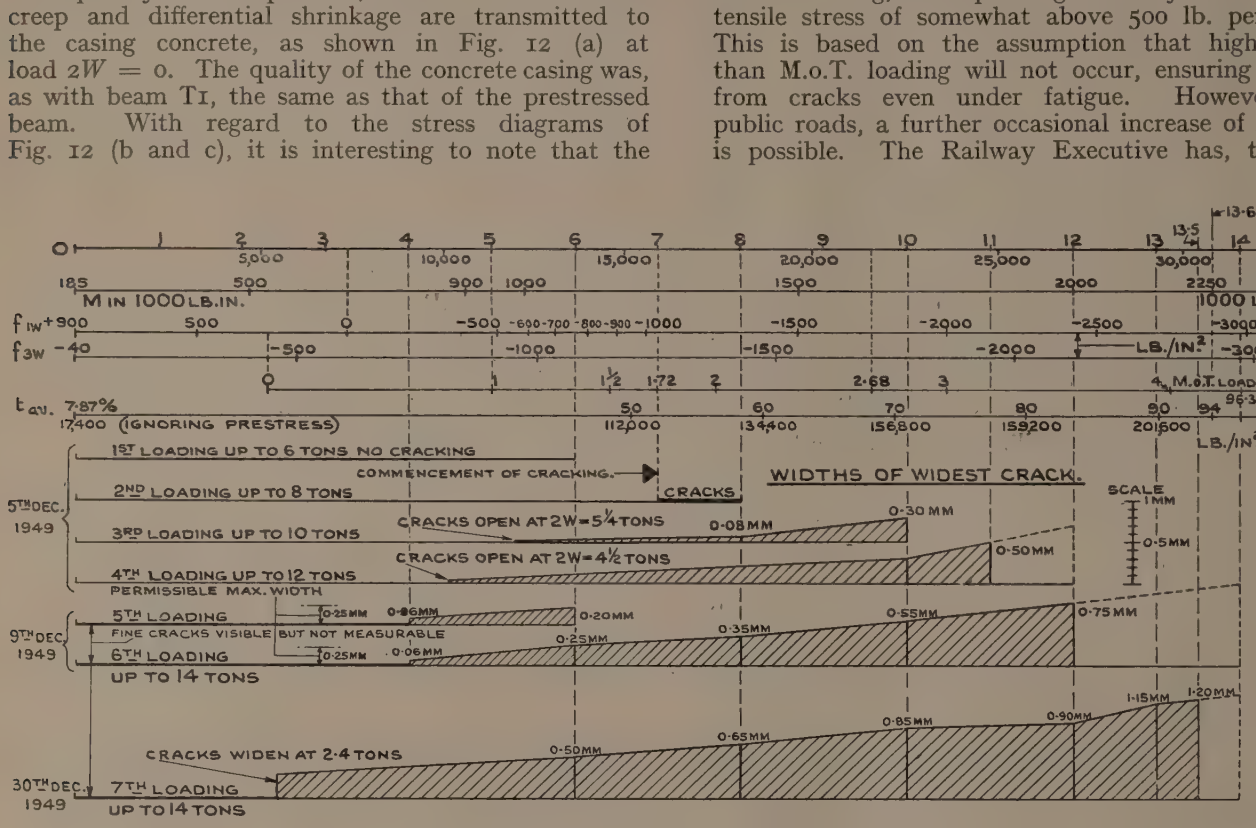

Fig. 11.—Maximum width of cracks. Test beam T2

tensile stresses in fibre 3 are greater than those in fibre 1 at commencement of cracking. Similar results were already indicated in Fig. 7 for test beam T1, and it was decided to pay particular attention to the commencement and development of any cracking. With very close observation, it has been found that the cracks become visible first at the bottom fibre of the beam, at a stress of approx. 1,000 lb. per sq. in., at which stage the stress in fibre 3 in the additional casing amounts to 1,250 lb. per sq. in. From these observations it might be assumed that cracks always develop from the bottom fibre, and that at the level where the non-prestressed casing is bonded to the prestressed beam, in spite of greater extensions, no cracks develop. However, Professor Evans has pointed out at the Unwin lecture (5) that very fine cracks occur at this level, as he was able to notice with a high-powered microscope, the width of the cracks being approx. 1/15,000 in. (0.0016 mm.). The observation made at test beams T1 and T2 are in agreement with Professor Evans' results, since no cracks were noticed with unaided eye, capable of detecting cracks of a width of approx. 0.02 mm. While a number of such very fine cracks may have developed, no noticeable reduction in the resistance moment occurred, since no change in slope of the deflection curve was noticed at that stage.

By investigating Fig. 10 it can be seen that, at the first loadings, smaller deflections occurred than with test beam T1 according to Fig. 7, which indicates the

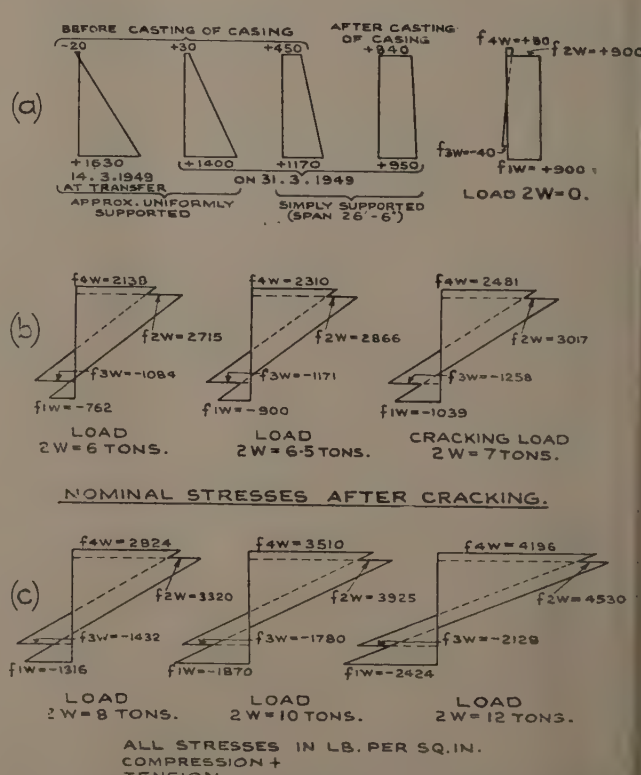

Fig. 12.—Concrete stresses at various loadings. Test beam T2

decided that such bridges should be designed for twice the M.o.T. load, with concrete tensile stresses at the bottom fibre of the prestressed concrete not exceeding 1,000 lb. per sq. in., as is indicated in a diagram corresponding to Fig. 10 (6). Such a design would ensure freedom from cracks for a very rare overloading slightly less than twice the M.o.T. load.

In this case the entire tensile resistance of both tensioned and untensioned wire has been taken into account, in order to have a measure of the maximum possible loading. Thus the maximum load is indicated in the graph, Fig. 10, as $2W = 14.55$ tons, as indicated in Fig. 8 for $t_m = 100$ tons per sq. in., whereas a load a_p increased to 11.54 in., resulting in a calculated maximum bending moment of 2,435,000 lb. in. and a maximum load $2W = 14.9$ tons instead of 14.55 tons, as indicated in Fig. 10.

From Fig. 10 it is seen that the second composite partially prestressed beam behaved, in principle, similarly to the first, but a much higher loading was reached. Cracking occurred at the second application of the T1 cracking load, *viz.* $2W = 7$ tons, though no cracks had been noticed at a load of $2W = 6.5$ tons. In Fig. 12 (b and c), the stresses in a homogeneous section are shown for each loading, e.g. at 6.5 tons, the calculated tensile stress at the bottom fibre of

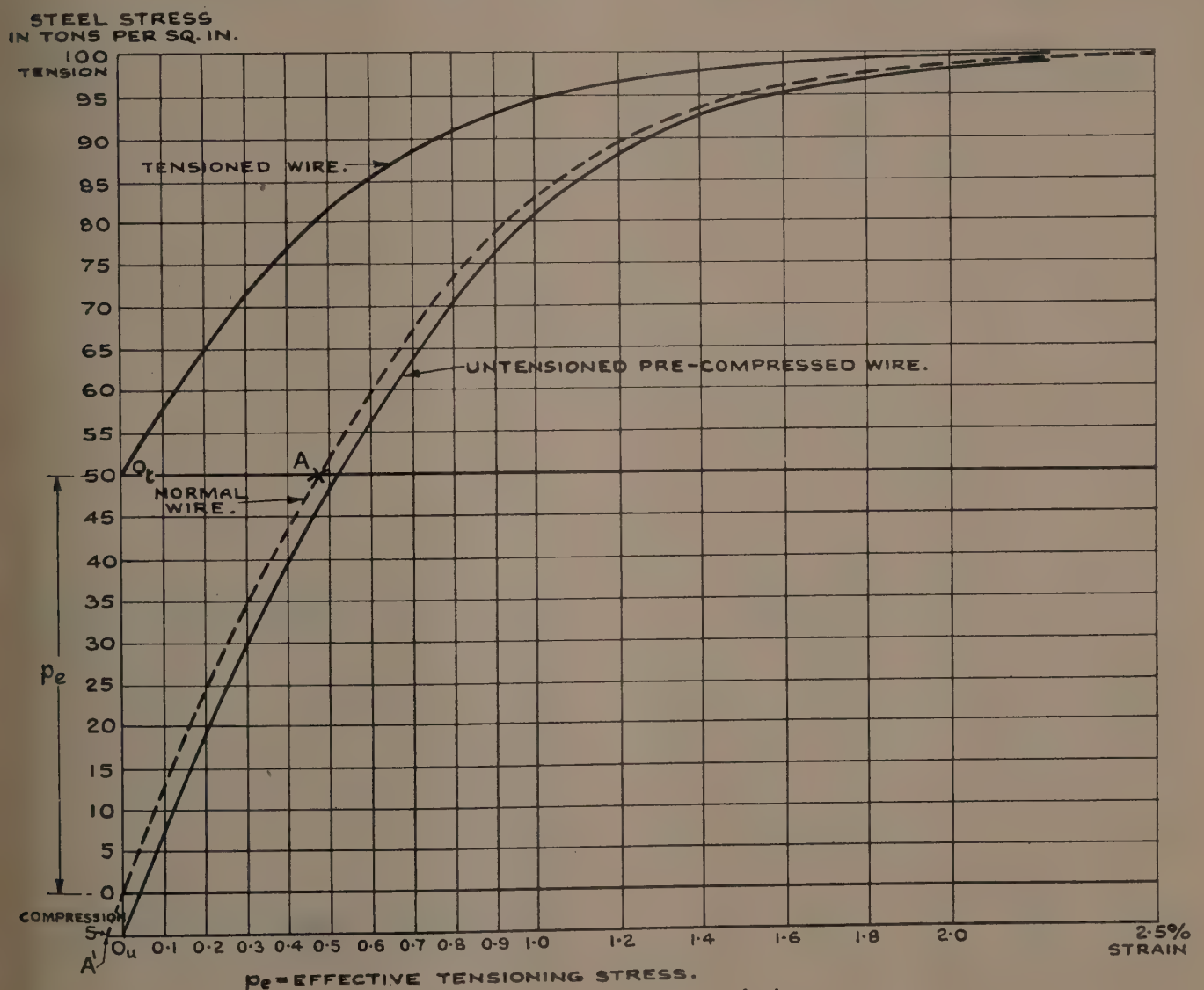

Fig. 13.—Stress strain diagram of wire

of $2W = 13.65$ tons has been considered for test beam T1 in Fig. 7, based on a stress of 85 per cent. of the ultimate strength in the untensioned wire. However, with good bond, the entire tensile strength should be reached also in the untensioned wires, as was the case with beam 4 in the tests carried out at the Brixton School of Building (3).

It may be mentioned that the calculation in Fig. 8 is based on a prism strength $c_p = 6{,}000$ lb. per sq. in. However, the concrete of T2 at the age of testing had a higher prism strength (approx. 7,200 lb. per sq. in. obtained on prism 4 in. by 4 in. by 16 in.). In such a case the value n_p would be reduced to 2.93 in. and the precast concrete was $f_1 = 900$, and that of the *in situ* concrete $f_3 = 1{,}171$ lb. per sq. in., and at cracking they amounted to 1,039 and 1,258 lb. per sq. in. respectively. No crack had been noticeable earlier, although the concrete tensile stress in the fibre 3 was in excess of values at which cracks normally develop. The third loading was carried up to a load of $2W = 10$ tons, corresponding to a nominal tensile stress of 1,870 lb. per sq. in. At the fourth loading, which followed immediately upon the third, the cracks remained closed, in spite of the previous high stresses until a load, corresponding to a tensile stress of approx. 320 lb. per sq. in. was reached. In Fig. 10, the load deflection curve is dotted between

o and $2W = 4.5$ tons, since no intermediate measurement was taken. After the load had been increased to $2W = 12$ tons (at which the first beam had failed due to separation of casing from precast concrete), closing of the cracks occurred only after the tensile stress had reversed into compressive stress, as seen in Fig. 11. At this loading of $2W = 12$ tons, the nominal tensile stress amounted to 2,424 lb. per sq. in.

So far as the ultimate resistance is concerned, reference is made to Fig. 13, in which the stress-strain diagrams of the tensioned and untensioned wires are shown. If material at a loading before the compressive stresses at the bottom fibres reverse into tensile stresses.

When the seventh loading was applied up to a loading $2W = 14$ tons, the adhesion was destroyed at one weak point in the connection between the prestressed and "added" concrete; a similar failure occurred to the first beam at a stage when the wires were still fully intact without permanent strain.

This fact was ascertained by special strain measurements shown in Fig. 14. Elongations of two tensioned and two untensioned wires were measured by means

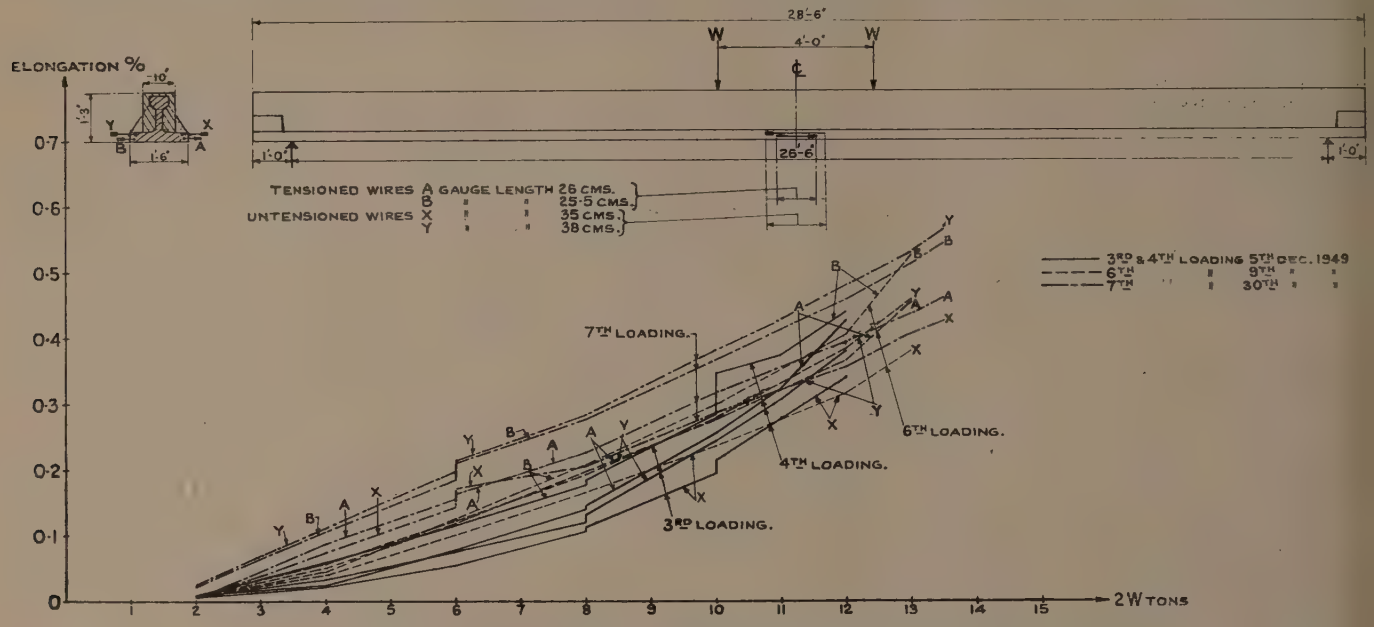

Fig. 14.—Measured strains for tensioned and untensioned wires. Test beam T2

the effective prestress in the tensioned steel is taken as 50 tons per sq. in. (point A in the curve), and the pre-compression in the untensioned wire is 5 tons per sq. in. (point A′ in the curve), the origins of these two stresses represent the zero stresses for wires in the composite beam. Two curves are plotted with the origins O_t corresponding to A and O_u corresponding to A′. Considering the same extension in the tensioned and untensioned wires at failure (e.g. 2.5 per cent.), it is seen that there will be very little difference in stress between the tensioned and untensioned wires. From Fig. 10 it is seen that the sixth loading was extended to a load of $2W = 14$ tons, at which the nominal concrete tensile stress amounted to 2,978 lb. per sq. in. and the expected maximum load for T1, *viz.* $2W = 13.64$ tons, was thus exceeded.

When the load of $2W = 14$ tons was reached at the sixth loading, the beam unfortunately slid down from the supports, as indicated in Fig. 10, whereby it was slightly damaged. A temporary deflection of $\frac{1}{8}$ in. remained as measured by a reduction of the camber from $\frac{3}{8}$ in. to $\frac{1}{4}$ in. However, after a rest of three weeks, this deflection recovered and the entire camber of $\frac{3}{8}$ in. was regained. This phenomenon is particularly notable, since previously a load had been applied corresponding to as much as 96.4 per cent. of the maximum loading which could be considered (tensioned and untensioned wires reaching the ultimate strength).

At this loading of 96.4 per cent. the deflection amounted to more than 4 in. and the width of the greatest cracks to 0.75 mm. At the sixth and last loading the deflection curve was much steeper, but this may still be considered as one relating to a homogeneous

Fig. 15.—Part view of test beam T2, showing the gauges for strain measurement at a load 2W = 12 tons

of dial gauges over lengths of between 10 and 15 in. as indicated in the Fig. 14, and seen in the photograph Fig. 15, showing also some of the cracks. In Fig. 16 some of these measurements are plotted as stresses in relation to the loading.

The strain measurements plotted in Fig. 14 were made over a length of approximately 25 cm. for the tensioned wires and 35 cm. for the untensioned wires, and two measurements were taken for each type of wire. There was hardly any difference in the average strain between the tensioned and untensioned wires. At one side of the test beam the strain in the tensioned

wire was higher and at the other side that in the untensioned wire, as seen from Fig. 14. However, for the sake of simplification, the same strains for tensioned and untensioned have been plotted in Fig. 16a for two different loadings. While there is no difference in strain in the two types of wires according to Fig. 16a, the stresses differ considerably, as is seen in Fig. 16b. Two strain-load curves have been plotted in diagram Fig. 16a: one relates to the fourth loading after the previous one had reached around two-thirds of the ultimate load. In this case it was noticed that the increase in strain approximated the sum of widths of cracks measured within the gauge lengths as shown

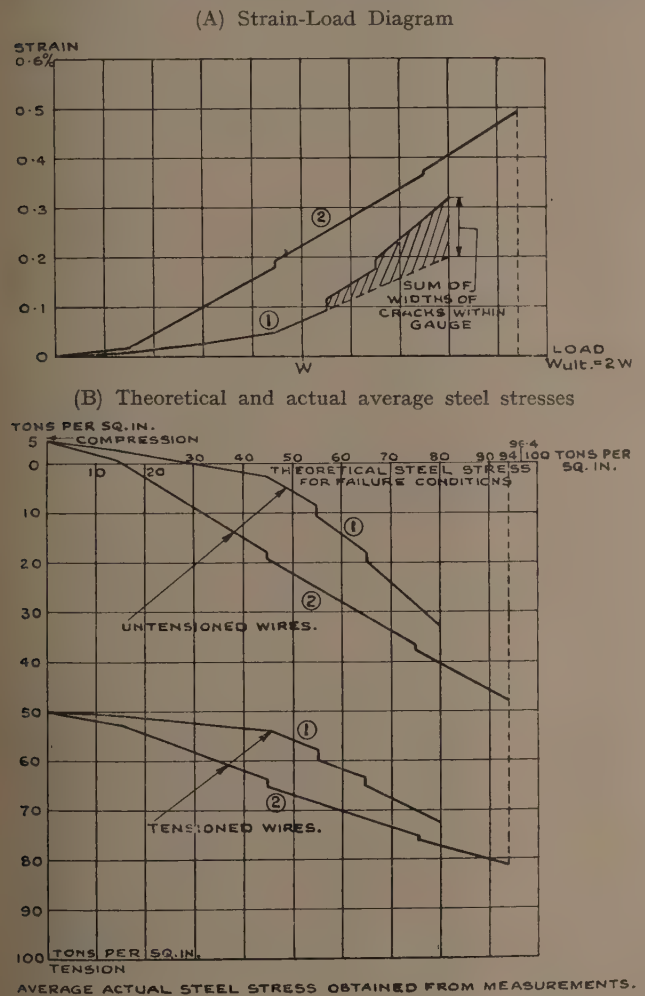

Fig. 16.—Strains and stresses in tensioned and untensioned wires, based on strain measurements in comparison with calculated steel stresses. Test beam T2

on the graph. This would prove that the bond is not, or only slightly, destroyed adjacent to the cracks. The second strain-loading curve corresponds to the seventh loading at a state after the structure has already been loaded to 96.4 per cent. of the expected maximum. This average stress of 96.4 tons per sq. in. corresponds to a stress of 91 per cent. of the ultimate tensile strength in the untensioned steel, assuming that the tensioned steel, which amounted to 60 per cent. of the total steel area, had already reached the ultimate strength. From Fig. 16b it is seen that a theoretical stress of 90 tons per sq. in. corresponds to an actual average steel stress of approximately 46 tons per sq. in. in the untensioned wire and of 80 tons per sq. in. in the tensioned steel. It must be noted that these are average stresses between those within the cracks and along those parts between the cracks where the steel is bonded to the concrete. The actual stresses within the cracks are not known, and such an investigation would be one of many interesting subjects for further research.

Conclusions

1. Commencement of cracking. The tests on all three beams have confirmed that commencement of cracking cannot be noticed before the loading has reached such a stage that a concrete tensile stress of approx. 1,000 lb. per sq. in. is obtained on the basis of a straight line stress distribution for a homogeneous material. At that stage the cracks are just visible, but not measurable, employing a Brinell microscope as seen in Fig. 9. With a high-powered microscope or with modern methods, such as ultrasonic methods, it might be possible to observe development of cracking at an earlier stage. However, if numerous fine, but invisible, cracks developed at an earlier stage, they would be of no essential influence on the carrying capacity, since an appreciable change in slope of the deflection line does not occur before distinct cracks have developed. Nor would such invisible cracks be of any harm from the point of view of corrosion, since even very fine visible cracks are harmless. Thus the development of any invisible cracks can be ignored, and it is sufficient to limit the consideration to visible cracks, commencing at a bending tensile stress of approx. 1,000 lb. per sq. in. for high grade concrete.

2. Cracking in a composite beam. In the test beams T1 and T2, in which there was a pre-compression in the bottom flange but no pre-stress in the *in situ* portion just above, commencement of visible cracks was noticed to occur at the bottom fibre, the cracks developing upwards. As Professor R. H. Evans has found, very fine cracks of a width of 1/15,000 in. (0.0016 mm.) commence previously at the bottom fibre of the *in situ* concrete. However, as pointed out under (1), such invisible fine cracks are of no harm and can be ignored. Thus, from the test results, it can be concluded that no visible cracks develop at that stage in this fibre where the non-prestressed additional concrete is adequately bonded to the prestressed concrete, in spite of the high theoretical concrete tensile stresses, exceeding the modulus of rupture. Apparently, the development of visible cracks is restrained by the bond between prestressed and additional concrete and confined to the development of very fine harmless cracks.

3. Closing of cracks. All three tests have confirmed that cracks close up entirely on reduction of load, as soon as the tension reverses into compression, the prestress acting again along the entire length of the tensile zone, provided that the loading has not previously reached a stage of approx. 80 per cent. of the maximum loading, when permanent deformation occurs and the cracks remain visible though not measurable, as shown with test beam T2 (Fig. 11).

4. Cracks becoming invisible. Cracks appear to close even at a higher load than that at which the tension reverses into compression, provided that the previous maximum loading has only slightly exceeded the load at which the commencement of cracking becomes noticeable. This phenomenon depends on the depth of the cracks. As long as they are rather shallow, this earlier closing takes place; but this closing may not be complete and it must, therefore, be assumed that at this stage the cracks become only invisible.

5. Co-operation of tensioned and untensioned wires. Both tests T1 and T2 have proved that there is hardly any difference with regard to cracking and resilience in the behaviour of beams in which all wires are tensioned and those in which a part of the wires is tensioned and the other part untensioned, provided only that an effective prestress in ensured. Even with a previous loading up to 70 per cent. of the maximum load, the cracks closed up to become invisible on removal of the load, and only after previous higher loading some of the cracks remained just visible (but not measurable).

6. Recovery. A remarkable recovery was obtained with beam T2. Even after the loading had already reached 96 per cent. of the maximum, a relatively small apparent permanent deflection of $\frac{1}{8}$ in. remained, but this disappeared after a rest of three weeks. This capability of recovery should be of great advantage in the case of repeated loading. It is usual to base the fatigue strength on tests carried out in quick repetition, say 500 per minute, so that one or two million loadings are reached in a test extending over less than three days and nights, whereas there will be almost always a time for recovery in the structure, and the time of loading, during which two million repetitions of the maximum load occur, will extend over a much longer period, at least over several years. Thus, it can be said that laboratory fatigue tests give results much more severe than the actual loading conditions.

7. The actual stress in the wires. The strain measurements of the tensioned and untensioned wires over a considerable gauge length have shown that measured strain is much less than that expected. In individual loadings it was ascertained that the rapid increase in strain approximated the sum of measured cracks within the gauge length, which proves that the bond was destroyed in the immediate vicinity of the cracks only. Thus, the main extension occurs only in the cracks themselves, and a strain measurement through a crack limited to a short length would, therefore, be entirely different from the mean strain measured over the cracks for a considerable length. The mean steel stresses obtained on the basis of the strain measurements are considerably smaller than the stresses calculated for a cracked section.

8. The ultimate resistance. The test beam P1 showed a remarkably good agreement between calculated and actual failure load, the former based on an estimated prism strength. It can only be assumed that with beams of relatively shallow compression flanges, the prism strength may well represent the mean maximum compressive stress. However, it would be important to obtain more test results with regard to the failure of over-reinforced beams like P1. With under-reinforced beams T1 and T2, the ultimate load was not reached, in view of destruction of the bond between prestressed concrete and casing. However, from the failure of beam T2, and other beams, it can be concluded that with under-reinforced beams, even if the tensile reinforcement consists of tensioned and untensioned wires, the maximum tensile resistance of the steel, calculated for the specified tensile strength, should be reached, or even exceeded. After a loading up to 96.4 per cent. of the maximum, no sign whatsoever of any approach to failure of the steel was noticed, and hardly any permanent elongation remained after the failure load was removed.

9. Advantages of constructions according to test beams T. When considering the properties of these beams, with regard to cracking, resilience and recovery, it would appear of technical and economical advantage to make use of them in the design of building construction. By reducing the prestressed concrete component to the minimum necessary, it is possible to reduce the cost and to take advantage of the tensile resistance of the non-prestressed additional concrete to a very high extent (in fact, in excess of its tensile strength) without the occurrence of visible cracking.

10. Future research. The author has suggested further tests in conclusion 13 of the paper (3) which may be repeated in the following, the purpose being to show :—

(a) effect of combined shear and bending ;
(b) the effect of variations in grouting post-tensioned wires ;
(c) the stress distribution at failure in over-reinforced beams ; and
(d) the influence of fatigue on cracking.

This last question would require especially wide investigations to obtain the fatigue resistance of prestressed concrete under different conditions, (i) where the maximum load occurs within certain intervals, e.g. for road bridges, and (ii) where the maximum load applies in quick repetitions as with railway bridges. In both cases it would be interesting to study also the behaviour of prestressed concrete under fatigue conditions at various stages, after cracks have already once developed.

A further study of importance would be to investigate the co-operation of prestressed and additional unprestressed concrete in composite beams under fatigue conditions, with particular attention to the development of cracking.

In view of the great number of problems still to be solved with prestressed concrete, it would be of great advantage if the studies of the various research workers could be combined and special arrangements made whereby they will be able to exchange their views. This would have the effect of avoiding duplication.

Acknowledgment

The author would like to express his thanks to The Railway Executive, and the Civil Engineer of the Eastern Region, Mr. J. I. Campbell, M.I.C.E., for permission to use the designs, diagrams and particulars of tests made in the New Works Office, King's Cross. He is particularly indebted to Mr. R. E. Sadler, A.M.I.C.E., Assistant Engineer (New Works), for his kind support. The prestressed beams were manufactured by Dow-Mac, Ltd., at Tallington, Lincs.

References

(1) Report on Prestressed Reinforced Concrete Sleepers Tested as Simply Supported Beams by L.N.E.R. Concrete and Constructional Engineer, April and May, 1947.

(2) Fully and Partly Prestressed Reinforced Concrete, by P. W. Abeles, Journal of the American Concrete Institute, January, 1945. Discussion and Closure, Supplement, November 1945.

(3) Small Scale Demonstration Tests on Prestressed Concrete Beams, by P. W. Abeles and C. H. Hockley, The Structural Engineer, June, 1950.

(4) Principles and Practice of Prestressed Concrete, by P. W. Abeles. Crosby Lockwood & Son, London, 1949. E. F. Ungar Publ. Ltd., New York, 1949.

(5) Research and Development in Prestressing, Unwin Lecture at the Inst. of Civil Engineers, by Prof. R. H. Evans, December 1950.

(6) Development in the use of Prestressed Concrete on British Railways, by A. Dean, Railway Division, Inst. of Civil Engineers to be read on 8th May, 1951.

Institution Notices and Proceedings

ORDINARY GENERAL MEETING

An Ordinary General Meeting of the Institution of Structural Engineers was held at 11, Upper Belgrave Street, London, S.W.1, on Thursday, March 29th, 1951, at 5.55 p.m., Mr. J. E. Swindlehurst, O.B.E., M.A., M.I.C.E., M.I.Struct.E. (President), in the Chair.

The Minutes of the Ordinary General Meeting held on January 25th, 1951, as published in the Journal in March, were taken as read, were confirmed and signed.

The following members were elected in accordance with the Bye-Laws. Will members kindly note that the elections, as tabulated below, should be referred to when consulting the Year Book for evidence of membership?

Students

ALCOCK, Bruce Lawrence, 5, Netley Road, Perridgevale, Port Elizabeth, South Africa.

BAILEY, John, 388, Warrington Road, Culcheth, Warrington, Lancs.

CHAPMAN, William Edwin, Greaves Cottage, Edale Road, Hope, nr. Sheffield.

COLEMAN, Peter Stanley, 16, Oakwood Avenue, Hutton, Nr. Brentwood, Essex.

DRAGE, John Frederick, 29, Meadow Way, Wembley, Middlesex.

DUERDEN, Thomas Brian, Chigwell Golf Club, Chigwell, Essex.

DUNN, Michael Roger, 95, Beechdale Road, Brixton Hill, S.W.2.

GENTRY, Leslie John, 31, Meadfoot Road, Streatham Vale, S.W.16.

GRABHAM, Frederick Robert, 14, Winterwell Road, Brixton, S.W.2.

JARVIS, Anthony Peter, 43, The Birches, Winchmore Hill, N.21.

JENKINSON, Anthony Richard, 52, North Street, Egham, Surrey.

KYCIA, Serafin, 32, The Drive, Walthamstow, E.17.

LAWSON, Desmond Kane, 20, Mount Charles, Belfast, Northern Ireland.

MCNALLY, Eneas, 118, White Moss Road, Hr. Blackley, Manchester, 9.

MANDY, Gordon William, 151, Cutenhoe Road, Luton, Beds.

NEWTON, Christopher John Broughton, Manningford, Wey Manor Road, West Weybridge, Surrey.

POWELL, Charles William, 50, Highfield Avenue, Scunthorpe, Lincs.

ROWE, Edmund Frederick, 66, Brixham Crescent, Ruislip Manor, Middlesex.

STANYON, Philip George, 166, College Street, Long Eaton, Notts.

WARRINER, Harold Maurice, 188, Derby Road, Salford 6, Lancs.

Graduates

BENBOW, Roland, 159, Kenley Road, Merton Park, S.W.19.

CHARD, Roy Creswell, B.Sc. (Civil) Wales, 1, Sandown Road, Port Talbot, Glams.

DEW, Raymond William, 50, Woodfield Drive, East Barnet, Herts.

DIVERALL, William, B.Sc. Hons. (Civil) Glasgow, c/o Caledonian Christian Club, 9-11, Endsleigh Gardens, W.C.1.

GILLOOLY, Lawrence, B.Sc. (Civil) Rand, c/o Structural Eng. P.W.D., Central Government Offices, Pretoria, South Africa.

GOSAVI, Prabhakar Keshao, B.Sc. Bombay, Damodar Nivas, Panchavats-Nasik, Bombay State, India.

LEWIS, Michael Robin, B.Sc. (Eng.) Rand, 69, Gilling Court, Belsize Grove, N.W.3.

LOUW, Timotheus, B.Sc. (Civil) Rand, 507, 24th Avenue, Villieria, Pretoria, South Africa.

MACDONALD, Alexander Thomas, 3, Denholm Drive, Giffnock, Glasgow.

MONCRIEFF, Malcolm Lawrence Anderson, B.Sc. (Eng.) London, 8, Gombards, St. Albans, Herts.

RUSHTON, Frank, c/o Sierra Leone Development Co., Ltd., P.O. Box 7, Freetown, Sierra Leone.

SADLER, William, 1, Cheltenham Avenue, Thornaby-on-Tees, Stockton-on-Tees, Co. Durham.

SAGAR, Naresh Chander, Sagar Mansions, Gopal Road, Ludhiana, Punjab, India.

SHROTRI, Ganesh Shankar, B.E. (Civil) Bombay, 115, Narayan Peth, Poona 2, India.

THEAN LIP THONG, B.Sc. (Eng.) London, 33, West Cromwell Road, S.W.5.

WELLS, Kenneth James, B.Sc. (Civil) Birmingham, 3, Primrose Hill Road, N.W.3.

WHITTLE, Geoffrey Farrow, B.Sc. (Tech.) Manchester, 17, Carlton Road, Derby.

Associate-Member

CHATTERJEE, Ranjit Mohan, B.E. (Civil) Calcutta, A.M.I.E. (India), c/o Chief Engineer, Calcutta Port Commissioners, 15, Strand Road, Calcutta, India.

Associate

BIRKS, Herbert, 3, Laurel Road, Saltburn-by-Sea, Yorks.

Transfers

Students to Graduates

ALLEN, John Derek, 11, Crwys Road, Roath, Cardiff.

HARVEY, Wreyford Frank Petrie, 7, Colbourne Road, Hove 2, Sussex.

HILL, Peter Henry, 8, Oakbury Road, Fulham, S.W.6.

LIGHTBAND, Michael Stuart, B.Sc. (Eng.) London, 2A, Palfrey Close, St. Albans.

O'CONNELL, William Noel, B.Eng., N.U.I., c/o Leslie Turner & Partners, 5A, The Pavement, Grove Road, Sutton, Surrey.

SCOTT, John Rodger, 15, Tweed Crescent, Riddrie, Glasgow, E.1.

Graduate to Associate-Member

KURZKE, Werner Walter Oskar, 9A, Cuthbert Buildings, Maitland Street, Bloemfontein, O.F.S., South Africa.

Associate-Members to Members

GUINSBOURG, David, c/o Head Office, Palestine Electric Corp., P.O. Box 57, Haifa, Israel.

KASS, Boris Morduch, P.O. Box 205, Salisbury, Southern Rhodesia.

OCKLESTON, Allan Joshua, B.E., Ph.D. (Civil) N.Z., A.M.I.C.E., Dept. of Civil Engineering, University of Witwatersrand, Milner Park, Johannesburg, South Africa.

Members to Retired Members

CLAXTON, Harold, M.C., A.M.I.Mech.E., "Corners," West Chiltington Common, Nr. Pulborough, Sussex.

MILLAR, John, F.R.I.C.S., 27, Park Hill, S.W.4.
SEAH ECK JIM, 4, Keng Chin Road, Singapore.

RE-ADMISSION

Graduate

WICKRAMASINGHE, Alastair Claude, E.E.'s Bungalow, Dimbula, Talawakelle, Ceylon.

OBITUARY

The Council regret to announce the deaths of Edward Charles BUCK, M.I.C.E., M.I.Mech.E., of Port of Spain, Trinidad, who was the Institution's Representative in the British West Indies from 1940 to 1949, and Chotolal Chunilal DALAL, B.E., M.I.C.E., of Hyderabad (Members).

RESIGNATION

Notification was given that the Council had accepted with regret the resignation of Leslie Harper BAKES, L.R.I.B.A. (Member).

EXAMINATIONS, JANUARY, 1951

The Examinations were held in January, 1951, at the usual centres in Great Britain, and at the following overseas centres: Baghdad, Beirut, Bombay, Cairo, Cape Town, Christchurch (N.Z.), Colombo, Durban, Georgetown, Hong Kong, Johannesburg, Lucknow, Lusaka, Madras, Port Elizabeth, Salisbury (S. Rhodesia), Singapore, and Wellington (N.Z.).

Seventy-four candidates took the Graduateship Examination, 57 at Home centres and 17 overseas.

Two hundred and twenty-four candidates took the Associate-Membership Examination, 187 at Home centres and 37 overseas.

The total number of entries for both examinations was thus 298.

Of these, 53 passed the Graduateship Examination and 93 passed the Associate-Membership Examination.

The names of the successful candidates are :—

GRADUATESHIP EXAMINATION

BAKER, Alexander John.
BHARUCHA, Sorab Hormusji.
BLACKBURN, Keith Chalkley.
BROTTON, Arthur Reginald.
CLARK, Raymond Bostock.
COTTON, Stanley Arthur Charles.
COX, Allan Thomas Charles.
CREIGHTON, Leslie.
DZIEWULSKI, Jerzy.
EASTON, Edgar.
FROST, Archibald Douglas.
GARTHWAITE, Charles Henry.
GOULD, Noel Brian.
HAIGH, Jack.
HALLOWS, Brian Hector.
HARRIS, Clifford James.
HICKS, Edward George.
HOARE, John Hubert.
HOPKINS, Donald McTaggart.
HUSKINS, Leslie Roy.
JAMES, Arthur Neville.
JAMES, Dennis Frederick.
KACZKOWSKI, Tadeusz Adam.
LODGE, Dudley William.
MANUELL, Herbert Joel.
MARTIN, Geoffrey Walter.
MCCARTE, John.
MEREDITH, William James.
MORGAN, Ivor Charles.
NEWMAN, James Herbert.
NICHOLS, George William.
NOLLER, Gerald Rous Allen.
PITT, Peter Hubert.
PRIDDEY, Harry Walter.
RAJAPAKSE, Rajapske Kankananalage Chandradasa.
RAYNER, Patrick Anthony.
RENDALL, William Cutt.
ROUSSEAU, Francis McGill.
ROWLEY, John Charles.
SALEEM, Muhammad Akbar.
SHERWIN, Albert.
SNOWDEN, George William.
SRINIVASAN, Kilnagar Vedachala Iyer.
SUTTON, Derek Harold.
TATT, Robert James.
TAYLOR, James Edward.
TRANFIELD, Raymond.
TRANFIELD, William.
TRUMP, Vernon Henry.
TYLMAN, George Anthony.
WHITE, Allen Robert.
WIDDRINGTON, Thomas Anthony.
WOJCIECHOWSKI, Jan.

ASSOCIATE-MEMBERSHIP EXAMINATION

ALLEN, Arthur Charles.
ARMSTRONG, Ian Fraser.
BANNATYNE, William.
BARNAS, Stanislaw Adam.
BARRON, Thomas.
BARRY, Maurice Henry Alfred.
BATH, Charles Frederick.
BOGEN, Alexander David.
BOWMAN, Arnold.
BROCKLEHURST, Frank.
BUCHAN, Ian.
BYRNE, Bernard John Le Cesne.
CANTLAY, William Gordon.
CASS, Ronald Harry.
CAUNT, George.
CHILTON, William Gordon Eley.
CLARKE, Godfrey Baynes.
COLLINS, Brian Thomas.
COOPER, John Reginald.
COOPER, Stanley Maurice.
COX, Dennis.
DALE, Hilary Michael.
DEAKIN, Brian.
DE SILVA, Muttuwa Sarukalige M.
DOWSETT, Victor Frederick.
DODSWORTH, Foster.
DOWNES, Arthur Harvey.
EATON, Kenneth Vernon.
ELIA, Panicos.
ELLIS, Lewis Geoffrey.
ESAKOF, Edgar Arnold.
FARROW, George Albert.
FINN, Edward Vivian.
FORSBREY, Leonard William.
FORTEY, Joseph William.
GHOSH, Arabinda.
GROOME, Leonard William Edward.
HANNA, Milad Mikhail.
HILL, David Alexander Howard.
HOLLENBACH, Carel Arnold.
HUMPHREYS, Robert.
HUSBAND, Richard.
JALINK, John Bernard.
KING, Charles William.
LANE, Alan Sidney.
LANGFIELD, Ronald Alexander.

LEE, Gwoh Liang.
LEVY, Joseph.
LIEBENBERG, Algernon Charles.
LOACH, Douglas Clifford.
LOCK, Kenneth Alfred.
LOWSON, Barry Christian.
MOSELEY, Arthur Raymond.
MULLINS, Leslie Herbert MacLeod.
NEWTON, Peter Dennis.
OGDEN, John.
ORME, Donald Harrison.
OSGERBY, Colin.
PAYNE, Richard James.
PEACOCK, Mark Beauchamp.
PENNINGTON, Duncan John.
POMFRET, Donald.
POOLEY, Noel.
QUINION, David William.
RAMSEY, Frank Cyril.
ROBERTSON, Alexander Gordon.
RUSSELL, John.
SALE, Desmond James.
SARGENT, Albert Reginald.
SHORTER, Christopher Thomas.
SIMMS, George Alfred.
SMITH, Richard Anthony.
SPEED, Cecil Harry.
STAKE, David Walshaw.
STANDEVEN, Alec.
STEEL, Richard William Charles.
TAN TIONG BENG.
TAYLOR, Alan.
TAYLOR, Geoffrey Bell.
THOMPSON, Leslie.
THOMSON, Frederick Murray.
THORP, Cyril Francis.
TWELVETREES, Walter Sydney.
VAN HEERDEN, William Jacobus Stephanus.
VAUGHAN, Thomas Henry.
WARD, Walter Burton.
WARK, Donald Hulme.
WEBBER, Norman Bruton.
WHITLAM, Edwin Frederick.
WHYTE, Basil Percy.
WILSON, William Barry.
WOLSTENCROFT, Derek.
WYATT, Christopher Terrel.

PRIZE LIST—JANUARY, 1951, EXAMINATIONS

The Council have awarded the following prizes in connection with the Examinations held in January, 1951 :—

WALLACE PREMIUM (SENIOR)—(For the candidate who takes the whole of the Associate-Membership Examination, passes in all subjects, and obtains the highest marks in the paper, "Theory of Structures (Advanced).")

Joseph LEVY, of Glasgow.

HUSBAND PRIZE—(For the candidate who takes the whole of the Associate-Membership Examination, passes in all subjects, and obtains the highest marks in the paper, "Structural Engineering Design and Drawing.")

Charles William KING, of Richmond, Surrey.

ANDREWS PRIZE—(For the candidate who obtains the highest aggregate of marks in the Associate-Membership Examination, passing in all subjects.)

Robert HUMPHREYS, of Middlesbrough.

WALLACE PREMIUM (JUNIOR)—(For the most successful candidate in the Graduateship Examination, passing in all subjects.)

James Edward TAYLOR (Jnr.), of Birmingham.

EXAMINATIONS—JULY, 1951

The Examinations of the Institution will next be held at centres in the United Kingdom and overseas on July 17th and 18th (Graduateship), and on July 19th and 20th (Associate-Membership).

FORTHCOMING MEETINGS

The following meetings will be held at 11, Upper Belgrave Street, London, S.W.1 :—

Thursday, May 17th, 1951

Ordinary Meeting at 6 p.m., when Dr. K. Hajnal-Konyi (Member) will present a paper on "Comparative Tests on Various Types of Bars as Reinforcement of Concrete Beams."

Members wishing to bring guests to this meeting are requested to apply to the Secretary for tickets of admission.

Thursday, May 24th, 1951

Ordinary General Meeting for the election of members, 5.55 p.m., followed by the Annual General Meeting at 6 p.m.

Thursday, June 28th, 1951

Ordinary General Meeting for the election of members only.

BENEVOLENT FUND

The Annual General Meeting of the Voting Contributors to the Institution of Structural Engineers Benevolent Fund will be held at 11, Upper Belgrave Street, London, S.W.1, on Thursday, May 24th, 1951, at 6.30 p.m.

SESSIONAL AWARDS

The Council have made the following awards for the Session 1949-50 :—

INSTITUTION SILVER MEDAL

Professor J. F. BAKER, O.B.E. (Member), for a paper entitled "The Design of Steel Frames."

INSTITUTION BRONZE MEDAL

Mr. R. GRAY (Member), for a paper entitled "Ridged Steel Arch Roofs."

BRANCH PRIZES

The following awards have been made in respect of papers given before the Branches during the Session 1949-50 :—

LANCASHIRE AND CHESHIRE BRANCH PRIZE

Mr. R. GRAY (Member) for a paper on "Ridged Steel Arch Roofs."

NORTHERN COUNTIES' BRANCH PRIZE

Mr. D. W. COOPER (Associate-Member) for a paper on "Shell Concrete."

SCOTTISH BRANCH PRIZE

Mr. W. SHEARER SMITH (Associate-Member), for a paper on "Cold Formed Sections in Structural Practice with a proposed Design Specification."

YORKSHIRE BRANCH PRIZE

Mr. Stewart CHAMPION (Associate-Member), for a paper on "Some Reasons for Failure in Concrete Structures."

REPRESENTATION

The Council have made the following nominations of members to represent the Institution :—

ARCHITECTS' REGISTRATION COUNCIL OF THE UNITED KINGDOM AND ADMISSION COMMITTEE
Lt.-Col. R. F. GALBRAITH, M.C. (Vice-President).

ENGINEERING JOINT COUNCIL

The PRESIDENT (Mr. J. E. Swindlehurst, O.B.E.), and Mr. Leslie TURNER (Past-President).

INTERNATIONAL ASSOCIATION FOR BRIDGE AND STRUCTURAL ENGINEERING

The President has accepted office as a Vice-President of the 1952 Congress of the International Association for Bridge and Structural Engineering, and the Secretary has been authorised to act as Joint Honorary Treasurer of the Congress with the Secretary of the Institution of Civil Engineers.

LONDON GRADUATES' AND STUDENTS' SECTION

A visit has been arranged to the Murex Welding Works on Saturday, May 5th. The party will meet at the works at 2.30 p.m., and members may travel by trolley-bus from Manor House Tube Station, Green Line No. 715 from Marble Arch, or by train from Liverpool Street Station. The journey takes about an hour.

At the Annual General Meeting held on March 13th, the following Honorary Officers and Committee members were elected :—

Chairman : R. E. Foot.

Vice-Chairman : P. L. Harvey.

Hon. Secretary : D. B. Rogers.

Hon. Assistant Secretary : D. S. M. Sampson.

Committee Members : C. A. Brown.
B. T. Hale.

Ex-officio Committee Member : R. Tranfield.

BRANCH NOTICES

LANCASHIRE AND CHESHIRE BRANCH

Hon. Secretary : G. Greenlees, A.M.I.Struct.E., 16, Mayfield Avenue, Stretford, Lancs.

GRADUATES' AND STUDENTS' SECTION

Hon. Secretary : F. M. Turner, 11, Colwell Avenue, Stretford, Lancs.

MIDLAND COUNTIES' BRANCH

Hon. Secretary : E. R. Deeley, A.M.I.Struct.E., Arranmoor, Adshead Road, Dudley, Worcs.

GRADUATES' AND STUDENTS' SECTION

The following meetings have been arranged :—

Saturday, May 5th, 1951

Visit to the Building Research Station, Watford.

Tuesday, May 29th, 1951

Joint Meeting with the Students' Section of the Institution of Civil Engineers. A talk on Code of Practice 114 : 1948—The Structural Use of Reinforced Concrete in Building—will be given by Mr. A. T. Clarke (Associate-Member) at the James Watt Memorial Institute, Birmingham, at 7 p.m.

Hon. Secretary : M. H. Evans, B.Sc., 107, Metchley Lane, Harborne, Birmingham, 17.

NORTHERN COUNTIES' BRANCH

Hon. Secretary : J. A. Williams, A.M.I.C.E., A.M.I. Struct.E., 57, Reid Park Road, Jesmond, Newcastle-upon-Tyne.

NORTHERN IRELAND BRANCH

Hon. Secretary : S. G. Duckworth, M.I.Struct.E., Lisleen, 13, Finaghy Road, North, Belfast.

SCOTTISH BRANCH

Hon. Secretary : D. G. Drummond, B.Sc., A.M.I.C.E., A.M.I.Struct.E., 11, Woodside Terrace, Glasgow, C.3.

SOUTH-WESTERN COUNTIES' BRANCH

Hon. Secretary : E. W. Howells, M.I.Struct.E., 10-12, Market Street, Torquay, Devon.

WALES AND MONMOUTHSHIRE BRANCH

The Annual General Meeting of the Branch will be held at the Mackworth Hotel, Swansea, on Tuesday, May 1st, 1951, at 6.30 p.m.

Hon. Secretary : E. R. Steward, A.M.I.Struct.E., Edrom, Ashleigh Road, Blackpill, Swansea.

WESTERN COUNTIES' BRANCH

Hon. Secretary : G. E. Saunders, M.I.Struct.E., Dunkery, Edward Road, Walton St. Mary, Clevedon, Somerset.

YORKSHIRE BRANCH

The Seventh Ordinary Meeting of the Session was held on Wednesday, March 21st, 1951, in the Chemistry Lecture Theatre of Leeds University, at 7 p.m., and took the form of a joint meeting with the Yorkshire Association of the Institution of Civil Engineers. Mr. H. C. Husband, B.Eng., M.I.C.E., M.I.Mech.E. (Member) was in the Chair, and the meeting was attended by 75 members and friends.

Mr. J. Guthrie Brown, M.I.C.E. (Member of Council) gave a lecture entitled " Hydro-Electric Development Pitlochry," which was followed by questions and discussion. A vote of thanks to the lecturer was proposed by Professor R. H. Evans, D.Sc., M.I.C.E., M.I.Mech.E. (Member), and seconded by Mr. T. Knight on behalf of the Yorkshire Association.

The Annual General Meeting of the Branch will be held at the Great Northern Hotel, Leeds, on Wednesday, May 9th, 1951, at 6.30 p.m. Mr. J. G. Faber, B.Sc., A.M.I.C.E. (Associate-Member), will give a lecture on " The Construction of 240-ft. clear span Sheds for Aircraft Production."

Hon. Secretary : E. Wrigley, A.M.I.Struct.E., City Engineer's Dept., Civic Hall, Leeds, 1.

UNION OF SOUTH AFRICA BRANCH

Branch Hon. Secretary : A. E. Tait, B.Sc., A.M.I.C.E., A.M.I.Struct.E., P.O. Box No. 3306, Johannesburg. During week-days Mr. Tait can be contacted in the City Engineer's Department, City Hall, Johannesburg 'phone 34-1111, ext. 257.

Natal Section Hon. Secretary : E. G. Bennet, A.M.I.Struct.E., c/o Reinforcing Steel Co., Ltd., P.O. Box 478, Durban.

Cape Section Hon. Secretary : R. Stubbs, M.I.Struct.E., P.O. Box 1692, Cape Town.

Cold Formed Sections in Structural Practice with a Proposed Design Specification*

By W. Shearer Smith, A.M.I.C.E., A.M.I.Struct.E.

Synopsis

The modern trend of structural development is to reduce weight, and simultaneously to increase structural efficiency. A powerful means for realising these aims is to be found in the use of sections of thin walled proportions. Such sections, on account of their lightness, together with outlines chosen for maximum stiffness, give a high load-carrying capacity to structural weight ratio.

Despite the increasingly widespread use of these types of sections in this country, little design information is available, and there is no Design Specification at present. It is to fill this gap that the paper and specification have been prepared. The specification has been drawn up on the bases of research work carried out during the past seven years in the Department of Civil and Mechanical Engineering of The Royal Technical College, Glasgow.

The paper starts with an introduction covering the cold rolling process, the range of application of cold formed sections, assembly methods and auxiliary processes.

This is followed by the Proposed Design Specification and a Commentary on this, indicating the theoretical basis, the experimental evidence and the practical limits which condition the drafting of the various clauses.

The paper culminates in the presentation of selected designs chosen from a wide range, demonstrating the economic practice of cold formed section construction.

Notation

l = Effective length.
r = Radius of gyration.
r_b = Radius of gyration of beam perpendicular to plane of bending.
b = Breadth of plate component.
b_p = Length of bearing measured parallel to the web.
d = Depth of web of girder.
t = Thickness of plate component.
f_y = Yield stress (14 tons/sq. in.).
f_c = Critical stress.
f_t = Actual axial tensile stress.
f_a = Actual axial compressive stress.
f_b = Maximum compressive bending stress co-existent with shear stress in the web.
f_{bt}† = Sum of tensile stresses due to bending about principal axis.
f_{bc}† = Sum of compressive stresses due to bending about principal axis.
f'_s = Actual shear stress in webs.
f_p = Actual bearing stress.
F_t = Permissible stress in tension.
F_a = Permissible stress in compression.
F_{bt} = Permissible tensile stress in bending.
F_{bc} = Permissible compressive stress in bending.
F'_{bc} = Permissible compressive stress in bending in web.
F_S = Permissible shear stress (other than webs).
F'_S = Permissible shear stress in webs of girders.
F_p = Permissible bearing stress.
C_1 = Constant depending on end conditions.
C_2 = Constant depending on edge conditions.
C_3 = Constant depending on end conditions (angle struts).
C_{TB} = Torsion-bending constant.
m = Constant depending on the ratio $(l/r_b)/(d/t)$
K_1 = Flange bending stress factor.
K_2 = Web bending stress factor.
K_3 = Web shear stress factor.
η = Column irregularity factor.
I_p = Polar moment of inertia.
σ = Poisson's ratio = .3 for mild steel.
E = Modulus of elasticity (11,000 tons/sq. in.).
B = Lateral stiffness in bending = $E \times I_{yy}$.
G = Modulus of rigidity = $E/2\ (1 + \sigma)$.
J = $1/3\ st^3$ where s = circumference of section outline.
C = GJ = Torsional rigidity.

Introduction

The use of light gauge structural sections, cold formed from steel strip or sheet by rolling mill or other means, is fast becoming commonplace in this country. Many good reasons may be put forward for this somewhat rapid development, but perhaps the most important is the ease with which a very wide range of sections can be produced.

(a) *Cold Rolling or Forming Process*

The shaping of steel sheet by at least one of several methods will no doubt be familiar to all, but, as by far the largest quantities of structural sections are now being cold rolled, a word or two about :

(i) the Rolling Mill, as compared with

(ii) the Press Brake,

will not be out of place.

For cold rolling, the material used is in the form of strip or coil, produced from the billet usually hot rolled to strip‡ form.

The strip varies in thickness and width according to the sections required, 16 gauge (1/16 in.) to 8 gauge (5/32 in.) being considered most suitable for structural purposes. Coils can be obtained up to 36 inches wide weighing nearly 3 tons, but the majority of practical strip widths used do not exceed 18 in. This hot rolled strip is usually supplied pickled and oiled to ensure scale-free surfaces. Not only is this essential to avoid undue roll-wear, but it provides a most excellent surface, after degreasing, for any subsequent protecting or finishing coat.

In the rolling process the strip, mounted on a drum or in a coil box, is fed into one end of the mill, where it passes between rollers or cones of required contour, resulting in the finished section leaving the other end.

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*Given before the Institution of Structural Engineers, at 11, Upper Belgrave Street, London, S.W.1, on Thursday, March 1st, 1951.

†These symbols have also been used, for simplicity's sake, to denote the actual maximum tensile or compressive stress due to bending with (or without) direct load.

‡The new Abbey Works of the Steel Company of Wales, Limited, with its huge " continuous hot strip mill " will have an estimated output of a million tons annually. This strip should have a minimum guaranteed yield stress of 14 tons per sq. in.

The mills, consisting of 6 to 15 sets of spindles with their suitably spaced forming cones, take from a few hours to several days to set, depending on the intricacy of the section.

As it leaves the mill, the section slides along a movable table, on which is mounted a jig stopper at the far end and a saw or cutting tool at the mill end.

When the section meets the stopper, it takes the table with it until the cutting operation is completed, after which the table moves back into position ready for another length of section. The high-speed friction disc is used for larger sections, but new specially-devised swing-shear tools are now proving invaluable for cutting the smaller sections to exact length.

As the strip being rolled nears its end, the mill is stopped and a new coil butt-welded to the old to ensure a continuous rolling operation. The portion containing the weld is always scrapped.

Apart from the fact that the length of coil may be as much as 600 feet, it will be obvious that almost any length of section can be rolled, but the ease with which it can be handled and conveniently transported are governing factors.

When rolling a channel, for instance, the base or web of the channel remains horizontal throughout, while the flanges are formed, each set of cones shaping them more towards their ultimate angle. The maximum breadth (i.e. depth when being rolled) of section or channel flange, including lip, is usually not more than 4 inches.

The press-brake is essentially a machine containing a long horizontal tool or former, which rises and falls with a pressure of about 150 tons, to meet a suitably formed lower fixed portion. This machine can produce relatively small quantities of simple sections, not justifying a mill setting, in lengths up to nearly 20 ft. Complicated sections requiring several operations and having a girth in excess of 18 in. can also be pressed by this method. Several smaller component parts may even be produced simultaneously.

(*b*) *Range of Application*

While there is scarcely any limit to shape or section, ordinary channels, along with outwardly lipped and inwardly lipped channels, are by far the most popular sections at present. The zed bar is also likely to play a large part as purlins.

The outwardly lipped or "top hat" section, with its splendid structural properties, is ideally suited for rafter members in roof trusses as it permits the internal members to fit into it, with a minimum use of gusset plates. Channels have at least one advantage over the angles in that they are all symmetrical about one axis, thus facilitating concentric loading and economical design.

Cold formed sections in modern steel constructions are being developed both independently and supplementary to the heavier conventional hot rolled structural sections.

Light occupancy building where the live loads are moderate, roof trusses, purlins, bracings, and floor constructions at relatively close spacing, form but one group in which cold formed sections have been used with complete success. Their overall efficiency has also been proved in storage racks, pallets, steel shuttering and automobile construction.

(*c*) *Auxiliary Processes and Assembly Methods*

Here, there is a very wide field open for the really expert production engineer. Modern methods can and have been introduced. Special multiple punches, forming tools and other appliances, combined with some measure of standardisation, play a large part.

For the shop assembly of various components in say, a roof truss, welding by electric arc is used almost exclusively. Jigs are generally used to locate the various assemblies during the preliminary welding operations to ensure accuracy and interchangeability. These may be either very simple or highly complicated ones, depending on the nature and quantities of assemblies being produced.

The customary care should be taken to avoid burning through the lighter gauges and, to accomplish this, No. 12 gauge electrodes are invariably employed.

Spot and projection welding have been used with some success for certain types of constructions.

Bolting is generally reserved for site connections.

(*d*) *Protection against Corrosion*

The possibility of corrosion of light gauge steel in permanent buildings was investigated some time ago by a committee embodying representatives of the Department of Scientific and Industrial Research and many other competent organisations, and their recommendations were published in British Standards Institution publication P.D.420 : 1945 (3).

Since then, the Council for Codes of Practice for Buildings has produced a draft Code of Practice for Painting Iron and Steel[6].

One of the big advantages of cold formed sections over ordinary hot rolled structural sections, in their resistance to corrosion, lies in the fact already mentioned, that the strip material used must necessarily have been cleaned by pickling and oiling ; the resulting scale-free surface provides, after degreasing, an excellent base for any subsequent rustproofing process.

The evidence collected by the Pittsburgh Testing Laboratory, in their very full report for the American Iron and Steel Institute[10] on the durability of such constructions installed during the past 35 years, is most reassuring. The condition of the steelwork in 50 different types of buildings, divided into three ratings, were as follows :—

45 buildings good ; 5 buildings fair ; and none poor.

Yet these structures relied, for the most part, on the standard commercial paints then available.

It is not intended to belittle the need for protecting steel by one of several methods, but rather to emphasise the importance of most careful preparation of the steel for protective coatings, rather than the need for expensive coatings as such.

I.—Permissible working Stresses for Light Gauge Structural Members

The following proposed specification has been compiled for structural members cold formed or rolled from light gauge sheet or strip steel having a yield stress f_y of 14 tons/sq. in. and Young's Modulus E of 11,000 tons/sq. in.

Axial Stresses in Tension—Ties

1. The permissible direct stress in pure tension on the net area of the section should not exceed

$$F_t = 0.59 f_y = 8.26 \text{ tons/sq. in.}$$

Axial Stresses in Compression—Struts (Uncased)

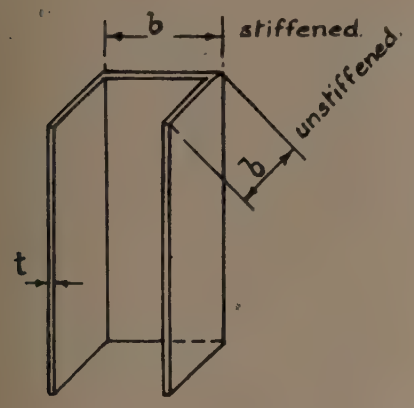
2. The permissible direct stress in pure compression on the gross sectional area should not exceed the lesser of the values given by (i) and (ii) as follows :—

(i) F_a selected from Table 1 to correspond to the slenderness ratio of the member, where—

l = effective length as defined in B.S. 449 : 1948.
r = radius of gyration.

(ii) F_a selected from Table 1 to correspond to the maximum plate ratio b/t of the member, where—

b = actual width of plate component of the cross section under consideration.
t = thickness of plate component.



Typical Dimension Notation

N.B.—In tables where only "stiffened" edge b/t values are recorded stress values for "unstiffened" edges may be obtained by selecting the stress value corresponding to a "stiffened" b/t value equal to *three* times the actual b/t ratio of the "unstiffened" edge plate.

A plate component may be considered to have "stiffened" edges if width of both its adjoining plate components is not less than the sizes given below—otherwise it should be considered as "unstiffened."

Table 2

Adjoining plate width for "Stiffened" edge

Thickness of plate	Minimum width of Adjoining plates
18g. (.048 ins.)	
16g. (.064 ins.)	
14g. (.080 ins.)	$\frac{3}{8}$"
12g. (.104 ins.)	
$\frac{1}{8}$ (app. 10g.) (.125 ins.)	
8g. (.160 ins.)	

Bending Stresses—Beams

3. The stresses are applicable to sections symmetrical about the plane of bending and to other types of sections if they are effectively prevented from twisting.

In calculating tensile and compressive stresses in bending the net moment of inertia of the cross-section should be used.

(*a*) *Tensile flange stress*

For beams other than girders with built-up flanges the bending stress in the extreme fibres should not exceed

$$F_{bt} = 0.65\ f_y = 9.10 \text{ tons/sq. in.}$$

For girders with built-up flanges the above value of F_{bt} should be reduced by 5 per cent.

(*b*) *Compressive flange stress* (*uncased beams*)

The bending stress in the extreme fibres should not exceed the least of the values given by (i), (ii), and (iii) as follows :—

(i) $F_{bc} = 0.65\ f_y = 9.10$ tons/sq. in.

Table I.

Permissible working stresses in tons/sq. in of gross section for axial loads

l/r	b/t unstiffened edge	b/t stiffened edge	F_a tons/sq. in.	l/r	b/t unstiffened edge	b/t stiffened edge	F_a tons/sq. in.
0			8.26	160	32.32	96.97	1.77
5	1.01	3.03	8.03	165	33.33	100.00	1.68
10	2.02	6.06	7.80	170	34.35	103.04	1.60
15	3.03	9.09	7.57	175	35.36	106.07	1.52
20	4.04	12.12	7.34	180	36.37	109.10	1.44
25	5.05	15.15	7.11	185	37.38	112.13	1.37
30	6.06	18.18	6.89	190	38.39	115.16	1.31
35	7.07	21.21	6.66	195	39.40	118.19	1.25
40	8.08	24.24	6.43	200	40.41	121.22	1.19
45	9.09	27.27	6.20	205	41.42	124.25	1.14
50	10.10	30.30	5.97	210	42.43	127.28	1.09
55	11.11	33.33	5.74	215	43.44	130.31	1.04
60	12.12	36.37	5.51	220	44.45	133.34	1.00
65	13.13	39.40	5.28	225	45.46	136.37	0.96
70	14.14	42.43	5.05	230	46.47	139.40	0.92
75	15.15	45.46	4.82	235	47.48	142.43	0.88
80	16.16	48.49	4.59	240	48.49	145.46	0.85
85	17.17	51.52	4.36	245	49.50	148.49	0.82
90	18.18	54.55	4.12	250	50.51	151.52	0.79
95	19.19	57.58	3.88	255	51.52	154.56	0.76
100	20.20	60.61	3.65	260	52.53	157.59	0.73
105	21.21	63.64	3.43	265	53.54	160.62	0.70
110	22.22	66.67	3.22	270	54.55	163.65	0.68
115	23.23	69.70	3.02	275	55.56	166.68	0.66
120	24.24	72.73	2.84	280	56.57	169.71	0.63
125	25.25	75.76	2.69	285	57.58	172.74	0.61
130	26.26	78.79	2.51	290	58.59	175.77	0.59
135	27.27	81.82	2.36	295	59.60	178.80	0.57
140	28.28	84.85	2.23	300	60.61	181.83	0.56
145	29.29	87.88	2.10	315	63.64	190.92	0.51
150	30.30	90.91	1.98	330	66.67	200.01	0.46
155	31.32	93.95	1.87	350	70.71	212.13	0.41

Having found l/r for member and b/t for plate component, the allowable stress is the *minimum* stress corresponding to either l/r or b/t from table.

Intermediate values may be determined by interpolation.

For girders with built-up flanges the above value of F_{bc} should be reduced by 5 per cent.

(ii) $F_{bc} = \dfrac{22750}{(l/r_b)(d/t)} K_1$ tons/sq. in. for sections of uniform thickness.

and $F_{bc} = \dfrac{37400}{(l/r_b \times d/t)} K_1$ tons/sq. in. for sections built-up of two channels back to back, securely attached together, where :—

l = the length between effective lateral restraints.
r_b = the radius of gyration of the beam section perpendicular to the plane of bending.
d = the depth of beam in the plane of bending.
t = the uniform gauge thickness of section.
K_1 = a factor dependent on the ratio given in Table 3.

Table 3

Bending stress factors K_1

Sections uniform thickness $l/r_b / d/t$	Sections back to back $l/r_b / d/t$	K_1
Over 10	6.10	1.00
,, 8	4.88	1.09
,, 6	3.66	1.15
,, 4	2.44	1.40
,, 3	1.83	1.65
,, 2	1.22	2.31
,, 1	0.61	6.59
,, 0.5	0.31	9.34

Intermediate values may be determined by linear interpolation.

(iii) $F_{bc} = \left(\dfrac{141}{b/t}\right)^2$ tons/sq. in. as in Table 4 to correspond to the effective plate ratio b/t of the compressive flange where b and t are defined as for struts in Clause 2 (ii).

Table 4

Compressive stresses in bending for flanges

b/t stiffened edge	F_{bc} tons/ sq. in.	b/t stiffened edge	F_{bc} tons/ sq. in.	b/t stiffened edge	F_{bc} tons sq. in.
46.82	9.10	58.24	5.86	93	2.30
48.00	8.63	60.00	5.52	105	1.80
49.00	8.28	63.00	5.01	120	1.38
50.00	7.95	66.00	4.56	138	1.04
51.00	7.64	69.00	4.18	159	.79
52.50	7.21	72.00	3.83	183	.59
54.00	6.82	75.00	3.53	216	.43
55.50	6.45	78.00	3.27	255	.31
57.00	6.12	84.00	2.82	300	.22

For b/t unstiffened edge multiply b/t stiffened edge by *three* and read corresponding F_{bc} value.

Intermediate values may be determined by linear interpolation.

(c) *Bending stress in the web*

The compressive stress in the web should not exceed 9.10 tons/sq. in. or F'_{bc} selected according to (i) or (ii) below, whichever is applicable, to correspond to the effective plate ratio d/t of the web, where :—

d = actual clear depth of the web for "stiffened" edge plate and *three* times the actual clear depth of the web for "unstiffened" edge conditions.
t = thickness of web.

(i) When the web is symmetrical about the axis of bending.

$F'_{bc} = \left(\dfrac{345}{d/t}\right)^2$ tons/sq. in. as in Table 5.

Table 5

Permissible compressive stresses in bending for webs

d/t stiff'n'd edge	F'_{bc} tons /sq. in.	d/t stiff'n'd edge	F'_{bc} tons /sq. in.	d/t stiff'n'd edge	F'_{bc} tons /sq. in.	d/t stiff'n'd edge	F'_{bc} tons /sq. in.
47.17	53.50	114	9.10	142.68	5.88	213	2.61
50	46.24	117	6.69	147	5.50	228	2.29
60	33.06	120	8.27	153	5.08	246	1.97
70	24.29	123	7.87	159	4.71	270	1.63
80	18.60	126	7.50	165	4.37	300	1.32
90	14.69	129	7.15	174	3.93	360	.92
100	11.90	132	6.83	183	3.55	450	.59
105	10.80	135	6.53	192	3.23	600	.33
110	9.84	141	5.98	201	2.95	750	.21

F'_{bc} stresses exceeding 9.10 tons/sq. in. in above table to be used only in conjunction with web factors K_2, Table 6.

Intermediate values may be determined by linear interpolation.

(ii) If the web is unsymmetrical about the axis of bending, F'_{bc} obtained from Table 5 is to be multiplied by a factor K_2 given in Table 6.

Table 6

Bending stress (for web) factors K_2

f_{bt}/f_{bc}	K_2
1 or >1	1.0
.75	.76
.5	.565
.25	.42
0	.326
—.25	.261
—.50	.213
—.75	.184
—1	.17

Where f_{bt} = actual maximum tensile stress due to bending or due to combined bending and direct loading in web.
In cases where the stress over the web is wholly compressive f_{bt} is taken as the numerically smaller extreme fibre stress in the web and of negative sign.
f_{bc} = actual maximum compressive stress due to bending or combined bending and direct loading in the web.

Intermediate values may be determined by linear interpolation.

Shear Stresses

4. (a) *Simple shear stress.* The shear stress in parts other than webs should not exceed
$F_s = 0.42 f_y = 5.88$ tons/sq. in.

(b) *Shear stress in webs due to bending.* The average shear stress F'_s on the net cross-section of the web should not exceed the lesser of the values given by (i) and (ii) below for webs without stiffeners :—

(i) $F'_s = 0.42 f_y = 5.88$ tons/sq. in.

(ii) $F'_s = \left(\dfrac{163}{d/t}\right)^2$ tons/sq. in. as in Table 7 to correspond to the effective plate ratio d/t of the web d and t being defined as in Clause 3c

Table 7

Permissible Shear Stress in Webs not accompanied by bending stress. If bending stresses are present in addition to shear stress these figures are to be reduced as stated in Clause 5(c).

d/t stiff'n'd edge	F'_s tons/ sq. in.	d/t stiff'n'd edge	F'_s tons/ sq. in.	d/t stiff'n'd edge	F'_s tons/ sq. in.	d/t stiff'n'd edge	F'_s tons/ sq. in.
45	13.12	67.25	5.88	110	2.20	160	1.04
50	10.63	70	5.42	120	1.84	170	.92
55	8.78	80	4.15	130	1.57	180	.82
60	7.38	90	3.28	140	1.36	190	.74
65	6.29	100	2.66	150	1.18	200	.66

F'_s stresses exceeding 5.88 tons/sq. in. in above table to be used only in conjunction with reducing factor, Clause 5c, Bending and Shear.

(c) **Web Stiffeners.* Bearing stiffeners shall be provided at all points of concentrated loads where the value of load in tons exceeds :

$$\left(7{\cdot}5 + 1{\cdot}6\sqrt{\frac{b_p}{t}}\right) t^2 f_y.$$

and at points of supports where the reaction in tons exceeds the value :

$$\left(5 + 0{\cdot}6\sqrt{\frac{b_p}{t}}\right) t^2 f_y.$$

where b_p = length of bearing measured parallel to the web in inches.
t = thickness of web in inches.

Combined Stresses

5. a. *Bending and axial compression.* Members subject to both axial compression and bending stresses shall be so proportioned that the quantity

$$\frac{f_a}{F_a} + \frac{f_{bc}}{F_{bc}}$$

does not exceed unity, where :

f_a = the axial compressive stress.
F_a = the permissible compressive stress in axial loaded struts (see Table 1).
f_{bc} = the sum of the compressive stresses due to bending about both principal axes.
F_{bc} = the permissible compressive stress for members subject to bending (see Clause 3). For the purpose of this clause, the allowances made in Clause 3b (ii) Table 3 should not be used where excessive flexibility will result.

b. *Bending and axial tension.* Members subject to both axial tension and bending stresses shall be so proportioned that the quantity

$$\frac{f_t}{F_t} + \frac{f_{bt}}{F_{bt}}$$

does not exceed unity, where :

f_t = the axial tensile stress.
F_t = the permissible axial tensile stress (see Clause 1).
f_{bt} = the sum of the tensile stresses due to bending about both principal axes.
F_{bt} = the permissible tensile stress in bending (see Clause 3a).

c. *Bending and Shear.* When the thickness of the web of a beam is less than one forty-fifth of its depth (Clause 3c), the permissible shear stress given in Table 7 shall be multiplied by the reducing factor $(1 - K_3\sqrt{f_b^5})$ in which $K_3 = [(d/t) \div 345]^5$ and f_b is the maximum co-existent compressive bending stress in tons/sq. in. in the web at the section in question.

Bearing Stress†

6. The permissible bearing stress for thicknesses not less than 16 gauge on the net area of contact should not exceed

$$F_p = 0{\cdot}8 f_y = 11{\cdot}2 \text{ tons./sq. in.}$$

II.—Commentary on the Proposed Design Specification

NOTE :—The clause numbers indicated refer to the proposed specification presented in Section I.

(*a*) TIES—*Clause* 1

Tensile tests carried out on specimens cut from both strip steel or coil and finished cold formed sections indicate behaviour similar to ordinary mild steel. The only departures manifested from steel to B.S.15 are the values of the yield stress f_y and Young's Modulus E. Minimum values of these quantities have been assessed on the basis of the experimental values of 14 tons/sq. in. and 11,000 tons/sq. in. respectively.

In view of the foregoing, the factor of .59 as given in B.S.449 : 1948, is used to determine the permissible stress in tension, giving $F_t = .59 \times 14 = 8.26$ tons/sq. in.

(*b*) STRUTS—*Clause* 2

Struts may fail either by "material" failure or by "shape" failure. The first is due to yielding of the material. The second is failure at a stress below the yield stress of the material, due to instability of structural shape.

Failure due to instability may occur as (i) primary failure or (ii) secondary failure.

Primary failure may be due to flexural instability only, referred to in the paper as overall bending, or it may be due to flexural-torsional instability, referred to as torsion-bending. Primary failure due to torsional instability alone is very rarely met with in practice, as it obtains only when there is no rotational end-restraint on the column.

The secondary failure relevant to thin walled struts is flexural instability of the plate components (flange or web), referred to in the paper as plate component instability in bending.

These are the fundamental cases of instability—it should be appreciated, however, that any combination of these may occur. The basis of efficient and economic design is the equality of strength under these various possible types of failure, i.e. best design should always aim at equal overall column and plate component strengths.

The relevant formulæ for critical stress corresponding to the above modes of failure are :—

Overall Bending Critical Stress $f_c = \dfrac{C_1 E}{(l/r)^2}$ . . . . . Eq. (1)

where constant C_1 depends on end conditions.

Overall Torsion-Bending Critical Stress $f_c = \dfrac{1}{I_p}\left(GJ + \dfrac{\pi^2}{l^2} E C_{TB}\right)$ . . Eq. (2)

where constant C_{TB} depends on section outline and the position of the centre of rotation of the cross-section.

For equal angles Eq. (2) can be reduced to :—

$$f_c = \left\{ C_3 \frac{\pi^2}{1 - \sigma^2} \frac{r^2}{l^2} + \frac{1}{2(1 - \sigma)} \right\} E \left(\frac{t}{b}\right)^2 \quad \text{Eq. (3)}$$

where constant C_3 depends on end conditions.

Plate Component Critical Stress in bending $f_c = \dfrac{C_2 E}{(1 - \sigma^2)} \left(\dfrac{t}{b}\right)^2$ . . . Eq. (4)

where constant C_2 depends on edge conditions.

(i) *Consideration of Torsion-Bending Failure*

Any strut form whose critical weakness is its torsional strength, e.g. wide thin angle sections, will fail by pure twisting if compressed by an axial compressive load—assuming the ends free to rotate relative to each other.

In such cases the sections rotate about the so-called "shear centre," which is the point in a section at which loads applied, as in beam action, cause no twisting.

* Based on work of Winter and Pian—Cornell Univ. Bulletin No. 35, Part 1, April, 1946.

† Based on joints, tested in tension.

The critical stress f_c (for the theoretical case having free ends) is given by

$$f_c = \frac{1}{I_p} \times GJ$$

If, however, as in a practical case, the ends are restrained rotationally, relative to each other, any twist of the column causes longitudinal bending stresses to arise in the flanges, tending to stiffen the column and so increase f_c to the value in Eq. (2)

$$f_c = \frac{1}{I_p}\left(GJ + \frac{\pi^2}{l^2} E\, C_{TB}\right)$$

The value of the torsion-bending constant C_{TB} depends exclusively on the assumed centre of rotation of the cross section, which is no longer the shear centre. The appreciable difference in f_c resulting from different assumed positions of this centre of rotation is shown in Fig. 1.

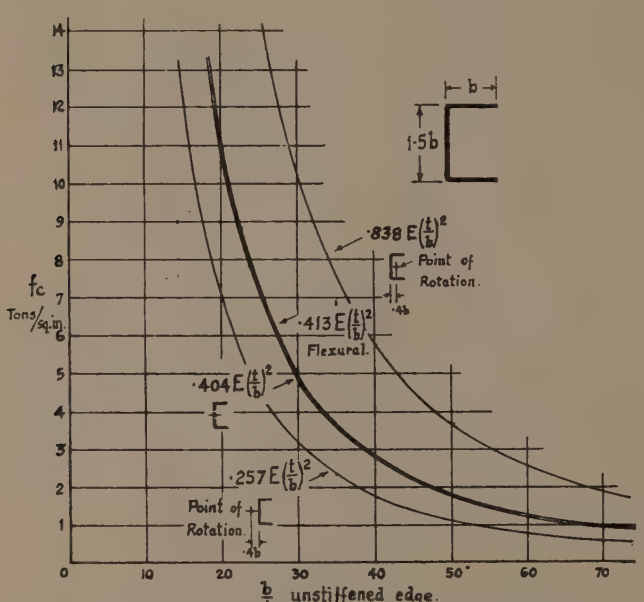


Fig. 1.—Effect of the position of the centre of rotation on the torsion bending critical stress f_c

This figure compares the torsion-bending critical stress f_c, Eq. (2), of a channel of the proportions shown, with the bending critical stress f_c, Eq. (4), of the flange plate component. It is seen that, when the centre of rotation coincides with the shear centre, the channel will fail by torsion bending. On the other hand, assuming the centre of rotation inside the web as shown, the torsion-bending strength increases to approximately double the flexural plate component strength. Thus it is seen that the location of the centre of rotation is a vitally important matter. The experimental work carried out[33] indicates that rotational end restraint very much less than that at present in actual structures is sufficient to prevent the development of torsional instability in sections, other than angles. For the present, therefore, it is proposed to disregard torsion when dealing with overall stability, and base the design procedure on bending actions only.

(ii) *Consideration of Failure of Struts in Bending*

Referring to Clause 2 of the specification, Table 1 has been drawn up on the basis of equal overall and plate component strength. Equating Eq. (1) for hinged ends to Eq. (4) for stiffened or unstiffened edge conditions, the following relations result :—

$$\left.\begin{aligned} \frac{b}{t} &= .606 - \frac{l}{r} \quad \ldots \text{stiffened edge} \\ \frac{b}{t} &= .202 - \frac{l}{r} \quad \ldots \text{unstiffened edge} \end{aligned}\right\} \text{Eq. (5)}$$

These are used to obtain the values in the two b/t columns of Table 1 corresponding to any given values of the slenderness ratio ranging from 0 to 350.

The permissible average compressive stress F_a shown in the last column is obtained from the Perry-Robertson (1 and 19) formula, as given in Appendix C of B.S.449 : 1948, using a yield stress of 14 tons/sq. in., and E of 11,000 tons/sq. in. (Fig. 2).

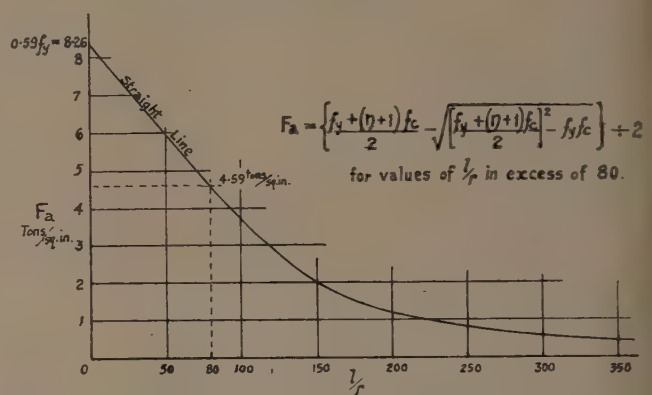


Fig. 2.—Formula from which values shown in Table 1 have been derived

Thus, reading across any line in Table 1 gives the slenderness ratio and corresponding plate component ratios for equal overall and component strength, together with the allowable stress in axial compression.

The table may also be used if no such correspondence exists between l/r and b/t, the permissible working stress being taken as the smaller of the stresses corresponding to the l/r or b/t ratios respectively. (See examples in Appendix).

For angles, the torsion bending strength, being the critical weakness, must as a rule be used for the design basis.

For equal angles of slenderness ratio greater than about 15, Eq. (3) reduces to

$$f_c = \frac{1}{2\,(1 - \sigma)} E \left(\frac{t}{b}\right)^2$$

(assuming the sections to rotate about the heel) as the term representing the influence of flexure and that of end conditions becomes negligible.

Thus for Poisson's Ratio $\sigma = .3$

the critical stress in torsional stability } $f_c = 0.385\, E \left(\frac{t}{b}\right)^2$

while the flexural strength of the legs } $f_c = 0.403\, E \left(\frac{t}{b}\right)^2$

when considered as plate components with simply supported and free edge conditions.

It would, therefore, appear that equal angles will fail due to torsional instability rather than due to plate component failure. The closeness of the critical stress values, however, do not justify differentiation from the point of view of design, particularly as a small decrease in the values of Poisson's Ratio will be sufficient to make the two values equal. For design,

therefore, it is recommended that Table 1 should be used, with the b/t values corresponding to simply supported and free edges.

Unequal angles are under consideration, but no information is offered at present. Their behaviour, although failing in torsion-bending, is different from equal angles due to the different position of the centre of rotation.

The values given in Table 2 are based on extensive experiments carried out, and represent the results of some 200 tests[33]. The practical equivalent to the type of edge conditions described is shown in Fig. 3.

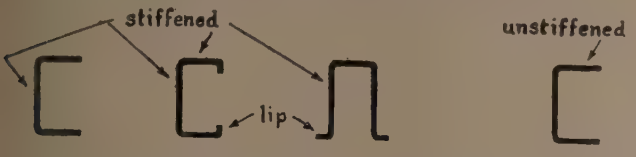


Fig. 3.—Practical edge conditions

It should be noted that the stress values given under F_a in Table 1 have been calculated for the optimum conditions of overall and plate component collapse. Experimentally, it has been shown — Moir and Kenedi[17] — that the edge conditions of the plate components under these (optimum) conditions correspond to simply supported edges.

When, however, there is no tendency to overall collapse, the plate components appear to receive increased support along their edges ; thereby increasing the value of their critical stress. In such cases an increase in the value of F_a is permissible — refer to Fig. 4.

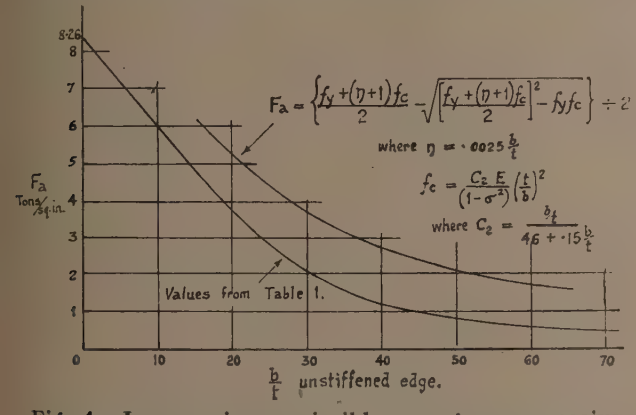


Fig. 4.—Increase in permissible average compressive stress F_a when plate component strength is the limiting factor. $1/r$ not greater than about 85.

This figure shows the increase in plate strength obtained for struts of slenderness ratio not more than about 85. As this matter is still being investigated, the results and corresponding increase of F_a values will be presented at a later date.

(c) BEAMS—*Clause* 3

The types of failure which may occur in a beam section which is prevented from developing a progressive twisting action under normal loading (but which is free to buckle laterally) may be as follows :—

(i) Due to material failure.
(ii) Torsion-bending failure due to lateral instability.
(iii) Instability of the compression flange in bending.
(iv) Instability of the web in bending.

Failure due to instability in shear and under combined stressing effects are treated separately in their appropriate sections later.

The relevant formulæ for critical stress corresponding to the modes of failure described under (ii) to (iv) are as follows :—

Lateral torsion-bending strength under pure moment :

$$f_c = \frac{md}{2\,I_{xx}} \sqrt{\frac{BC}{l}} \quad \ldots\ldots\ldots \text{Eq. (6)}$$

Flexural or bending strength of plate component (web or flange) :

$$f_c = \frac{C_2\,E}{(1-\sigma^2)} \left(\frac{t}{b}\right)^2 \quad \ldots\ldots\ldots \text{Eq. (4)}$$

where constant C_2 depends on edge conditions.

(i) *Material Failure*

Clauses 3a and 3b(i) have been obtained as in B.S.449 : 1948, using the value of .65 times the yield stress, giving

$$F_{bc} = F_{bt} = .65 \times 14 = 9.10 \text{ tons/sq. in.}$$

(ii) *Failure due to Lateral Instability*

Equation (6) may be reduced to the form given in Clause 3b(ii) by substituting $B = E\,I_{yy}$; $C = G \times 1/3\,st^3$ (where s is circumference) and noting that depth of section $\frac{}{r_{xx}}$ is approximately 2.65 for a very wide range of these walled sections.

This yields

$$f_c = \frac{45{,}500}{\left(\frac{l}{r_b}\right)\left(\frac{d}{t}\right)} K_1 \text{ tons/sq. in.}$$

and using a load factor of 2 on the permissible maximum compressive stress in the flange becomes

$$F_{bc} = \frac{22{,}750}{\left(\frac{l}{r_b}\right)\left(\frac{d}{t}\right)} K_1 \text{ tons/sq. in.} \quad \text{Eq. (7)}$$

The value of K_1 given in Table 3 depends on the ratio of $l/r_b \div d/t$ and represents the strengthening effect of flange bending when the ends of the beam are prevented from rotating relative to each other. This effect is exactly analogous to that described in the torsion-bending failure of columns.

It is to be noted that Eq. (7) applies only to sections of uniform thickness. In cold rolled sections, thickness variation is usually encountered only in composite sections such as two channels back to back. The permissible working stress for double channel sections is of the same form as Eq. (7), the numerical constant, however, changing from 22,750 to 37,400. It is emphasised that this latter value is only applicable if the two channels are so connected that they truly strain together.

(iii) *Instability of the Compression Flange in Bending*

Clause 3b(iii) gives the permissible compressive stress values in bending for beam flanges. These are based on the critical stress form of Eq. (4) :

$$f_c = \frac{C_2\,E}{(1-\sigma^2)} \left(\frac{t}{b}\right)^2$$

It should noteb de that this gives the critical stress for plate component action under *uniform* flange bending stress.

Incorporating a load factor of 2 and taking C_2 as 3.3, i.e. corresponding to stiffened edges, the formula

$$F_{bc} = \left(\frac{141}{b/t}\right)^2 \text{ tons/sq. in.} \quad \ldots\ldots\ldots \text{Eq. (8)}$$

is obtained. The values presented in Table 4 have been calculated from this formula.

(iv) *Instability of the Web in Bending* (*Clause* 3c)

The behaviour of webs of beams introduces the problem of a plate component under a varying longitudinal stress due to bending. The stress distribution under pure bending is the well-known triangular variation, the compressive stress being equal to the tensile stress if the web is symmetrical about the neutral axis. For such a case the formula giving the critical stress is as given by Eq. (4), the constant C_2 being 19.66. Incorporating a load factor of 2 as before, the value

$$F^1_{bc} = \left(\frac{345}{d/t}\right)^2 \text{ tons/sq. in.} \quad \text{Eq. (9)}$$

is obtained. Corresponding values are given in Table 5.

It should be noted that this table is extended beyond 9.10 tons/sq. in. The values exceeding this amount are only of significance when used in conjunction with the web factor K_2 as explained hereafter.

It was stated that Eq. (9) applies only when the longitudinal compressive and tensile bending stresses are equal in magnitude. If the web is unsymmetrical about the neutral axis, i.e. the actual tensile stress due to bending, f_{bt} differs from the actual compressive stress due to bending f_{bc}, the values given by Eq. (9) are to be multiplied by the web factor K_2 given in Table 6 for different ratios of $\frac{f_{bt}}{f_{bc}}$.

These values have been derived from the theory given by Timoshenko[26]. It should be realised that this also covers the case of combined bending and direct stressing in a web, as the only effect of such combination on the web is to alter the ratio f_{bt} to f_{bc}. The negative sign in Table 6 is an artifice to allow for the case when the bending stress in the web is compressive over the whole depth. In such a case, f_{bt} is taken as the smaller of the compressive extreme fibre stresses and negative in sign.

If f_{bt} is greater than f_{bc}, i.e. f_{bt}/f_{bc} is greater than 1, the stresses given in Table 5 could be increased — theoretically speaking — this increase, however, is not thought practicable.

(d) Shear Stresses—*Clause* 4

The modes of collapse under shearing action may be due, as before, to material or instability failure. In addition, the crushing action of concentrated load on beam webs is considered in this section.

(i) *Material Failure in Shear*

In conforming to the method of approach of the previous sections, pure shearing action is taken as the factored yield stress indicated in Clause 4a. $F_s = .42 f_y = 5.88$ tons/sq. in., the factor being as indicated in B.S.449 : 1948.

(ii) *Instability of the Web Component in Shear*

Clause 4b, dealing with the shear stress in webs, differentiates between material failure (Clause 4b(i)), and failure due to plate instability (Clause 4b(ii)). The characteristic stress expression for failure due to plate instability is the form given in Eq. (4), the constant C_2 however being 4.4 and, when a load factor of 2 is incorporated as before, the expression becomes

$$F^1_s = \left(\frac{163}{d/t}\right)^2 \text{ tons/sq. in.} \quad \text{Eq. (10)}$$

Corresponding values are given in Table 7.

It should be noted that the values stated can only be applied when the bending stresses in the web are negligible by comparison, e.g. at the regions such as the supports of a simply supported beam.

(iii) *Crushing*

One form of failure peculiar to thin webs is local crushing at the point of application of a concentrated load. This aspect is being investigated, but results are not yet available on a sufficiently wide range for definite recommendations to be made.

It is suggested, meantime, that the empirical formulæ for P_c, the concentrated load causing crushing failure as given by Winter[32] be used, *viz* :—

$$P_c = \left(15 + 3.25\sqrt{\frac{b_p}{t}}\right) t^2 f_y \text{ tons}$$

for intermediate concentrated loads,

and

$$P_c = \left(10 + 1.25\sqrt{\frac{b_p}{t}}\right) t^2 f_y \text{ tons}$$

for end reactions.

Using a load factor of 2, these reduce to the forms given in the specification.

If the actual loads exceed these permissible values for any given bearing length, it is recommended that bearing stiffeners be introduced at the points of action of these loads.

(e) Combined Stresses—*Clause* 5

(i) *Combined Bending and Axial Stresses*

Clauses 5a and 5b give the combination of bending with axial compression and bending with axial tension respectively. The connection proposed by the B.S.449 : 1948, has been investigated in respect of thin walled sections and found to be satisfactory. Consequently, it is proposed that the same form be used for thin walled sections.

(ii) *Combined Bending and Shear—Clause* 5c

The relationship proposed to design between the fractional shear stress f_s/F^1_s and the fractional bending stress f_b/F^1_{bc} for combined bending and shear is

$$\frac{f_s}{F^1_s} - \sqrt{\left(\frac{f_b}{F^1_{bc}}\right)^5} = 1 \quad \text{Eq. (11)}$$

A comparison between the curve in Eq. (11) and the theoretical relationship by Timoshenko[26] is shown in Fig. 5.

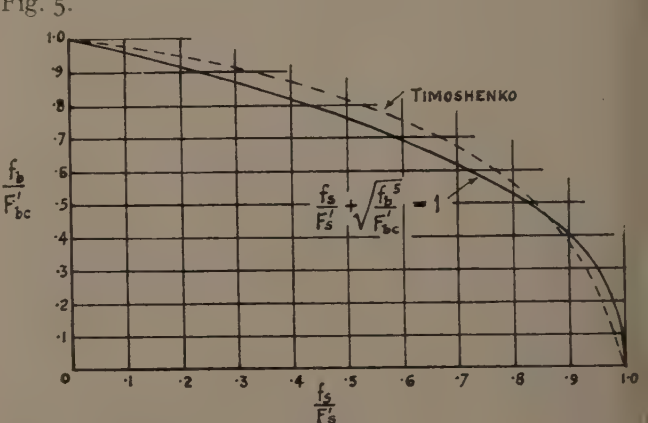


Fig. 5.—Comparison between theoretical and proposed combined shear and bending relationship

Note : For $\frac{f_s}{F'_s} + \sqrt{\frac{f_b^5}{F'^5_{bc}}} = 1$ read $\frac{f_s}{F'_s} + \sqrt{\left(\frac{f_b}{F'_{bc}}\right)^5} = 1$

Substituting for F^1_{bc} in Eq. (11)

the value $F^1_{bc} = \left(\frac{345}{d/t}\right)^2$ tons/sq. in. obtained in Eq. (9

$\frac{f_s}{F^1_s}$ can be expressed as

$$\frac{f_s}{F'_s} = 1 - K_3 \sqrt{f_b^5} \quad \text{. . . . . Eq. (12)}$$

where $K_3 = \left[\left(\frac{d}{t}\right) \div 345\right]^5$

When the thickness of the web is less than one forty-fifth of its depth, the term $K_3\sqrt{f_b^5}$ is greater than .01, and the consequent reduction in f_s/F'_s should be allowed for. This is effected by multiplying the permissible shear stress in Table 7 by the reducing factor $(1 - K_3\sqrt{f_b^5})$.

(f) BEARING STRESS—*Clause* 6

Experimental investigation has established that the mechanism of failure for bolted joints in tension, for plate thicknesses above 16 gauge, is in no way different from the usual conception. Consequently, the use of the factored yield stress as given by B.S.449 is fully permissible and safe :—

i.e. $F_p = 0.8 f_y = 11.2$ tons/sq. in.

If, however, the plate thickness is below 16 gauge, secondary buckling effects arise, and ultimate carrying capacity is difficult to predict. Research is at present proceeding on this matter. Meantime, it is suggested that bolted joints should not be used with plate thicknesses below 16 gauge.

III.—Cold Formed Section Design

PRACTICE AND ECONOMY ILLUSTRATED BY DESIGN EXAMPLES

Design Examples

(a) *Four-Way Stacking Pallet*

The steel pallet 4 feet square shown in Fig. 6 is transported and stacked by means of forked lift trucks. Sides are removable to facilitate loading from the inside of a railway wagon. Pallets have a carrying capacity of 2 tons each and are to be stacked five high.

The following tables give a stress analysis of the main components.

POSTS

Section	Area sq. in.	Load tons	f_a tons/sq.in.	Overall Column l/r	Overall Column F_a tons/sq.in.	Plate Component b/t web	Plate Component F_a tons/sq.in.	End Bearing f_a tons/sq.in.	End Bearing F_a tons/sq.in.	
		inc. shock						Eff. .4 sq."		
□ 2" × 2" × ⅜" × 14g	.501	2.5	5.00	18	7.43	25	6.35	6.25	6.35	—

SLATS

Section	Load tons	B. M ft. tons	b/t top flg.	F_{bc} tons/sq.in.	Z Reqd.	min. Z Given	f_s tons/sq.in.	d/t	F'_s tons/sq.in.	F'_{bc} tons/sq.in.
				Table 4					Table 7	
□ 6" × 1" × ½" × 16g	.33 dist.	.044	94	2.3	.23	.276	1.29	15.6	5.88	> 9.10

The importance of palletisation will be, perhaps, better understood if it is realised that as much as 45 per cent. of the cost of certain materials is spent on handling.

(b) 35 *ft.* 0 *in. Span Roof Truss*

The roof truss shown in Fig. 7 has a pitch of one-fifth and is placed at 12 ft. 6 in. centres. The roof covering consists of large corrugated asbestos cement sheets and the roof is designed for a wind load corresponding to 75 m.p.h. at 20 ft. (13.9 lbs./sq. ft.)—B.S.449 ; 1948.

The following tables give a stress analysis of selected members and weight comparison.

RAFTER

Section	Area sq. in.	Load tons	f_a tons/sq. in.	Overall Column l/r	Overall Column F_a tons/sq. in.	Plate Component b/t web	Plate Component F_a tons/sq. in.
□ 1½" I.D. × 2¼" × ½" × 12g	.684	3.9	5.7	47	6.11	21.6	6.62

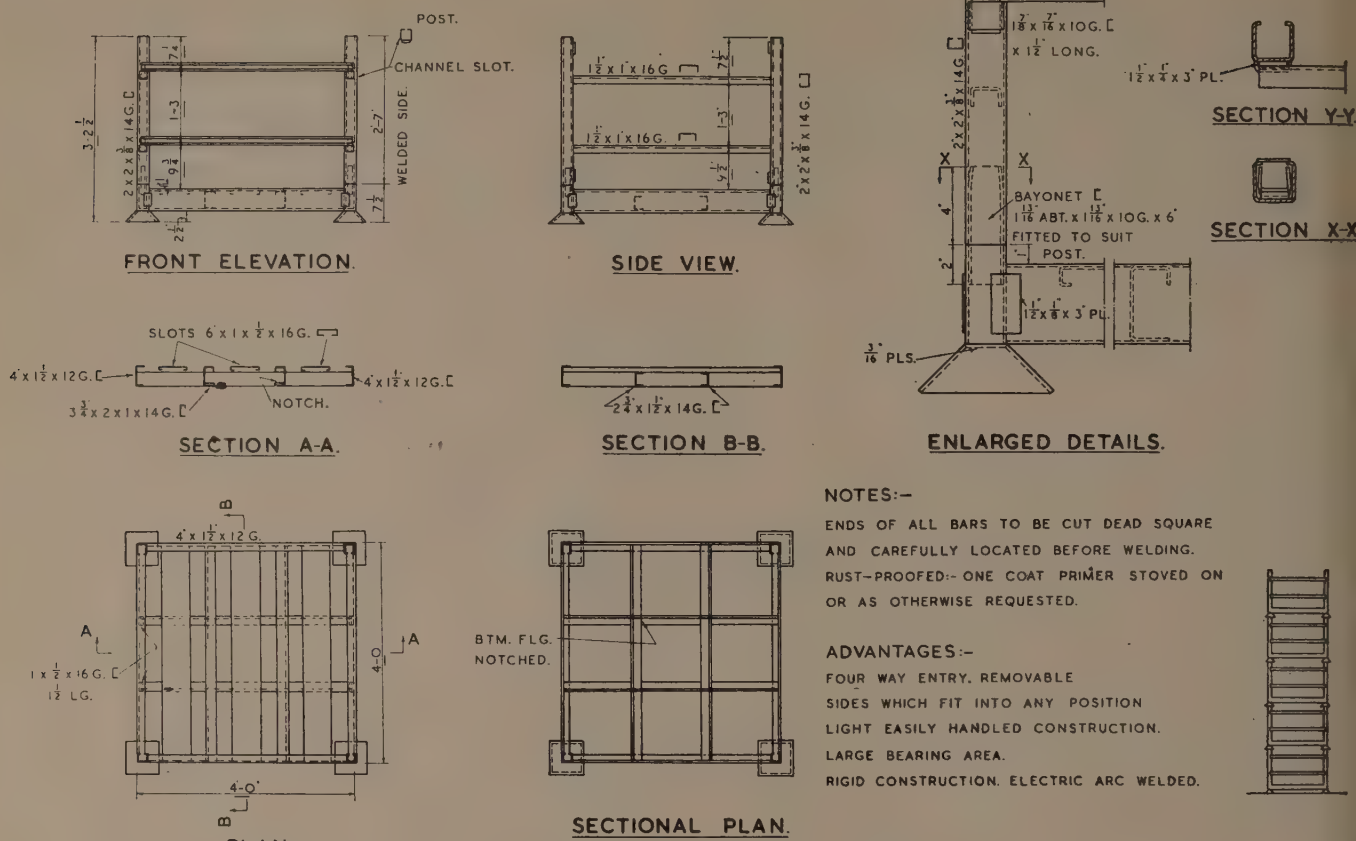
MAIN-TIE

Section	Area sq. in.	Load tons Tension	Load tons Compn.	f_t tons/sq.in.	F_t tons/sq.in.	f_c tons/sq.in.	l/r	F_a tons/sq.in.	b/t	F_a tons/sq.in.
1½" · 2" · ⅜" · 14g	.474	3.62	.83	7.63	8.26	1.75	140	2.23 excl. wind allowance	25	6.35

Comparison of Weights (both based on welded construction)

	Hot rolled construction	Cold formed sections	Saving in weight
Roof truss	500	230	54%
Purlins (12 ft. 6 in. bay)... ...	745	381	49%
Longl. main tie bracing	100	46	54%
	1,345 lb.	657 lb.	51%

FOUR-WAY STACKING PALLET.



NOTES:–

ENDS OF ALL BARS TO BE CUT DEAD SQUARE AND CAREFULLY LOCATED BEFORE WELDING.
RUST-PROOFED:- ONE COAT PRIMER STOVED ON OR AS OTHERWISE REQUESTED.

ADVANTAGES:–

FOUR WAY ENTRY. REMOVABLE SIDES WHICH FIT INTO ANY POSITION
LIGHT EASILY HANDLED CONSTRUCTION.
LARGE BEARING AREA.
RIGID CONSTRUCTION. ELECTRIC ARC WELDED.

Fig. 6.—Four-way stacking pallet

STANDARD 35 ft. ROOF TRUSS

SPAN	35'-0" CRS OF BEARINGS.		
CENTRES	12'-6" MAX. AS DRAWN.		
• TRUSSES AT INCREASED CENTRES CAN BE DESIGNED TO CLIENTS REQUIREMENTS			
PURLINS	10'-0"	CENTRES OF TRUSSES	3½"×2"×⅜"×14G [WITH 1"×10G. SAG FLAT
	12'-6"	DO	4"×2¼"×⅜"×14G [DO.,
	* 15'-0"	DO	5"×2½"×⅜"×14G [DO.,
LONG[CTIES	10'-0"	DO	2½"×1½"×14G [
	12'-6"	DO	3"×1¾"×14G [
	15'-0"	DO	3½"×2"×14G [

ROOF COVERING. LARGE CORR. ASBESTOS CEMENT SHEETS; GALV[D] CORR. STEEL OR R.P.M. SHEETS.

DESIGNED TO B.S. 449: 1948. WIND 75 M.P.H. AT 20 FT
FABRICATION ELECTRIC ARC WELDED IN HALVES AS SHOWN
WEIGHT OF TRUSS.– – – – – – – 230 LBS.

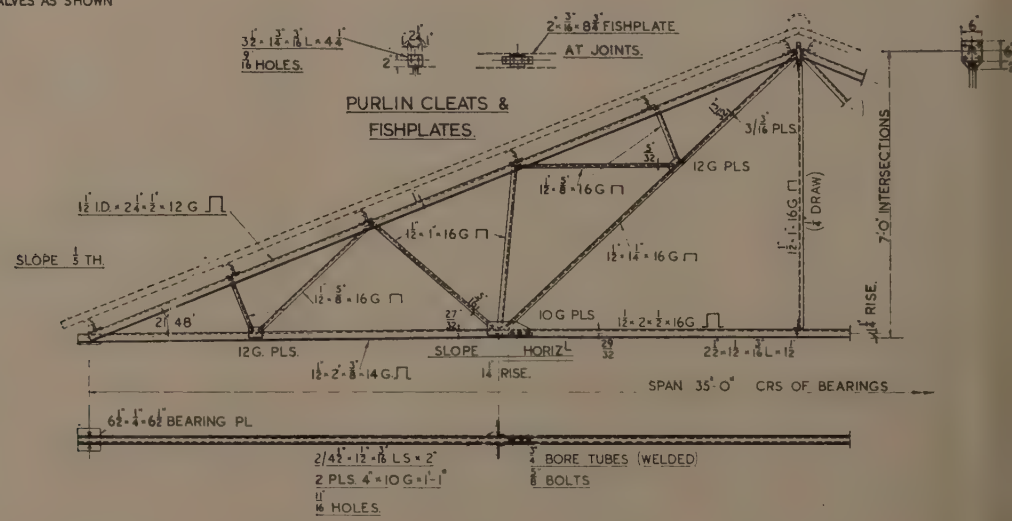


Fig. 7.—35 ft. span roof truss

ALL HOLES 11/16 DIA FOR ⅝ BOLTS UNLESS OTHERWISE STATED. RUST-PROOFING COAT.

It is seen that a saving in weight of approximately 51 per cent. has been effected, being equivalent to a reduction of steel weight per square foot (of floor area) from 3.1 lb./sq. ft. for hot rolled construction (a purposely selected *low* value), to 1.5 lb./sq. ft. for cold formed. In point of fact, savings in weight for roof trusses may be over 60 per cent. when the comparison is made with ordinary hot rolled bolted construction.

The cost of cold formed structures is in no case more than the corresponding hot rolled, and is considerably cheaper, where repetition work is involved. Additional savings in transport and erection should also be noted.

(c) "Cor-Ten" High Tensile Steel Floor Beams and Stanchions

The design shown in Fig. 8 was evolved to save steel in office block construction where large quantities were proposed.

The comparison of weights is as follows:

	Hot rolled construction	Cold formed construction	Saving in weight
Beams or lattice girders	1,197	552	54%
Stanchions	308	160	48%
Tee beams	154	57	63%
	1,659	769	54%

In this case there was no difference in cost.

When applied to light roof construction for schools, this type of lattice girder shows up favourably.

Girders to span 24 ft. at 8 ft. 3 in. crs. to carry 62 lb./sq. ft.

Type of roof beam	Weight per unit
R.S. joist, 10 ft. × 4½ in. × 25 lb.* ...	600 lb. approx.
"Proprietary Brand" bar joist	360 lb. ,,
Lattice girder in C.F.S. sections (M.S.) ...	300 lb. ,,
,, ,, ,, ,, ,, (Cor-Ten) ...	210 lb. ,,

*This section on light side on account of deflection.

Savings in cost also could be effected, when applied to the more or less standard components at present under consideration for modern schools.

(d) Battery Electric Vehicle—Chassis

The chassis frame designed in Cor-Ten (high tensile corrosion-resisting steel) for a 2-ton battery electric vehicle is shown in Fig. 9.

Cold formed sections have, for many years, been used as one of the basic structural materials in automobile construction, and have now been tested and accepted by long records of service. In addition to their high structural efficiency, under all types of static and dynamic loading, the "pay load" of commercial vehicles may be increased appreciably, due to the savings in structural weight effected by this form of construction.

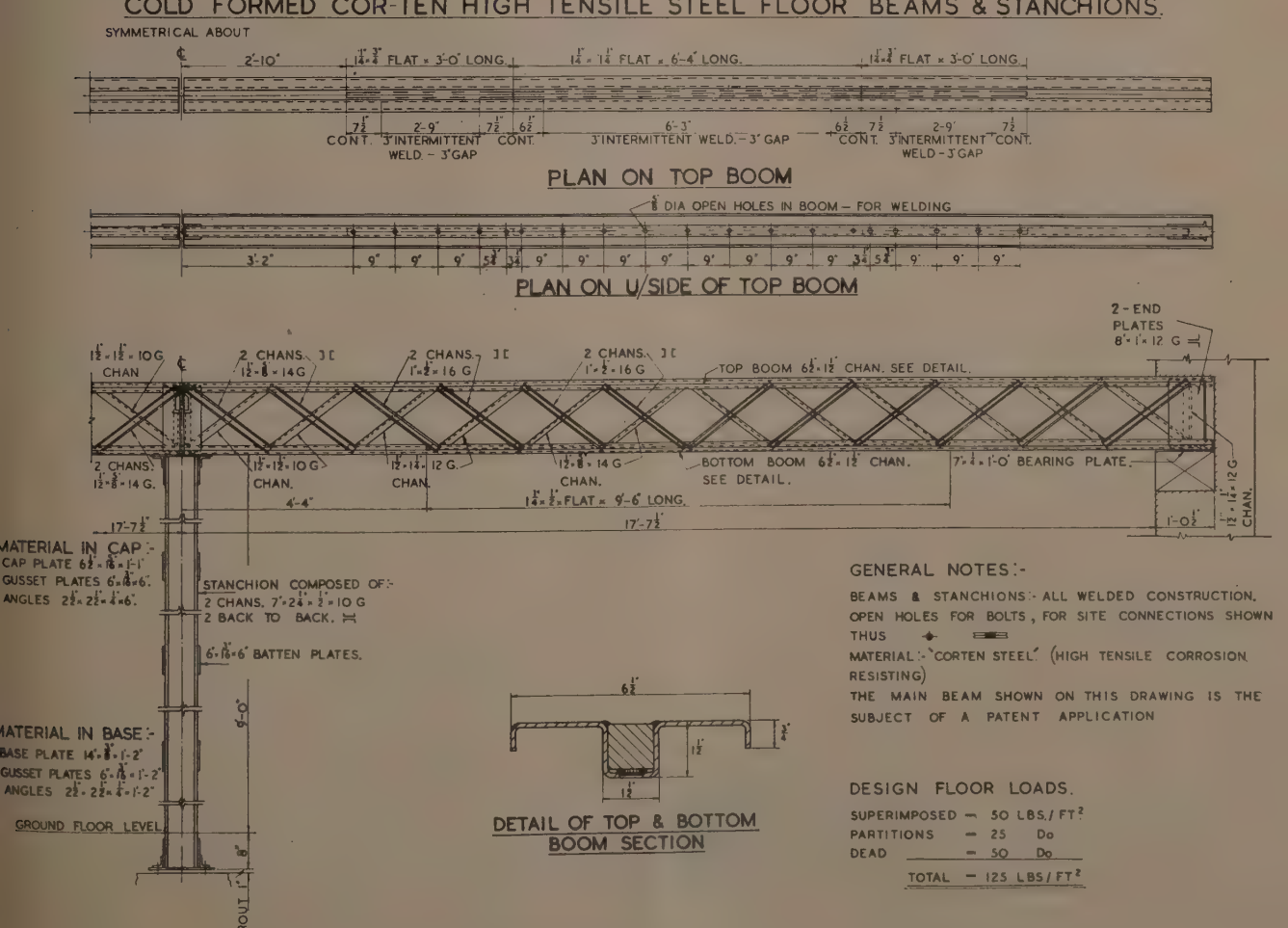

COLD FORMED 'COR-TEN' HIGH TENSILE STEEL FLOOR BEAMS & STANCHIONS
SYMMETRICAL ABOUT
PLAN ON TOP BOOM
PLAN ON U/SIDE OF TOP BOOM
2 - END PLATES 8" · 1" × 12 G
TOP BOOM 6½" · 1½" CHAN. SEE DETAIL.
BOTTOM BOOM 6½" · 1½" CHAN. SEE DETAIL.
7" · ¾" · 1'-0" BEARING PLATE.
MATERIAL IN CAP :-
MATERIAL IN BASE :-
STANCHION COMPOSED OF :-
2 CHANS. 7" · 2¼" · ½" · 10 G
2 BACK TO BACK.
6" · ⅜" · 6" BATTEN PLATES.
GROUND FLOOR LEVEL
GROUT
DETAIL OF TOP & BOTTOM BOOM SECTION
GENERAL NOTES :-
BEAMS & STANCHIONS :- ALL WELDED CONSTRUCTION.
OPEN HOLES FOR BOLTS, FOR SITE CONNECTIONS SHOWN THUS
MATERIAL :- 'CORTEN STEEL' (HIGH TENSILE CORROSION RESISTING)
THE MAIN BEAM SHOWN ON THIS DRAWING IS THE SUBJECT OF A PATENT APPLICATION
DESIGN FLOOR LOADS.
SUPERIMPOSED — 50 LBS./FT²
PARTITIONS — 25 Do
DEAD — 50 Do
TOTAL — 125 LBS/FT²

Fig. 8.—"Cor-Ten" high tensile steel floor beams and stanchions

2-ton battery electric vehicle—"Cor-Ten" high tensile steel frame

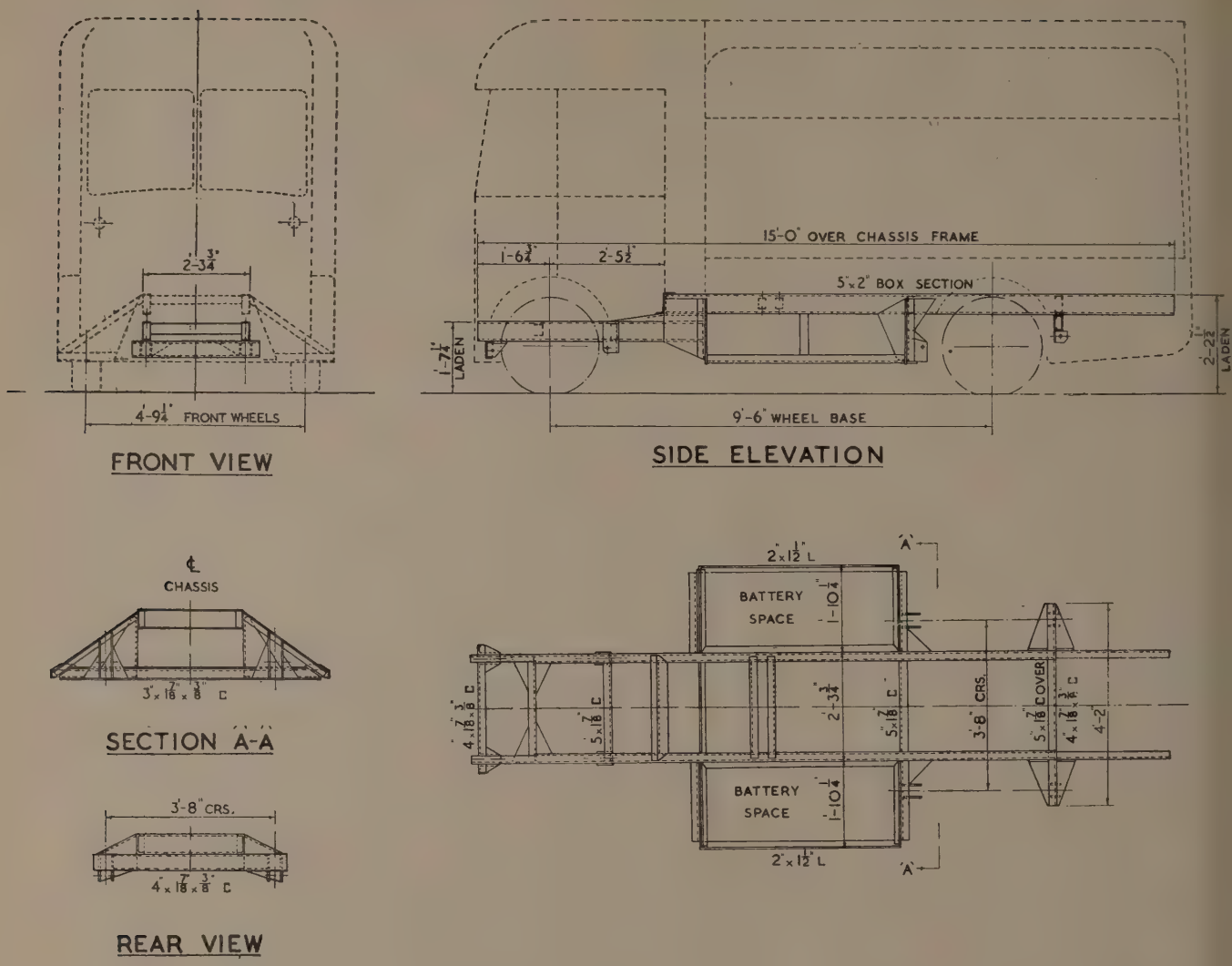


Fig. 9.—Battery electric vehicle—chassis

Summary

The subject matter presented forms, in effect, a survey of present-day knowledge. Care has been taken that the proposed design specification in lay-out and approach should follow the accustomed forms, and thereby ensure that designers experienced in the design of hot rolled constructions will have no difficulty in using it.

Cold formed sections in structural practice have proved themselves in several spheres as being most efficient and economical, either used alone or in conjunction with hot rolled structural shapes. This subject is a very rapidly expanding one, active research being carried on both in this country and America.

It is hoped that the paper will stimulate free and full discussion of this vitally important development, and presently lead to a very much needed Standard Specification.

Acknowledgements

The specification and commentary are based on research work carried out in the Department of Civil and Mechanical Engineering of the Royal Technical College, Glasgow. The author wishes to record his appreciative thanks for co-operation received from members of the College Staff.

Permission from The Ayrshire Dockyard Co. Ltd., Irvine, for use of drawings, photographs, specimens and slides is gratefully acknowledged.

Appendix

Design Examples illustrating Application of Specification.

Ties.

	Section	W/ft. lbs.	Area Sq. ins. Gross		Nett	F_t tons/sq. in.	Permissible Axial Load
1.	⊓ 2 I.D. × 2½ × ⅜ × 14g	1.990	.594	less 2 9/16 holes	.504	8.26	4.16 ton

Appendix—*continued*

Struts.

	Section	W/ft. lbs.	Area sq. ins.	Overall Column l/r	Overall Column F_a tons/sq. in.	Unstiffened Edge b/t flange	Unstiffened Edge F_a tons/sq. in.	Stiffened Edge b/t web	Stiffened Edge F_a tons/sq. in.	Permissible Axial Load
2.	[$3\frac{1}{2} \times 1\frac{1}{2} \times 14g$	1.686	.50	85	4.36	19	3.88	38	5.40	1.94 tons
	Maximum Permissible Stress : smallest of these equal to 3.88									
3.	[$4\frac{1}{2} \times 1\frac{1}{2} \times 14g$	1.957	.58	95	3.88	19	3.88	56	4.00	2.25 ton
	Equal Strength Section									
4.	[$5\frac{1}{2}'' \times 1'' \times 14g$	1.957	.58	95	3.88	$12\frac{1}{2}$	5.40	60	3.10	1.82 tons
	This section would fail with collapse of web									
5.	⊓ 2"I.D. $\times 2\frac{1}{2}'' \times \frac{3}{8}'' \times 14g$	1.990	.549	28	6.98			31.3	5.89	3.24 tons
	⊓ Has splendid structural properties as a strut									

Equal Angle Struts

	Section	W/ft. lbs.	Area sq. ins.	l ins.	r ins.	l/r	F_a tons/sq. in.	b/t	F_a tons/sq. in.	Permissible Axial Lead
6.	$\llcorner 1'' \times 1'' \times 10g$	.760	.224	28	.193	145	2.10	8	6.44	.47 tons
	Column Failure									
7.	$\llcorner 1\frac{1}{2}'' \times 1\frac{1}{2}'' \times 14g$	.775	.230	28	.294	95	3.88	$18\frac{3}{4}$	4.01	.89 tons
	Approx. Equal Strength Section									

Beams.

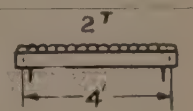
	Section	Span feet	Load tons	Lateral Supports feet	b/t flange	F_{bc} tons/sq.in.	b/t web	F_{bc} tons/sq. in.	Z Reqd. ins.^{3}	Z Given ins.^{3}
8	[$7'' \times 3\frac{1}{4}'' \times \frac{3}{8}'' \times 14g$	12	1.0	1.25	Table 4 41	9.1	Table 5 $87\frac{1}{2}$	9.1	1.98	2.49

Shear checked Table 7 and *Cl.* 5e $F'_v = 3.25$ tons/sq. in. at $\frac{1}{4}$ pt. Defln. within 1/325.

	Section	Span feet	Load tons	Lateral Supports feet	Clause 3b*ii* F_{bc} tons/sq.in.	b/t flange	F_{bc} tons/sq.in.	Z Reqd. ins.^{3}	Z Given ins.^{3}
9	2[$7'' \times 1\frac{1}{2}'' \times 14g$ back to back][	12	10	4	$K_1 = 3.0$ \ 12.8 \ 9.1	18.75 56.3	6.28	2.87	2.82
	Maximum permissible stress due $1\frac{1}{2}''$ flange.								
10	2[$7'' \times 1\frac{1}{4}'' \times 14g$ back to back][	12	10	4	$K_1 = 2.21$	7.43	15.66 47.0	9.0	2.43	2.54
	By adopting $1\frac{1}{4}''$ flange permissible stress increased.									

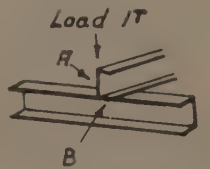
Shear in web.

	Section	Span feet	Load tons	Reaction tons	d/t	F'_s tons/sq. ft.	Web Area sq. ins.	f'_s tons/sq. ft.
11	[$6'' \times 2\frac{1}{2}'' \times \frac{3}{8} \times 16g$	4	2.0 dist.	1.0	93·8	3.04	.376	2.66



Crushing at Reaction and Concentrated Load.

	Section	Load tons	b_p ins.	l ins.	Max. Permissible Reaction or Concentrated Load
12	Top Beam A [$5 \times 2\frac{1}{2}'' \times \frac{3}{8}'' \times 14g$	1.0	2.25	.08	Max. Permissible Reaction at A = .733 ton
13	Bottom Boom B [$4'' \times 2\frac{1}{4}'' \times \frac{3}{8}'' \times 14g$	1.0	2.5	.08	Max. Permissible Con. Load at B = 1.47 tons



Stiffener required at end of top beam only.

Appendix—*continued*

Combined bending and axial compression.

	Section	Load tons	Area sq. ins.	*B M* ins. tons	*Zxx* Comp. Flg.	*l/r*	F_a min. tons/sq. in.	f_a tons/sq. in.	F_{bc} tons/sq. in.	f_{bc} tons/sq. in.
14	⌐ 1½″ ID × 3″ × ½″ × 12g	3.45	.840	1.8	.661	50	5.97	4.11	9.1	2.72

$$\frac{fa}{Fa} + \frac{f_{bc}}{F_{bc}} < \text{Unity.}$$

Combined bending and shear.

	Section	Fixed Span Laterally Supported	Load tons	*Zxx* Given ins³	*d/t*	K_3	f_{bc}	$\sqrt{f_b^5}$	f'_s tons/sq.in.	F'_s × Reducing Factor tons/sq. in.
15	□ 8″ × 2″ × ⅜″ × 14g	8	2.0 dist.	2.175	100	.002	7.36	146.7	1.59	2.86 × .7066 = 1.87

Bearing—Tension Joint.

	Section	Load tons	Diameter of Black Bolts	Bearing Values on Black Bolts as B.S. 449 : 1948		Number of Black Bolts Required
				14g (.08″)	10g (.125″)	
16	2 □ 8″ × 1″ × 14g][with central 10g gusset	2.0	⅝″	2 × .4 = .8	.624	say 4

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Annual Dinner

The Annual Dinner of the Institution of Structural Engineers was held at the Dorchester Hotel, London, W., on March 28th, 1951. The President, Mr. J. E. Swindlehurst, O.B.E., M.A., M.I.C.E., was in the Chair.

The loyal toasts having been honoured,

The Rt. Hon. The Earl Mountbatten of Burma, K.G., P.C., G.C.S.I., G.C.I.E., G.C.V.O., D.S.O., proposed the toast of the Institution, and voiced his sense of great honour and privilege in so doing. It was always slightly embarrassing on such occasions, he said, to decide what to say. There being so many members present who knew more than he did about the Institution, it might appear rather ridiculous that he should give them particulars of their organisation. On the other hand, there were so many guests present who knew probably less about it than he did, so that it might not be out of place to spend a few minutes reviewing the position of the Institution.

Recalling that it was founded in 1908, he said that after attaining its Silver Jubilee it received a Royal Charter, and thus joined the great Chartered bodies in this country. Its membership had increased steadily and healthily, and this year it had achieved the record figure of nearly 6,000. There were nine Branches in the United Kingdom and there were representatives of the Institution in every part of the Commonwealth. Each year some 600 candidates were examined in the United Kingdom, and in up to 35 centres overseas.

Those facts were sufficient to indicate the important position the Institution held. In the 43 years of its existence its advance had coincided with the advances that had taken place scientifically and industrially, and it was to a large extent because of those advances that the Institution had prospered. It could not have achieved its national stature, however, without extremely able guidance from its officers, and in particular, he had no doubt, from its present President.

Commenting that he had no technical qualifications to talk about structural engineers, Earl Mountbatten said the only connection he had had with them was in Combined Operations during the war. During most of the first two years of the war he was at sea in a destroyer flotilla, and after most of the ships had been sunk or damaged he had taken command of the aircraft carrier H.M.S. *Illustrious*, in America. But before her refit was completed he was called back by telegram, and was told to report to the Minister of Defence, Mr. Winston Churchill, now an Honorary Member of the Institution.

That was in October, 1941. Recalling the situation that had prevailed at that time, nine and a half years ago, he said that all our Allies had been overrun by the Germans, and their territories were in enemy hands, except Russia, who had become our Ally only just before, and she was in process of being rapidly overrun by the Germans. At that time America had not come into the war, nor did there seem the slightest likelihood of her coming into it. It was then that that remarkable war leader, Mr. Winston Churchill, in giving Lord Mountbatten his instructions for his new post, had made a statement which was sufficiently remarkable to be repeated ; and so far as he knew, it had not previously been made public. Earl Mountbatten was told that he was to relieve Admiral of the Fleet Lord Keyes ; and he remarked, " That was quite awkward, because Admiral Keyes had been Commander-in-Chief of the Mediterranean Fleet when I was a two-striper on his staff, but the Admiral was most kind and helpful." Mr. Churchill had added : " I want you to take over Combined Operations, to carry out raids with Commandos on the enemy-held coasts. But your primary job is to prepare for the invasion of the Continent, because it is time we thought about taking the offensive."

There could hardly have been a moment in our history when it looked less likely that we should be able to turn to the offensive than in October, 1941 ; and it was a measure of Mr. Churchill's greatness that that was the moment he had chosen to turn his mind to the offensive.

The preparation for the offensive was a gigantic task. The technique of an assault on an enemy-held coast had yet to be devised, because there had been no cases of really successful assaults in recent history. Arrangements had also to be made for the follow-up of troops when the assault had taken place, so that our build-up would be quicker than that of the enemy, and for our supplies to be continued across the beaches until a port was captured.

For the purpose of breaching a strongly defended coast quite obviously air power was essential. The tendency was naturally to stress the advantage from that point of view of an attack on the Pas de Calais, because we could then bring our fighters easily to bear ; indeed, it was so obvious that the Germans had come to the same conclusion and had defended that part of the coast so strongly that an attack there would have been quite unprofitable.

The part of the coast which was completely undefended was the Atlantic coast, in the Bay of Biscay. Earl Mountbatten said his mind had turned to the possibility of an assault landing where there would be no opposition ; but we must have air cover continuously on a very large scale, and aircraft carriers could not have provided it, because it was precisely in the Bay of Biscay that enemy submarines could operate in the greatest numbers, and they could have sunk or damaged all the aircraft carriers ; so we could not have sustained the invasion long enough to be able to build up.

We wanted an island near the coast of France, in the Atlantic, from which aircraft could be operated ; and a brilliant man on his staff, Mr. Geoffrey Pyke, had put up a solution which would interest structural engineers technically. The suggestion was to build a floating airfield of huge dimensions, displacing one million tons and having a runway 3,000 ft. long and 300 ft. wide, fitted with electric motors in nascelles projecting on each side to give power for about three to four knots. It was to be virtually unsinkable, that is, capable of withstanding at least 50 or 60 torpedoes before it could be put out of action, and proof against being put out of action by bombing.

Mr. Pyke had put forward the idea of building the island or ship out of a form of ice. The scheme had the name of " Habakkuk," and the material was called " Pykerete," after the inventor. It consisted of about 95 per cent. water, the remaining 5 per cent. consisting

of very cheap ingredients mixed up and frozen together, producing a mixture practically as strong as concrete.

The next thing was to "sell" the idea to the right people. "I went to Chequers," he said, "arriving at about the time Mr. Churchill was getting ready for dinner, and I was told that he was in his bath. I said, 'Good, that is exactly where I want him to be.' I nipped up to his dressing-room and called out to him that I had a block of new material which I would like him to put in his bath. I gave him a block of Pykerete."

One of the curious properties of that material was that, after the surface had melted, the melting practically stopped, because the ingredients formed a furry insulating coating on the outside. The fact that the block did not melt appreciably in a hot bath impressed the Prime Minister so much that he backed further investigation.

Details were worked out, including building sites in Newfoundland, where the material would freeze on being poured on.

At the suggestion of Mr. Churchill, he had arranged to submit the Habakkuk Project to the Quebec Conference in August, 1943, when there was a meeting of the Combined Chiefs of Staff. The difficulty was to sell the idea to the Americans. But an opportunity occurred when, quite early in the conference, a disagreement had arisen between the British and American Chiefs of Staff. At those conferences the Principals of the two Chiefs of Staff Committees sat around the table, and behind them the members of the Staffs, very senior officers, looked on with stony eyes when they felt their seniors were selling the pass. The Chief of the Imperial General Staff, Field-Marshal Lord Alanbrooke, had suggested that the members of the Staffs should leave the room, and that the Principals should sit around the table and continue their talk alone. The Staff did leave, wondering what their bosses would do in their absence; but it was not long before harmony was restored.

It was then that he had asked permission to introduce "Habakkuk." He had a sample of Pykerete available to compare with a sample of ice, and he asked that he might bring it in. Permission was given, and in came a member of his staff with a dumb-waiter, on which he wheeled a large block of ordinary ice and a large block of Pykerete, plus a kitchen chopper. Earl Mountbatten asked which of the Chiefs of Staff was the strongest, and General Arnold (then Chief of the United States Army Air Forces), took off his coat, seized the chopper and brought it down on the block of ice, which was satisfactorily split. Then he spat on his hands, seized the chopper again and advanced on the Pykerete, little knowing what he was in for. He brought the chopper down, but did not even dent the Pykerete; instead, he jarred his elbows and gave a shriek of pain. The members of the staffs outside had heard the blow and had recognised the voice of one of their Chiefs in pain, and they became rather worried.

Then Lord Mountbatten drew his Service revolver and fired a shot at the ice, which was shattered. To demonstrate that Pykerete would stand up to gun-fire, he fired a shot at that. The bullet bounced off and nearly hit Sir Charles Portal (then Chief of the Air Staff); and outside, the white-faced staff were heard to say, "My God, they've started shooting!"

However, the idea of landing on the Biscay coast was turned down in favour of a landing between Cherbourg and Le Havre, and so the idea of using Pykerete platforms was not taken up, although it had appeared not nearly so far-fetched as some of those that were taken up. The idea that was adopted—the Mulberry harbour—had sounded rather more crazy than Habakkuk when he first put it forward.

The story of Mulberry was rather like the story of the tank; no one person had thought of it. When considering the problem of the assault on France it was realised, from weather statistics covering a period of many years, that even in the best of summers there were rarely more than three or four days on which the weather was completely calm; so that we could not count on a period of more than three or four days during which to bring in further waves of assault troops and maintain supplies across the beaches.

The first obvious answer was that we had to capture a port. We had tried that at Dieppe, and we discovered that we could not do it; to have tried it again and failed would have been a disaster of the first magnitude. At the end of 1941 or the beginning of 1942, one of his staff, Captain Hussey, had put up the idea of a Bubble breakwater composed of a line of pipes in the sea, with holes in them fed by an air compressor; air bubbles would be pushed out into the sea, producing a sort of elastic medium through which the wave motion was not transmitted. Earl Mountbatten had mentioned it to the Prime Minister, who had added the idea of having piers. Then Captain Hughes Hallett, another member of his team, had suggested block-ships; and from there it was but a step to the Mulberry harbour, which presented the most difficult structural engineering problems during the war and was the greatest contribution made by structural engineering in that period. He put another member of his staff, Major-General Sir Harold Wernher, himself a great industrialist, in charge of organising the administrative arrangements for its construction. Without his great drive and ability, Mulberry could never have been completed in time. It was the greatest single effort of construction in this or any other war, and he paid tribute to the fact that it was a structural engineering effort.

Finally, Lord Mountbatten expressed his interest in the account of the construction of the Royal Festival Hall, by Mr. E. Roland Hole and Mr. F. C. Eales, published in the March number of THE STRUCTURAL ENGINEER, which he described as fascinating, and he commented particularly on the measures that were taken to ensure that the workmen engaged there were kept in touch with day-to-day developments and were made to feel the importance of their part in the project. That, he said, was terribly important, and it was perhaps a remarkable fact that structural engineers, who had to concern themselves with so many materials, had kept right abreast of the times and realised the importance of the human element; he congratulated them on that fact.

The PRESIDENT, replying to the toast, said:—

My Lords, Your Excellencies, My Lord Mayor, Ladies and Gentlemen:

My first duty this evening is to say on your behalf and my own, a very sincere "thank you" to our Guest of Honour for his most inspiring and interesting speech.

Men of action are said to be slow to speak, and it is further said of them that they only speak when they have something to say. I submit to you to-night that our Guest of Honour falls into the second category, while I, your spokesman on this memorable occasion, regretfully admit that I fall into the first.

All my fellow-members here to-night may, I think with justice, claim to be men of action, but none of us can compare in that sphere with our distinguished guest. Action has been the keynote of his life from early days at Osborne, at the Royal Naval College, Dartmouth, and throughout his service in the First War, followed by a short sojourn at Cambridge University, to the present day.

Mention of Dartmouth reminds me of a little story, alleged to be true, which is told of a certain cadet.

The weather in 1916 was as freakish in that part of the world as it has been this Easter, and contrary to all precedent, there was a heavy snowfall which covered the hills in the vicinity of the College. There was an immediate rush by the cadets with improvised sledges, tea-trays, in fact, anything that would slide, and one of them had the misfortune to sustain an accident which necessitated enforced inactivity in sick bay. Even the accident, which was of a painful nature, could not damp the spirits of that young man. Getting rather bored, he conceived the brilliant idea of inserting an advertisement in the National Press, giving his name and inviting correspondence. The response was rapid; within 48 hours a whole mail-van of letters arrived for him. The next day three mail-vans arrived. That was too much for the Captain of the College, who promptly issued orders diverting all further correspondence—presumably to salvage.

Wild horses would not drag from me the name of that cadet. I rather think, however, that at least one other person in this room beside myself might hazard a good guess at his identity.

Entering the Royal Navy as a naval cadet in 1913, he became a midshipman in 1916, lieutenant 1920, and lieutenant-commander in 1928, and achieved his "brass hat" as commander in 1932. So far he had attained the most rapid rate of promotion possible to any naval officer, but when I tell you he was promoted to captain in 1937, to commodore in 1941, and rear-admiral in 1946, you have some measure of his outstanding quality.

During the First War he served in H.M.S. *Lion* and in H.M.S. *Elizabeth*, and between the wars in H.M.S. *Repulse*, H.M.S. *Renown*, and H.M.S. *Revenge*. In 1929 he became Senior Wireless Instructor, Signal School, Portsmouth, Fleet Wireless Officer to the Mediterranean Fleet in 1931-1933, later commanding H.M.S. *Daring* and H.M.S. *Wishart*, before going to the Admiralty in the Naval Air Division in 1936. Of his service in the last war, I would mention that he commanded H.M.S. *Kelly* and the Fifth Destoyer Flotilla in 1939, and H.M.S. *Illustrious* in 1941. Appointed Chief of Combined Operations in 1942-1943, he reached acting rank of vice-admiral, and held at the same time acting rank of lieutenant-general and air-marshal. What a combination!

On becoming Supreme Allied Commander South-East Asia Command, he was advanced to acting admiral, and was still holding that command at the victorious conclusion of the war in Burma, with which his name will be for ever linked.

The year 1946 found him in command of the First Cruiser Squadron, with the substantive rank of rear-admiral, but he was not long to remain in that position, for, in 1947, he succeeded the late Lord Wavell as Viceroy of India. He took a prominent part in the procedural plan for the transfer of power to India on a Dominion status basis in 1947, and by the choice of the Indian Congress, later became Governor-General of the new Dominion of India, holding office until June, 1948. What a record for any one man!

It is told of him that, during the war, when in command of His Majesty's ships, he invariably fought the enemy to a standstill, and had more ships sunk under him than any other commander of that time. I would not mind wagering that he, as an old Cantab., was never so completely sunk as was poor old Oxford last Saturday!

It is a great pleasure to me to be able to tell you that the stroke of the victorious Cambridge boat is present among us to-night—Mr. Jennens, the son of the Chairman of our Midland Counties' Branch. At the risk of being accused of promoting international discord, I am wondering whether the fact that the Cambridge crew took their orders through an English cox as distinct from the Oxford crew, who were coxed by an American, had anything to do with the outcome of the race.

At the risk of embarrassing our guest, which, if I do so, I trust he will forgive, may I just enlarge for a few moments on the theme of the so-called Silent Service. Such a career could only come to a man of exceptional talents, but I feel sure that he would be the first to admit that no small credit is due to the Service which nurtured him, and to which he has given life-long allegiance.

What is the Navy's secret? Can we learn from it any lessons from which we as a profession may gain benefit? Perhaps we can.

The Navy is a very highly trained service from the technical point of view, but before all technical achievement they place leadership. Responsibility is thrust upon them from an early age. They must show initiative and yet remain loyal members of a team. They must learn to obey orders as well as on occasion to disregard them. They must be tactful, quick in decision, but above all, must never lose the human touch.

Humour is never missing from the Navy. I would like to tell you a little story of a young lieutenant who found himself in charge at Tobruk, in the course of the North African campaign, with refugees, troops, prisoners, and a thousand and one other people crowding in upon him. Rations were reaching a somewhat parlous condition, so parlous indeed that he felt impelled to send out a signal setting out the situation in which he found himself, and requesting advice and help. Judge of his astonishment when the return signal came back in just these few words: "Over to you: Take care you do not get eaten."

With all this, therefore, I feel we can to-night derive real inspiration from the very personality of our guest himself, from what he has had to say, and from the traditions of the Service that he adorns.

Let us go forward to our own tasks, determined that every command with which we are entrusted shall be a happy ship.

That is what we have always sought in our Institution, and what we like to feel we have attained. Our relations with our sister Institutions, and the Government Departments with which our work brings us into close contact, have been uniformly happy, and as the years have gone by, have progressed from strength to strength.

This evening we are gathered together for a social occasion, and it does not appear to me that this is an appropriate moment in which to launch into a sea of

statistics. Let it suffice, therefore, to say that our numbers have increased considerably since we last met for a similar evening's enjoyment, and that the number of persons desirous of sitting for our examinations has reached new records.

I submit that there must be something very fine in the traditions of an Institution for such consistent progress to be possible.

To return to the Navy theme for a moment before concluding, I do want, as your President, to thank the Chairmen and the Honorary Secretaries, the Captains and Commanders as it were of the ships that comprise our Fleet, for their tact, guidance and diplomacy when showing the Institution flag in what are, at times, somewhat troubled waters, and last but not least, I want publicly to thank the Commander of our Flagship, Major Maitland, and Jimmy I, the indefatigable Mr. H. R. Gray, and all associated with them, for their sterling work throughout the year, and for the way in which they have organised to-night's function and ministered to our enjoyment.

The theme of my remarks has been "Action," and I cannot sit down without again thanking our distinguished guest for his most inspiring speech and for his graciousness in honouring us with his presence to-night.

May I, as my last word this evening, give you a thought for to-morrow: Think as men of action; act as men of thought."

Mr. Walter C. Andrews, O.B.E., M.I.C.E. (Vice-President), proposed the toast of The Guests, and this was responded to by Dr. W. H. Glanville, C.B.E., D.F.C., M.I.C.E., M.I.Struct.E. (President of The Institution of Civil Engineers).

Institution Notices and Proceedings

ORDINARY GENERAL MEETING

An Ordinary General Meeting of the Institution of Structural Engineers was held at 11, Upper Belgrave Street, London, S.W.1, on Thursday, April 26th, 1951, at 5.55 p.m., Mr. Walter C. Andrews, O.B.E., M.I.C.E., M.I.Struct.E. (Vice-President), in the Chair.

The Minutes of the Ordinary General Meeting held on February 22nd, 1951, as published in the Journal in April, were taken as read, were confirmed and signed.

The following members were elected in accordance with the Bye-Laws. Will members kindly note that the elections, as tabulated below, should be referred to when consulting the Year Book for evidence of membership?

Students

Brockbank, Edwin, 17, Burleigh Road, Gorse Hill, Stretford, Lancs.

Chappell, Ronald Charles, "Sayesmore," 79, Liberty Lane, Addlestone, Surrey.

Davidson, Gerald Keith, 46, Fairfield Crescent, Huyton-cum-Roby, Liverpool.

Doyle, William Sherwood, "Aldbury," Grove Road, Rondebosch, Cape Town, South Africa.

Freiter, Ian Neville Ferguson, c/o Mrs. Cassells, 229, North Street, Glasgow, C.3.

Jefferies, Robert Lionel, c/o Dorman Long (Africa), Ltd., Craigs Battery, Woodstock, Cape, South Africa.

Marriott, Philip Stanley, 171, Cranston Road, Forest Hill, London, S.E.23.

Nettleton, John Clive, 115, Nottingham Road, Kensington, Johannesburg, South Africa.

Norman, Kenneth Peter, 189, Burntwood Lane, Wandsworth Common, London, S.W.17.

Rogers, Kenneth Jordan, "Harpenden," Burton Road, Findern, Nr. Derby.

Steele, Alan David, 74, Evering Road, London, N.16.

Strachan, Alan, 29, George Street, Altrincham, Cheshire.

Wild, Peter, 11, Brumby Wood Lane, Scunthorpe, Lincs.

Wilkinson, Herbert, 57, Smith Drive, Orford, Warrington, Lancs.

Williams, Robert Raymond, 253, Townsend Avenue, Liverpool, 11.

Graduates

Adasko, Stanislaw, 96, Victoria Road, Salford, 6, Lancs.

Anderson, Joseph, 48, Langcroft Road, Glasgow, S.W.1.

Ausden, Alistair Charles, 78, Herkomer Road, Bushey, Herts.

Baker, Alexander John, 17, Overhill Road, East Dulwich, London, S.E.22.

Bharucha, Sorab, 103, Fellows Road, London, N.W.3.

Boggon, William George, B.Sc.(Civil), Bristol, 20, Lawn Road, Rowley Park, Stafford.

Burslem, John Austin, 1, St. Brannocks Road, Cheadle Hulme, Cheshire.

Carter, William Geoffrey, 8, Lyndene Gardens, Kingsway, Cheadle, Cheshire.

Daly, James Gerard, B.E.(N.U.I.), Lammascote House, Lammascote Road, Stafford.

Doyle, Patrick Joyce, B.E.(N.U.I.), A.M.I.C.E., c/o Taylor Woodrow (E. A.), Ltd., P.O. Box 2256, Nairobi, E. Africa.

Easton, Edgar, 21, Farmhouse Road, Streatham Vale, London, S.W.16.

Fagan, John George, Delivery Counter, G.P.O. Wellington, New Zealand.

Garthwaite, Charles Henry, 187, Stoke Newington High Street, London, N.16.

Gillbanks, William James, 80, Heys Road, Prestwich, Manchester.

Gould, Noel Brian, 6, Hope Avenue, Brakpan, Transvaal, South Africa.

Hancock, Trevor Gordon, B.A.(Cambridge), c/o Messrs. Scott & Wilson, 47, Victoria Street, S.W.1.

Harris, Clifford James, P.O. Box 645, Port Elizabeth, South Africa.

Hughes, John Tudor, B.Sc.(Eng.), Rand, 21, Burton Manor Road, Stafford.

Huskins, Leslie Roy, 30, Angus Mansions, 266, Jeppe Street, Johannesburg, South Africa.

Kershaw, Albert, 22, Roxholme Avenue, Harehills, Leeds, 7.

Kinnaird, George Alfred, 15, Ailsa Drive, Giffnock, Renfrewshire.

Lineker, Leslie, 69, Beam Hill, Burton-on-Trent, Staffs.

Manuell, Herbert Joel, 78, Mentz Street, Booysens, Johannesburg, South Africa.

Meredith, William James, 44, Watersfield Way, Canons Park, Edgware, Middlesex.

Mitra, Asit Kumar, 64, Fern Avenue, Newcastle upon Tyne, 2.

Morgan, Ivor Charles, c/o Reinforcing Steel Co., Ltd., P.O. Box 117, Salisbury, Southern Rhodesia.

NICHOLS, George William, 63, Usher Road, Bow, London, E.3.
NZEGWU, Alexander Chukwuemeka, B.Sc.(Hons.)(Civil), Wales, c/o University College, Singleton Park, Swansea.
OLDHAM, Norman Arthur, A.M.I.Mun.E., c/o Miss Tarvin, 2, Avenue Road, New Malden, Surrey.
PITT, Peter Hubert, 10, Reynolds Close, London, N.W.11
PREECE, Stanley George, 235, Hale End Road, Woodford Green, Essex.
RAYNER, Patrick Anthony, "April Cottage," Gateland Lane, Shadwell, Leeds.
RENDALL, William Cutt, 7, Victoria Gardens, Wembley, Middlesex.
ROGERS, Arthur David, B.Sc.(Eng.), London, Station Road, Netley Abbey, Nr. Southampton, Hampshire.
SALEEM, Muhammad Akbar, 57, Abingdon Villas, London, W.8.
SARKAR, Sujit Kumar, B.E., Calcutta, c/o C. W. Glover & Partners, 54, Victoria Street, London, S.W.1.
SHEARCROFT, Richard, B.Sc.(Eng.), Edinburgh, 74, Common Road, Kensworth, Beds.
SHERWIN, Albert, 31, Hamilton Road, Walthamstow, E.17.
SHINDE, Kashinath Madhav, B.E., Bombay, 210, Budhwar Peth, Poona 2, India.
STOCH, Solly Gerald, B.Sc.(Eng.), Rand, 18, Linden Road, Emmarentia Ext., Johannesburg, South Africa.
STONEBRIDGE, Marcus Allan, A.M.I.C.E., A.M.I.Mun.E., 1, High Street, Lowestoft, Suffolk.
TAYLOR, James Edward, 38, Edwards Road, Erdington, Birmingham, 24.
TIBBETTS, William Arthur, 75, Ralph Road, Shirley, Nr. Birmingham.
TYLMAN, George Anthony, 14, Cambalt Road, London, S.W.15.
WESTALL, Thomas Rimmer, Franks Farm, Culcheth, Nr. Warrington, Lancs.

ASSOCIATE-MEMBERS

ARMSTRONG, Ian Fraser, 1, Rykneld Drive, Littleover, Derby.
BATH, Charles Frederick, 6, Queensgate, South Kensington, London, S.W.7.
BOGEN, Alexander David, 25, Chardmore Road, London, N.16.
CASS, Ronald Harry, A.M.I.C.E., A.M.I.Mun.E., Frears Way, Morley St. Botolph, Nr. Wymondham, Norfolk.
JALINK, John Bernard, P.O. Box 1091, Johannesburg, South Africa.
LEE GWOH LIANG, 26, Flower Market Road, 3rd Floor, Kowloon, Hong Kong.
MULLINS, Leslie Herbert MacLeod, B.Sc.(Eng.), London, 271, Nether Street, West Finchley, London, N.3.
NEWTON, Peter Dennis, B.Sc.(Hons.) (Eng.), London, A.C.G.I., 6, Clabon Road, Norwich.
OGDEN, John, "Lynwood," Hill Cliffe Road, Walton, Warrington, Lancs.
OSGERBY, Colin, B.Sc.(Eng.), London, A.M.I.C.E., A.M.I.Mun.E., 9, Park Avenue, Roker, Sunderland.
RAMSEY, Frank Cyril, c/o Electricity Supply Commission, Box 1091, Johannesburg, South Africa.
SHORTER, Christopher Thomas, 42, Webb Street, Northmead, Ext. 3, Benoni, Transvaal, South Africa.
STAKE, David Walshaw, B.Sc.(Civil), Manchester, Brown O'Sullivan & Partners, 16, Gloucester Place, London, W.1.
WARD, Walter Burton, A.M.I.Mech.E., 83, Hale Gate Road, Halebank, Widnes, Lancs.
WEBBER, Norman Bruton, B.Sc.(Hons.) (Eng.), London, A.M.I.Mun.E., c/o Borough Engineer's Department, Town Hall, Worthing.
WHITLAM, Edwin Frederick, M.Sc.(Eng.), London, A.M.I.C.E., 58, Hillcrest Road, Orpington, Kent.

MEMBER

CRERAR, Finlay, C.B.E., Merrywood, Ellerton Road, West Wimbledon, London, S.W.20.

TRANSFERS

Students to Graduates

BROTTON, Arthur Reginald, 42, Roman Road, Linthorpe, Middlesbrough.
CHRISTIE, Dennis, 37, Grasmere Crescent, Sinfin, Derby.
CLARK, Raymond Bostock, c/o Dorman Long (Africa), Ltd., P.O. Box 68, Germiston, South Africa.
DUNN, William Nicoll, 43, Banstead Road, Carshalton, Surrey.
DZIEWULSKI, Jerzy, 26, Spruce Hills Road, Walthamstow, London, E.17.
FROST, Archibald William Douglas, 15, Peckham High Road, Peckham, London, S.E.15.
HALLOWS, Brian Hector, 23, Dorset Avenue, Romford, Essex.
HICKS, Edward George, 67, Hamilton Avenue, Ilford, Essex.
HOARE, John Hubert, 16, Rivercourt Road, London, W.6
JAMES, Arthur Neville, 20, Birkett Drive (off North Ridge Road), Durban, South Africa.
JAMES, Dennis Frederick, 29, Barriedale, New Cross, S.E.14.
JAMES, John Fraser, 47, Castle Street, Port Talbot, Glams.
KACZKOWSKI, Tadeusz Adam, 43, Bromley Road, Walthamstow, London, E.17.
MCCARTE, John, 314, Carntynehall Road, Glasgow, E.2.
MILGATE, Peter, 3, Saxon Avenue, Crumpsall, Manchester, 8.
NOLLER, Gerald Rous Allen, 27, Franciscan Road, London, S.W.17.
PRIDDEY, Harry Walter, 75, Ira Street, Miramar, Wellington, New Zealand.
ROWLEY, John Charles, 63, Pelham Road, Wimbledon, S.W.19.
SUTTON, Derek Harold, 65, Beresford Road, Chingford, E.4.
TATT, Robert James, 58, St. Marks Road, Bush Hill Park, Enfield.
TRANFIELD, Raymond, 16, Rivercourt Road, London, W.6.
WHITE, Allen Robert, 1, Blenheim Lodge, Hadley Road, New Barnet, Herts.
WIDDRINGTON, Thomas Anthony, c/o Wm. Bain & Co., Ltd., Lochrim Works, Boksburg, Johannesburg, South Africa.
WOJCIECHOWSKI, Jan, 63, Rectory Road, Walthamstow, E.17.

Student to Associate-Member

RUSSELL, John, 33, Loomsway, Irby, Heswall, Wirral, Cheshire.

Graduates to Associate-Members

BANNATYNE, William, B.Sc.(Hons.) (Civil), Glasgow, 20, Killearn Drive, Ralston, Paisley, Renfrewshire.
BARRON, Thomas, 18, Edith Street, Middlesbrough.
BARRY, Maurice Henry Alfred, 70, Minet Road, Brixton, S.W.9.
BROCKLEHURST, Frank, 46, Boston Road, Holbeach, Spalding, Lincs.
BUCHAN, Ian, B.Sc.(Civil), Glasgow, c/o P.W.D., P.O. Box 137, Lusaka, Northern Rhodesia.

CANTLAY, William Gordon, B.Sc.(Civil), Glasgow, 53, Queen Square, Strathbungo, Glasgow, S.1.
CAUNT, George, 31, Downing Street, Sutton-in-Ashfield, Nottingham.
CLARKE, Godrey Baynes, 110, Mayfield Road, Sanderstead, Surrey.
COOPER, John Reginald, Flat 5, "Longcroft," Southdown Road, Shoreham-by-Sea, Sussex.
COX, Dennis, B.Sc., Wales, 103, North Road, Longsight, Manchester, 12.
DALE, Hiliary Michael, A.M.I.C.E., 6, Wilson Road, Sheffield, 11.
DE SILVA, Muttuwa Sarukalige Maitripala, B.Sc.(Eng.), London, 83, Emmanuel Road, London, S.W.12.
DODSWORTH, Foster, 8, Palm Street, Middlesbrough, Yorks.
DOWNES, Arthur Harvey, 7, Kensington Road, Manchester, 21.
EATON, Kenneth Vernon, 66, Windmill Road, Hemel Hempstead, Herts.
ELIA, Panicos, c/o Reinforcing Steel Co., Ltd., Nairobi, Kenya.
ERULKAR, Jonathan Eliab, B.E.(Civil), Bombay, Sarah Lodge, 48, Shivaji Park, Bombay 28, India.
ESAKOF, Edgar Arnold, B.Sc.(Eng.), Rand, A.M.I.C.E., 16, St. Georges Street, Yeoville, Johannesburg, South Africa.
FARROW, George Albert, "Locksley," Etherley Lane, Bishop Auckland, Co. Durham.
FORTEY, Joseph William, 10A, Aston Street, Birmingham, 4.
GHOSH, Aravinda, B.Sc., B.E., Calcutta, c/o S.T.U.P. (Procedes Freyssinet), 42, Avenue de Friedland, Paris 8e.
HANNA, Milad Mikhail, 9, El-Eskandarani Street, Shubra, Cairo, Egypt.
HEIKEL, Bernard Hugo, 528, Gorton Road, Reddish, Stockport, Cheshire.
HUSBAND, Richard, 83, Roman Road, Middlesbrough.
KING, Charles William, 33, Old Deer Park Gardens, Richmond, Surrey.
LANGFIELD, Ronald Alexander, M.Eng., Liverpool, A.M.I.C.E., "Beechwood," 11, Rock Park, Rock Ferry, Cheshire.
LOCK, Kenneth Alfred, 83, Longlands, Adeyfield, Hemel Hempstead, Herts.
LOWSON, Barry Christian, 42, Paris Court, 89, Juta Street, Braamfontein, Johannesburg, South Africa.
MOSELEY, Arthur Raymond, 3, Belmont Road, Penn, Wolverhampton.
PAYNE, Richard James, B.Sc.(Eng.), London, 16, Dufton Road, Linthorpe, Middlesbrough.
PEACOCK, Mark Beauchamp, M.A., Oxford, "San Salvatore," Rhodes Avenue, Westville, Natal, South Africa.
POOLEY, Noel, A.M.I.Mun.E., Borough Engineer's Department, Capitol Works Section, Upton Towers, Slough, Bucks.
ROBERTSON, Alexander Gordon, B.Sc. (Hons.) (Eng.), Glasgow, c/o 128, Brondesbury Park, N.W.2.
SALE, Desmond James, 74, Church Road, Worcester Park, Surrey.
SIMMS, George Alfred, 114, Crestway, Roehampton, S.W.15.
SMITH, Richard Anthony, 7, Park Terrace, Garstang Road, Fulwood, Preston, Lancs.
STEEL, Richard William Charles, 15, Bramley Road, Sutton, Surrey.
TAN TIONG BENG, 23, Kuo Chuan Avenue, Singapore, 15.
TAYLOR, Geoffrey Bell, 27, Church Street, Heanor, Notts.
THOMPSON, Leslie, A.M.I.Mech.E., 39, Hendham Road, Upper Tooting, London, S.W.17.
THOMSON, Frederick Murray, 23, Caledonia Street, Wellington, E.4, New Zealand.
THORP, Cyril Francis, 16, Wilmar Drive, Salendine Nook, Huddersfield, Yorks.
VAN HEERDEN, Willem Jacobus Stephanus, B.Sc.(Eng.), Rand, P.O. Box 653, Pretoria, South Africa.
VAUGHAN, Thomas Henry, 207, Castle Road, Salisbury, Wilts.
WHYTE, Basil Percy, c/o H. L. & H. Concrete Engineers (Pty.), Ltd., Box 943, Durban, South Africa.
WILSON, Raymond, 20, Park Avenue, Thornaby-on-Tees, Stockton-on-Tees, Co. Durham.
WILSON, William Barry, B.Sc.(Eng.), London, 2, Parkers Hill, Ashstead, Surrey.
WINCH, Eric Douglas, B.Sc.(Eng.), London, 46A, Auriol Road, West Kensington, London, W.14.
YOUNG, Joseph Anthony, A.M.I.Mun.E., 40, Oaklands Avenue, Oxhey, Herts.

Associate-Members to Members

BOURNE, Kenneth Lowick, "The Outlook," Olive Grove, Stourport-on-Severn, Worcs.
COLLINSON, Kenneth, 11, Gipsy Lane, Marton, Middlesbrough.
DRUMMOND, David Gordon, B.Sc.(Eng.), London, A.M.I.C.E., F. A. Macdonald & Partners, 11, Woodside Terrace, Glasgow, C.3.
FAULKNER, Geoffrey Lawrence, B.Eng., Sheffield, Messrs. Kennedy & Donkin, Delmore Road House, Inverness.
GRIFFITHS, Harold Caine, Crossways, 19, Coningsby Road, Woodthorpe, Nottingham.
ROBB, Alexander, B.Sc.(Eng.), St. Andrews, A.M.I.C.E., Concrete Design, Ltd., 21, Bond Street, Leeds, 1.
ROGERS, Frank, Box 136, Accra, Gold Coast, West Africa.
THOROWGOOD, Wilfrid Ernest, M.A.(Cantab.), A.M.I.C.E., A.M.I.Mun.E., Downend, Fort Road, Guildford, Surrey.

RE-ADMISSION

Associate-Member

SHIRPURKAR, Krishna Appaji, Nagpur Municipal Committee, Nagpur, C.P., India.

OBITUARY

The Council regret to announce the deaths of Marcel Ernest GERARD, Siegfried HERZ and Sir Percy Malcolm STEWART, Bt. (Members).

JUNE MEETING

An Ordinary General Meeting for the election of members only will be held at 11, Upper Belgrave Street, London, S.W.1, on Thursday, June 28th, 1951, at 5 p.m.

JULY EXAMINATIONS

The Examinations of the Institution will next be held at centres in the United Kingdom and overseas on July 17th and 18th (Graduateship), and on July 19th and 20th (Associate-Membership).

OVERSEAS REPRESENTATIVE

The Council have appointed Mr. E. Snowden, B.Eng., A.M.I.C.E., A.M.I.Struct.E., to be the Institution's Representative in Brisbane, Queensland, Australia.

REPRESENTATION

The Council have made the following nominations of members to represent the Institution :—

INSTITUTION OF CIVIL ENGINEERS JOINT COMMITTEE ON VIBRATED CONCRETE :
Mr. S. Vaughan (Vice-President).
Mr. W. C. Andrews, O.B.E. (Vice-President).
Mr. W. H. Woodcock (Hon. Editor).

MINISTRY OF EDUCATION. JOINT COMMITTEE ON HIGHER NATIONAL CERTIFICATES IN CIVIL ENGINEERING :
Dr. S. B. Hamilton (Vice-President).

MINISTRY OF EDUCATION. JOINT COMMITTEE ON NATIONAL DIPLOMAS AND CERTIFICATES IN BUILDING :
Major F. G. Etches (Member).

MINISTRY OF HEALTH. ADVISORY COMMITTEE TO ASSIST IN REVISING THE MODEL SERIES OF BUILDING BYE-LAWS :
Mr. L. E. Kent (Hon. Librarian).

BRITISH INSTRUMENT INDUSTRIES EXHIBITION,

NATIONAL HALL, OLYMPIA, LONDON, JULY 4TH-14TH, 1951

Thanks to the courtesy of the organisers, a certain number of free tickets are being made available for use by members of the Institution. Members who desire to obtain a ticket are asked to communicate with the Secretary at an early date.

INTERNATIONAL ASSOCIATION FOR BRIDGE AND STRUCTURAL ENGINEERING

The International Association for Bridge and Structural Engineering is holding its Fourth Congress in Cambridge and London from August 25th to September 5th, 1952.

The Association, whose Headquarters is in Zurich, was founded in 1929 for the purpose of promoting international co-operation among scientists, engineers and manufacturers ; the interchange of knowledge, ideas and the results of research work in the sphere of bridge and structural engineering in general, whether in steel, concrete, or another material.

The British Section of the Association, under the Chairmanship of Mr. Ewart S. Andrews (Past-President of the Institution and a Vice-President of the Association), is responsible for organising the 1952 Congress, and a provisional programme of events has been arranged. During the period August 25th to August 30th, the Congress will be centred at Cambridge, where the programme will consist of technical meetings, at which papers will be read and discussed, a Reception by the University, and visits to places of interest in and around Cambridge. On August 30th, the Congress will move to London, and from September 1st to 5th, various visits and tours to places in and around London will be held, in addition to a Government Reception, a Service in Westminster Abbey, and the Congress Dinner. Three-day visits to South Wales and Scotland are also being arranged during this period.

The Lord President of the Council, The Rt. Hon. Viscount Addison, K.G., has accepted office as the President of the Congress, which is being sponsored by the Institution of Civil Engineers and the Institution of Structural Engineers. The Presidents of the two Institutions, Dr. W. H. Glanville and Mr. J. E. Swindlehurst, have been appointed as two of the Vice-Presidents of the Congress, and the Secretaries, Mr. E. Graham Clark and Major R. F. Maitland, as Joint Honorary Treasurers.

Membership of the Association may be acquired by any person interested in bridge and structural engineering, provided that he belongs to a recognised scientific or technical society, or possesses the requisite scientific or technical knowledge, and Government or local authorities, scientific institutes, associations, syndicates and firms may become collective members of the Association.

Members of the Institution desirous of applying for membership of the Association, for which the annual subscription is £1 1s., may obtain further details of the Association's activities and application forms upon application to the Chairman of the British Section, Mr. Ewart S. Andrews, 318, Bank Chambers, 329, High Holborn, London, W.C.1.

LONDON GRADUATES' AND STUDENTS' SECTION

A visit to the West India Docks has been arranged for June 16th, meeting at the Main Entrance at 9.45 a.m.

Directions : Aldgate East Station on the District Line, and trolley-bus to Burdett Road.

Hon. Secretary : D. B. Rogers, 85, Bramfield Road, S.W.11.

BRANCH NOTICES

LANCASHIRE AND CHESHIRE BRANCH

Hon. Secretary : A. S. Sinclair, 28, Kenwood Road, Stretford, Lancs.

GRADUATES' AND STUDENTS' SECTION

Hon. Secretary : F. M. Turner, 11, Colwell Avenue, Stretford, Lancs.

MIDLAND COUNTIES' BRANCH

Hon. Secretary : E. R. Deeley, A.M.I.Struct.E., Arranmoor, Adshead Road, Dudley, Worcs.

GRADUATES' AND STUDENTS' SECTION

Hon. Secretary : M. H. Evans, B.Sc., 107, Metchley Lane, Harborne, Birmingham, 17.

NORTHERN COUNTIES' BRANCH

Hon. Secretary : J. A. Williams, A.M.I.C.E., A.M.I.Struct.E., 57, Reid Park Road, Jesmond, Newcastle-upon-Tyne.

NORTHERN IRELAND BRANCH

Hon. Secretary : S. G. Duckworth, M.I.Struct.E., Lisleen, 13, Finaghy Road North, Belfast.

SCOTTISH BRANCH

The Annual General Meeting of the Branch was held on Tuesday, April 17th, when the following Honorary Officers and Committee members were elected :—

Chairman : R. Summers.
Vice-Chairman : R. H. Sharpe.
Hon. Treasurer : J. Camerson.
Hon. Secretary : D. G. Drummond, 11, Woodside Terrace, Glasgow, C.3.
Immediate Past Chairman : Dr. C. M. Moir.
Committee Members : N. M. Brydon, G. M. Dingwall, W. Girvan, W. Heigh, R. M. Howatt, H. P. Johnston, A. MacLean, H. McClusky, J.P., A. G. F. Russell, W. Basil Scott.
Hon. Auditors : J. Laird, H. B. Sutherland.

SOUTH-WESTERN COUNTIES' BRANCH

Hon. Secretary : E. W. Howells, M.I.Struct.E., 10-12, Market Street, Torquay, Devon.

WALES AND MONMOUTHSHIRE BRANCH

A very interesting afternoon was spent on April 11th, when about 50 members of the Branch visited the New Strip Mill at Abbey Works, Margam, by kind

permission of The Steel Company of Wales. Every facility was given to inspect the buildings, which are now practically complete, also the portion of the plant already in production and the remaining plant which is being rapidly assembled. Tea was taken in the works' canteen, and the Chairman afterwards expressed the thanks of the members to the Steel Company and to the Consulting Engineer.

The party then travelled to the Seabank Hotel, Porthcawl, where the Annual Dinner was held. It was a pleasure to have the President and Mrs. Swindlehurst with us on this occasion, and we regretted that Major Maitland was prevented by illness from attending.

Hon. Secretary : E. R. Steward, A.M.I.Struct.E., Edrom, Ashleigh Road, Blackpill, Swansea.

WESTERN COUNTIES' BRANCH

The Annual Dinner was held at the Royal Hotel, Bristol, on Wednesday, March 14th. Mr. P. C. Girdlestone (Branch Chairman) presided, and Mr. Stanley Vaughan (Vice-President) and Major R. F. Maitland were present, together with a number of other guests representing kindred societies.

The Dinner was followed by a programme of musical entertainment.

Hon. Secretary : C. E. Saunders, M.I.Struct.E., Dunkery, Edward Road, Walton St. Mary, Clevedon, Som.

YORKSHIRE BRANCH

The Eighth Ordinary Meeting was held on Wednesday, April 18th, at the Great Northern Hotel, Leeds, at 6.30 p.m. The Chairman was Mr. A. Robb, B.Sc., A.M.I.C.E. (Member), and the meeting was attended by 45 members and friends. A lecture on " The Institution of Structural Engineers' Report on Prestressed Concrete " was given by Professor R. H. Evans, D.Sc., Ph.D., M.I.C.E., M.I.Mech.E. (Member). Questions and discussion followed and a vote of thanks, proposed by Mr. D. R. S. Wilson (Member), and seconded by Mr. J. Dossor, A.M.I.C.E., M.I.Mech.E. (Member), was accorded to the Lecturer.

Hon. Secretary : E. Wrigley, A.M.I.Struct.E., City Engineer's Department, Civic Hall, Leeds, 1.

UNION OF SOUTH AFRICA BRANCH

Branch Hon. Secretary : A. E. Tait, B.Sc., A.M.I.C.E., A.M.I.Struct.E., P.O. Box No. 3306, Johannesburg. During week-days Mr. Tait can be contacted in the City Engineer's Department, City Hall, Johannesburg. 'Phone : 34-111, ext. 257.

Natal Section Hon. Secretary : E. G. Bennett, A.M.I.Struct.E., c/o Reinforcing Steel Co., Ltd., P.O. Box 478, Durban.

Cape Section Hon. Secretary : R. Stubbs, M.I.Struct.E., P.O. Box 1692, Cape Town.

Book Reviews

Asphalte in Modern Building Construction, by G. J. Hancock. Edited by A. A. Chant and F. E. Drury. (London, Newnes, 1950.) 230 pages, 9¼ in. by 6½ in. 30s.

This is an authoritative reference book which deals in a very clear manner with most of the points upon which information might be sought within the acknowledged limits of its scope. The limitation is accounted for by the fact that the book is published under the ægis of the Natural Asphalte Industry, the matter having been largely gathered together and prepared by the late Secretary to the Mine Owners' Council. The text shows some signs of changed viewpoint during the process of compilation.

The nature and varieties of asphalte, raw materials, manufacturing processes and testing are dealt with in four chapters (80 pages), and five more are devoted to the uses, technique of asphalting, details of application to specific problems and specification, though not in this order.

Two excellent features are the clear illustrations of the method of application and details of use of asphalte and copious reference to publications for further information upon matters dealt with in the text. The book may be confidently recommended to structural engineers and others engaged in building work.

We are reminded that one function of asphalte is to remedy our defects since " Although really good and dependable concrete for resisting the effects of the weather can be produced, there are operational uncertainties which make it worth while to introduce protection. . . ."

The book is very well produced. S. C. G.

Lexique Technique Francais-Anglais et Anglais-Francais Concernant le Materiel de Travaux Publics (French-English Technical Glossary of Engineering Equipment Terms) (Paris : Institut Technique du Batiment et des Travaux Publics, 1949). 184 pp., 8 in. by 5 in. 700 francs.

This glossary of French, English and American technical terms relating to engineering equipment, which is intended primarily for the use of French contractors who wish to order plant and spare parts manufactured in Great Britain and the United States, will also be found useful by all those who require the French equivalents of English technical terms, many of which are not found in current bilingual dictionaries.

The glossary covers plant used in building and public works and also engineering equipment in general. Detailed conversion tables complete the glossary proper and facilitate conversion from British to metric units and vice versa.

The book is bound in red cloth.

Strength and Elasticity of Materials : Theory of Structures. Solutions to B.Sc.(Eng.) Examination Questions (Internal), by W. H. Brooks. (London, Macdonald, 1950.) 141 pages, 8½ in. by 5½ in. ; 117 illus. 12s. 6d.

A cursory glance at the title may suggest this is a further " text-book " on two familiar subjects, but on closer inspection it will be realised that any such impression is erroneous.

It is, in fact, considered a well-arranged series of solutions (" worked-out " answers) to the question-papers set by the University of London Examination Board for Part I of the B.Sc.(Eng.) Internal Examination.

The author is to be congratulated on his successful grouping of the subjects under their relative headings, together with his solutions.

The book is well illustrated by 117 line diagrams and the solutions are clearly expressed in such a manner

as to enable them to be easily understood and followed. Where questions call for definitions these are expressed in concise, yet clear forms.

A glossary of symbols used throughout the book is also incorporated, thereby clarifying formulæ in which other symbols might—in certain cases—have been used by authors of various books with which the student may have become acquainted during his earlier tuition.

The book includes sections covering examples of :— Stresses and Strains, Shear Forces and Bending Moments, Deflection of Beams and Cantilevers, Resilience, Torsion of Shafts, Helical Springs, Struts and Columns, Composite Bars and Beams, Temperature Stresses, Forces in Plane Frames, Deflection of Plane Frames, Stability of Retaining Walls and Dams, Suspension Bridges, Three-pinned Arches, Rolling Loads on Simple Beams, Influence lines—Shear and Bending, Miscellaneous Examples, etc.

It is well indexed under its various sections, in addition to which a separate index is provided giving the examination papers set during each year (1938-1948 inclusive).

The book should prove invaluable to students preparing for the London B.Sc. examination previously mentioned, also for those studying privately for the A.M.I.C.E., A.M.I.Struct.E., or R.I.B.A. Examinations since the basic principles involved in many of the questions are of a similar character.

It will also be found suitable as a reference book by students desirous of making a quick revision prior to their sitting for any of the above examinations, especially for those who have had to recently concentrate on other subjects and wish finally to make a refresher study.

The book is of convenient size, its price is considered attractive, and should be within the means of most students ; a fact which often has to be borne well in mind.

A. E. P.

Der Stahlbeton, by R. Saliger, 7th Edition. (Vienna, Deuticke, 1950.) 644 pages, 6½ in. by 9½ in., 670 figures.

This is, generally speaking, one of the best textbooks on reinforced concrete which should be of interest not only to the student but also to the designer and research worker. As in former editions, the material is clearly presented and illustrated, and contains a survey on research, theory and practice, and a considerable part dealing also with the design of rigid frames. The new edition embodies studies on the development of high strength material, particularly work-hardened steel, on the formation and widths of cracks and on the plastic behaviour of reinforced concrete at failure.

There are, however, some points to which certain objection must be made in spite of the lucidity of presentation. Firstly, the survey on investigations is not quite complete with regard to high strength materials bond resistance and formation of cracks, since there is no reference to most of the work on these subjects by certain experts which must be known to the author (e.g., that of the late F. v. Emperger, Hon.M.I.Struct.E.).

Secondly, certain assumptions have been made about stress distributions, and Saliger derives therefrom a theory indicating the width and spacing of cracks under definite loading. This is shown as an exact solution, while it is in fact only an assumption which will agree with the actual behaviour in certain instances. The author shows the influence of plastic behaviour on failure in an impressive manner ; however, everything is based on his own, rather complicated theory, based on certain assumptions, while Whitney in America suggested a very simple approximation apparently just as suitable as the author's more theoretical solution. Whitney, however, who published his method in the same year as the author (1937) is only mentioned in the bibliography.

Further, as in former editions, the author, as a keen nationalist, replaces many words, having foreign roots, by newly created German expressions ; e.g., instead of " exzentrisch " (eccentric) he uses " ausmittig," instead of " elastisch " (elastic) he uses " federnd," or " plastisch " (plastic) is mostly referred to as " bildsam". By omitting words of Latin or Greek origin which represent common links between various languages in scientific publications, the author prevents many readers from easily understanding the essence of the matter. On the other hand, some of these international words have already been reintroduced in the 7th Edition such as " symmetrisch " (symmetrical), which Saliger previously called " spiegelgleich " (mirror-equal), and it may therefore be hoped that the author will in future employ other internationally recognisable words and not limit artificially the use of his book.

P. W. A.

Sewage Treatment : Design and Specification, by L. B. Escritt. (London, The Contractors Record and Municipal Engineering, 1950). 257 pages, 9½ in × 6 in. 30s.

With the publication of this volume the author has fulfilled his desire of producing a complementary work to his " Sewerage Design and Specification."

The subject is dealt with in a most comprehensive manner and presents the designing engineer with a great deal of useful practical data in a most readable form. The needs of the student are by no means overlooked, being well catered for by descriptions of plant and processes in general use.

The selected subject matter is modern, its treatment clear and concise, and the usual historical preamble has been dispensed with. Structural design is not introduced and the book is essentially non-mathematical, the only exception being an example calculation for a percolating filter scheme. There is, however, no lack of the more vital facts and figures necessary for the successful design of sewage treatment works, such as ideal rates of flow for almost any stage of a scheme and the capacities for all types of equipment.

The author has given particular attention to detail in the chapters on sedimentation, sludge treatment and percolating filters, and in the latter attempts to consolidate the findings of the Royal Commission on Sewage Disposal by giving rates of treatment for medium-to-coarse filter media.

Activated sludge treatment occupies very little space, this being limited to an introductory description of the various proprietary processes and systems. The reader interested in less spectacular projects, however, will appreciate the chapter devoted to isolated buildings and small communities and be warned of the dangers in cesspool design.

Although quite useful, too much should not be expected of the three end chapters on specification, the rather brief handling of which scarcely justifies such a degree of equality with " Design " as is implied by the title.

The book is well illustrated with detailed drawings and plates of actual works.

L. J. G.

The Economic Design of Rectangular Reinforced Concrete Sections, by T. P. O'Sullivan. (London, Pitman, 1950). 8½ in. × 5½ in, 76 pages. 12s. 6d.

In major structures of the portal or arched type, the variation, both in magnitude and position of the line of thrust is often so great as to involve an enormous amount of calculation, if the most economical arrangement of reinforcement is to be achieved. Whatever the eventual detail of the cross-section, final calculations of stress must be based on an assumed arrangement and quantity of reinforcement, and to decide this curves are most helpful.

The author has analysed the variation of stress which occurs as the eccentricity of a load changes from nothing to infinity, that is, pure bending, and derives the formulae for each of a number of " phases " in accordance with normal assumptions as this change proceeds, the equations being so stated that the majority can be plotted as straight lines; consequently the reader, using his own particular data, can plot them with comparative ease.

One point to which the author calls particular attention is the fact that it may be possible in some cases, if the eccentricity of the load is virtually constant and certain, to achieve appreciable economy by the use of reinforcement on one side of the cross-section only.

There are one or two minor errors which will, no doubt, be amended in subsequent editions. For instance, the numerical values of percentages stated on Figs. 23, 24 and 25, and it might be advisable to clarify the statement as to the purpose of the study, particularly item (d).

P. G. B.

Fortschritte im Stahlbeton, by Professor R. Saliger. Vienna, 1950. 138 pages.

Last year's new edition of the author's textbook was amplified by more recent theories of creep, plastic flow and pretensioning, restating the validity of his 1936 design method based on the plastic theory. The present report is based on the results of ten years' research and practice in concrete and steel, the first part dealing with a comparison of cold-worked steels. Among these, TOR steel, which now constitutes about 80 per cent. of the total consumption in Austria, in an improved shape devised by Saliger himself, shows the best results.

Tests by Dr. Hajnal-Konyi (1943) show that the so-called yield point and the ultimate stress of square bars can be raised through cold twisting by over 80 per cent. of the figures for untwisted steel. Saliger submits that his slightly ribbed round bars can be improved by 100 per cent. and over, through twisting against one fixed end. The improvement increases inversely with the angle of twisting of a straight bar from 90° down to 25°. (Square bars were twisted to about 33°). Hardness diagrams reveal a more even distribution of improvements in TOR steel; toughness, ductility and workability were also tested. He claims that the advantage of cold working, lost through reheating in a welded joint, can be regained by subsequent re-twisting.

High quality concrete, the corollary of improved steel, is dealt with in the second part of the report. The bursting action of non-prismatic steels in concrete is explained. A reasoned explanation of the resistance to slipping of steels at high tensional stress is given. The paramount importance of the spacing of reinforcement is emphasised, and the width of concrete cracks is related to steel percentage.

The superiority of TOR steel may be disputed by competing steels. However, this well documented report is stimulating reading. The ample local and Swiss bibliography includes some English and U.S.A. contributions.

P. M. T.

Explanatory Handbook on the B.S. Code of Practice for Reinforced Concrete (CP.114 : 1948), by W. L. Scott, W. H. Granville and F. G. Thomas. (London, Concrete Publications). Price 9s.

This handbook follows the lines of the Explanatory Handbook on the D.S.I.R. Code (1938) prepared by the first two authors of the present book, its aim being to explain and amplify the Code of Practice for the Structural Use of Normal Reinforced Concrete (1948) drafted by the Code Committee of the Institution of Structural Engineers. The latter Code has been in general use for two years and the Explanatory Handbook provides much valuable supplementary information for the practical designer and for the student.

The clauses of the Code are reproduced in their entirety, the author's explanatory matter being interspersed as required. The section on Design Considerations occupies some two-thirds of the book and contains, among other valuable information, some very practical tables, giving bending moment coefficients for continuous beams and for various arrangements of continuous panels and also permissible loads for a range of square and octagonal columns.

This Handbook should prove a popular and valuable addition to the Code.

An Introduction to Distribution Methods of Structural Analysis, by R. W. Steed. (London, Pitman). 116 pages, 8¾ in. × 5½ in., 140 illustrations. 12s. 6d.

This little volume deals principally with the application of the moment distribution method to continuous beams and simple portal frames. As an introduction to the subject, the book will be of value to engineering students and to engineers who have not previously found the time to master the Hardy Cross method.

Minor criticisms apart, the text has been carefully and clearly written, and the diagrams well presented.

As the author points out in the preface, no student is likely to understand fully moment distribution without having first gained confidence with the solution of problems relating to fixed beams. The first chapter has, accordingly, been devoted to the detailed calculation of end fixing moments. Some of the rules and formulae given in this chapter have not been derived by the most direct means, and with a strange inconsistency, the bending moment diagrams are drawn below the base line, whereas in later chapters they appear above.

The chapters dealing with continuous beams and portal frames are more lucid, and the general principles of the distribution method are clearly shown by a number of worked examples. The calculation of sway moments is explained in detail, emphasis being rightly placed on the importance of a sign convention. Examples showing the use of some of the well-established " short cuts " might well have been included, particularly when considering symmetrical portal frames.

C. H. H.

The Analysis of Single-Storey Multi-Bay Gabled Rigid Frames

By A. J. Francis, M.Sc., Ph.D., A.M.I.C.E., A.M.I.Struct.E.

Abstract

Gabled rigid frames are becoming increasingly common in single-storey factory construction, but where there are two or more bays the analysis by the usual methods of structural theory presents considerable difficulties in computation. This paper presents a method, based on the general conception of moment and thrust distribution, which greatly simplifies such problems. It is particularly advantageous where the structure consists of a large number of bays, where the roof members are in several straight portions or are curved, and where the members are of variable cross-section. It is also applicable to multi-span portal frame bridges.

Introduction

Multi-bay gabled rigid frames are being increasingly adopted for single storey factory construction, and it has been shown in a recent paper to the Institution[1] that their use generally leads to an economical design. They are, however, difficult to analyse by the classical methods of structural theory, a fact which may well be an obstacle to their more general use. If, for instance, the method of slope-deflection is applied to the framework shown in Fig. 1(a), then except in the symmetrical

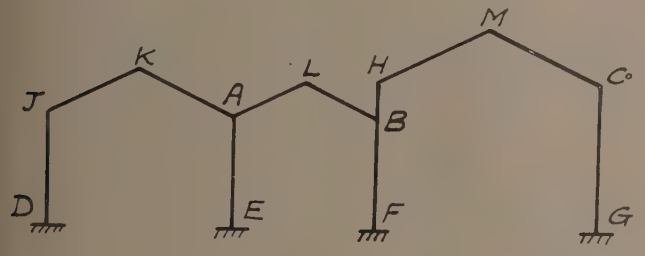

Fig. 1 (a)

case the analysis leads to eleven simultaneous equations, the solution of which is a very laborious process. If the moment-distribution method is employed in the manner described by Matheson[2], the degrees of freedom of the structure being considered separately, the problem is simpler, but there are still four simultaneous equations to be solved before the bending moments in the members due to loads on the frame are obtained.

The writer has shown elsewhere[3] how certain problems on complicated rigid frameworks involving multiple sway may be solved by the moment-distribution method in a single distribution table, no simultaneous equations being introduced. The procedure for the analysis of single-storey multi-bay gabled frames, which is given in the present paper, is similar in principle but involves the direct distribution of horizontal forces as well as of moments. The fundamentally similar problem of the multi-span arch on elastic piers has already received some attention in the U.S.A., notably from Hardy Cross[4] and Hrennikoff[5], Cross's treatment being preferable to that of Hrennikoff. The writer's procedure differs from these in certain respects, particularly in the manner in which horizontal forces are distributed.

It is assumed that the behaviour of the structure under consideration is elastic, and that deformations due to axial and shearing forces may be neglected.

Procedure

The method is best described by reference to a particular framework. Consider the three-bay frame shown in Fig. 1(a), and let the roof of the middle bay be loaded in some way.

The frame is split up into the components shown in Fig. 1(b), the three members DJ, JK and KA, the two members AL and LB, and so on, being considered as single composite members. This greatly simplifies the problem, since it is now only necessary to consider the equilibrium of two joints—A and B—instead of eight when, for instance, the slope-deflection method is employed.

1. The joints A and B are first restrained by external moments and forces against rotation and translation, and the fixed-end moments and thrusts at A and B due to the applied loads are calculated by the elastic centre method described later.

2. The fixed-end horizontal thrusts at A and B are removed and A and B are allowed to translate *without rotation*.

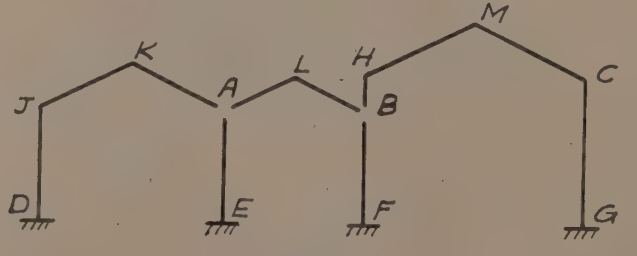

Fig. 1 (b)

3. Owing to the translation of A and B, moments are set up at the ends of the members meeting at each joint. There is now at each joint an unbalanced moment preventing rotation, equal to the algebraic sum of the original fixed-end moment and the moments induced by the translation of the joints. A and B are now allowed to rotate one at a time, *without further translation*, until the unbalanced moment at either joint is negligible.

4. The rotation of each member produces restraining forces at A and B. The vertical components of these forces are carried directly by the columns, but the horizontal components are unbalanced. The unbalanced horizontal force at each joint is now removed and the joints are allowed to move sideways without rotation, as in Step 1.

5. The unbalanced moments set up at each joint in Step 4 are distributed as in Step 3.

The process of alternately allowing the joints to translate and rotate is continued until the residual (unbalanced) moment and horizontal force at each joint are negligible. The moments at the ends of the members

meeting at A and B are then obtained by adding all fixed-end, translation, balancing and carry-over moments introduced during the analysis.

Symbols

E : Young's Modulus.
I : relevant moment of inertia of a member (or of element of member).
s : length of member.
δs : element of length of member.
L : horizontal length of member.
x, y : co-ordinates of element of member referred to rectangular axes Ox, Oy through O.
$\frac{\delta s}{EI}$: elastic weight of element.
W : elastic weight of complete member.
O : elastic centre of member (i.e. the centroid of its elastic weight).
$I_x \left(= \Sigma \frac{y^2 \delta s}{EI} \right)$: moment of inertia of elastic weight of member about a horizontal axis Ox through O.
$I_y \left(= \Sigma \frac{x^2 \delta s}{EI} \right)$: moment of inertia of elastic weight of member about a vertical axis Oy through O.
$I_{xy} \left(= \Sigma \frac{xy \delta s}{EI} \right)$: product of inertia of elastic weight of member about axes Ox, Oy.
$I_x' = I_x - I^2_{xy}/I_y$
$I_y' = I_y - I^2_{xy}/I_x$
$\psi = \tan^{-1} I_{xy}/I_x$

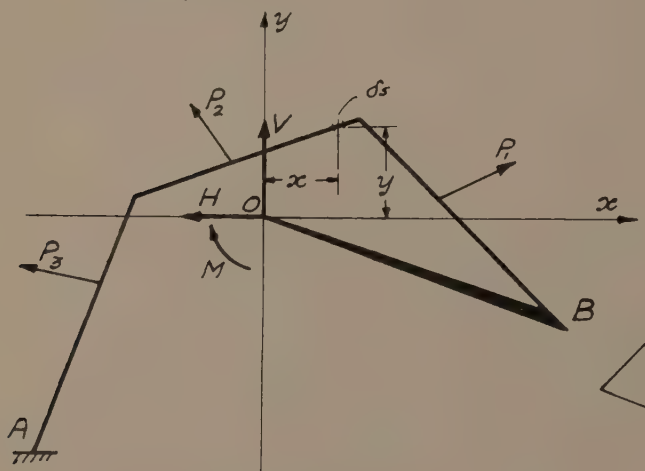

Fig. 2 (a)

$\theta = \tan^{-1} I_{xy}/I_y$
$\bar{x}$, $\bar{y}$: co-ordinates of mid-point of a straight portion of member *or* co-ordinates of centroid of a μ/EI diagram.
P_1, P_2 etc. : applied loads.
μ : bending moment due to applied loads.
H, V and R : horizontal, vertical and resultant forces, respectively, applied to one end, or to the elastic centre, of member.
M : moment applied to the elastic centre of member.
M_{BA} : moment applied by the joint B to the end B of member AB.
$A \left(= \Sigma \frac{\mu \delta s}{EI} \right)$: area of a μ/EI diagram.
$M_x \left(= \Sigma \frac{\mu y \delta s}{EI} \right)$: moment of area of a μ/EI diagram about Ox.
$M_y \left(= \Sigma \frac{\mu x \delta s}{EI} \right)$: moment of area of a μ/EI diagram about Oy.

Φ_B : angle of rotation of the end B of the member AB.
Δ_B : displacement of the end B of the member AB.
$T \left(= \frac{I}{I^1_x} \right)$: thrust stiffness of member.
$S_B \left(= \frac{I}{W} + \frac{h^2}{I^1_x} + \frac{x^2_b}{I_y} \right)$: moment stiffness of end B of member AB.
α : angle of inclination of resultant force accompanying a moment applied to one end of a member.

Conventions of Signs

1. A special sign convention, which is explained in the following, is used in the calculation of the fixed-end thrusts and moments due to applied loading on a member.

2. In carrying out the distribution of moments and thrusts, the following convention of signs is employed :

(a) a moment applied by a joint to the end of a member is positive when anti-clockwise ;

(b) a horizontal thrust applied to the elastic centre of a member, or by a joint to the end of a member, is positive when acting to the left.

Fixed-End Moments and Thrusts in an Unsymmetrical Member due to Applied Loads

AB (Fig. 2(a)) is an unsymmetrical member, fixed at A and B and subjected to a system of loads P_1, P_2 etc. Considering a small element of the member, of length δs

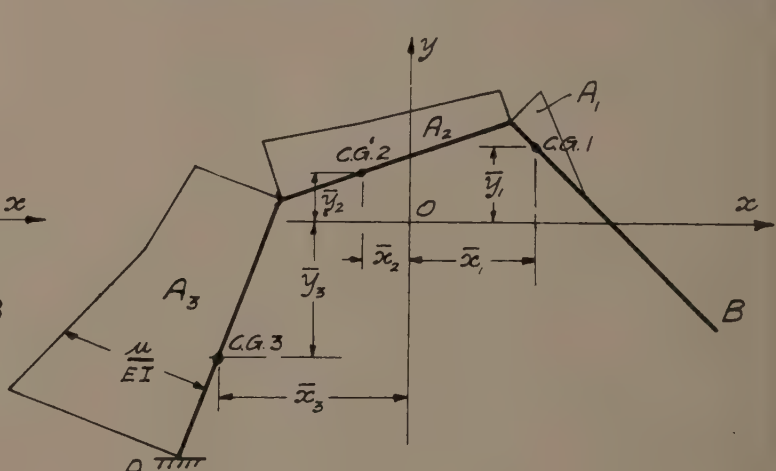

Fig. 2 (b)

and moment of inertia I, the expression $\delta s/EI$ is defined as the elastic weight of the element. The centroid of the elastic weight W of the complete member is located at O. O is also termed the elastic centre of the member. It follows from the definition of O that, if horizontal and vertical axes Ox, Oy are drawn through O as origin,

$$\Sigma \frac{x \, \delta s}{EI} = \Sigma \frac{y \, \delta s}{EI} = 0 \text{ (zero).}$$

The positive directions of x and y are as shown in Fig. 2(a).

If the member AB is cut just above B, fixity of B may be preserved by applying to B, in general, horizontal and vertical forces and a couple. It is found more convenient, however, to apply the necessary forces and couple (H, V and M) at O, instead of at B, and it may be imagined that their effects are transmitted to B through a perfectly rigid bar OB fixed to the member at B. AB is now a cantilever fixed at A and

subjected to the applied loads P_1, P_2, etc. and H, V and M.

The bending moment μ at any point (x, y) due to the applied loads P is considered positive when it produces tension on the inside face of the member. The positive directions of H, V and M are as shown in Fig. 2(a). Then it may be shown that (Appendix I)

$$M = \frac{A}{W}$$

$$H = \frac{M_x - (I_{xy}/I_y) \cdot M_y}{I_x'}$$

$$\text{and } V = \frac{M_y - (I_{xy}/I_x) \cdot M_x}{I_y'}$$

The meaning of the various symbols is explained on p.190. The fixed-end moments and thrusts at A and B may now be obtained by statics.

In order to evaluate:

$$A\left(=\Sigma\frac{\mu\,\delta s}{EI}\right),\ M_x\left(=\Sigma\frac{\mu y\,\delta s}{EI}\right) \text{ and } M_y\left(=\Sigma\frac{\mu x\,\delta s}{EI}\right)$$

imagine a diagram to be constructed along the member perpendicular to the plane of the paper, such that the ordinate at each point is μ/EI. Then A is the area of this diagram and M_x and M_y are the moments of this area about the x and y axes respectively.

Taking each straight length of the member in turn, determine for each length the co-ordinates of the centroid of the μ/EI diagram constructed on it. Then, referring to Fig. 2(b),

$$M_x = \Sigma\frac{\mu y\,\delta s}{EI} = A_1\bar{y}_1 + A_2\bar{y}_2 + \ldots\ldots\ldots$$

with due regard to the signs of $\bar{y}_1$, $\bar{y}_2$, etc. M_y is obtained in a similar manner.

If the straight lengths of the member are each of uniform moment of inertia, it is necessary to draw only the bending moment diagram, since for each individual length μ/EI is everywhere proportional to the bending moment μ. In this case also the following formulae for I_x, I_y and I_{xy} for a straight prismatic member of length s apply (see Fig. 3):

$$I_x = \frac{s}{3EI}\left(y_1^2 + y_1y_2 + y_2^2\right)$$

$$I_y = \frac{s}{3EI}\left(x_1^2 + x_1x_2 + x_2^2\right)$$

$$\text{and } I_{xy} = \frac{s}{12EI} \cdot x'y' + \frac{s}{EI} \cdot x\bar{y}$$

In the formula for I_{xy}, the first term represents the product of inertia of the length about its own elastic centre (its mid-point) and is positive when the slope of the length, dy/dx, is positive, and *vice versa*. The second term represents the additional product of inertia when axes are taken through O.

In all the above formulae it is most important to give the x's and y's their correct signs according to the convention indicated above.

With curved members it is necessary to divide the member into a sufficient number of parts and use graphical integration. Where the moment of inertia of a member changes abruptly at some point along its length, as is common in steel construction, the portions having different moments of inertia are taken separately.

Distribution of Horizontal Thrusts

If the member AB (Fig. 4(b)) is fixed at A, the end B may be given a purely horizontal displacement Δ (i.e. without rotation or vertical displacement) by applying at the elastic centre a force R inclined at an angle $\theta = \tan^{-1} I_{xy}/I_y$ to the horizontal Ox (see Appendix II). The horizontal component H of this force is given by

$$H = \frac{\Delta}{I_x'} = T \cdot \Delta,$$

where T is defined as the thrust stiffness of the member. The moment produced at B by this force R is *H.h.* For members having a vertical axis of symmetry, I_{xy} and therefore θ are equal to zero, and

$$R = H = \frac{\Delta}{I_x}.$$

Now consider a three-bay frame (Fig. 5). Let joint A be given a displacement Δ_A to the right, without rotation. If B is prevented from moving, then this displacement of A can be produced by applying, at

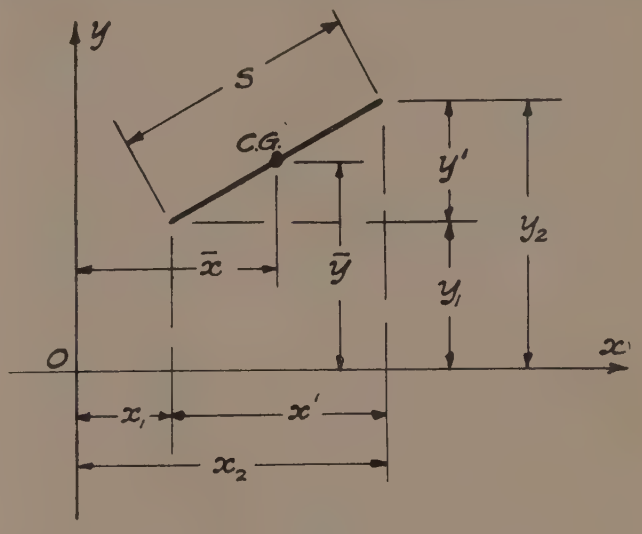

Fig. 3

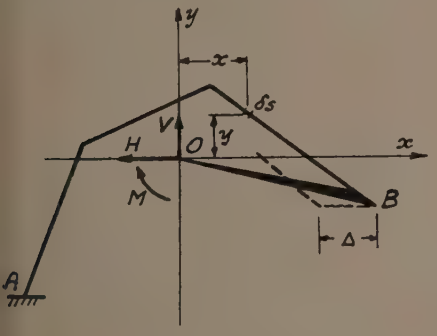

Fig. 4 (a)

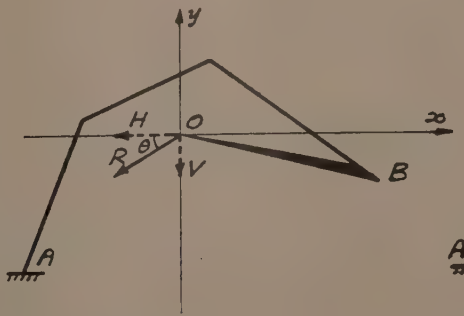

Fig. 4 (b)

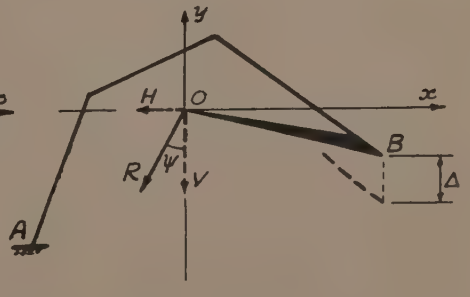

Fig. 4 (c)

the elastic centres of *AKJD*, *AE* and *ALB*, forces R_1, R_2 and R_3, whose horizontal components are H_1, H_2 and H_3 respectively. Then

$$\frac{H_1}{T_1} = \frac{H_2}{T_2} = \frac{H_3}{T_3} = \Delta_A.$$

Hence $H_1 : H_2 : H_3 = T_1 : T_2 : T_3$.

The total horizontal force H is the sum of H_1, H_2 and H_3, and divides between the three members in the ratio of their thrust stiffness; H_1, for instance, is equal to $H.T_1(T_1 + T_2 + T_3)$.

B is not, however, fixed, but is also permitted to translate without rotation, and the effective thrust stiffness of *ALB* is thereby reduced. The two members *BF* and *BHMCG*, whose thrust stiffnesses are T_4 and T_5 respectively, must deform by the same amount at *B*, and their combined thrust stiffness is therefore $T_4 + T_5$. Since the force H_3 carried by *ALB* is transmitted to *B* and thence to *BF* and *BHMCG*, the displacement Δ_B of *B* is $H_3(T_4 + T_5)$ and H_3 divides between *BF* and *BHMCG* in the ratio $T_4 : T_5$. Now *A* moves a distance H_3/T_3 to the right relative to *B*, so that the total displacement of *A* is

$$\Delta_A = \frac{H_3}{T_3} + \frac{H_3}{T_4 + T_5} = H_3\left(\frac{1}{T_3} + \frac{1}{T_4 + T_5}\right).$$

If the reciprocal of the thrust stiffness is termed the thrust flexibility, the expression in brackets above is the sum of the flexibility of *ALB* and the combined flexibility of *BF* and *BHMCG*. The effective thrust stiffness of *ALB* therefore becomes $\left(\frac{1}{\frac{1}{T_3} + \frac{1}{T_4+T_5}}\right)$.

Denoting this by T_3', the H's are now in the ratio

$$H_1 : H_2 : H_3 = T_1 : T_2 : T_3'.$$

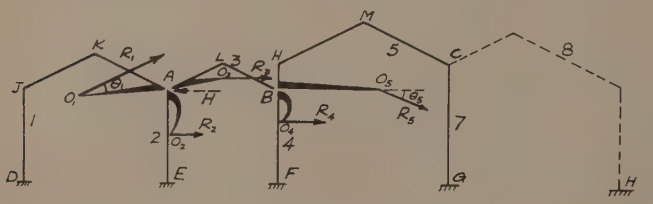

Fig. 5

If there were another bay to the right of *C*, as shown dotted in Fig. 5, and the individual thrust stiffnesses of *BHMC*, *CG* and *CH* were T_6, T_7 and T_8 respectively, the effective thrust stiffness of the part of the frame to the right of *A*, with *A*, *B* and *C* permitted to translate without rotation, would be determined as follows. The combined thrust stiffness of *CG* and *CH* being $T_7 + T_8$, the effective thrust stiffness of *BC* would be reduced from T_6 to $\left(\frac{1}{\frac{1}{T_6} + \frac{1}{T_7+T_8}}\right)$. Denoting this by T_6', the effective stiffness to the right of *A* would then be $\left(\frac{1}{\frac{1}{T_3} + \frac{1}{T_4+T_6'}}\right)$.

In this manner the effective thrust stiffness of any portion of a multi-bay frame may be determined.

Now suppose *ALB* to be loaded in such a manner that when *A* and *B* are initially held fixed there is a fixed-end horizontal thrust $+ H$ at *A* (i.e. acting to the left) and zero horizontal thrust at *B*. It will be recalled that in Step 2 of the method the fixed-end thrusts are removed and *A* and *B* are permitted to translate without rotation. The effect of removing the fixed-end thrust at *A* is the same as if a force $-H$ were superimposed upon it. It has been shown above how to find how this latter force divides between the members. When this has been done the moments produced at the ends of the members by the translation of the joints may be calculated.

Normally, the applied loading on a member such as *ALB* causes fixed-end thrusts at both ends of the member. It is best, however, to consider the fixed-end thrust at *B* independently of that at *A*, and determine the effects of releasing them separately, the results being combined.

An unbalanced thrust at a joint may, alternatively, be distributed by allowing only one joint at a time to translate. This method involves less preliminary calculation, but the relative proportions of columns and roof members in factory frames are usually such that the rate of convergence is very slow. The writer has found the direct method of distribution outlined above much more convenient and rapid.

Distribution of Moments

The end *A* of the unsymmetrical member *AB* (Fig. 6) is fixed, and the end *B* is free to rotate but is prevented

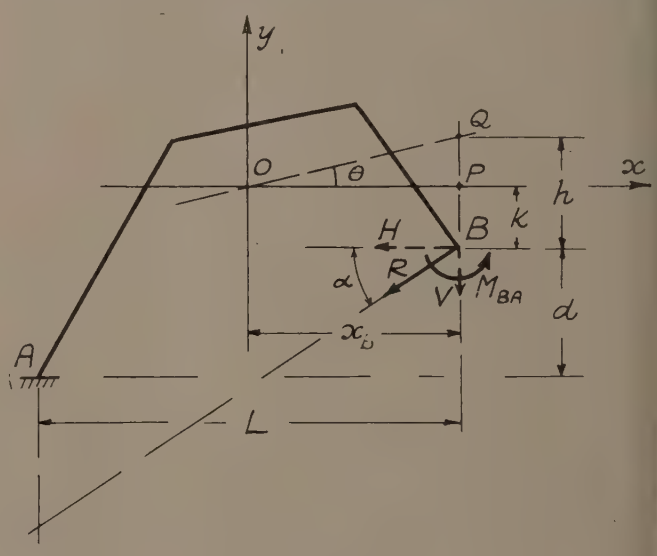

Fig. 6

from displacing. If an anticlockwise moment M_{BA} is applied to *B*, then it may be shown (Appendix III) that

$$M_{BA} = \Phi_B\left(\frac{1}{W} + \frac{h^2}{I'_x} + \frac{x_b^2}{I_y}\right) = S_B.\Phi_B,$$

Φ_B being the angle of rotation of *B* under the moment M_{BA}. S_B is defined as the moment stiffness of the end *B* of the member. *h* and x_b are as shown in the figure.

To prevent translation of *B*, it is necessary to apply a force *R* at *B*. This force is inclined to the horizontal at an angle α where

$$\tan\alpha = \frac{k.I_{xy} + x_b.I_x}{h.I_y}.$$

The positive direction of α is downwards to the left as shown. The line *OQ* is inclined to the horizontal at an angle θ ($= \tan^{-1} I_{xy}/I_y$), *OQ* lying in the first quadrant if I_{xy}/I_y has a positive value. The vertical distances *k* and *h* are positive if *P* and *Q* are above *B*. The horizontal component of *R* is given by

$$H = \frac{M_{BA}.h}{S_B.I'_x}.$$

If this expression is positive, R acts to the left. If Q is below B, however, then h is negative according to the above convention and R then acts to the right.

The moment produced at A (the carry-over moment) is

$$M_{AB} = -(M_{BA} + H.(d - L.\tan\alpha)).$$

In members which have a vertical axis of symmetry, A and B are on the same level, I_{xy} and θ are zero, and the above expressions become

$$M_{BA} = \left(\frac{1}{W} + \frac{h^2}{I_x} + \frac{(L/2)^2}{I_y}\right).\varphi_B, \quad \tan\alpha = \frac{L\,I_x}{2hI_y}$$

$$H = \frac{M_{BA}.h}{S_B.I_x} \text{ and } M_{AB} = -M_{BA}\left(1 - \frac{L^2}{2S_BI_y}\right).$$

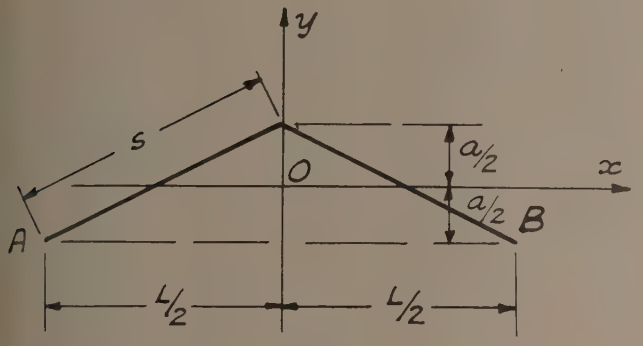

Fig. 7

In the simplest form of roof member, shown in Fig. 7, the sides are frequently of equal uniform moment of inertia I. Then with the symbols shown in the figures,

$$W = \frac{2s}{I}, \quad I_x = \frac{sa^2}{6I} \text{ and } I_y = \frac{sL^2}{6I}.$$

The thrust stiffness $T = 6I/sa^2$,

and for a moment M_{BA} applied at B,

$$S = \frac{7I}{2s}, \quad H = \frac{6M_{BA}}{7a},$$

$$\tan\alpha = \frac{a}{L} \text{ and } M_{BA} = -\frac{M_{AB}}{7}.$$

The factors for the end A are the same as those for end B.

The general formulae for a member pinned at A are the same as when A is fixed.

The application of the method is illustrated by two examples.

Example 1

The symmetrical two-bay rigid-jointed frame shown in Fig. 8 is fixed at the bases of the columns. The

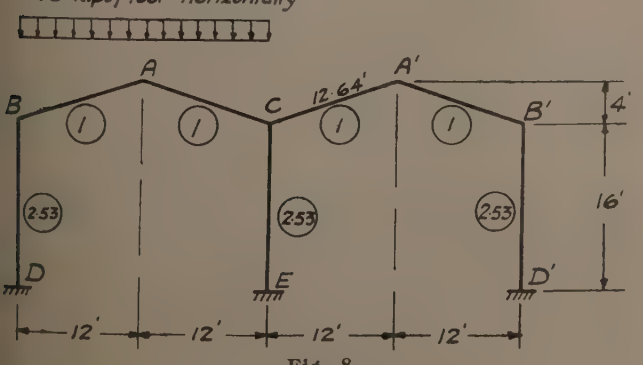

Fig. 8

material of the members is the same throughout, and E will be taken as unity. The relative moments of inertia of the members are given in circles. AB and AC carry a uniform vertical load of 10 kips per foot horizontally. In the analysis, DB, BA and AC, and similarly $D'B'$, $B'A'$ and $A'C$ are regarded as single composite members. It is therefore necessary to consider only the equilibrium of joint C.

Properties of the members (*Fig.* 9)

Table 1 contains the principal dimensions and derived properties of members $DBAC$ and CE. Those of $D'B'A'C$ and $DBAC$ are identical. Considering $DBAC$

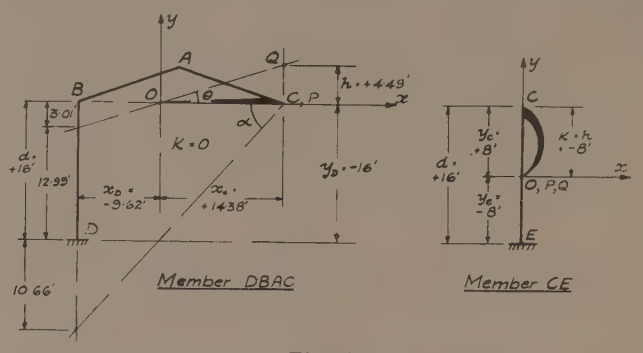

Fig. 9

and taking moments of the elastic weights about a horizontal axis through D, to obtain the position of the elastic centre O,

$$6.33 \times 8 + 12.64 \times 18 + 12.64 \times 18 = 31.61y_D,$$

from which $y_D = 16.00$ ft.

Similarly, by taking moments about DB,

$$x_D = 9.62 \text{ ft.}$$

O now becomes the origin of co-ordinates. In Table 1 the co-ordinates of the ends of DB, BA and AC, working from D round to C, are recorded. For DB, x_1 is the

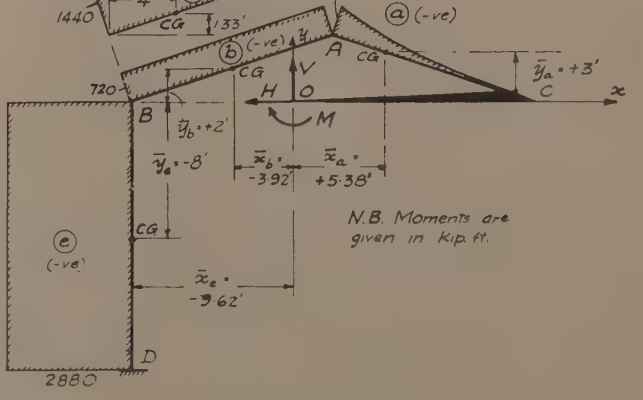

Fig. 10

co-ordinate of D, x_2 that of B, and so on. Using the expressions previously derived, I_x, I_y, I_{xy}, I'_x, I'_y, T, $\tan\theta$, h, k, S, the horizontal component of the thrust accompanying a moment applied at C, α (the inclination of this thrust) and the carry-over factors to D and E are then worked out. Substitution in the formulae is automatic, but care must everywhere be taken of the signs of the x's and y's and other dimensions. The properties of the member CE may actually be obtained more simply by slope-deflection methods, since the member is of uniform cross-section. Columns carrying crane loads, however, are frequently of varying cross-

	DBAC			CE
	DB	BA	AC	
S	16	12·64	12·64	16
I	2·53	1	1	2·53
$\frac{S}{I} = W$	6·33	12·64	12·64	6·33
x_1	(D) −9·62	(B) −9·62	(A) +2·38	0
x_2	(B) −9·62	(A) +2·38	(C) +14·38	0
y_1	(D) −16·00	(B) 0	(A) +4·00	−8
y_2	(B) 0	(A) +4·00	(C) 0	+8
I_x	540	67·4	67·4	134·9
I_y	585	317·5	1041	0
x'	0	12	12	0
y_1'	16	4	4	16
I_{xy} about centroid of member	0	+50·6	−50·6	0
$\bar{x}$	−9·62	−3·62	+8·38	0
$\bar{y}$	−8·00	+2·00	+2·00	0
I_{xy} about O	+487	−91·8	+21·2	0

Table 1 (part)

	DBAC	CE
W	31·61	6·33
I_x	674·8	134·9
I_y	1943·5	0
I_{xy}	+607·2	0
$I_x' (= I_x - \frac{I_{xy}^2}{I_y})$	485·1	134·9
$I_y' (= I_y - \frac{I_{xy}^2}{I_x})$	1396·5	0
$T (= \frac{1}{I_x'})$	0·00206	0·00742
$\tan\theta (= \frac{I_{xy}}{I_y})$	+0·3125	0
x_c	+14·38	0
K	0	−8
h	+4·49	−8
$\frac{1}{W}$	0·0316	0·158
h^2/I_x'	0·0416	0·474
x_c^2/I_y	0·1063	0
S	0·1795	0·632
H accompanying unit moment at $C (= h/S.I_x)$	+0·0515	−0·0938
$\tan\alpha = \frac{K.I_{xy} + x_c.I_x}{h.I_y}$	+1·111	0
Moment Carry-Over Factor $= -[1 + \frac{h}{S I_x}(d - L\tan\alpha)]$	−0·451	+0·5

Table 1 (cont.)

section, and the null-point method is then to be preferred.

An unbalanced thrust at C distributes between $DBAC$, $D'B'A'C$ and CE in the ratio of their respective thrust stiffnesses T, giving the thrust distribution factors for the three members entered at the beginning of the distribution table (Table 3). The moment distribution factors are obtained from the moment-stiffnesses of the members in a similar way (see also Table 3).

Fixed-end thrusts and moments at D and C due to the applied loading on DBAC

The bending moment diagrams (all negative) due to the loading on $DBAC$, which is considered as a cantilever fixed at D, are shown in Fig. 10. These diagrams must be imagined as standing on the member perpendicular to the plane of the paper. The diagrams (*b*) and (*c*) on BA represent the moment produced by the loading on AC, and diagram (*d*) the moment due to the loading on BA itself. (It is convenient to split up the complete diagram for BA in this way into simple figures whose properties are known: the area of a parabola-like diagram (*d*) is one-third the area of the enclosing rectangle and its centroid is at a distance three-quarters of the base from the apex).

The calculations for H, V and M are set out in Table 2. H is positive (i.e. it acts to the left). The fixed-end horizontal thrust at C is, therefore, positive and equal to H, while the thrust at D is $-H$, since there are no other horizontal forces on the member. M is negative, i.e. according to the sign convention adopted in determining H, V and M it acts in an anticlockwise sense. In finding the moments at C and D, it is necessary to use the sign convention adopted in the distribution

able (a positive moment is anticlockwise). Taking moments about C,

$M_{CA} = +1{,}342 - 121.8 \times 14.38 = -411$ kip. ft.

Cutting the member just above D and taking moments about D,

$M_{DB} + 1{,}342 + 121.8 \times 9.62 + 43.1 \times 16 - 2{,}880 = 0$

or $M_{DB} = -321$ kip. ft.

	A	$\bar{x}$	$\bar{y}$	M_x $(= A\bar{y})$	M_y $(= A\bar{x})$
(a)	-3036	+5·38	+3·00	-9110	-16320
(b)	-9110	-3·62	+2·00	-18220	+32900
(c)	-9110	-5·62	+1·33	-12147	+51100
(d)	-3036	-6·62	+1·00	-3036	+20070
(e)	-18220	-9·62	-8·00	+145760	+174800
Σ	-42512			+103247	+262640

$M_x' = M_x - \frac{I_{xy}}{I_y} M_y = +20919. \therefore H = \frac{M_x'}{I_x'} = +43.1$ (kips)

$M_y' = M_y - \frac{I_{xy}}{I_x} M_x = +170067 \therefore V = \frac{M_y'}{I_y'} = +121.8$ (kips)

$M = A/w = -\frac{42512}{31.61} = -1342$ (kip. ft.)

Table 2

Distribution of moments and thrusts (*Table* 3)

Step 1. The external loading is applied to $DBAC$ and C is held fixed by applying there a horizontal thrust of + 43.1 kips and a moment of — 411 kip. ft. (The necessary vertical force of 121.8 kips is carried directly by CE.)

Step 2. The horizontal thrust at C is removed and the joint is allowed to translate without rotation. To balance the fixed-end thrust of + 43.1 kips, forces are applied, at the elastic centres of the three members $DBAC$, $D'B'A'C$ and CE, such that their horizontal components total — 43.1 kips and are in the ratio of the thrust distribution factors of the members (line 7).

Step 3. The balancing forces, applied to the centroids of the members and inclined at angles $\tan^{-1} I_{xy}/I_y$ to the horizontal, produce moments about C, D, D' and E. These moments are the horizontal components H of the balancing forces times the appropriate value of $\bar{x}$ (line 5). There is now an unbalanced moment of — 258.6 kip. ft. at C. The joint is allowed to rotate without translation and the unbalanced moment distributes among the three members in the ratio of their moment distribution factors (line 10). The balancing moment at the end of each member produces a carry-over moment at the other end of the member (line 11).

Step 4. The balancing moments at C are multiplied by the appropriate factor in line 3 to give the horizontal component of the thrust induced at the end C of each member by the balancing moment (line 12). This gives a resultant unbalanced thrust of — 10.63 kips, which is distributed as in Step 2 (line 13), by allowing C to translate without rotation.

The balancing thrusts in line 13 cause moments at the ends of the members and there is once more an unbalanced moment of — 37.7 kip. ft. at C (line 14). This is distributed by allowing C to rotate without translation (line 15).

A further distribution of thrusts and moments is carried out, after which the unbalanced thrust at C is small enough to be neglected. The moments at the ends of the members are then obtained by adding up all moments in the appropriate columns. It will be seen that the rate of convergence is rapid. For a preliminary design it would have been sufficient to stop at line 16, and a check on the strength of the members could then be carried out. The solution agrees well with that given by Amerikian[6], who solved this problem by the method of slope-deflection.

The moments at other points in the frame may be determined in the manner given below for point B.

With C initially held fixed, the equation

$M_{BA} + 1{,}342 + 121.8 \times 9.62 - 2{,}880 = 0$

results from taking moments about B.

$\therefore M_{BA} = +369$ kip. ft.

During the analysis, the additional forces and moments shown in Fig. 11 are applied to C. Taking moments about C,

$M_{BA} + 5.49 \times 3.01 - 2.72 \times 26.66 + 54.8 = 0$,

from which $M_{BA} = +1.6$ kip. ft.

The final value of M_{BA} is therefore $+ 369 + 1.6 = + 370.6$ kip. ft.

This problem could have been approached by splitting up the frame into two roof members, BAC and $B'A'C$, and the three columns DB, CE and $D'B'$. The determination of the properties of the members would be much

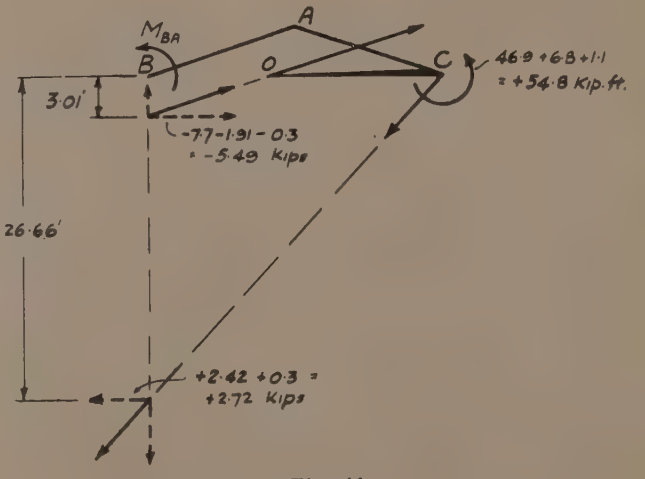

Fig. 11

simpler, but thrusts and moments would have to be distributed at three joints, B, C and B', instead of only at C. This procedure is probably to be preferred, however, for frames of many bays, where the number of joints at which distributions are required is not appreciably reduced by treating the end bays as in the above example.

Example 2

The symmetrical frame of Fig. 12 is pinned at the feet of the columns. The figures in circles are the

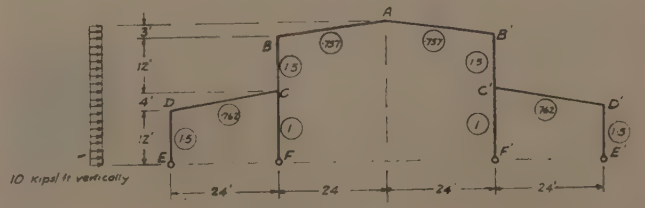

Fig. 12

	DB	EC	CA	CE	CA′	D′B′	
1.			·181	·638	·181		Moment Distribution Factor
2.			-·451	½	-·451		Moment Carry-over Factor
3.			+0·0515	-0·0938	+0·0515		To convert moments to thrusts, multiply by:
4.			·179	·642	·179		Thrust Distribution Factor
5.	-12·99	-8	+4·49	-8	+4·49	-12·99	To convert thrusts to moments, multiply by:
6.			+43·1				Fixed-end horizontal thrust
7.			- 7·7	-27·7	- 7·7		Balancing thrusts
8.	-321		-411				Fixed-end moments
9.	+100	+221·6	- 34·6	+221·6	-34·6	+100	Moments due to balancing thrusts
10.			+46·9	+164·8	+46·9		Balancing moments
11.	- 21·2	+82·4				- 21·2	Carry-over moments
12.			+2·42	-15·47	+2·42		Thrusts due to balancing moments
13.			+1·91	+ 6·84	+1·91		Balancing thrusts
14.	-24·8	-54·9	+8·6	-54·9	+8·6	-24·8	Moments due to balancing thrusts
15.			+6·8	+24·1	+6·8		Balancing moments
16.	- 3·0	+12·0				- 3·0	Carry-over moments
17.			+0·3	- 2·3	+0·3		Thrusts due to balancing moments
18.			+0·3	+1·1	+0·3		Balancing thrusts
19.	- 3·9	- 5·6	+1·3	- 8·8	+1·3	- 3·9	Moments due to balancing thrusts
20.			+1·1	+4·0	+1·1		Balancing moments
21.	- 0·5	+2·0				- 0·5	Carry-over moments
22.	-274·4	+257·5	-380·9	+350·8	+ 30·1	+46·6	Final moments (Σ lines 8,9,10,11, 14,15,16, 19,20,21
23.	-279·2	+256·5	-375·8	+348·8	+26·9	+50·5	Amerikian's solution (kip.ft).

Table 3

relative moments of inertia. E is the same throughout and is taken as unity. The members DB, DC, CB and BA are subjected to a horizontal uniformly distributed load of 10 kips per foot measured vertically.

In the analysis which follows, EDC, $BCAB'C'$, and $C'D'E'$ are treated as single composite members, and it is therefore only necessary to consider the equilibrium of two joints, C and C'.

The properties of the members are given in Table 4. Considering the member EDC (Fig. 13), the elastic weight of the hinge E is infinite and the elastic centre coincides with E, which becomes the origin of co-ordinates. The moment carry-over factor from C to E is merely worked out as a check from the formulae; it must obviously be zero.

Member $CBAB'C'$ is symmetrical about a vertical axis through the apex A, and the elastic centre lies on this axis. Taking moments about CC' for the height of the elastic centre above CC',

$$8 \times 6 + 32.13 \times 5 + 32 \times 13.5 + 8 \times 6 = 80.\bar{y}, \text{ or}$$
$$\bar{y} = 12 \text{ ft.}$$

O therefore lies on the line BB'

The thrust and moment stiffness formulae for symmetrical members apply to $CBAB'C'$. In Fig. 13 the direction of the thrust is shown for a moment applied at C'. When a moment is applied at C the thrust line is inclined downwards from C to the right at the same angle.

The elastic centre O of member CF coincides with the hinge F.

Fixed-end moments and thrusts for the loading on EDC and CBAB'C'

Member EDC (Fig. 14):

H, V and M are applied to the end C through an imaginary rigid bracket extending from C to O. M is statically determinate, and by taking moments about C

$$M + 10 \times 16 \times 8 = 0, \text{ or } M = -1{,}280 \text{ kip. ft}$$

(i.e. opposite to the direction shown in Fig. 14).

The positive bending moment due to M (Fig. 14 (e) and (f)) must be included in the complete bending moment diagram for EDC. The bending moment diagram due to the applied load is obtained by considering the member as fixed at E, and is split up for convenience into the diagrams (a) to (d). The calculations for H and V are given in Table 5, from which $H = +104.6$ kips and $V = -16.65$ kips. Then the fixed-end moment at C, considered positive if anti-clockwise, is

$+1{,}280 - 104.6 \times 16 + 16.65 \times 24 = +7.5$ kip. ft.

Member $CBAB'C'$ (Fig. 14) :

The calculations are arranged in Table 5. Using the values obtained for H, V and M, the fixed-end moment at C' is

$+52.5 + 31.87 \times 12 - 4.55 \times 24 = +325.5$ kip. ft.

Cutting the frame just above C and taking moments

	EDC		CBAB'C'				CE
	ED	DC	CB	BA	AB'	B'C'	
S	12	24·35	12	24·18	24·18	12	16
I	1·5	·762	1·5	·757	·757	1·5	1
S/I (=W)	∞		8	32	32	8	∞
x_1	0	0	-24	-24	0	+24	0
x_2	0	+24	-24	0	+24	+24	0
y_1	0	+12	-12	0	+3	0	0
y_2	+12	+16	0	+3	0	-12	+16
I_x	384	6310	384	96	96	384	1365
I_y	0	6130	4608	6144	6144	4608	0
x'	0	24	I_{xy} of CBAB'C' is zero				I_{xy} of CE is zero
y'	12	4					
I_{xy} about own centroid	0	+255					
$\bar{x}$	0	+12					
$\bar{y}$	+6	+14					
I_{xy} about O	0	+5370					

Table 4 (part)

about this point for the equilibrium of this portion of the member,

$M_{FCB} + 52.5 + 31.87 \times 12 + 4.55 \times 24 - 1{,}125 = 0$,

or M_{FCB} (the fixed-end moment at C) = $+581.1$ kip. ft.

Distribution of horizontal thrusts at C

The thrust stiffnesses of the individual members of the frame are shown in Fig. 15. Then,

Combined stiffness of $E'D'C'$ and $C'F'$ ... ... $= 0.001385$

Combined flexibility of $E'D'C'$ and $C'F'$ ... $= \dfrac{1}{0.001385} = 723$

Flexibility of $CBAB'C'$ ... $= \dfrac{1}{0.001042} = 960$

Hence effective flexibility to right of C = 1,683

Effective stiffness to right of C ... ... $= \dfrac{1}{1{,}683} = 0.0005945$

Stiffness of CF ... ... $= 0.000732$

Stiffness of EDC ... ... $= 0.000653$

An unbalanced thrust at C distributes between EDC, CF and the part of the frame to the right of C in the ratio of the above stiffnesses. A thrust of $+100$ therefore divides as follows :

	EDC	CBAB'C'	CF
W	∞	80	∞
I_x	6694	960	1365
I_y	6130	21504	0
I_{xy}	+5625	0	0
$I_{x'}$ (= $I_x - I_{xy}^2/I_y$)	1534	960	1365
$I_{y'}$ (= $I_y - I_{xy}^2/I_x$)	1405	21504	0
T (= $1/I_{x'}$)	0·000653	0·001042	0·000732
tan θ (= I_{xy}/I_y)	+0·918	0	0
x_c (or $L/2$)	+24	24	0
K	-16		
h	+6	+12	-16
1/W	0	0·0125	0
$h^2/I_{x'}$	0·02345	0·1500	0·1875
x_c^2/I_y (or $(L/2)^2/I_y$)	0·0942	0·0268	0
S	0·11765	0·1893	0·1875
tan α (= $\dfrac{K.I_{xy}+x_c.I_x}{h.I_y}$ or $L.I_{xy}/2h.I_y$)	+1·917	+0·0894	0
H accompanying unit moment at C (= $h/S.I_x$)	+0·0333	+0·0661	-0·0625
d	+16		
Moment Carry-Over Factor (= $-[1+\frac{h}{S.I_x}(d-L\tan\alpha)]$ or $-[1-L^2/2SI_y]$)	0	-0·717	0

Table 4 (cont.)

EDC	CF	To right of C
+33.0	+37.0	+30.0

The thrust of $+30.0$ to the right of C distributes into $E'D'C'$ and $C'F'$ in the ratio of their respective stiffnesses, i.e. $+14.15$ to $E'D'C'$ and $+15.85$ to $C'F'$. It should be particularly noted that a thrust of -30.0 is exerted on the end C' of the central roof member by the joint C', since the horizontal forces at the ends C and C' of this member must be in equilibrium.

The distribution of a thrust at C' is a mirror image of the above, owing to the symmetry of the frame.

The complete distribution of thrusts and moments is given in Table 6. The procedure is essentially as in Example 1, and need not be described in detail. One point in connection with the distribution of horizontal thrusts must, however, be emphasised. When a moment is applied to, say, the end C of the central roof member and it rotates without translation, a horizontal thrust is exerted by the joint on the member. It must not be forgotten that this thrust is transmitted to the other end C' of the member, where an equal and opposite reactive thrust is exerted by the joint C' on the member. This latter thrust must be included in the unbalanced thrusts at C' (see lines 26 and 40).

The moments after three successive distributions of thrusts and moments are given in line 50. The agreement with the solution of Amerikian[7] is very close.

Deformation of the frame

The controlling factor in the design of a frame is, frequently, not the maximum stress but the deformation of the frame, and particularly the movement of the tops of the columns, under the working loads. To obtain the displacement Δ_c of C in Example 2, above, first find the sum of the balancing thrusts in CF. This is — 82.5 — 13.1 — 5.0 — 2.5 — 1.6 — 0.3 or 105.0 kips. Then

$$\Delta_c = -105.0 \quad I_x$$

In working out the value of I_x for CF, the actual (not the relative) moment of inertia of the member must be used and the correct value of E for the material must also be inserted.

Rate of convergence

In the two above examples, which represent frame of typical practical proportions, the convergence i seen to be rapid. Only when the columns are ver slender compared with the other members does th rate of convergence become slow. As an extreme case Example 2 was worked out with columns having relative moment of inertia of 0.1 instead of 1 as in th

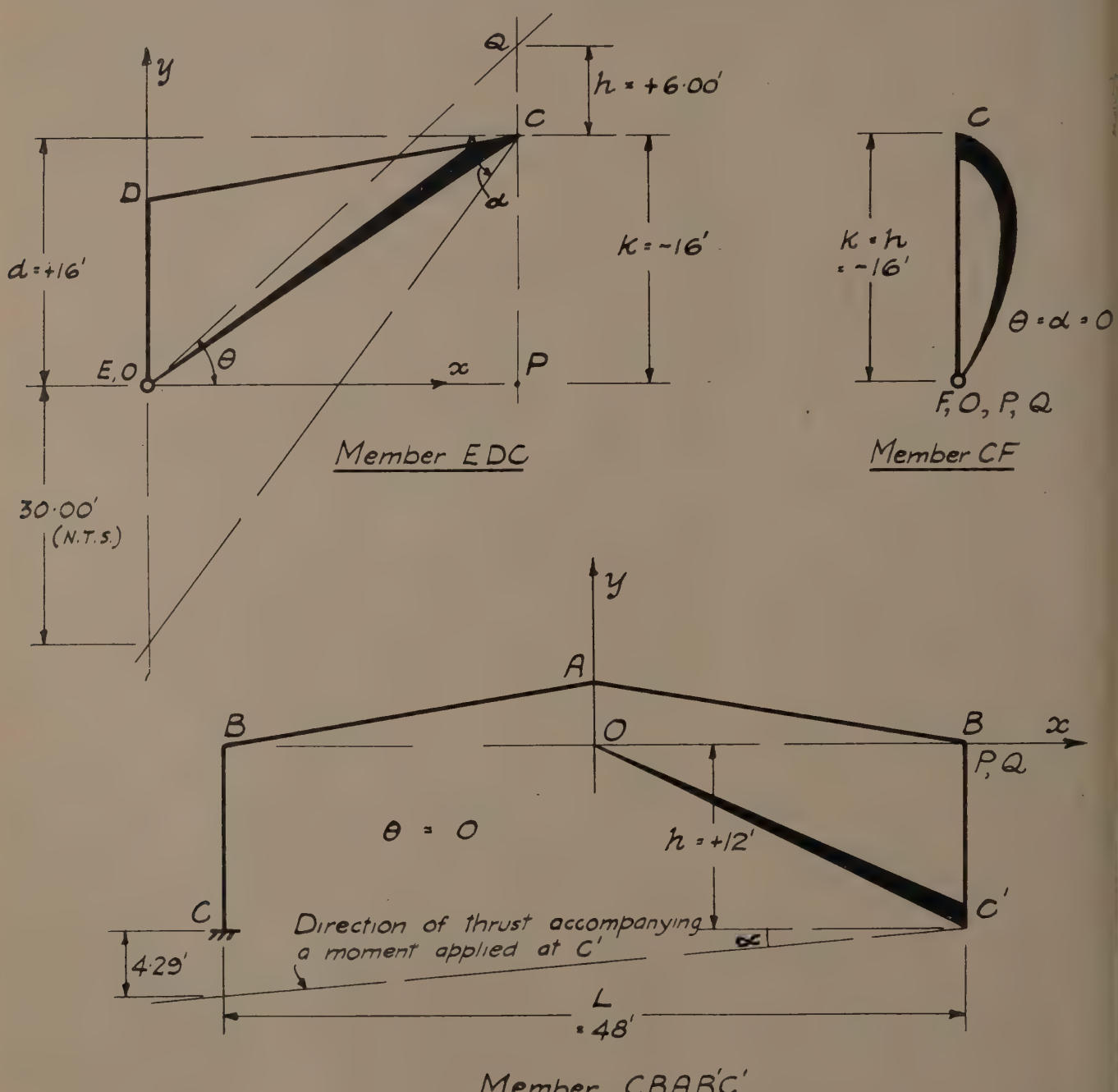

Fig. 13

Example. It was found that seven successive distributions of thrusts and moments were then required for a satisfactory solution. In most practical problems, however, three successive distributions should be sufficient.

Influence line for moment at a point

To find an influence line it is first necessary to determine, by a number of separate analyses, the moment at the point due to (*a*) unit thrust and (*b*) unit moment applied at each, in turn, of the joints where distributions are necessary. A unit load in any position on one of the members of the frame will produce fixed-end thrusts and moments at one or two of these joints, and these moments and thrusts can be expressed in terms of the distance x or y of the load from one end of the loaded member. By using the results obtained in the separate distributions, it is then possible to obtain the moment at the point for which an influence line is required in terms of the distance x or y, thus giving the required influence line for unit load on the particular member.

Settlement of a column base

The effect of, for instance, a vertical settlement Δ of the base F in the frame of Example 2 may be obtained as follows. Joint C' is initially held fixed, and the members EDC and $CBAB'C'$ are displaced downwards without rotation by applying forces Δ/I_y' at their elastic centres (see Appendix II). These forces produce unbalanced moments at C and C', and since the force in EDC is inclined to the vertical it also produces an unbalanced horizontal thrust at C. The unbalanced thrust and moments are then distributed in the usual way.

Appendix I

End moments and thrusts in an unsymmetrical member AB with fixed ends, due to any applied loading

Referring to Fig. 2(a) and the explanatory text of page 190, the bending moment m at the point (x, y) is

$$m = \mu - M - Hy - Vx.$$

Since B is fixed, the rotation and the horizontal and vertical displacements of O, which is connected to B by a rigid bracket, are zero. Therefore, summating along the whole length of the member,

$$\Sigma \frac{m\,\delta s}{EI} = \Sigma \frac{my\,\delta s}{EI} = \Sigma \frac{mx\,\delta s}{EI} = 0.$$

Hence, substituting for m,

$$\Sigma \frac{\mu\,\delta s}{EI} - M\,\Sigma \frac{\delta s}{EI} - H\,\Sigma \frac{y\,\delta s}{EI} - V\,\Sigma \frac{x\,\delta s}{EI} = 0 \qquad (1)$$

$$\Sigma \frac{\mu y\,\delta s}{EI} - M\,\Sigma \frac{y\,\delta s}{EI} - H\,\Sigma \frac{y^2\,\delta s}{EI} - V\,\Sigma \frac{xy\,\delta s}{EI} = 0 \qquad (2)$$

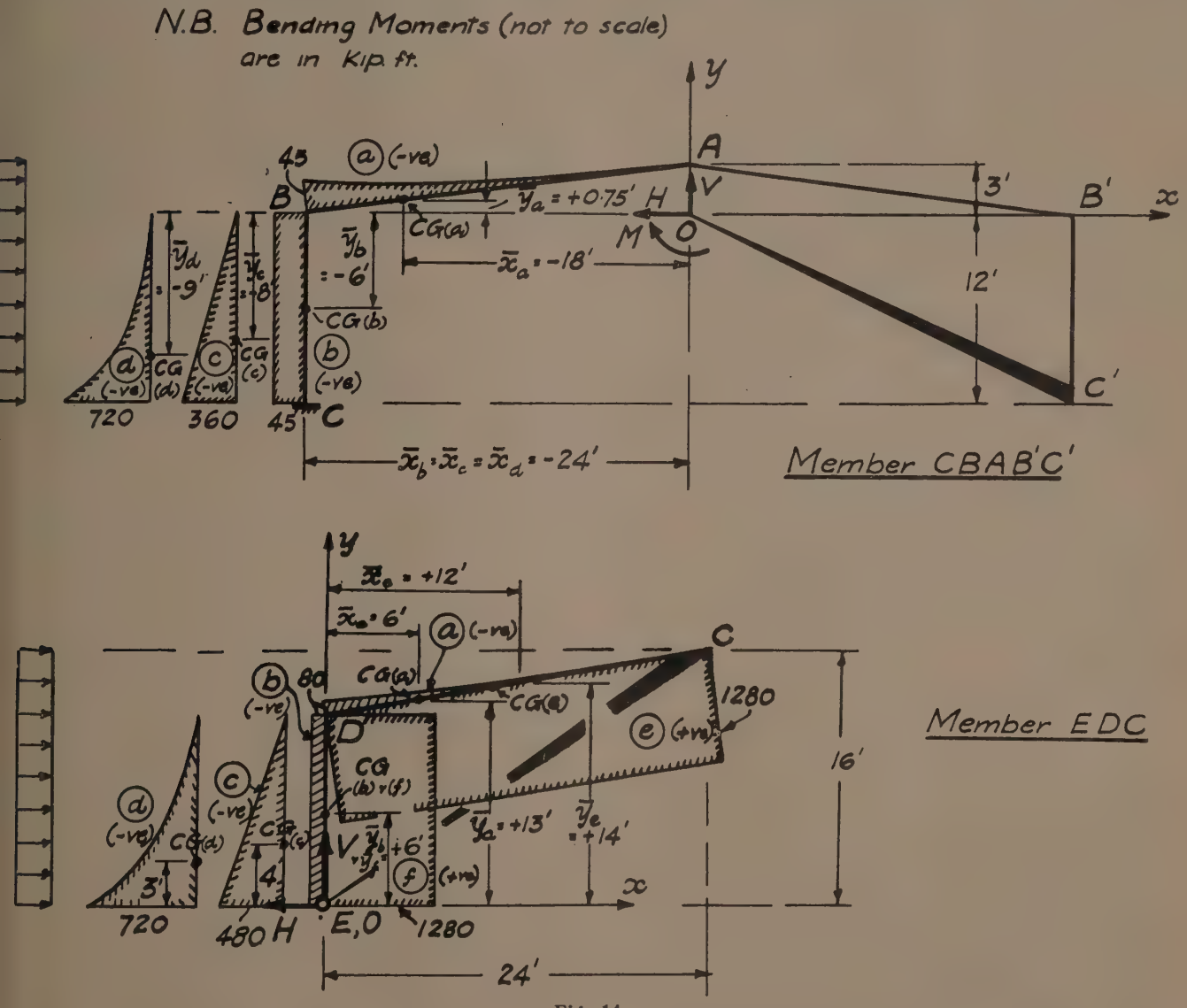

Fig. 14

	A	$\bar{x}$	$\bar{y}$	$M_x (=A\bar{y})$	$M_y (=A\bar{x})$	
(a)	-852	+6	+13	-11060	-5112	
(b)	-640	0	+6	-3840	0	
(c)	-1920	0	+4	-7680	0	
(d)	-1920	0	+3	-5760	0	
(e)	+40960	+12	+14	+573440	+491520	
(f)	+10240	0	+6	+61440	0	Member EDC
			Σ	+606520	+486408	
$M_{x'} = 606520 - 486408 \cdot \frac{5625}{6130} = +160600$ $\therefore H = \frac{+160600}{1534} = +104.6$ kips						
$M_{y'} = 486408 - 606520 \cdot \frac{5625}{6694} = -23330.$ $\therefore V = -\frac{23330}{1405} = -16.65$ kips						
(a)	-480	-18	+75	-360	+8640	
(b)	-360	-24	-6	+2160	+8640	
(c)	-1440	-24	-8	+11520	+34560	
(d)	-1920	-24	-9	+17280	+46080	Member CBAB'C'
Σ	-4200			+30600	+97920	
$H = M_x/I_x = +30600/960 = +31.87$ kips.						
$V = M_y/I_y = +97920/21504 = +4.55$ kips.						
$M = A/W = -4200/8 = 52.5$ kip. ft.						

Table 5

and

$$\Sigma \frac{\mu x\, \delta s}{EI} - M \Sigma \frac{x\, \delta s}{EI} - H \Sigma \frac{xy\, \delta s}{EI} - V \Sigma \frac{x^2 \delta s}{EI} = 0 \qquad (3)$$

Also, by the definition of O, the elastic centre of the member,

$$\Sigma \frac{x\, \delta s}{EI} = \Sigma \frac{y\, \delta s}{EI} = 0.$$

Therefore from (1),

$$M = \frac{A}{W} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (4)$$

and from (2) and (3),

$$H = \frac{M_x - \frac{I_{xy}}{I_y} \cdot M_y}{I_x'} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (5)$$

and

$$V = \frac{M_y - \frac{I_{xy}}{I_x} \cdot M_x}{I_y'} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (6)$$

An unsymmetrical member pinned at A

In this case the pin may be regarded as a very short length of member of infinitely small moment of inertia. The elastic weight of the hinge, and therefore of the member, is infinite, and the elastic centre coincides with the hinge. M may be found by statics, and is equal and opposite to the moment of the applied loading about the hinge.

Equations (2) and (3) now become, respectively,

$$\Sigma (\mu - M) \frac{y\, \delta s}{EI} - H \Sigma \frac{y^2 \delta s}{EI} - V \Sigma \frac{xy\, \delta s}{EI} = 0 \quad \ldots \qquad (2')$$

and

$$\Sigma (\mu - M) \frac{x\, \delta s}{EI} - H \Sigma \frac{xy\, \delta s}{EI} - V \Sigma \frac{x^2 \delta s}{EI} = 0 \quad \ldots \qquad (3')$$

The resulting expressions for H and V are the same as for a member with fixed ends, except that $(\mu - M)$ is substituted for μ. M_x and M_y are now, therefore, the moments of the $(\mu - M)$ diagram about Ox and Oy respectively. μ is the moment of the applied loading at the point (x, y), the member being considered as a cantilever fixed at A, as before.

Appendix II

Translation of one end of a member

Consider the unsymmetrical member AB, fixed at A (Fig. 4(a)).

Let the forces required to produce a purely horizontal movement Δ of B without rotation be H, V and M, applied at the elastic centre O of the member. The moment m at the point (x, y) is

$$m = M + Hy + Vx.$$

Since O is attached to B by a rigid bar, O also displaces an amount Δ horizontally without rotation.

Hence

$$\Sigma \frac{m\, \delta s}{EI} = 0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (7)$$

$$\Sigma \frac{my\, \delta s}{EI} = \Delta \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (8)$$

$$\Sigma \frac{mx\, \delta s}{EI} = 0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (9)$$

Substituting in (7),

$$M \Sigma \frac{\delta s}{EI} + H \Sigma \frac{y\, \delta s}{EI} + V \Sigma \frac{x\, \delta s}{EI} = 0$$

from which

$$M = 0.$$

From (9),

$$H \Sigma \frac{xy\, \delta s}{EI} + V \Sigma \frac{x^2 \delta s}{EI} = 0,$$

or

$$\frac{V}{H} = -\frac{I_{xy}}{I_y}.$$

From (8),

$$H \Sigma \frac{y^2 \delta s}{EI} + V \Sigma \frac{xy\, \delta s}{EI} = \Delta,$$

from which

$$H = \frac{\Delta}{I_x - \frac{I_{xy}^2}{I_y}} = \frac{\Delta}{I_x'}.$$

	CD	CF	CB	C'B'	C'F'	C'D'	
1.	·237	·379	382	·382	·379	237	Mt Distr. Factors
2			-·717	-·717			Mt C O Factors
3.	+·0333	-·0625	+·0661	+·0661	-·0625	+·0333	To convert Mts to Thrusts, mult by
4.	+33·0	+37·0	+30·0	-30·0	+15·85	+14·15	Distn. of Thrust +100 at C
5.	+14·15	+15·85	-30·0	+30·0	+37·0	+33·0	" " " +100 at C'
6.	+6	-16	+12	+12	-16	+6	To convert Thrusts to Mts., mult by
7.	+104·6		+118·1	+31·9			Fixed-end Hor.l Thrusts
8	-73·5	-82·5	-66·7	+66·7	-35·3	-31·5	Balance thrust at C
9.	-4·5	-5·0	+9·6	-9·6	-11·8	-10·5	" " " C'
10	-468	+1400	-686	+686	+755	-251	Mts due to balancing thrusts
11.	+8		+581	+325			F-E. Mts.
12.	-197	-318	-320	-580	-575	-360	Balancing Mts.
13			+418	+230			C.O. Mts.
14	-99	-159	-160	-88	-87	-55	Bal.g Mts.
15			+63	+115			C.O. Mts.
16	-15	-24	-24	-44	-44	-27	Bal.g Mts.
17			+32	+17			C.O. Mts.
18	-8	-12	-12	-7	-6	-4	Bal.g Mts
19			+5	+9			C.O. Mts.
20	-1	-2	-2	-4	-3	-2	Bal.g Mts.
21			+3	+1			C.O. Mts
22.	-1	-1	-1	-1			Bal.g Mts
23.	-321	-516	-519	-724	-716	-448	Sum of Bal.g Mts.
24.	-781	+884	-103	+659	+41	-669	Sum of Mts (ΣI)
25.	-10·7	+32·2	-34·2	-48·0	+44·7	-14·9	Hor.l Thrusts due to Bal.g Mts
26.			+48·0	+34·2			
27.	-11·6	-13·1	-10·6	+10·6	-5·6	-5·0	Balance thrust at C
28.	-2·3	-2·5	+4·8	-4·8	-5·9	-5·3	Balance thrust at C'

	CD	CF	CB	C'B'	C'F'	C'D'	
29	-83	+230	-70	+70	+184	-62	Mts due to Bal.g Thrusts
30	-23	-37	-37	-73	-73	-46	Bal.g Mts
31			+53	+27			C O Mts
32	-13	-20	-20	-10	-10	-7	Bal.g Mts
33			+7	+14			C O Mts
34	-2	-2	-3	-6	-5	-3	Bal.g Mts
35			+4	+2			C O Mts
36		-2	-2	-1	-1		Bal.g Mts
37	-38	-61	-62	-90	-89	-56	Sum of Bal.g Mts
38	-121	+189	-68	+23	+95	-118	Sum of Mts (ΣII)
39	-13	+38	-41	-60	+56	-19	Hor.l Thrusts due
40			+60	+41			to Bal.g Mts
41	-15	-16	-13	+13	-07	-06	Balance thrust at C
42	-02	-03	+05	-05	-07	-06	Balance thrust at C'
43	-10	+30	-10	+10	+22	-7	Mts due to Bal.g Thrusts
44	-2	-4	-4	-10	-9	-6	Bal.g Thrusts
45			+7	+3			C.O. Mts.
46.	-1	-3	-3	-1	-1	-1	Bal.g Mts
47			+1	+2			C.O. Mts
48.			-1	-1	-1		Bal.g Mts
49.	-13	+23	-10	+3	+11	-14	Sum of Mts. (ΣIII)
50	-915	+1096	-181	+685	+147	-831	Final Mts (ΣI + ΣII + ΣIII)
51.	-919	+1100	-182	+687	+147	-835	American's solution (Kip. ft)

Table 6

V acts downwards if I_{xy} is positive, and the resultant R of V and H is inclined at an angle θ ($= \tan^{-1} I_{xy}/I_y$) to the horizontal (Fig. 4(b)).

Similarly, it may be shown that to produce a vertical movement Δ of B without rotation, a force R at O is required which is inclined at an angle ψ ($= \tan^{-1} I_{xy}/I_x$) to the vertical, and whose vertical component V is equal to Δ/I_y' (Fig. 4(c)).

The corresponding expressions for a member pinned at A are the same as the above.

Appendix III

Rotation of one end of a member

Consider first the case where the end A of the unsymmetrical member AB is fixed (Fig. 16). Let a moment M'_{BA} be applied to the end B, and let this end be allowed, for the present, to translate freely. Then the relation between the moment M'_{BA} and the rotation Φ_B of B is given by

$$\Phi_B = M'_{BA}\, \Sigma \frac{\delta s}{EI} = M'_{BA}.\, W.$$

The vertical movement of B is $M'_{BA}\, \Sigma \dfrac{(x_b - x)\ \delta s}{EI}$,

or $M'_{BA}.\, x_B.\, W$ (upwards), and the horizontal movement of B is $M'_{BA}\, \Sigma \dfrac{(y + k)\ \delta s}{EI}$ or $M'_{BA}.\, k.\, W.$ (to the right).

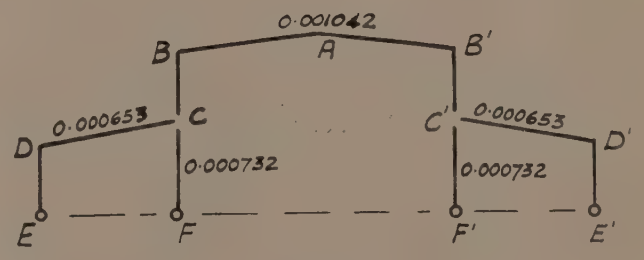

Fig. 15

To restore the horizontal displacement of B to zero, it is necessary to apply a force R_1, at the elastic centre O, inclined at an angle θ ($= \tan^{-1} I_{xy}/I_y$) to the axis Ox. From Appendix II it follows that the horizontal component H_1 of R_1 is given by $M'_{BA}.\, k.\, W./I_x'$. Similarly, to restore the vertical displacement of B to zero, it is necessary to apply at the elastic centre a force R_2, which is inclined to the vertical at an angle ψ ($= \tan^{-1} I_{xy}/I_x$) to the axis Oy, and whose vertical component V_2 is given by $M'_{BA}.\, x_b.\, W/I_y'$. The horizontal component H_2 is equal to $V_2.\, \dfrac{I_{xy}}{I_x}$.

The angle of rotation B is unaffected by the application of the forces R_1 and R_2, which produce purely horizontal and vertical movements of B respectively. R_1 and R_2, however, increase the moment at B to M_{BA}, where

$$M_{BA} = M'_{BA} + H_1.\, h + H_2.\, k + V_2.\, x_b.$$

Substituting the expressions obtained above for H_1, H_2 and V_2 and reducing to the simplest terms,

$$M_{BA} = M_{BA}.\ W\ \left[\frac{1}{W} + \frac{h^2}{I_x'} + \frac{x_b^2}{I_y}\right].$$

But $M'_{BA}.\ W = \theta_B$.

Therefore

$$M_{BA} = \left[\frac{1}{W} + \frac{h^2}{I_x'} + \frac{x^2_B}{I_y}\right].\ \theta_B.$$

The expression $\left[\frac{1}{W} + \frac{h^2}{I_x'} + \frac{x_B^2}{I_y}\right]$ is termed the moment stiffness (S_B) of the end B of the member AB.

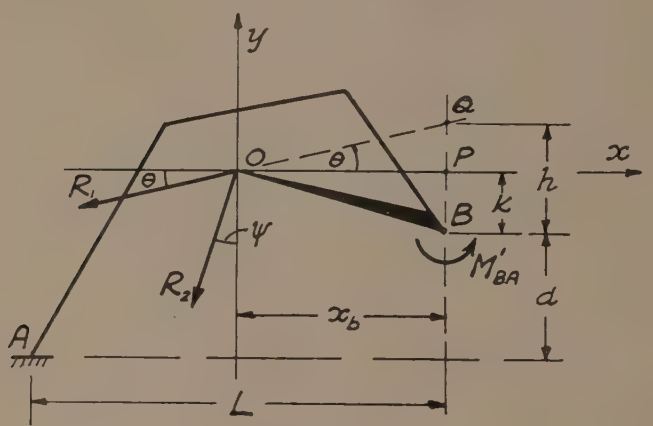

Fig. 16

It is also necessary to know the value of the force R which must accompany M_{BA} in order to keep the end B fixed in position. The horizontal component of this force is H where

$$H = H_1 + H_2.$$

Substituting for H_1 and H_2 and reducing,

$$H = \frac{M_{BA}.\ h}{S_B.\ I'_x}.$$

The vertical component V of R is $V_1 + V_2$, or

$$V = H_1.\frac{I_{xy}}{I_y} + V_2.$$

On substituting for V_2 and H_1, this expression reduces to

$$V = \frac{M_{BA}}{S_B}\left[\frac{k.\ I_{xy} + x_b.\ I_x}{I'_x.\ I_y}\right]$$

The angle of inclination of R to the horizontal is $\tan^{-1} V/H$,

$$= \tan^{-1}\left[\frac{k.\ I_{xy} + x_b.\ I_x}{h.\ I_y}\right] \text{ (see Fig. 6).}$$

R acts downwards and to the left if the expression in brackets is positive.

The moment produced at A may be obtained by imagining the member AB to be cut just above A. Then, considering the moment M_{AB} at A to be positive when anticlockwise, the expression

$$M_{AB} = -\left[M_{BA} + H\ (d - L.\ \tan\alpha)\right]$$

results from taking moments about A for the equilibrium of the member. M_{AB} is known as the "carry-over" moment from B to A.

If the member is reduced in cross-section for a very short length near A (A being fixed) it is clear that, although the elastic centre will move somewhat towards A, the validity of the theory developed above is unaffected. A hinge at A being regarded as a very short length of member of infinitely small cross-section, the formulae will, therefore, still apply if A is hinged, though it must be remembered that the elastic weight of the hinge, and therefore of the complete member, is infinite and the elastic centre is now at A.

Conclusion

The classical methods of structural theory, and even the moment-distribution method in its usual form, are very unwieldy when applied to the type of structure dealt with in this paper. The method outlined in the foregoing renders the more complicated types of factory frames much more amenable to analysis, and it is hoped that designers will find it of assistance. Its advantages are most marked where the structure under consideration has a number of bays, where the roof members are built up from a number of straight portions or are curved, and where the members are of non-uniform cross-section. The frames analysed in the paper are, in fact, fairly simple and do not demonstrate the full power of the method, but it was convenient to choose them since the problems were the most complicated ones for which the writer was able to discover published solutions by other methods. It is immaterial whether the structure is symmetrical, as in these problems, or not.

However complicated the frame, the solution is gradually approached by a process which entirely obviates the necessity of solving simultaneous equations. Every step in the analysis has a definite physical meaning, and the designer can see clearly how the structure gradually assumes its final deformed shape under the applied loads. The ordinary 10-inch slide rule is quite accurate enough for the calculations.

Acknowledgements

The writer wishes to record his indebtedness to Professor C. Batho, D.Sc., M.I.C.E., M.I.Struct.E., for his advice and encouragement.

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Prestressed Steel Structures

Discussion on Professor Magnel's Paper★

THE CHAIRMAN introduced Professor MAGNEL, who then summarised his paper. At the conclusion, he said he did not claim that we had arrived at the final stage of the developments discussed. But, pointing to the necessity for economy in steel, in view of the large quantities needed for re-armament, he said the Americans were putting a lot of people out of work because there was not sufficient steel available for buildings, and so on, and in his own country and ours the situation would be the same. As a sound policy to effect saving in steel he suggested, first, that we should use much more reinforced concrete than steel; secondly, we should use more and more prestressed concrete instead of reinforced concrete; and thirdly, for big spans, where prestressed concrete could not do the job, we should try to use prestressed steel.

THE CHAIRMAN, thanking Professor Magnel for his paper and for the manner of its presentation, said they were grateful to hear a lecture from so distinguished an engineer, and for the touches of humour which he had brought into its presentation. One felt sure some of the steelwork people would be disappointed that they could not have what Professor Magnel had referred to as "negative sections." The subject of the lecture was new, and those who had not had time to read the paper, or who had perhaps found it difficult, had probably felt, after listening to its presentation, that it was extremely easy; that was due to the magnificent way in which Professor Magnel had explained the problems involved.

Whilst increased deflections might be permissible in some structures, one was glad that Professor Magnel had not favoured increased deflections for bridges, for that would be a retrograde step.

(The vote of thanks was heartily accorded).

Mr. DONOVAN H. LEE (Member), said that perhaps he had a slight advantage in that he had read Professor Magnel's paper published in Belgium on the same subject; and it had been most enjoyable also to listen to his presentation of the paper that evening. He believed the two papers corresponded fairly closely, although he had not had time to read the English one very carefully, and he felt the way in which the fundamental basis of comparison was obtained between ordinary steel construction and prestressed steelwork was in itself indeed excellent, whatever value the rest of the paper might have. As some of those present would readily understand, he also had previously considered the possibilities of prestressing steelwork, though by using high tensile alloy steel bars instead of the wire Professor Magnel had used.

With regard to the use of 0.2 in. diameter wires, Mr. Lee said that to leave them exposed was to put us at the mercy of the electrician who crawled about a roof, trying to put in new electrical fittings and cutting away anything which obstructed him. Again, the use of exposed 0.2 in. diameter wires was rather frightening from the point of view of corrosion. There were some very polluted atmospheres in some parts of this country—although, from the little experience he had of Belgium, he felt that Charleroi might equal Widnes—and it would be interesting to hear Professor Magnel's comments concerning the protection of such wires against accident and corrosion.

The results, however, when considered in the overall way of result in comparison with cost, were not so favourable as the results obtained from Professor Magnel's fundamental comparisons and the graphs which he had shewn. And, in fact, it did not make much difference whether it was hard-drawn plain carbon wire or alloy steel bars which were used. Taking for example an ordinary steel lattice girder which would need seven square inches of cross section of mild steel in the bottom boom, and say ten square inches in the top boom, Mr. Lee showed, by diagrams, that this could be replaced by a prestressed mild steel bottom boom of 2.8 square inches with 0.625 square inches of high tensile steel in two bars, one each side of the bottom boom and working at a stress of 42 tons per square inch. However, while the saving of weight was quite considerable, the deflection was increased 50 per cent., and the saving in cost, by the time the end connections had been made and the prestressing done, was about nothing. There was no advantage to be obtained to the compression flange, while if for example the prestressing steel was situated below the bottom boom, some small benefit could be obtained to the compression boom by induced tension from the prestressing, but the overall depth of the girder would be increased. Of course, where the girder itself had a dead weight which was quite large in relation to the load it carried, the saving of weight of the girder itself would lead to some saving of load which Professor Magnel had shown could be cumulative.

Referring to Professor Magnel's fundamental comparison, Mr. Lee suggested that on the first graph there was a mistake in the scale.

Professor MAGNEL said that he had noticed it.

Mr. Lee said he presumed that the comparison was a fundamental one between a latticed girder prestressed and one which was not prestressed. In the case of the girder not prestressed it was assumed that the top and bottom flanges were the same throughout; that was of course contrary to usual practice, which was that the top and bottom booms would be built up as the bending moment built up. Mr. Lee wondered whether that would invalidate some of the graphs. He had noticed that there was apparently no saving in cost if the initial cost of the wire was more than 4/1. It therefore appeared important not to overlook including the labour cost of prestressing in with the wire.

Of course, he agreed that, where there was little live load, there was best opportunity to benefit by prestressing, whereas, if there was a high ratio of live load to total load, deflection was a trouble. For example, he could not imagine the use of long span prestressed lattice girders in Canada or the United States, where a snow loading of 40 lb. per sq. ft. was often assumed, because the weight of the girder itself became almost insignificant in that case.

With regard to the 500-ft. span latticed girders, he said it appeared quite impossible for us to do this at a depth/span ratio of 33/1 in structural steel. But

★*Read before the Institution of Structural Engineers at* 11, *Upper Belgrave Street, London, S.W.*1, *on Thursday, November* 9*th*, 1950, *Mr. E. Granter, B.Sc., M.I.C.E., M.I.Struct.E. (Vice-President) in the Chair. Published in* THE STRUCTURAL ENGINEER, *Vol. XXVIII, No.* 11, *p.* 285. (*November*, 1950).

it seemed that in aircraft hangar design, if the roof were raised from say 22 to 32 feet it did not add a lot to the cost; therefore, he asked why Professor Magnel assumed that there should be a depth of only 15 ft. Perhaps if he admitted it might have more height, it would benefit prestressed steelwork even more than conventional construction, as it would reduce the deflection more.

Deflection became most serious immediately the live load was any substantial proportion of the whole. We should not have latticed girders and camber them up, and then, when there was a fall of snow, find that they bowed downwards; he had known trouble to arise with conventional steelwork roofs of hangars, due to distortion during gales.

Professor MAGNEL, referring to Mr. Lee's comments on corrosion, said the problem existed, but he was expounding the principles independently of the diameter of the wires. They were making high tensile wire up to a diameter of $\frac{1}{2}$-in., which was already much better from the point of view of corrosion. Also, the system he had explained could be applied by using alloy steels up to 3 in. diameter, if one cared to do that. He had a small laboratory team to check the theory. He had never dreamed of making things such as Mr. Lee had in mind in aggressive atmospheres; in such atmospheres it was necessary to paint the wire, just as one painted tubes, and so on.

Mr. LEE, repeating his point concerning the graphs, said he was concerned about the stopping off of plates at the top and bottom booms. In other words, the section of a latticed girder became smaller as the moment became smaller, but in the graphs it was assumed to be constant.

Professor MAGNEL agreed and said that in his graphs he was also providing very big economies; even if they were halved we should still gain something which from the practical point of view was quite interesting.

With regard to the 15 ft. depth of beam for the hangar, he said that that was chosen for reasons which had nothing to do with economy. The specification asked for hangars of 250 ft. span, and the beams and girders were not to be of more than a certain height, and one had to stick to that height, although the work could have been done more economically.

Professor R. H. EVANS (Member) said he was glad to have been responsible for Professor Magnel's presence on that occasion. It had been his privilege to see the erection of the girder described in the paper, and he had taken advantage of the opportunity to ask Professor Magnel to come to London; and the Professor had agreed. The matter was put to the Secretary, and the Institution had made arrangements for the visit.

The first paper in this country on prestressed concrete, he recalled, had been presented to the Institution in 1936, and now the Institution had the privilege of the first paper on prestressed steel. Professor Magnel had certainly indicated the principles of prestressed steel very clearly.

With regard to the first girder, when the bottom boom was prestressed, Professor Evans asked what was the magnitude of the stresses induced in the other members; there must be some effect, owing to the rigidity of the riveted joints. In the case of the second girder, this question would be more important and the stress analysis due to prestressing should take into account the rigidity of the welded joints.

Again, in the first girder buckling was impossible, as the wires were positioned in the bottom member. In the case of the second girder he was not sure whether there was just a parabolic cable from the centre point right up half way to the end of the girder. He asked what looked after buckling in that case, and whether it was due to the wires being positioned in suitable brackets attached to the web members.

As had been pointed out by Professor Magnel, the benefit arose when the dead load was high compared with live load. He asked what was the limiting span.

In connection with strain gauge measurements, he said that quite a lot of work had been done at Leeds on girders and trusses, and the experience there was that the stress distribution across the flanges was nothing like so variable as was indicated by Professor Magnel's graph. He asked whether, in Professor Magnel's work, the strains had been measured on both sides, because at Leeds it had been found essential to record the average strain. Strain measurement on one side could result in a very serious error.

Many engineers, he continued, would be pleased to know the relative positions of prestressed concrete and prestressed steel in the case of, say, hangars and he asked if Professor Magnel could give an indication of the relative costs.

Finally, he asked for further elucidation of the type of structure illustrated for the hangar illustrated in Fig. 12. As he understood the sketch, the structure was a two-hinged portal but no diagonal members were shown in the two legs and he would like to know the reason for this omission.

Professor MAGNEL first thanked Professor Evans for his kind remarks.

Dealing with the question concerning the effect of prestressing the lower member of the first girder, the stresses induced in the other members, he said he had given some comparisons, showing strain gauge measurements after prestressing. Very little stress was induced in members other than the lower member, at least, where he had taken measurements. His strain gauges were all concentrated in the middle of the length between two adjacent junctions of bars, and he was quite sure that he had measured nowhere a stress greater than ten per cent. of what was in the lower member. So that it meant very little.

With regard to the buckling of the second girder, he said that on the web of the lower member he had welded steel plates with holes through which the wires passed; it was a vertical element, so that there was perfect unity between the cables and the lower member. The arrangement was equivalent to that made in the case of the first girder tested, where in the "U" shape he had also welded steel plates with holes through which the cable was threaded.

As to indicating a limit to the use of prestressed steel in relation to superload and dead load, he said it would be difficult to give a limit at present; he had not thought sufficiently about it.

But he had two sources of economy. The first source was in the fact that he was replacing section A in mild steel by A_r and A_c, the weight and cost of which was small. He had shown in one of his first diagrams that the economy in cost was only about 13 per cent. Then he had pointed out that the 13 per cent. became much higher, when we took into account that the forces acting on the whole beam became smaller. When the lower member became 33 per cent. lighter by reason of the use of prestressed steel there was an economy of 11 per cent. in the weight of the girder; when an additional load of 0.25 per cent. was to be added, the 11 became 38 per cent. economy in weight. If a load equal to the dead load were added, the 11 per cent economy became 20 per cent. The second reason for

economy, in the total weight to be carried by the girders, was only of great significance when, as was nearly always the case in hangars, the beams were very close.

The problem of comparative tests was always very difficult where there were not comparative standards. For prestressed concrete he could give very accurate figures based on experience in Belgium, but he was sorry he could not do so in respect of prestressed steel.

With regard to hangars, he said that firstly the beam was made prestressed, secondly it was a portal frame, and thirdly the portal frame was also prestressed; one of the hinges, before it was fixed after the stretcher was completely put up, was pushed inwards by a jack to give further prestress to the prestress imposed by the cable.

Mr. F. Walley asked Professor Magnel whether he had considered the use of prestressing in relation to the reduction of secondary stresses in steel trusses. He had in mind a design described by Professor Dr–Ing. H. Rusch of a large reinforced concrete truss carrying a dead load moment of 13,000 ton/ft. and a live load moment of 23,000 ton/ft. where the secondary stresses amounted to 100—150 per cent. of the primary ones. To reduce these secondary stresses Dr. Rusch had post-tensioned the bottom boom. It was doubtful if the design could have been economically carried out without prestressing, and he wondered whether something similar might be possible in steel trusses to secure advantages other than the ones to which Professor Magnel had referred.

Commenting on the reference in Professor Magnel's paper to the work on Professor Dr–Ing. Dischinger, he said that not every engineer had the magnanimity to refer to the work of other engineers in the same field. Dr. Dischinger had designed prestressed composite constructions of steel and reinforced concrete and he had been able to achieve something which it was difficult to do with either separately, whereby simply supported bridges could be built of spans of 300 ft. and continuous bridges up to 600 ft., and flat arched bridges of up to 900 ft. span. Mr. Walley suggested that prestressing operations might lie in that region as well as in steel alone.

Professor Magnel, dealing with the question concerning secondary stresses, considered a classical girder, without prestressing, which was loaded to impose secondary stresses due to bending moments, and so on; those stresses had a certain sign, no matter what it was. He suggested that, if we had prestressed the lower member, which normally was under tension, we were introducing secondary stresses probably of the opposite sign to the primary. So that it might be that the two more or less cancelled each other out, and the effect of the secondary stresses was small. That was only his impression and he could not put it forward as a certainty; it should be checked.

When he had written his French paper on prestressed steel he had not known of the earlier paper by Dr. Dischinger. He had since obtained it, and had found it difficult to read and to understand thoroughly. On the other hand, it was very interesting. But Dr. Dischinger had considered prestressed steel on rather different lines from those on which he had studied the subject. Dr. Dischinger had dealt with prestressed composite structures, composite girders of concrete and steel, and it was necessary to prestress the concrete to be able to rely on it for the resistance of moments in both directions. Professor Magnel believed there was a future for composite concrete and steel prestressed. Since reinforced concrete was introduced as a practical method of construction, more than fifty years ago, there had been patents on prestressing. There were such patents in the last century, but it was only during the last ten years or so that prestressing had become of general interest. There were many directions in which its application was possible. Only that day he had been told that a British engineer was thinking of prestressing cast iron, which was quite a sensible idea.

Mr. S. R. Banks said that some time ago he had had the privilege of arranging a meeting of the Engineering Institute of Canada in Montreal to be addressed by Professor Magnel; and he had been so impressed by Professor Magnel's abilities as a lecturer that he had felt he could not miss the present London meeting, the more so as it was to deal with so fresh a subject as prestressed steel. He was very interested in the use of a more expensive material in lesser quantity in order to make a more economical structure, for it was the cost of structures that usually mattered in civil engineering.

He was engaged at the moment in somewhat similar investigations in connection with the use of light metals, where the same phenomenon arose, that by using the more expensive materials, but in lesser degree, economy could be shown.

Mr. Banks asked if Professor Magnel could see any advantage in the use of steel for prestressing aluminium structures, bearing in mind two possible drawbacks, one being the low modulus of aluminium, which was about one third that of steel and approximately that of concrete; and the other that the coefficient of expansion of aluminium was twice that of steel.

Professor Magnel replied that he had thought of prestressing aluminium alloys, such as Duralumin, which materials had been used in one or two cases for making bridges; at Arvida, in Canada, there was an arched bridge in aluminium. But, comparing an aluminium tie-bar with a compound one made partly of aluminium and partly of high tensile steel, he said the weight of the high tensile steel was dead against the use of the compound tie-bar; in terms of specific gravity there was a factor of something like three between steel and aluminium. He was quite sure that the saving in weight would be far less than it would be if prestressed steel were used.

But more important than that, a very careful enquiry should be made into the fatigue of aluminium alloys. Perhaps the aeroplane constructors had made tests, but he knew of no publications in that connection, and unless a very thorough study was made of the fatigue properties of those alloys it would be difficult to say anything serious about prestressing aluminium.

Mr. O. J. Masterman (Associate-Member) commented that it was a mark of genius to make complicated matters appear simple; Professor Magnel had done it, as he always did. The Institution should be extremely grateful to have had the chance, so early in the development of the technique of prestressed steel, to have it explained so simply.

It seemed that there might be a possibility of making still further economies if the prestressing cable could be arranged in such a way that an eccentricity was provided which would have the effect of changing the stresses in both the top and the bottom booms, just as was done in prestressing concrete. It seemed that in the paper only the economies in the lower boom had been considered; and although Mr. Masterman realised that diagonals and other intermediate members might present difficulties, he asked if Professor Magnel considered it possible to prestress in such a way as to reverse the stresses in both booms that would be imposed by the loading.

It was a little disappointing that Professor Magnel had not the opportunity at the meeting, no doubt due to the shortness of the time available, to say anything more about his most interesting idea of the progressive stressing of a steel structure. It was most intriguing; but from the paper he had gained the impression that it was very difficult to make it a practical possibility.

Professor MAGNEL said the idea of trying to cancel out the stresses in both the top and the bottom members of a girder at the same time was very attractive, but the difficulty was to know how to do that in the top member; he was speaking all the time of a simply supported beam, the top of which was compressed. By means of the cable he was applying the greatest possible eccentricity by dealing with the lower members. If one were dealing with a girder with a solid web, it would give tensile stress in the top member, and we could never increase that tensile stress by the cable so long as we kept the cable inside the web. Theoretically, we could also relieve the stress in the top by putting the top members into tensile stress initially, but we should put on each side of it a member which was compressed. The advantage of dealing with the lower member was that we could put it under compression by using a single buckle where there was no danger of buckling, but for the top members we could not do anything. He did not think we could do anything better than was being done now.

With regard to progressive stressing, which he had mentioned, he said the idea was as follows: "In the case of a member which had to carry a load in pure tension, he would put the reduced section under a compression of 20,000 lb. per sq. in., and the load was applied to give a tensile stress of 20,000 lb. per sq. in.; i.e., he was using stresses of minus 20,000 and plus 20,000. But he could carry the same load in different ways. If he first prestressed up to 20,000 lb. per sq. in. in compression, and then applied half the load ($F/2$), he could, when the load was applied, prestress further so that there was again a compressive stress of 20,000 lb. per sq. in. Then he could apply the second half of the load; and if one did that, not only with $F/2$, but with F/n, one could reduce the section very considerably.

But there were factors against it. First there was the cost of repetitions of prestressing. Secondly, force F could no longer be removed completely. We could only remove the last bit, F/n; if we removed another part of the force, the compression in the member became too great. If cost were not an objection, repetitive prestressing could only be done when F/n of the load was a movable part.

A VISITOR asked for Professor Magnel's views on the use of glass fibre instead of high tensile steel wires, with special reference to the corrosion resistance and the high tensile strength of the glass fibre.

Professor MAGNEL said he believed M. Freyssinet had spoken of glass wires. Glass had indeed a very high tensile strength; but it had one great disadvantage, that as soon as the skin was scratched, however slightly, it would break "like glass." It became quite brittle, the distribution of stress became different, and he did not think it would ever be used. The idea was very attractive, but he did not think it was one which the practical engineer would try; and in any case he did not suppose that any capitalist would invest even £1 in a company which proposed to use glass for tensile members!

At the conclusion of the discussion, the Chairman expressed gratitude to Professor Magnel for the added information he had given in his replies.

WRITTEN COMMUNICATION

To the Editor,

From Mr. J. E. KINDLER, (Deputy Chief Engineer—Structures, Co-ordinator-General's Dept., Queensland, Australia.

"On page 295, referring to the use of a concrete slab for the support of deck loads in conjunction with steel girders, there is this statement:

> 'This concrete slab exists in any case in a steel bridge, but up to the present it has never been taken into account as working together with main steel girders.'

"In January, 1934, a paper by Mr. A. W. Knight, M.I.E.Aust., was published in the Journal of the Institution of Engineers, Aust., in which the development of composite steel and concrete bridges was presented in some detail; this paper reports tests which appear to offer substantial reassurance upon any doubts which might be entertained as to possible disturbances to composite functioning resulting from shrinkage cracking. Since that date numerous Australian bridges have been built incorporating Mr. Knight's practices, and in some cases employing details which represent considerable extensions of Mr. Knight's work. The service record of all of these bridges appears so far to have been entirely satisfactory.

"More recently, American technical literature has contained a number of comprehensive reports upon work relative to the development of composite steel and concrete bridges; of these contributions, the most noteworthy is possibly that published in the Proceedings of the American Society of Civil Engineers for March, 1948. The results of the American research programmes are, at least in some instances, now being applied in Australian practice."

Book Review

Junior Principles of Quantity Surveying, by R. D. Wood. (London, THE ESTATES GAZETTE 1950). 392 pages, $8\frac{1}{4}$ in. $\times$ $6\frac{1}{4}$ in. 35s.

The author of "Junior Principles of Quantity Surveying"—surely he means "Principles of Quantity Surveying for Juniors"—is to be congratulated on producing a work that must appeal to the youngsters who have decided to make this profession their career. The concise, clear and lucid manner in which he presents the several steps from the beginning, known as squaring, through abstracting and billing up to taking-off, cannot fail to enlighten and assist any pupil or student who applies himself intelligently to the subject.

Mr. Wood's book even goes further and gives examples of how to deal with variations on a contract, and final account. His appendix of abbreviations used in taking-off will prove very helpful to the beginner, for, to the uninitiated the dimensions are just hieroglyphs, and it is on these that the student gets his introduction to squaring and abstracting.

Of course, there is much more in Quantity Surveying than this book can possibly cover, even in addition to the author's previous work "Principles of Quantity Surveying." The student will need practical experience, but with this book at his side he should make rapid progress.

The work conforms to the rules laid down in the Standard Method of Measurement of Building Works authorised by agreement between the R.I.C.S. and the N.F.B.T.E., and this becomes the stepping stone to the professional examinations. H. H. B. S.

Institution Notices and Proceedings

ORDINARY GENERAL MEETING

An Ordinary General Meeting of the Institution of Structural Engineers was held at 11, Upper Belgrave Street, London, S.W.1, on Thursday, May 24th, 1951, at 5.55 p.m., Mr. J. E. Swindlehurst, O.B.E., M.A., M.I.C.E. M.I.Struct.E. (President), in the Chair.

The Minutes of the Ordinary General Meeting held on March 29th, 1951, as published in the Journal in May, were taken as read, confirmed and signed.

The following members were elected in accordance with the Bye-Laws. Will members kindly note that the elections as tabulated below should be referred to when consulting the Year Book for evidence of membership.

Students

Bowley, Robert, Pendley Manor Lodge, Tring, Herts.
Briscoe, William Harry, 3, Langley Fields, Nr. Wellington, Shropshire.
Brown, David Henry, 12, Buxton Road, Hazel Grove, Nr. Stockport, Cheshire.
Collins, Michael Spencer, 123, Carisbrooke Road, Newport, Isle of Wight.
Dawes, John Gordon, Newlands, Forest Lane, Walsall.
Frischmann, William Wilem, 9, Agamemnon Road, London, N.W.6.
Harwood, Frank, 9, Green End Road, Kingsway, Levenshulme, Manchester, 19.
Higgs, Derek Edward, 63, Chelsham Road, Clapham North, S.W.4.
Jermy, Barrie Stuart, Thornton Hotel, 15, Bouverie Square, Fokestone, Kent.
Oke, Abraham Adebayo, 31, Ducie Street, Clapham, S.W.4.
Salamat, Muslim Purwez, Essex Lodge, Ashby Road, Loughborough, Leicester.
Underwood, Bernard George, 2, Dellfield Avenue, Berkhamsted, Herts.
Woodhead, Desmond Bertram, 23, St. Johns Road, Sea Point, Cape Town, South Africa.

Graduates

Anchor, Robert Dudley, B.Sc.(Civil), Birmingham, 128, Woodlands Road, Birmingham, 11.
Asher, Peter John, B.Sc.(Eng.), South Africa, 34, Devonshire Court, Esplanade, Durban, South Africa.
Booth, Robert Stuart, 99, Bramley Road, Acocks Green, Birmingham, 27.
Butterfield, Roy, B.Sc.(Eng.), London, 15, Rushton Road, Thornbury, Bradford, Yorkshire.
Clark, Eric Smith, 67, Church Street, Harpurhey, Manchester, 9.
Clayton, Roy Victor, Kia-Ora, Albert Road, Bognor Regis, Sussex.
Doshi, Pravinchandra Hiralal, B.E.(Civil), Bombay, 26, Imperial Chambers, Wilson Road, Ballard Estate, Bombay, India.
Fulwood, Alan Frederick, B.Sc.(Eng.), London, 10, Winton Road, North End, Portsmouth, Hants.
Ganguly, Jagadish Chandra, B.Sc.(Eng.), London, A.C.G.I., 28, St. Dunstans Road, Barons Court, London, W.6.
Hayter, Maurice Anthony, B.Sc.(Eng.), London, A.C.G.I., 12, Elvaston Place, S.W.7.
Heaviside, Donald, 32, Broad Street, Syston, Leicester.
Hill, Alan Ronald, A.M.I.Mun.E., 15, Lewell Avenue, Old Marston, Oxford.
Marden, Edwin David, B.A.(Cantab.), 50, Elm Park Gardens, Chelsea, London, S.W.10.
Messervy, George Keith, A.R.I.B.A., 340, Latymer Court, London, W.6.
Murray, Michael Graeme, M.A.(Cantab.), A.R.I.B.A., 14, Westbourne Terrace, London, W.2.
Parton, Girvan Meir, B.Sc.(Tech.), Manchester, 37, High Street, Pittshill, Stoke-on-Trent, Staffs.
Pearson, James Leslie, c/o Messrs. Dorman Long (Africa), Ltd., Escom House, Rissik Street, Johannesburg, South Africa.
Rowland, Vernon Roy, B.Sc.(Civil), Bristol, 123, Hartswood Road, London, W.12.
Saravanapavananthan, Nagilingam, B.Sc.(Eng.), London, 35, Gresham Road, London, S.W.9.
Shirlin, Gilbert Thomas, B.Sc.(Hons.), Manchester, 20, Lawn Road, Rowley Park, Stafford.
Stewart, James Campbell, B.Sc.(Eng.), Glasgow, 10, Mansionhouse Road, Langside, Glasgow, S.1.
Uren, John Michael Leal, B.Sc.(Eng.), London, A.C.G.I., Dalkeith, 17, High View, Pinner, Middlesex.

Members

Bölckow, John Dorman, Broom House, Jenny Lane, Woodford, Cheshire.
Clark, Eric George, 25, Cranford Gardens, Acklam, Middlesbrough, Yorkshire.
Davenport, Frank Wallace, The Appleby-Frodingham Steel Co., Scunthorpe, Lincolnshire.

Transfers

Students to Graduates

Davis, John, 29, Wood Lane, Shepherds Bush, London, W.12.
Lord, Peter Bamford, 232, Portland Street, Derby.
Manning, John Bernard, 73, Athenlay Road, S.E.15.

Graduates to Associate Members

Ellis, Lewis Geoffrey, B.Sc., 125, Amesbury Avenue, S.W.2.
Pennington, Duncan John, B.Sc.(Eng.), 30, Hurst Street, Morris Green, Bolton, Lancs.
Turner, Frederick Henry, 93, Shakespeare Drive, Cheadle, Cheshire.

Associate Member to Member

MacGregor, Ian, L. G. Mouchel & Partners, Ltd., 9, Ellison Place, Newcastle-upon-Tyne, 4.

Member to Retired Member

Fritchley, George Bowen, F.R.I.B.A., 31, Farnaby Road, Bromley, Kent.

Re-Admission

Graduate

Spencer, Herbert Harry, The Judge's Eye Qibets, Hills of Ephrayim, P.O. Ha Shofet, Israel.

OBITUARY

The Council regret to announce the deaths of William Ralph Low (Member), Arthur de Beauchamp Breton and Wilbert Chauncy Butcher (Associate-Members).

RESIGNATION

Notification was given that the Council had accepted with regret the resignation of Gordon Frederick Kent (Associate-Member).

ANNUAL GENERAL MEETING

The Annual General Meeting of the Institution of Structural Engineers was held at 11, Upper Belgrave Street, London, S.W.1, on Thursday, May 24th, 1951, at 6 p m. (Mr. J. E. Swindlehurst, O.B.E., M.A., M.I.C.E. M.I.Struct.E.), in the Chair.

The Secretary (Major R. F. Maitland, O.B.E.), read the notice convening the meeting.

The Minutes of the Annual General Meeting held on May 25th, 1950, as published in the STRUCTURAL ENGINEER, July, 1950, were taken as read and were confirmed and signed.

Mr. E. Granter moved the adoption of the Sessional Report of the Council and the accounts for the financial year, 1950. Lt.-Col. R. F. Galbraith seconded the motion, which was carried unanimously.

Mr. F. S. Snow proposed the re-election of Messrs. James Meston & Co., Chartered Accountants, as Auditors for the ensuing year. Mr. S. Vaughan seconded the motion, which was carried unanimously.

The Secretary then read the report of the Scrutineers on the ballot for the election of the President, the Honorary Officers and the ordinary members of Council for the Session 1951-52, as follows :—

"To the Council of the Institution of Structural Engineers :

Gentlemen,

We, the undersigned, report that at the request of the President, we have duly carried out the duties of Scrutineers of the Ballot for the election of Honorary Officers and Council for the Session 1951-1952, and we report accordingly as follows :—

We received 649 ballot papers, of which we rejected 41 as wholly spoiled and 25 as partly spoiled. We have attached a separate sheet showing the number of votes received by each candidate.

We declare the result of the Ballot to be as follows :—

ELECTED

President—Mr. Walter C. Andrews, O.B.E., M.I.C.E.

Vice-Presidents—Mr. E. Granter, B.Sc.(Eng.), M.I.C.E, Lt.-Col. R. F. Galbraith, R.E., M.C., B.Sc.(Eng.), A.M.I.C.E., Dr. S. B. Hamilton, M.Sc., A.R.C.S., A.M.I.C.E., Mr. S. Vaughan, B.Sc., M.I.C.E., A.C.G.I., Mr. L. E. Kent, B.Sc.(Eng.), M.I.C.E., Mr. T. B. Macaulay, B.Sc., M.I.C.E.

Honorary Treasurer—Professor W. Fisher Cassie, M.Sc., Ph.D., F.R.S.E., M.I.C.E.

Honorary Secretary—Mr. J. Mason, B.A., A.M.I.C.E.

Honorary Librarian—Mr. G. S. McDonald, M.I.C.E.

Honorary Editor—Mr. W. H. Woodcock, F.C.S.

Honorary Curator—Mr. J. Singleton-Green, M.Sc., A.M.I.C.E., A.M.I.Mech.E.

The above are elected for one year.

ELECTED AS ORDINARY MEMBERS OF COUNCIL (LONDON)

Mr. Donovan H. Lee, Mr. R. W. Schofield, Lt.-Col. G. W. Kirkland.

The above are elected for three years.

ELECTED AS ORDINARY MEMBER OF COUNCIL (COUNTRY)

Mr. H. C. Husband.

The above is elected for three years.

ELECTED AS ASSOCIATE OF COUNCIL (LONDON)

Mr. A. H. Ley.

The above is elected for three years.

ELECTED AS ASSOCIATE OF COUNCIL (COUNTRY)

Mr. B. C. Britton.

The above is elected for three years.

ELECTED AS ASSOCIATE-MEMBER OF COUNCIL (LONDON)

Mr. W. E. Thorowgood.

The above is elected for three years.

ELECTED AS ASSOCIATE-MEMBER OF COUNCIL (COUNTRY)

Mr. L. J. Griffiths.

The above is elected for three years.

We are, Gentlemen,

Yours faithfully,

(Signed) Thos. Bedford
H. Brompton
Malcolm Glover
S. M. Reisser,
Scrutineers."

On a motion proposed by the President, a vote of thanks was unanimously passed to the scrutineers.

JULY EXAMINATIONS

The Examinations of the Institution will next be held at centres in the United Kingdom and overseas on July 17th and 18th (Graduateship), and on July 19th and 20th (Associate-Membership).

REPRESENTATION

The Council have made the following nominations of members to represent the Institution :—

SCOTTISH EDUCATION DEPARTMENT. JOINT COMMITTEE ON HIGHER NATIONAL CERTIFICATES IN CIVIL ENGINEERING (INCLUDING STRUCTURAL)

Mr. H. B. Sutherland (Associate-Member).

BUILDING APPRENTICESHIP AND TRAINING COUNCIL

Mr. D. A. G. Reid (Associate-Member)—re-appointment.

PROFESSIONAL CLASSES AID COUNCIL

Mr. F. S. Snow (Past-President) (Re-appointed for a further period of three years).

BUILDING TEACHERS' CONFERENCE

Mr. D. Winston Aldred (Associate-Member).

INTERNATIONAL ASSOCIATION FOR BRIDGE AND STRUCTURAL ENGINEERING

Reference was made in the last issue of the Journal to the Fourth Congress of the International Association for Bridge and Structural Engineering, which is to be held in Cambridge and London from August 25th to September 5th, 1952.

It was stated that members of the Institution might obtain further details of the Association's activities upon application to the Chairman of the British Section, Mr. Ewart S. Andrews. It would, however, be appreciated if members of the Institution would in future address all such enquiries and requests for application forms to the Honorary Treasurer, British Section, International Association for Bridge and Structural Engineering, Major R. F. Maitland, O.B.E., 11, Upper Belgrave Street, London, S.W.1.

PERSONAL

Lt.-Colonel R. F. Galbraith, B.Sc., A.M.I.C.E. (Vice-President), has resigned his office as a Director of Galbraith Brothers, Ltd., of High Holborn.

Major R. A. B. Smith, M.C., A.M.I.C.E. (Member), has entered into partnership with Mr. G. P. Manning, M.Eng., M.I.C.E., as Consulting Structural and Civil Engineers.

Mr. C. B. Brown, B.A.I., M.I.C.E. (Member), has been joined in partnership by Dr. T. P. O'Sullivan, B.Sc., A.M.I.C.E., and the practice is now known as Brown, O'Sullivan and Partners.

Mr. P. M. Otway, M.I.Mech.E. (Member), has retired from the Ministry of Transport and is practising as a Consulting Engineer in collaboration with Major H. W. Aldington, C.B., M.I.C.E., at 14, Howick Place, Westminster, S.W.1.

The Secretary would be glad to hear from any member or visitor who may find that he came away from the meeting on April 26th with the wrong raincoat and pair of brown gloves.

THE MACLACHLAN LECTURE, 1950

The MacLachlan Lecture for 1950 was published in the June issue of the JOURNAL under the title "Cold Formed Sections in Structural Practice, with a proposed Design Specification." The Lecture was given by Mr. W. Shearer Smith, A.M.I.C.E. (Associate-Member). Owing to a printing error, the heading, "The MacLachlan Lecture, 1950," was omitted from the publication, as were also the following corrections :—

P. 165, *col.* 1, *para.* 2, *line* 6. For "bases" read "basis."

P. 166, *col.* 1, *para.* 11, *line* 1. For "building" read "buildings."

P. 166, *col.* 2, *line* 24. For "1945 (3)" read "1945^{3}."

P. 170, *col.* 2, *line* 8. For "(1 and 19)" read "(^{1} and ^{19})."

P. 171, *col.* 2. Para. 2, under Section (iii) should read "It should be noted . . ."

PP. 172-3. In Equations 9-12 and references thereto, a dash should have been used instead of the index "1," e.g. :—

"F'_{bc}" etc.

P. 173. Equation (12) should read : $\frac{f_s}{F'_s} = 1 - K_3\sqrt{f_b^5}$

P. 175, *col.* 1 (Table). For "Tee beams" read "Tie beams."

P. 175, *col.* 2 (Table). For "R.S. joist, 10 ft. × 4½ in. × 25 lb." read "R.S. joist, 10 in. × 4½ in. × 25 lb."

The Literature Committee would be glad to consider the publication of correspondence in connection with this Lecture. Communications on this subject intended for publication should be received by September 1st, 1951.

EXAMINATIONS

Preparation for the Examinations of the Institution by Attendance at Technical Colleges

A candidate for Graduateship or Associate-Membership may be able to attend a technical college ; these notes are intended to guide him in choosing the most suitable instruction.

Preparation for the Graduateship Examination

Technical colleges offer :

(a) Full-time courses for degrees or Higher National Diplomas in Building or Engineering.

(b) Part-time day or evening courses for Higher National Certificates in Building or Engineering.

If he obtains a Higher National Certificate or Diploma complying with Appendix II, Section V, of the Regulations Governing Admission to Membership, the candidate will be exempted from the Graduateship Examination.

Alternatively, he may study subjects selected from the available courses and sit the Graduateship Examination. At technical colleges courses are usually available in Building Science or Engineering Science, Strength of Materials, Theory of Structures and Surveying, but students are not normally allowed to select subjects from National Diploma or Certificate courses unless they can show evidence of sound training in more elementary studies. The advice of the College Authorities should be followed.

Preparation for the Associate-Membership Examination

At some technical colleges there are part-time courses in Structural Engineering which cover the syllabus of the Associate-Membership Examination. At other colleges the candidate must rely on Higher National Certificate courses or on advanced courses in Building, Civil Engineering or Municipal Engineering : these cover only part of the requirements for the Associate-Membership Examination.

Colleges in the first category provide at least two years of instruction in Theory of Structures and in Structural Engineering Design and Drawing up to Associate-Membership standard. They also give instruction in Structural Specifications, Quantities and Estimates.

The Colleges which have informed the Institution that courses in Structural Engineering are available are :—

Belfast College of Technology.
Birmingham Central Technical College.
Bolton Municipal Technical College.
Bradford Technical College.
Derby Technical College.
Dudley and Staffordshire Technical College.
Glasgow Royal Technical College.
Liverpool Technical College.
L.C.C. Brixton School of Building.
L.C.C. Hammersmith School of Building and Arts and Crafts.
Manchester College of Technology.
Middlesbrough Constantine Technical College.
Salford Royal Technical College.
South-West Essex Technical College, Walthamstow, E.17.

Colleges in the second category provide instruction in Theory of Structures from which the student may reach Associate-Membership standard, but instruction in Structural Engineering Design and Drawing and in Structural Specifications, Quantities and Estimates is not usually so complete. The colleges which have informed the Institution that such courses are available are :—

Brighton Technical College.
Cardiff Technical College.
Huddersfield Technical College.
Leeds College of Technology.
London Battersea Polytechnic.
London Northampton Polytechnic.
L.C.C. Westminster Technical College.
Plymouth and Devonport Technical College.
Preston Harris Institute.
Wigan Mining and Technical College.
Woolwich Polytechnic.

Students attending colleges in the first category are advised to take the organised courses in Structural Engineering. Students of Graduate Membership standard will usually be allowed to select subjects from courses provided by colleges in the second category.

ADDITIONS TO THE LIBRARY

The following books have been presented by Mr. A. Anderson :—

ADAMS, H., and MATTHEWS, E. R. Reinforced Concrete Construction. 2nd Edition. London, 1916.

BROWN, Hanbury. Irrigation : Its Principles and Practice as a Branch of Engineering. 2nd Edition revised. London, 1912.

BROWNLIE, T. A. M. Further Notes on Tube Wells : Boring, Sinking and Working. 2nd Edition. Calcutta and Simla, 1914.

COCHRAN, J. A Treatise on the Inspection of Concrete Construction. Chicago, 1913.

DAVIES, J. Engineering Office Systems and Methods. New York and London, 1915.

DAVIS, A. P., and WILSON, H. M. Irrigation Engineering. 7th Edition. New York, 1919.

ETCHEVERRY, B. A. Irrigation Practice and Engineering. Vols. II and III. New York and London, 1916.

FIDLER, T. C. Calculations in Hydraulic Engineering, Pts. I and II. London, 1907 and 1914.

HOOL, G. A. Reinforced Concrete Construction, Vols. I, II and III. New York and London, 1917, 1913 and 1916.

Hool, G. A., and Johnson, N. C. Concrete Engineers' Handbook. New York and London, 1918.
Hool, G. A., and Whitney, C. S. Concrete Designers' Manual. New York and London, 1921.
McCulloch, W. Conservation of Water. New Haven and London, 1913.
Newell, F. H., and Murphy, D. W. Principles of Irrigation Engineering. New York and London, 1913.
Reynolds, C. E. Reinforced Concrete Designers' Handbook. London, 1932.
Van Ornum, J. L. The Regulation of Rivers. New York and London, 1914.

LONDON GRADUATES' AND STUDENTS' SECTION

A visit has been arranged by kind permission of the British Concrete Development Association to their Prestressed works at Iver, Buckinghamshire, on July 14th. The party will meet on the platform at Paddington Station to travel by the 9.30 a.m. train to Iver. Railway fare (day return), 3s. 2d.

BRANCH NOTICES

Lancashire and Cheshire Branch

Hon. Secretary : A. S. Sinclair, A.M.I.Struct.E., 28, Kenwood Road, Stretford, Lancashire.

Graduates' and Students' Section

Hon. Secretary : F. H. Turner, 11, Colwell Avenue, Stretford, Lancashire.

Midland Counties' Branch

Hon. Secretary : E. R. Deeley, A.M.I.Struct.E., Arranmoor, Adshead Road, Dudley, Worcs.

Graduates' and Students' Section

Hon. Secretary : M. H. Evans, B.Sc., 42, Church Hill Road, Handsworth, Birmingham, 20.

Northern Counties' Branch

The Annual General Meeting of the Branch was held on Tuesday, April 3rd, at the Cleveland Scientific and Technical Institution, Middlesbrough, when the following honorary officers and committee members were elected for the session 1951-52 :—

Chairman : J. Gerrard, A.M.I.Mech.E. (Member).
Vice-Chairman : A. V. Buttress (Member).
Immediate Past-Chairman : W. H. Saxton, B.Eng., A.M.I.C.E. (Member).
Branch Hon. Secretary : J. A. Williams, A.M.I.C.E. (Associate-Member), 57, Reid Park Road, Jesmond, Newcastle upon Tyne, 2.
Tees Centre Hon. Secretary : O. Lithgow (Associate-Member).
Hon. Treasurer : L. Dobson (Associate-Member).
Committee : Tyne—E. Atkinson (Member), D. W. Cooper, B.Sc. (Associate-Member), A. G. Lyon, T.D., A.M.I.Mech.E. (Member), I. MacGregor (Associate-Member), E. A. Parsons (Member), T. L. Usherwood (Member).
Additional Member, ex-officio : Professor W. F. Cassie, M.Sc., Ph.D., F.R.S.E., M.I.C.E. (Member).
Mr. Cooper will continue to act as Hon. Assistant Secretary (Tyne Centre).
Committee : Tees—W. Fitton, B.Sc.(Eng.), A.M.I. Mech.E. (Member)., S. D. Hodgson (Associate-Member), T. Johnson (Member), T. A. Kennedy (Member), J. E. Nettleton (Associate-Member), J. Pringle (Member).
Hon. Auditors : C. A. Harding, F.R.I.B.A. (Member), E. R. Fryer (Associate-Member).

Northern Ireland Branch

Hon. Secretary : S. G. Duckworth, M.I.Struct.E., Lisleen, 13, Finaghy Road, North, Belfast.

Scottish Branch

Hon. Secretary : D. G. Drummond, B.Sc., A.M.I.C.E., A.M.I.Struct.E., 11, Woodside Terrace, Glasgow, C.3.

South-Western Counties' Branch

Hon. Secretary : E. W. Howells, M.I.Struct.E., 10-12, Market Street, Torquay, Devon.

Wales and Monmouthshire Branch

Hon. Secretary : E. R. Steward, A.M.I.Struct.E., Edrom, Ashleigh Road, Blackpill, Swansea.

Western Counties' Branch

The Annual General Meeting was held at the Grand Hotel, Bristol, on Friday, April 6th, under the Chairmanship of Mr. P. C. Girdlestone. The Annual Reports of the Honorary Secretary and Honorary Treasurer were read and adopted and the following honorary officers and committee members were elected for the session 1951-52.:—

Chairman : Professor A. G. Pugsley, O.B.E., D.Sc., M.I.C.E., F.R.Ae.S. (Member).
Vice-Chairman : E. N. Underwood, B.Sc.(Eng.), A.M.I.C.E. (Member).
Hon. Vice-Chairman : Ewart S. Andrews, B.Sc., M.I.C.E. (Member).
Hon. Secretary : C. E. Saunders, " Dunkery," Edward Road, Walton St. Mary, Clevedon, Somerset (Member).
Hon. Treasurer : F. K. Fennell (Associate-Member).
Committee : P. C. Girdlestone, M.A., A.M.I.C.E., M.I.Mech.E. (Member) (Past-Chairman, ex-officio), L. F. Summerfield, M.C. (Member) (Past-Chairman, ex-officio), C. J. D. Boxall, M.Eng., A.M.I.Mech.E. (Member) (Past-Chairman), C. H. Williams (Member) (Past-Chairman), N. G. T. Ball (Associate-Member), R. H. Barnett (Member), R. L. Bourqui, A.M.I.C.E. (Associate-Member), F. G. Clarke (Associate-Member), F. A. Long, A.R.I.C.S. (Member), J. W. Lorraine (Member), G. C. Mander, M.B.E., B.Sc., A.M.I.C.E. (Associate-Member), G. F. Poppleton, A.M.I.C.E. (Associate-Member), J. M. Rome (Member), A. L. Saunders (Member).
Hon. Auditors : J. M. Rome, F. A. Long.
Branch Delegate to Science Committee : Professor A. G. Pugsley.

Yorkshire Branch

The Twenty-seventh Annual General Meeting was held at the Great Northern Hotel, Leeds, on Wednesday, May 9th, 1951, at 6.30 p.m., under the Chairmanship of Mr. A. Robb, B.Sc., A.M.I.C.E. (M.), in the absence, owing to illness, of Mr. H. C. Husband. The Hon. Secretary's report and the financial statement for 1950 were adopted. The election of honorary officers and Committee members for the forthcoming session took place.

After the Business Meeting, a lecture entitled " Construction of 240-ft. Clear Span Sheds for Aircraft Production," was given by Mr. John G. Faber, B.Sc., A.M.I.C.E. (A.M.). Mr. Faber was accorded a vote of thanks, proposed by Mr. J. Jenkins, M.Sc. (M.), and seconded by Mr. H. E. Manning, B.Sc., M.I.C.E. (M.).

Hon. Secretary : E. Wrigley, A.M.I.Struct.E., City Engineer's Department, Civic Hall, Leeds, 1.

Union of South Africa Branch

Branch Hon. Secretary : A. E. Tait, B.Sc., A.M.I.C.E., M.I.Struct.E. P.O. Box No. 3306, Johannesburg. During week-days Mr. Tait can be contacted in the City Engineer's Department, City Hall, Johannesburg. 'phone 34-1111, ext. 257.

Natal Section

Hon. Secretary : E. G. Bennett, A.M.I.Struct.E., c/o Reinforcing Steel Co., Ltd., P.O. Box 478, Durban.

Cape Section

Hon. Secretary : R. Stubbs, M.I.Struct.E., P.O. Box 1692, Cape Town.

Concrete Control for Small Works*

By N. J. Ruffle, B.Sc. (Eng.), A.M.I.Struct.E.

Introduction

The concrete control methods described in this paper, although not new in principle, originated in their particular form during the construction for the Sunderland and South Shields Water Company of a small reinforced concrete reservoir, and were later used on the construction for the same Company of barrel vault roofs to cover two existing service reservoirs. The paper therefore deals with concrete control as carried out in these instances, with the implication that such methods may be directly applicable to other similar works. The title of the paper refers to "small works" but as the adjective is, of course, only relative, it may help to fix them on the scale of magnitude if it is stated that the two structures respectively cost £45,000 and £20,000, and involved the placing of 1,600 cu. yd. and 600 cu. yd. of concrete. In each case the concrete was mixed in a 14/10 mixer.

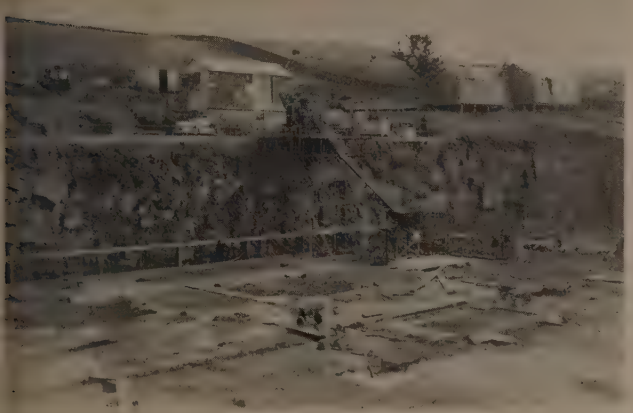

Fig. 1.—Break pressure tank No. 1
Lower layer of floor partially complete; concrete mixer, cement shed and aggregate bins above face of excavation

Both works were executed by contract, and in each instance the construction was supervised on site by one inspector.

Description of Works

Burnhope Pipeline — Break Pressure Tank No. 1

This tank has a capacity of one million gallons and terminates the first 11½ miles of the 44-miles-long Burnhope pipeline. The pipeline, when completed, will be a gravitational main designed to carry a normal flow of five million gallons per day from the Burnhope Reservoir near the head of the Wear Valley, down to two of the Water Company's service reservoirs at Stoneygate and Easington respectively.

The water falls from a top water level of 1,305 ft. O.D. at Burnhope, to a level of 512 ft. O.D. at Easington. At one point, where the pipeline crosses the River Wear near Durham, it is as low as 75 ft. O.D., and in order to limit the static pressures which would result from the Burnhope elevation, the pipeline has, in effect, been divided into three pipelines by the insertion of two small reservoirs, each of which provides a free water surface and is consequently known as a break pressure tank. It is the upper of these two tanks, sited on an excavated benching in a steeply-sloping hillside at approximately 1,000 ft. O.D. to which reference is now made. The site is shown in Fig. 1.

The dimensions are 163 ft. × 87 ft. × 13 ft. internal height. Its capacity is one million gallons, and it has

Fig. 2.—Break pressure tank No. 1
Upper layer of floor partially complete; construction of walls and columns commenced

a division wall which forms it into two compartments each holding half a million gallons, the valve work being so arranged that the two halves can be used together or one at a time, should it be desired to empty and inspect the other. The floor is constructed in two layers. The lower layer is of 3 in. thick plain 1-3-6 concrete laid in 15 ft. square bays, chessboard fashion, while the upper layer is of 6 in. thick reinforced 1-1¾-3½

Fig. 3.—Break pressure tank No. 1
Walls and columns completed; roof partially complete

concrete, again laid in 15 ft. square bays, but arranged so that the joints are staggered with those of the lower layer (Fig. 2). The walls span between the floor and the roof, and were designed as slabs fixed at the floor and freely supported at the roof. The roof is of flat slab construction, supported by the walls and by circular columns (Fig. 3). At one end is the inlet valve

*Paper read before the Northern Counties Branch of the Institution of Structural Engineers at Middlesbrough on the 5th February, 1951, and at Newcastle on the 7th February, 1951.

house, where the inflowing water is gauged, and where are housed the valves which control the water level in the tank and the flow in the pipeline.

The design generally conforms to the recommendations of the Institution of Civil Engineers' Code of Practice for Reinforced Concrete Structures for the Storage of Liquids.

As a matter of interest, when the tank was tested for watertightness, each half being under test for one month, the apparent leakage from each half was $\frac{3}{4}$-gallon per hour.

Roofing of Humbledon Reservoirs

For the purpose of preventing both atmospheric pollution and, by excluding sunlight, the growth of troublesome organisms, barrel vault roofs have been constructed over two open pure water service reservoirs in Sunderland. The reservoirs were built 102 and 77 years ago respectively, one having a concrete floor and the other an earthenware slab floor. They are waterproofed by puddle clay under the floors and behind the masonry walls, and, since the provision of columns to support a roof would have necessitated disturbing the puddle clay floor, it was decided to use

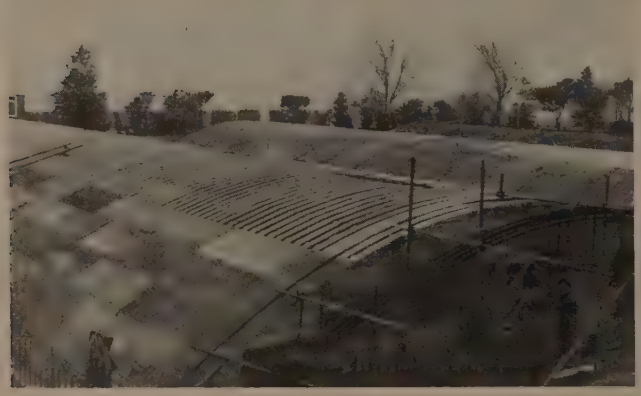

Fig. 4.—Roofing of Humbledon low reservoir
Boarding and steel sheeting

barrel vault roofing, spanning 100 ft., from wall to wall. Each roof consists of three barrels, which are reinforced with cold-drawn wire mesh, square twisted bars and round mild steel bars. The upper portions of the shells are $3\frac{1}{2}$ in. thick, but this increases to 10 in. in the valleys (Figs. 4, 5 and 6).

Concreting in Cold Weather

Concreting of the break pressure tank was carried on when the shade temperature on a rising thermometer reached 34°F. But for the barrel vault roofs with their small thickness of concrete, a much more stringent cold weather clause was enforced. This provided that concrete was not to be placed within a period of 24 hours of a temperature of 32°F. or lower being registered, and then only when the shade temperature was above 40°F.

Aggregates

The concrete aggregates in most common use in County Durham are probably crushed whinstone and pit or river sands and gravels which often contain a proportion of crushed material.

Crushed whinstone is a clean, strong aggregate of consistent quality, but it suffers in comparison with most gravels by virtue of its angular shape, especially in the smaller sizes where the particles tend to be flaky. Hence a higher water/cement ratio is generally necessary to reach the required workability, and the strength of the concrete may as a consequence be less. On the other hand, clean, hard gravels are not easy to obtain. Natural sands are usually used with both types of coarse aggregate, and again it is not easy

Fig. 6.—Roofing of Humbledon low reservoir
Structure virtually complete

in the Durham area to obtain a sand which is clean and well graded. The presence of coal particles is especially common. For the break pressure tank the aggregates used were crushed whinstone and a pit sand, and for the Humbledon roofs, river gravel and river sand.

Grading of Aggregates

In order to be able to produce consistently a concrete which has the degree of workability most suited to the proposed method of compaction, it is necessary to

Fig. 5.—Roofing of Humbledon low reservoir
Concreting of a barrel in progress

control the distribution of particle sizes in the combined aggregate. To achieve this, the coarse aggregate has normally to be delivered to the site in several sizes which can be re-combined in the desired proportions. In the case of the break pressure tank, the whinstone coarse aggregate was delivered in 3 sizes for a mix containing up to $\frac{5}{8}$ in. material, and in 4 sizes for a mix of up to $1\frac{1}{4}$ in. material ; while for Humbledon roofs the gravel aggregate was delivered in 2 sizes for a mix

of up to ¾ in. material, and in only one size, *viz.* ⅜ in. to 3/16 in. for a mix of ⅜ in. maximum size.

Proportioning of Mixes

The specification laid down a nominal mix by volume, e.g. 1-2-4, but indicated that the actual mix would be made up of the several sized aggregates, in proportions which would be decided when sieve analyses had been made.

For the purpose of proportioning, the volume mix was converted to that mix by weight which would give the weight of cement per cu. yd. to be expected from the volume mix specified. The sieve analyses were combined to form several trial gradings, generally of the type given in the Road Research Laboratory's Road Note No. 4. Hand mixes were then made, and with the help of the compacting factor apparatus (to which reference will be made later), the mix was chosen which required the minimum water for the necessary workability, and which had a good appearance with no tendency to segregate. These requirements usually result in using the coarsest grading which does not tend to segregate.

Although the hand mixes seemed to give good comparisons between the gradings for their appearance and for their relative water requirements, they did not show the actual water required by the machine mixer. It was found that, owing to the much greater efficiency of machine mixing, the water required was considerably less than with hand mixing.

Sieve analyses of the aggregate were made periodically throughout the jobs to check that the mix gradings were remaining correct.

Storage of Aggregates

Each size of aggregate was stored in a separate bin, where it could be kept clean and be allowed to drain. On the contracts described, each bin was specified to have two sides, a back, and a concrete floor which sloped towards the back so that the water could drain away from the mouth, and away from the aggregate which would be used first. The bins should hold at least two days' supply and it is a great advantage, especially in the case of the sand, for the bins to have a division wall. This enables freshly delivered material to drain for at least 16 hours before use.

Batching

Accuracy in the batching of all the various constituents of the concrete is of primary importance. In the case of the cement, accuracy has been assured by obtaining the Ordinary Portland cement in 1-cwt. bags, and by always using "whole bag" gaugings. Batching of aggregates may be carried out by weight or by volume, the latter being chosen for the works described in this paper. The loose densities of the aggregates were obtained by test and for batching, the various weights of the trial hand mixes were reconverted into volumes.

Gauge Boxes

The gauge boxes were each mounted on a Decauville rail bogie, and after being filled by shovelling from their respective bins, they were pushed along to the 14-10 non-tilting drum mixer and side-tipped into the loading hopper. For the coarse aggregate the boxes were of wood, rectangular in plan, and with as large a height in relation to the plan dimensions as was practicable. They were fitted inside with a lath screwed to the sides at a few inches from the top. The underside of the lath gave the level to which the box had to be filled, and the freeboard allowed for subsequent alteration in level if the grading of the coarse aggregates varied.

Sand, however, is not as amenable to accurate volume batching, owing to the bulking effect of its moisture content. For the 1-3-6 mixes, which were used at the break pressure tank for the lower floor layer and for general mass concrete, an allowance was made for bulking. It was found that the sand used, after having at least 16 hours to drain in the bin, rarely had a moisture content outside the limits of 3½ and 6½ per cent. A "bulking curve," indicating the variation of bulking with variation of moisture content was drawn from test results, and with the lath in the gauge box set for a mean bulking allowance, the theoretical error of batching weight was small. Nevertheless, the personal effect in filling a container with damp sand would appear to be considerable, and for the reinforced concrete mixes the more precise method of "inundation" was adopted.

Fig. 7.—Roofing of Humbledon low reservoir
Watertight sand gauge box

Watertight Sand Gauge Box

The gauge box for the sand was again of wooden construction, but was lined internally with a waterproof welded sheet steel lining (Fig. 7). To make the emptying of the box easier, the sides were slightly battered. A measured quantity of water was poured into the box (Fig. 8), and sand was then shovelled in and worked with the shovel until the water level reached a sliding indicator which was fixed to one of the inside faces. As the "working" released the entrapped air, the box remained full up to the indicator with sand and water only. The water had come from two sources.

The larger quantity was that which was poured into the box initially, and the other quantity was the moisture which was held by the sand. With a knowledge of the specific gravity and the moisture content of the sand, the height of the indicator could be adjusted so that the desired weight of sand was obtained. Usually, the moisture content of the sand was measured once a day by means of a pycnometer.

Speed in the operations of control is essential on site, and the calculation for determining the height of the indicator in the sand gauge box was reduced to an alignment chart (Chart 1).

For the purpose of measuring the sand, the water poured into the sand box can, of course, be any accurately known quantity provided that, when added to the moisture content of the sand there is sufficient water to inundate the sand — i.e. provided that after "working" the sand, all the voids in the sand are filled with water and some excess of water shows above the sand. But in practice, too much free water in the loading hopper of the mixer was found to be a nuisance, as it caused the materials to adhere to the hopper when it was discharging into the drum. The water was therefore kept to a reasonable minimum, and for a 1-2-4 mix was usually fixed at 3 or 3½ gallons. For the same reason, it was found to be best for the sand to be tipped into the loading hopper after the coarse aggregates and before the cement.

Pycnometer

The pycnometer, used for determining the moisture content of the sand is essentially a 2-lb. fruit preserving jar, fitted with a brass conical top which has a small

Fig. 8.—Roofing of Humbledon low reservoir
Water being poured into sand box from gauge bottle

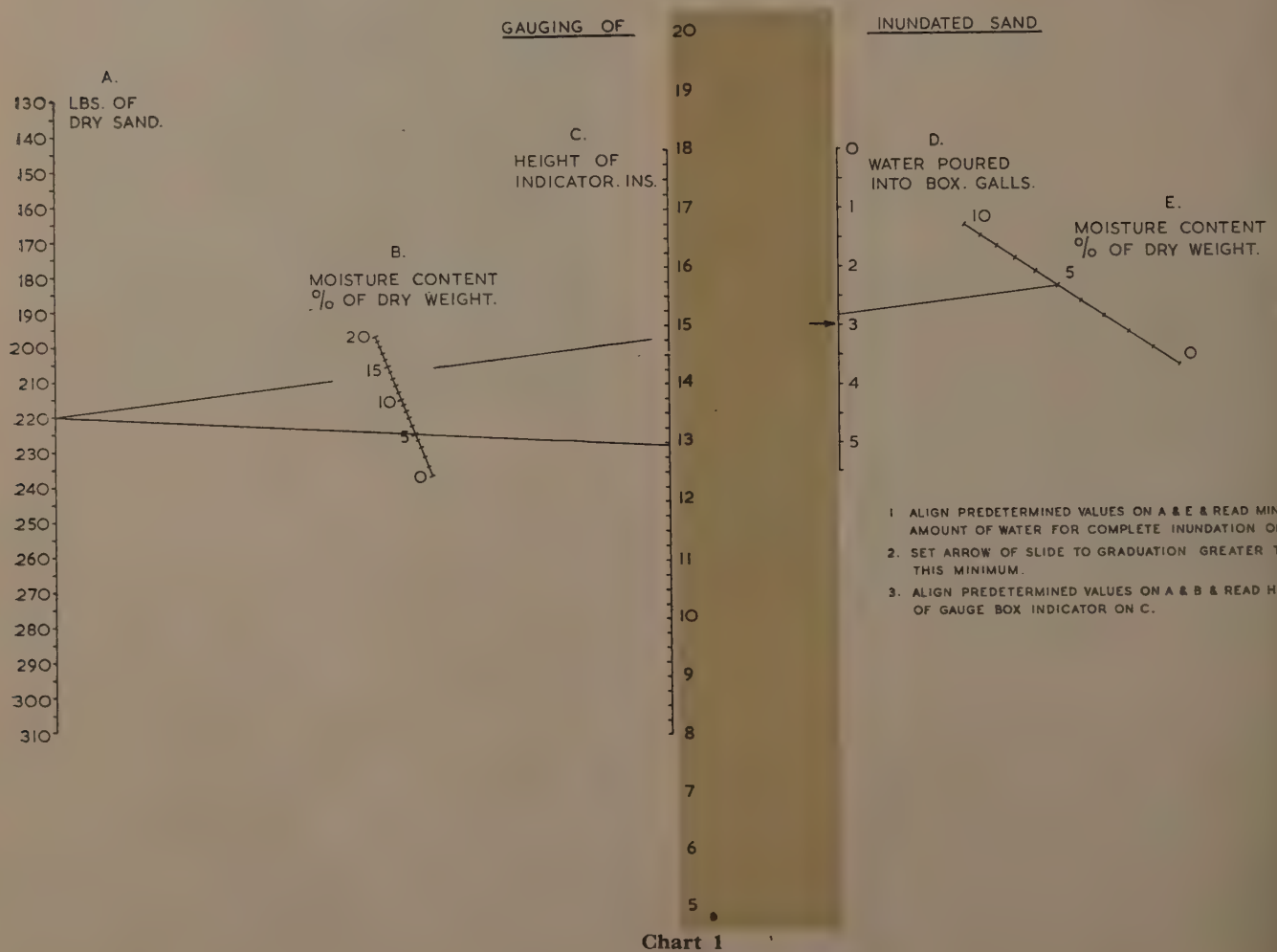

Chart 1

hole at the summit. The pycnometer is weighed containing an arbitrary amount of the damp sand, and is then filled with water and rolled to release the air. When all air is expelled and the water level is at the top of the cone, the pycnometer is re-weighed. From the two weighings, which have been made to the nearest dram, i.e. 1/16 oz., the moisture content may be calculated. The specific gravity of the sand is a constant

Fig. 9.—Break pressure tank No. 1
Concrete mixer, water gauging bottles, cement shed, sand bins

in this calculation, and is found from two similar weighings using dry sand. An alignment chart was constructed, embodying the constants of the pycnometer in use, to give the moisture content of the sample quickly and easily (Chart 2). A pycnometer test could be carried out comfortably in 10 minutes.

Water Gauging

It had been found that the type of water gauging apparatus usually fitted to concrete mixers was insufficiently accurate or adjustable for close control, and special water gauging bottles were used. Measurement was made by volume in vertical tubes of 8 in. diameter.

The small cross-sectional area of the bottles allowed the accurate adjustment of a glass indicator cum overflow tube which was clamped to the bottles. The top of the glass tube acted as an indicator on a scale graduated in one-tenths of a gallon. Two of these bottles were provided. One was used to measure the water required for the watertight sand gauge box, and the other to measure the water to be added to the drum of the mixer (Fig. 9).

The foregoing methods were used for ensuring that the amounts of coarse aggregate, sand, cement and added water were measured accurately. The ingredient remaining to be measured was the water present as moisture in the aggregates.

Although the total amount of water in a mix may be only 8 per cent. by weight of all the ingredients, small alterations can radically alter the appearance of the mix and affect its ultimate strength. The precise measurement of the total water is therefore of paramount importance, and at the same time the most difficult to achieve. The moisture content of the aggregate could be estimated by carrying out tests on each of the various sizes, either by pycnometer or by a drying out test. But while these tests are being made, many batches

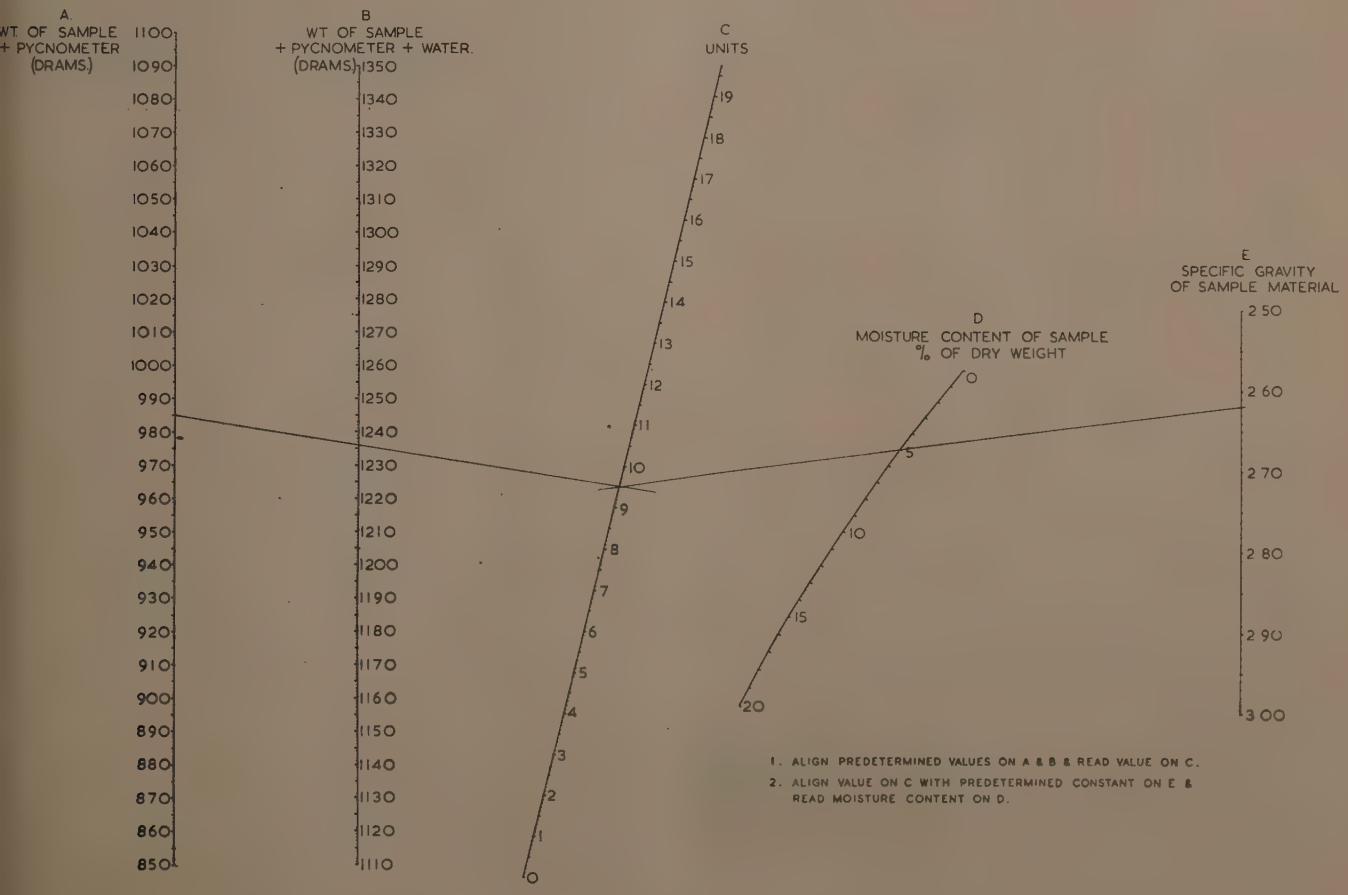

Chart 2

could be mixed, conditions in the stock piles could alter, and the results, especially using the slower drying out tests, would be in the nature of post mortems.

Compacting Factor Tests

However, there is another approach which can be made when all other materials have been gauged carefully. This is to measure the total water by inference from the quality of workability. Workability has been defined as that property of the concrete which determines the amount of useful internal work required to compact the concrete. It can be measured accurately by an apparatus which has been developed by the Road Research Laboratory, and which is known as the compacting factor apparatus. In this test, the weight of a cylinder-full of concrete, on which a certain standard compaction has been carried out, is divided by the weight which the cylinder-full would have been, had the concrete been fully compacted and all entrapped air released. The fraction thus found is known as the "compacting factor." Its value varies from 0.85 for a concrete which can be consolidated by hand only if it is placed in large masses, to a value of 0.95 for a fluid concrete which should be used only if the concrete has to be manually compacted among very congested reinforcement. For normal reinforced concrete work, where the concrete is manually tamped, values in the range of 0.89 to 0.92 are suitable.

The denominator of the compacting factor can be calculated from the weight of each constituent in the mix and their specific gravities. It is necessary to know how much water will be used, because water is by far the lightest constituent (s.g. of 1, whereas s.g. for most gravel aggregates is about 2.6, and for cement is 3.1), and an alteration in w./c. ratio therefore appreciably affects the weight per cu. ft. of the fully-compacted concrete. The amount of water required can be estimated from published data, or it can be ascertained more accurately by a trial run with the machine mixer. Again, from published data or from direct test, can be found the relation between the compacting factor and the water content of the mix. A chart may then be constructed with a scale representing the actual weight of the concrete in the cylinder, and giving by alignment to other scales the value of the compacting factor, and of the alteration required to the mixer water to correct the compacting factor to a desired standard value (Chart 3).

A development of this which is now in use, and which is more rapid in operation, entails the placing of the cylinder of concrete on one side of a pair of scales, while on the other side reside weights which balance the cylinder of concrete if the compacting factor is correct. Should the compacting factor be too high, i.e. the concrete is too wet, the concrete outbalances the weights, and extra weights are added until the scales tip. If on the other hand the compacting factor is too low, i.e. the concrete is too dry, the weights outbalance the concrete, and extra weights are placed on the concrete side of the scales until the scales tip. The extra weights used are $\frac{1}{4}$-lb. weights, because it was noticed that an 0.01 increment of compacting factor was closely equivalent to a change of $\frac{1}{4}$-lb. in the weight of the concrete. For a 1-2-4 mix at an average compacting factor of 0.905, each $\frac{1}{4}$-lb. weight represents approximately 1-lb. of water in the mix, and the mixer water is adjusted accordingly (Table 1). A typical preliminary calculation for the compacting factor test is given in Table 2.

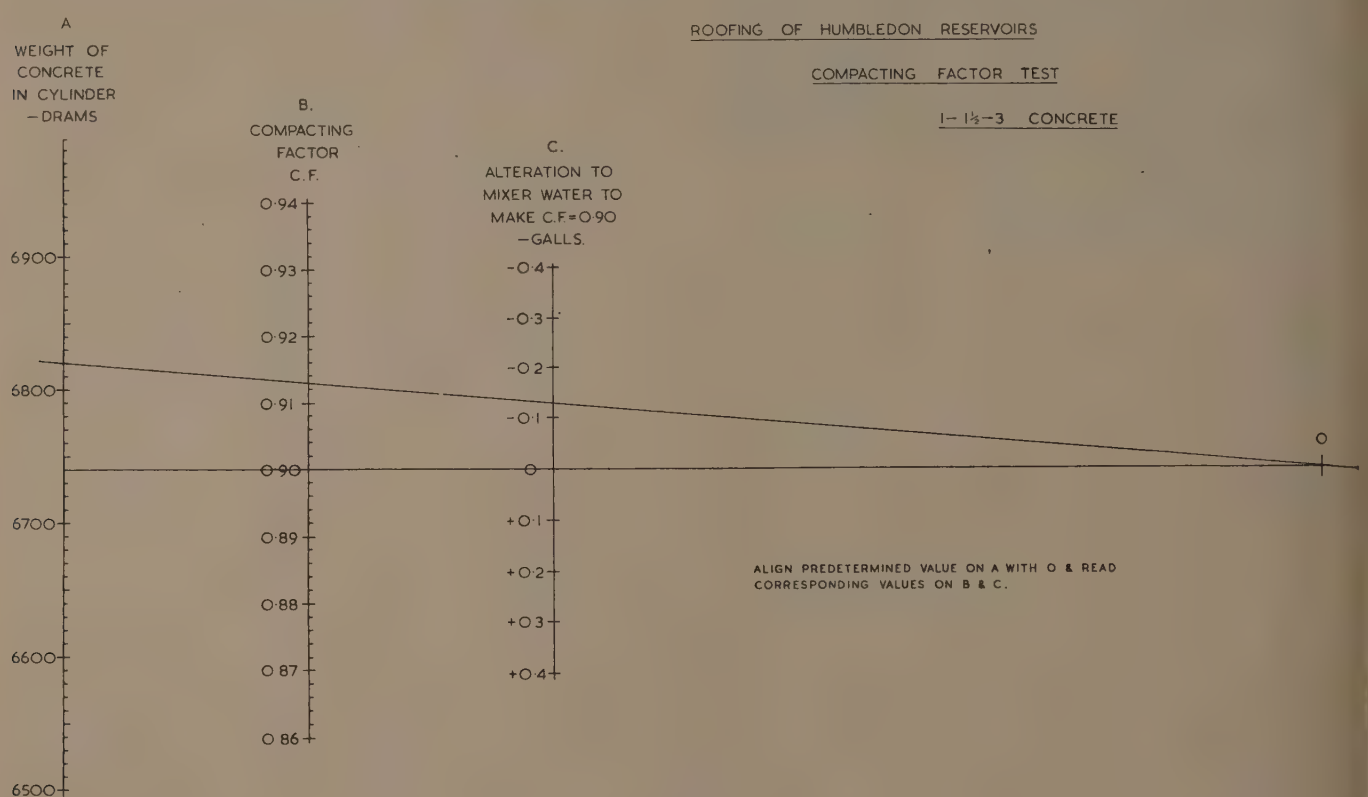

Chart 3

Much has been said and written about the necessity for judging the quantity of water by eye, owing to the impossibility of conducting repeatedly and quickly, tests for ascertaining the moisture contents of the various aggregates. The slump test must be classed as only a rough guide in this respect, on account of its liability to random variations. Furthermore, it

does not provide a basis for comparison of the workability of different mixes, i.e. it is not a true measure of workability. But experience has shown that, when batching is carefully performed, the means of keeping the water/cement ratio constant is provided by the compacting factor test. The test can be performed

TABLE 1
Compacting factor test
1 - 2 - 4 Concrete

Extra ¼-lb. weights to tip scale		Nominal C.F.	Alteration to mixer water-gall.
Weight side	Concrete side		
4		0.94	— 0.3
3		0.93	— 0.2
2		0.92	— 0.1
1		0.91	0
Balanced		0.905	0
	1	0.90	0
	2	0.89	+ 0.1
	3	0.88	+ 0.2
	4	0.87	+ 0.3

TABLE 2
Compacting factor test
Calculation of weight of fully compacted concrete

Material	Weight per batch lb.		Specific gravity	Absolute volume cu. ft.
Cement		112	3.10	0.58
Gravel ¾ in.—⅜ in. ...		308	2.60	1.90
Gravel ⅜ in.—3/16 in. ...		168	2.60	1.04
Sand		224	2.64	1.36
Moisture in aggregates ...	19.2			
Water poured into sand box	30.0			
Water added to mixer ...	15.0			
Total water		64	1.00	1.03
Total per batch ...		876		5.91

Weight of fully compacted concrete ... ... = $\frac{876}{5.91}$ = 148 lb./cu. ft.

Volume of compacting factor cylinder ... ... = 0.192 cu. ft.

Weight of fully compacted concrete in cylinder ... = 148 × 0.192 = 28.4 lb.

Weight of concrete in cylinder if C.F. = 0.905 = 28.4 × 0.905 ... ... = 25.7 lb.

Weight of C.F. cylinder empty = 18.0 lb.

Total balance weight required on scales ... ... = 43.7 lb. for C.F. = 0.905

rapidly, and when the aggregates are of fairly uniform moisture content, a test every half-hour may be all that is necessary to maintain the compacting factor, and hence the water/cement ratio, within close limits.

Occasions do arise, however, when quick variations of moisture contents occur, e.g. when rain has caused the moisture content of the aggregate piles to become variable. It is then that the water addition to the mixer has been determined by eye; but, by constant practice in looking at samples of concrete for which the compacting factor is known, the eye can judge a compacting factor quite accurately. It is therefore possible, for a period, to determine the water required on the visual appearance of the concrete, preferably when the concrete is in a standard position, e.g. by observing the appearance of the concrete as it piles up on the outlet chute of the mixer. But it is essential for this visual standard to be refreshed by frequent reference to the impersonal compacting factor apparatus.

As an indication of what it is possible to achieve, a copy is shown (Table 3) of an actual day's concrete report from the Humbledon High Reservoir Roof. It shows clearly the way in which the water bottle, calibrated in one-tenths of a gallon, can be used to

TABLE 3
Sunderland and South Shields Water Company
Humbledon Low Reservoir
Concrete Report No. 58
Date : 12th July, 1950
Weather : Fine, warm
Class of Concrete : 1 - 1½ - 3
Where placed : Valley beam No. 9, end sections

Time	Compacting factor	Mixer water gallons	Remarks
8 — 45	0.905	1.4	
9 — 15	0.900	1.4	
9 — 45	0.906	1.2	
10 — 25	0.911	1.3	
10 — 55	0.908	1.3	
11 — 30	0.900	1.3	
12 — 05	0.900	1.3	
1 — 15	0.906	1.3	
1 — 45	0.905	1.3	
2 — 15	0.907	0.9	Fresh bin of ⅜ in. to 3/16 in. commenced
2 — 45	0.910	0.9	
3 — 00	0.909	0.8	
3 — 30	0.908	0.8	

maintain a constant compacting factor, and it also shows the alteration which was necessary when loading from a second bin of ⅜ in. aggregate was commenced. It is suggested that, in addition to showing uniformity in batching, this set of results indicates the repeatability of the compacting factor test.

Test Cubes

It should be noted that the concrete for compacting factor testing and for the making of test cubes, was taken directly from the mixer, from the middle of batches during discharge from the drum. It is felt that, in the case of the controlling compacting factor test, it is essential for the concrete to be tested as it leaves the mixer. As the time of transporting the concrete to the shutters was small, and as it is not possible in any case to imitate exactly the conditions of placing, it was considered that the concrete for cubes should

also be taken at the mixer. The cubes were cast in sets of three, all three being made from one sample of concrete. Before casting, the compacting factor of the concrete was measured, and, as near to the time of casting as possible, pycnometer tests were carried out on the sand and coarse aggregates, and the water/cement ratio was assessed. The cubes were cured in a bath of damp sand, out of doors, until the day before they were required for the 28-day test.

Chronological Chart of Compressive Strengths (Chart 4)

Sets of cubes were made at irregular intervals, depending on the amount of concreting in progress. The means of each set are shown plotted against time.

Work on Break Pressure Tank No. 1 was closed for 3 months during the winter of 1949, with the consequent splitting of the job into Period 1 and Period 2, the results for which have been analysed separately. Similarly, at Humbledon, the job was split into two defined periods by the interval of 3 months between the concreting of the roofs at the Low and the High Reservoirs.

Histograms (Chart 5)

An impression of the scatter of the results is given by these diagrams, e.g. the better grouping of the

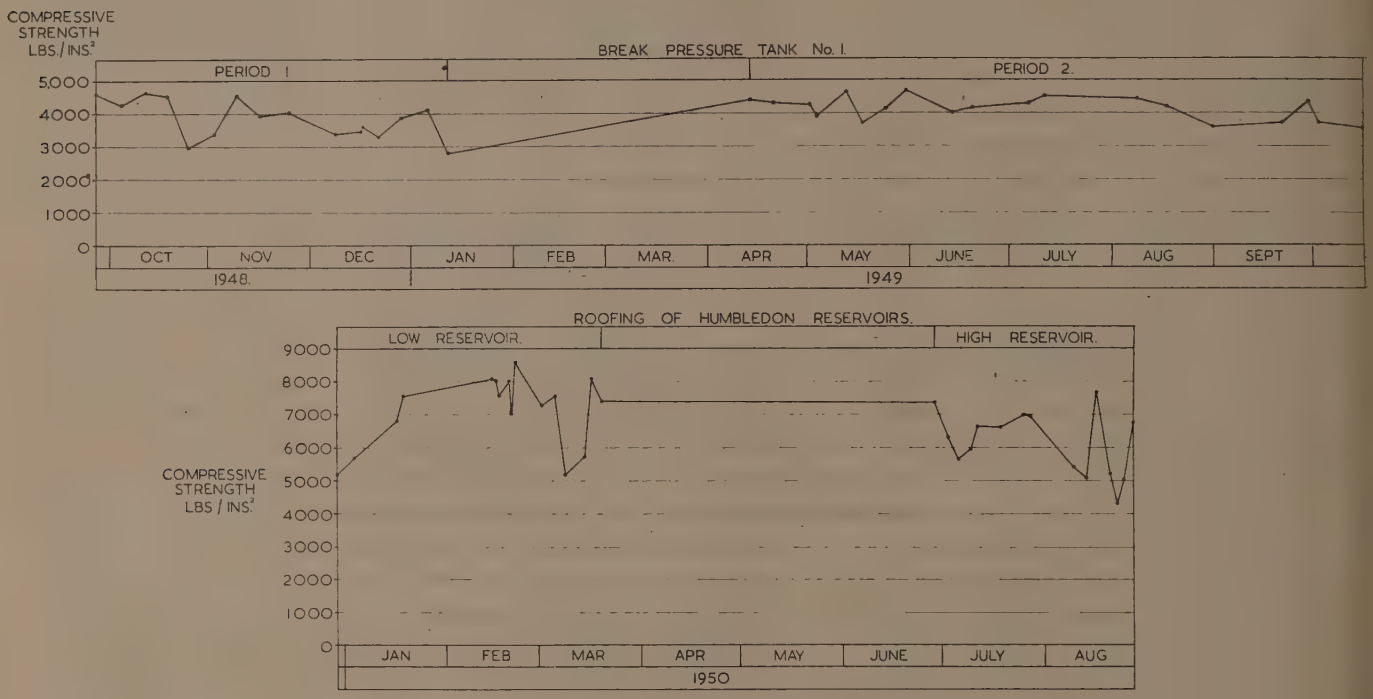

Chart 4

TABLE 4
Summarised results of 6 in. concrete cube tests

Structure		Break pressure tank No. 1			Roofing of Humbledon reservoir		
Nominal mix (vol.)		1 — 1¾ — 3½			1 — 1½ — 3		
Periods		1	2	Whole	Low reservoir	High reservoir	Whole
Number of sets of cubes		16	19	35	16	15	31
Compressive strength	Average lb./in.²	3,830	4,130	4,000	7,090	6,090	6,600
	Lowest / Average %	72	86	69	73	70	65
	Standard deviation lb./in.²	598	394	516	1,090	986	1,145
	Coefficient of variation %	15.6	9.5	12.9	15.3	16.2	17.3
Water / Cement ratio	Average	0.556	0.559	0.558	0.452	0.460	0.457
	Standard deviation	0.020	0.015	0.017	0.023	0.017	0.020
Compacting factor	Average	0.903	0.915	0.911	0.897	0.897	0.897
	Standard deviation	0.014	0.014	0.014	0.016	0.014	0.015

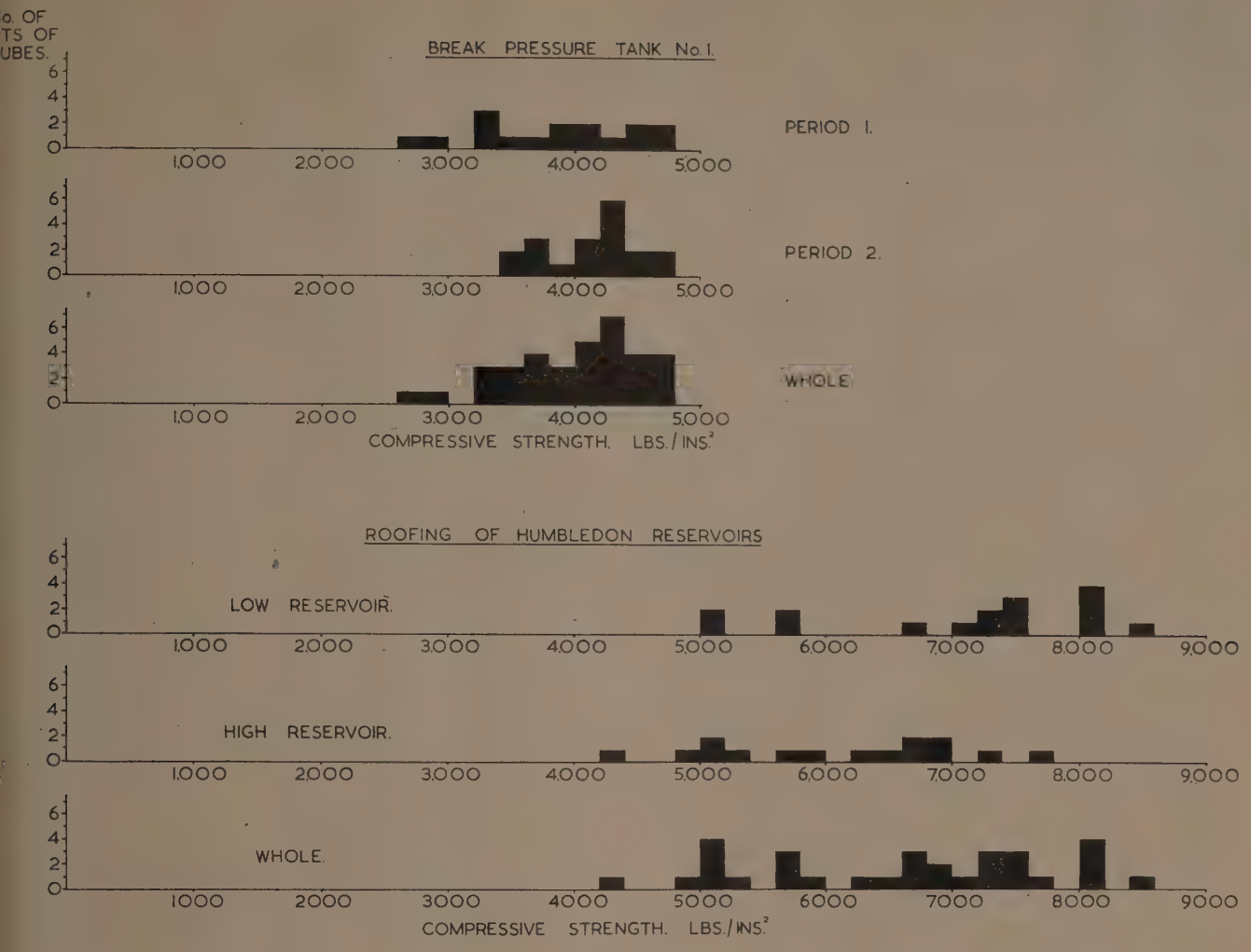

Chart 5

esults for Period 2 at the Break Pressure Tank, as ompared with that for Period 1, gives an indication of the more consistent strengths obtained during the Period 2. Similarly, the greater dispersion of the results at Humbledon, in comparison with the results at the Break Pressure Tank is evident.

TABLE 5

Humbledon High Reservoir
(Analysis of variance of cube test results)

Cause of variation	Standard deviation S. lb. /in.²	Variance S^2	Remarks
Water/cement ratio	250	63×10^3	Due to std. dev. of 0.017 in w./c. ratio. (Includes experimental error variance of pycnometer tests.
Making and testing cubes	100	10×10^3	Average variance of group means. (From within groups of 3 cubes.
Sampling and making cubes	200	40×10^3	Between groups of 3 cubes (Assumed value.
Mixing time	70	5×10^3	(Assumed value)
	(343)	118×10^3	Sum.
Total variation	986	970×10^3	Variation of Group means about grand mean.
Other causes, including cement strength	(923)	852×10^3	Difference.

Summarised Results (Table 4)

The table shows various statistics of the cube test results.

The Standard Deviation $S = \sqrt{\dfrac{\Sigma (x - \bar{x})^2}{n - 1}}$

where x is the mean of a set of 3 cubes,

$\bar{x}$ is the arithmetic mean of the results considered, i.e. the average strength, and

n is the number of sets of cubes considered.

The Coefficient of Variation $= \dfrac{S}{\bar{x}}$

It is worthy of note that the average strength for the Low Reservoir, which was constructed during the winter, was 1,000 lb. per sq. in. higher than that for the High Reservoir, which was constructed during the summer.

The standard deviation of water/cement ratio was calculated from the values of water/cement ratio which were obtained whenever a set of cubes was cast. As it therefore includes the results of pycnometer tests on the aggregates, the true standard deviation is likely to be less than that shown, owing to the pycnometer

experimental errors, and the errors in sampling for the pycnometer tests.

The standard deviation of compacting factor can also give a value of water/cement ratio variation, since the two are related. But the true variation is likely to be greater than that estimated from the variation of the compacting factor, as errors in inferring the water/cement ratio from the compacting factor (i.e., due to compacting factor experimental error and errors in batching) are not included.

Analysis of Variance (Table 5)

This table is put forward rather tentatively, in an endeavour to examine the effect of some of the factors which can cause variations in the strengths of the test cubes. The most variable of the four periods of construction was chosen for analysis, i.e. the roofing of Humbledon High Reservoir, where the coefficient of variation was 16.2 per cent.

Values for the first two causes of variation were calculated from the cube test results, but as no data were available for assessing the effect of the last two causes, values have been inserted which are only assumed, but which are thought to be in the correct order of magnitude.

By summing the variances, an estimate is obtained of the combined variance due to the causes listed, and the difference between this sum and the total variance appears to indicate that the effect of the causes considered is relatively insignificant. In other words, it is considered that these causes were not major contributors to the total variation in strength.

The temperature of curing the cubes is another factor which can cause variation in strength. For instance, cubes cured at a temperature of 36°F. are likely to have a strength of 28 days 8 per cent. less than if they had been cured at 64°F. During the construction of the High Reservoir roof the average curing temperature for the 28 days, calculated as the mean of the average daily maximum and the average daily minimum air temperature, varied between 57°F. and 63°F. The variation is not random but, as might be expected, shows a trend related to the time of year. The effect cannot therefore be included directly in the variance analysis. However, a difference between 57°F. and 63°F. appears likely to cause a total variation in strength of only 60 lb. per sq. in.

A process of elimination seems to suggest that variations in cement quality have had an important influence on the cube strengths. It is possible that to obtain the cement in bulk would reduce the variations, but this procedure would hardly be practicable for the small jobs considered in this paper. In any case, the method of purchasing cement in small quantities and packed in 1-cwt. paper bags has considerable advantages for small works in the convenience for handling and batching.

Conclusions

The full value of concrete control can be gained only if the mix can be designed with a sure knowledge of the mean strength which will be obtained, and also of the variations which will occur about the mean, i.e. the coefficient of variation which will result in the cube tests. One of the advantages of using strict control can then be financial. For instance, if the coefficient of variation of a concrete from which one set of cubes in 100 are to fall below a strength of 3,500 lb. per sq. in. at 28 days, can be taken as 10 per cent. instead of 20 per cent., then a saving of 5s. per cu. yd. of concrete can be effected. In this connection, the use of a cement with a guaranteed average strength subject to a known coefficient of variation would be of great benefit. A very high proportion of the cement manufactured is no doubt used in situations where high strength and low variations are of little value, and it is therefore suggested, especially in view of the increasing demand for high quality concrete for prestressed work, that a need exists for a special engineering cement.

The quality of concrete has to be considered when it is in two states, i.e. when freshly mixed and when hardened. The first state is important, as on this may depend many of the qualities desired for it in its second and more permanent state. A well graded mix which provides the necessary workability with the minimum of water, and which will not segregate during handling or work up too much 'fat' when tamped, is a prerequisite if a dense and uniform concrete is to be obtained. Thorough consolidation of the concrete within the shutters is essential, and it is an incentive to the workman to do this efficiently if he knows that he is being provided with a first class material :— a material which does not change from batch to batch through varying degrees of wetness, dryness, harshness or fattyness, but one which is maintained uniform throughout.

It is hoped that sufficient has been said to show that rigid quality control of concrete can be achieved by the methods described. The advantages which accrue in the freshly mixed state alone, with the favourable conditions thereby created for good placing and compaction, are, it is contended, sufficient justification for these or other equally effective measures.

There can be little doubt that strict quality control of concrete has come to stay, and can be viewed as the natural outcome of the unending quest for improved materials which is common to all industries. Probably because the conditions under which concrete is usually produced make unfavourable comparison with those which obtain in most factories, the control of concrete making as other chemical engineering processes are controlled has been adopted rather slowly. However, those engaged in research have been pressing the matter for many years, and the benefits obtained by putting into effect their recommendations surely cannot nowadays be ignored by those engaged in concrete construction.

Acknowledgements

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Prestressed Concrete Developments at the Field Test Unit, Thatched Barn

Discussion on Mr. O. J. Masterman's Paper*

Discussion

Professor A. L. L. BAKER (Member of Council), proposing a vote of thanks to the lecturer, said it seemed to him, on looking at the tables, that the most significant results were that in beams having a factor of safety of about 1.5 against cracking, a factor of safety against failure of about 3 had been obtained fairly consistently. That seemed to be very satisfactory, because if prestressed work was carried out under factory conditions of control of concrete, it should be possible to maintain consistently a standard of concrete which would justify designing to a safety-factor of 3. In the course of time, he hoped it would be possible to have such control of concreting in the field that working to a factor of safety of 3 would be justified.

Another point which had interested him was the slight reduction of strength of one or two of the beams after a period of time had elapsed. The strength increased up to a period of about ten months, which Mr. Masterman suggested was due to increases in the strength of the concrete ; and then the strength reduced gradually. It seemed to him that there were only two possible causes. One might be a deterioration in the strength of the concrete which, he thought, could be ruled out. The other might be a slight rise in the neutral axis taking place, which would concentrate the compression stresses in a much shallower compression zone. There were two possible causes of rise in the position of the neutral axis. One was creep of the steel under high load, and the other an increase in the modulus of elasticity of the concrete at the top of the beam. The latter could, of course, occur, but creep of the steel was more likely.

The results of tests on high tensile steel showed he believed, so far, that creep did not go on seriously increasing, but gradually came to a stop.

He would like to know if Mr. Masterman had got the records of the depth of the neutral axis, in the case of the beams which failed by compression of the concrete. Thereby, they could get the basic information for a general design theory for ultimate strength. Unless they knew the position of the neutral axis, they did not know how far the compression force in the beam was due to the strength of the concrete, or to the depth of the neutral axis.

Mr. A. C. VIVIAN (Member of Council), seconding the vote, said he had been interested to notice a reference to the use of pressure as well as vibration for the purpose of increasing the strength of concrete. Many years ago, he had put down a bit of concrete, and a learned friend of his had commented that it would disintegrate during the next frost because it was much too thin. What his friend did not know was that it had been put down with the back of a spade. If one hit concrete hard enough, his experience was that one did increase its strength. Vibration was one way of doing that, and he wondered if it was possible to make more use of pressure for producing a uniform and consistently high quality of concrete.

Before opening the meeting to discussion, the Chairman called on the Secretary to make an announcement.

THE SECRETARY said the Institution had received a letter which claimed that the L.C.C. type of prestressed precast beam was an infringement of an existing patent. The matter was being investigated, and meanwhile, any members or others interested in the matter were advised to bear that in mind.

THE CHAIRMAN said he had to rule that any discussion of that question of infringement was outside the purview of the Institution, and would therefore be inappropriate to the discussion on the paper which they had just heard.

Mr. DONOVAN LEE (Member), said that with regard to prestressed floor units, he would like to ask Mr. Masterman if he had made or was contemplating any tests with long span floor units ; as they all knew, 23 ft. was nowadays becoming quite an ordinary span, but we did not have much experience of greater spans than this, and it would seem desirable to ascertain what limitations, if any, there might be.

With regard to the Lee-McCall system, it should perhaps be mentioned that, although the tests referred to by Mr. Masterman were mostly beams having only straight bars, the bars being so disposed as to bring the centre of pressure at the desired height, using bars at the top in some cases, this was not essential where the beam webs were wide enough to allow bars to be bent up and not prejudice the placing of the concrete. Curved bars were, in fact, more generally used than straight bars, and there was little or no limitation on the curvature due to the physical properties of the steel, but more than moderate curvature of course involved increased friction.

He would like to answer the question Mr. Masterman had raised, as to why the high efficiency nuts do develop the full ultimate strength of the unthreaded part of the bar, so that no loss of sectional area need be taken into account in determining the prestress in the steel, but felt this must be deferred, in view of the lateness of the meeting.

With regard to the prestressing jacks, it should be mentioned those shown were of 35-ton capacity and mounted on rollers, and he would like to show a few slides, in one of which the standard mobile 42-ton jacking unit could be seen.

Mr. Donovan Lee then showed three slides, one of which depicted a 42-ton jack in use ; the second showed

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*Read before the Institution of Structural Engineers at 11, Upper Belgrave Street, London, S.W.1, on Thursday, 26th October, 1950, Lt.-Col. R. F. Galbraith, M.C., B.Sc.(Eng.), M.I.Struct.E., A.M.I.C.E., (Vice-President) in the Chair. Published in THE STRUCTURAL ENGINEER, Vol. XXVIII, No. 10, pp. 246-264 (October, 1950).

the vertical feature of the Leeds exhibition, representing a sword on end, and the third showed the sword being erected.

Mr. F. WALLEY said that during the past two years people who had been round the Thatched Barn had mentioned that they never understood why the wires in the small joist were distributed over the cross section instead of being grouped at the top and the bottom of the beam. They went on to claim that one would get a higher ultimate resistance if they were arranged in that way, but this had not been found to be the case. The ultimate resistance of the beams had proved to be approximately the same as if all the wire had been concentrated at the centroid of the steel. The sole argument for using it with small units, particularly where end blocks were not desirable, was to prevent longitudinal cracking. It was a method which he called the " Guyon " method, because he had first come across it in some of M. Guyon's excellent writings on prestressed concrete.

The slides which Mr. Masterman had shown seemed to indicate that the straightening of the wire had very little effect on the properties of it. In fact, looking at Table H, the proof stress was, on the whole, slightly increased, and the ultimate was slightly decreased, but both differences were not very significant. A word of warning was necessary, because this would seem to depend on the characteristics of the straightening machine used. They had not always had that same experience on every job. On one job in Scotland where they had to straighten wire, the proof stress had, in some cases, dropped as low as 55½ tons per sq. in. after straightening, and the average over the whole job was about 60 tons per sq. in. So, he would point out that the figures shown were all right for the type of straightener used, but they would not necessarily be correct for all types of straighteners. He thought that since wire in 8 ft. diameter coils which ran off straight was now available, straighteners on the site were a thing of the past. The properties of the steel actually put into the job were known, and they were not altered by any site work.

Mr. Masterman had said they always used 12-gauge wire, and he personally had never been satisfied that they should use anything larger in smooth wire for pre-tensioned work. Indented wire of 0.2 in. diameter was now coming off the production lines, and if it could be used, it would no doubt simplify the production of joists, since fewer wires would be used, and there would be no danger of bond slip in using them.

Referring to the long-term tests, particularly the tests shown in Fig. 17 and Table D, he thought he could say that the original purpose of those tests was of reassurance. In earlier tests, it had been found that when load was left on over-night, or even for a few hours, the deflection of a prestressed member increased rapidly. They began to wonder how long it would go on increasing. They did not know if failure would take place prematurely, and the tests were carried out to determine this.

He would like to draw attention particularly to the beams loaded to 150 lb. per sq. ft. This was equivalent to over 90% of the ultimate load. The load was left on for ten months, and the deflection of the beam increased from 2.30 to 3.92 in. When it was off-loaded it recovered to 0.84 in. If it was assumed that the initial deflection was elastic, and the subsequent deflection was due to creep, then it could be shown that about 50 per cent. of the creep deflection was recoverable, which was a remarkable fact. The increase of deflection at normal loading was very small, and did not give cause for alarm.

Before closing, he would like to express his appreciation on behalf of the Development Engineer's Section of the Ministry of Works to Mr. Masterman and his staff for all the work which they had given them during the past few years. It was not every designer who had a testing station at his back, and he was fully conscious of the advantages which they had had, and was grateful for and appreciated the help which they had received, and the way in which the tests had been carried out in the past few years.

Mr. J. H. HUMPHREYS said, with regard to the estimation of the amount of prestress imparted to a wire, it was rather disappointing that it had been found impractical to do that while listening to the musical note emitted by twanging the wire. Some such method had always appealed to him, as being easier and a more accurate way of determining prestressing. If possible, he would like to see it given a further trial.

He understood there was already an instrument used which measured the stress in rod by the frequency of a vibrating reed.

He was responsible for the design of the L.C.C. floor described by Mr. Masterman in his paper, and he recorded his thanks for the enthusiasm and the help which Mr. Masterman and his staff gave him in the design and testing of it. The units themselves were designed to be capable of resisting dead and live loads without any assistance. Although the tests indicated an increase of strength after filling over that of the individual units, he did not feel like allowing for the increased strength provided by the concrete filling. It was very poor concrete on the site, and one could not get that kind of strength which Mr. Masterman got at his testing station.

He did not think he had invented anything; the job was a straightforward one and, after all, they were using I-sections in cast iron 200 years ago.

Mr. S. C. C. BATE said that in the tests made at the Building Research Station on the 5 in. × 2½ in. joists, they had been interested in the ultimate strength of these members. A satisfactory method of calculating the strength of the joists was given on page 258 of the JOURNAL. It was assumed that at failure the compression zone of the concrete was uniformly stressed to 85 per cent. of the cube strength, and that the wires below the neutral axis were stressed to their tensile strength. This calculation gave a result which was, on the average, 10 per cent. higher than the experimental results. In some of the tests, measurements of strain were obtained on the joists, and the stress-strain characteristics of the concrete and the steel were determined. It had been found that this latter information could be used to calculate the position of the neutral axis in the joist during the stages of loading between cracking and failure with reasonable accuracy, but that the failing load was under-estimated by 15 per cent. on this basis. When failure of the joist occurred by crushing of the concrete, the neutral axis had then risen to about 1¼ in. from the top. From these preliminary tests, a fundamental study of the failure of pre-tensioned concrete members had been commenced.

He would also refer to another section of their work which dealt with the behaviour of the steel wire. A point of interest was the variation in properties which might be experienced, even within one coil of wire.

For tensile strength, they had obtained variations agreeing with those given by Mr. Masterman, but they had also found that, at the stresses normally adopted in prestressed concrete, the strain might vary along the wire. If the stress in the wires on the prestressing bed was judged from their extension, then, appreciable errors might occur, as the extension was usually calculated from a few tests on short specimens. Their tests were being continued, to examine the influence of these variations in properties on stressing operations.

Mr. R. H. SQUIRE (Member), referring to the graph which Mr. Masterman had shown comparing the calculated deflection of a joist with the actual, said that two points calculated on the curve between the point of the first crack and the ultimate load were not actually shown. One of them was of interest, he thought, for the reason that this point was calculated for a value of 0.7 of the ultimate strain of the concrete, and that was what he would expect as the ultimate strength under repetition loading based on Dr. Probst's* work. The value was 2,144 lb., load and deflection was 2.50 in.

Another point on which he would comment concerned the two graphs in the paper, Figs. 21 and 22.

Fig. 21 dealt with two groups of beams, and he would ask Mr. Masterman if each group was from the one batch of concrete; because he thought it rather surprising that there should be quite as much scatter, especially in the series FJ III.

That figure was an attempt to arrive at an acceptance test, which should be non-destructive, and that he understood to mean the loading should stop well short of that required to cause the first crack. Under those conditions, one could only get information on the quality of the concrete — if the position of the wires and the cross-section of the beam was exactly as intended they would get a very good " E " value, which was the measure of the quality of the concrete. That was one of the things which might go wrong in construction. The other thing was the amount of prestressing, and on that particular point, a non-destructive test would give no information. The only information that could be got as to the amount of prestressing which they had locked up was by the load required to produce the first crack.

About the other diagram, Fig. 22, it showed the load value at first crack and at failure. The loading on first crack increases almost directly with the amount of prestress, but the ultimate load does not, and so there is no direct relationship. On the right hand side there is a group of four tests on the six-inch joist. The ratio between first crack and the ultimate there was about 1.75. If the prestress was increased to about 212,000 lb./in.^{2}, the ratio between first crack and the ultimate would be only about 1.35. There was one suggestion he would make, and that was that when these tests were plotted, it might be helpful to segregate the tests which failed in compression and those that failed in shear on separate diagrams.

Reply to the Discussion

Mr. MASTERMAN, whose replies to questions had to be very brief owing to shortage of time at the meeting, has amplified his answers in writing.

He said that he greatly appreciated the kind things said by the proposer and seconder and others, and he knew that they would wish him to share these congratulations with the members of the staff at Thatched Barn who had done the work.

He felt that Professor Baker had been unduly cautious in regard to the factor of safety against failure which designers could accept for prestressed work. His own view was that a factor of safety of 2½ was adequate, in view of the fact that both the concrete and the steel were subjected during production to stresses higher than those which they were ever likely to experience afterwards, and in addition, because special care was necessarily taken in the manufacture of both materials because of the high strengths required. As mentioned in the paper, no conclusions had yet been drawn on the results of the long-term tests which were not yet complete. What was noticeable was that the strengths appeared to increase for a time after the prestress was effected, although it was generally expected that they would fall, owing to creep and shrinkage of the concrete. It almost seemed as if in the first few months the unit was gaining strength by the hardening of the concrete at a faster rate than it was losing strength by creep. When failure occurred by crushing of the concrete in these 5½ in. × 2 in. joists, the neutral axis had risen to about 1¼ in. from the top, as was stated by Mr. Bate during the discussion.

He was in full agreement with Mr. Vivian's comment that one would expect to obtain economies by prestressing, even with quite small components. The data given in the paper showed that substantial savings in steel and in concrete had been obtained, as compared with equivalent designs in reinforced concrete.

As regards the use of pressure for making high-strength concrete, Mr. Masterman said he was strongly in favour of this, and had shown the advantages of combining pressure and vibration when casting concrete products in a commercial development of prestressed joist manufacture, which was described in the current (October, 1950) number of CIVIL ENGINEERING. In this work, cube strengths of 5,000 lb. per sq. in. had been consistently achieved at 4 hours by combining pressure, vibration, and heat curing. The pressure of some 20 lb. per sq. in. applied to the concrete surface had been shown to increase the 4-hour-strength by about 1,000 lb. per sq. in. It gave the added advantages that the top of the casting was as well compacted as the bottom, that surplus water was squeezed out, and that if the cover was kept in place the casting could be immediately immersed in steam or water at 100°C without harming the concrete.

He was grateful to Mr. Donovan Lee for showing slides of the prestressed sword which he had designed to erect at the recent Leeds Exhibition. There had not been time to discuss the vast possibilities inherent in prestressing, but this long and slender component, with its thin curved hand guard, the whole of which had been cast in small pieces and unified by post-tensioning, was a striking example of what could be done, even in these early days of using the technique.

Mr. Masterman had long felt that the post-tensioning of factory-made pieces in this way was a very important development, and one which would quickly be exploited commercially, because it simplified the handling and transport of large units, and enabled the concrete to be cast in factories under conditions favourable for close control.

He was sorry that he could at present contribute no information on thin prestressed floors on spans in excess of 23 ft., though he was inclined to agree with Mr. Lee that the possibility of danger from dynamic loads harmonising with the natural frequency of the floor might need to be examined.

He was glad that Mr. Walley, in referring to the results of the long term tests, had brought out the

*THE STRUCTURAL ENGINEER, 1931, pp. 326 and 410.

point that they were reassuring. He (the author) felt that all the development work done at Thatched Barn had pointed the same way. It had shown that the required high concrete strengths were not difficult to attain, that the stretching and anchoring of the tendons presented no undue problems, that the bond was effective in pre-tensioned work using small wires, and that the tests showed ample margins of safety in the products.

Mr. Humphrey's hopes of using aural methods of checking the stress in tensioned wires were, he thought, doomed to disappointment for various practical reasons, the chief of which was that the human ear is not sufficiently sensitive. It would be a great advantage to have a simple means of measuring the load in a stretched wire, and the author and his staff were constantly trying to solve this problem.

Mr. Masterman was grateful to Mr. Bate for the information which he put forward so clearly during the discussion. The analytical work done by Mr. Bate on the tests which were made at the Building Research Station was most valuable in filling out the story of the development work, and in confirming to designers that a simplified method of calculation gave ultimate strengths in reasonable agreement with test results.

Mr. Squire's careful work had led to the same result, and had enabled him to forecast very closely, not only the first crack and the failing load of a prestressed joist, but the deflection at all loads up to failure.

In Fig. 35, the author has superimposed on Mr. Squire's calculated curve the deflections recorded from an actual joist. The close agreement would be seen. Up to first crack the curves coincided and the deflections were identical. (The figure did not quite show this, because at first crack the joist was unloaded, and the graph showed the second loading). The failing load as calculated was 5 per cent. on the safe side. The calculations, based on first principles, used only the properties of the concrete (cube strength and modulus), and its actual dimensions and the dimensions, pre-tension and stress-strain curve for the wire. The stress distribution in the concrete at failure was taken to be represented by a hyperbola with eccentricity 2, as was advocated by Mr. Squire in his paper to this Institution of June, 1943.

As regards the scatter of points on Fig. 21, it should be noted that each series of joists was made from four different batches of concrete. On Fig. 22, joists which failed in tension had originally been separated from those which failed in shear, but as no trend was distinguishable, they had all been plotted together in that figure.

Mr. Masterman appreciated the valuable comments made by Mr. Squire concerning non-destructive tests.

Written Communications

From: Dr. P. W. Abeles (Member)

Mr. Masterman's paper contains some valuable test results. However, there are several points requiring clarification. For example, the author estimates the concrete bending tensile strength from the recorded difference in load at the first and second opening of cracks. This, in the writer's view, is bound to lead to wrong conclusions. Tests on prestressed concrete beams have shown that, on reducing the load after first cracking, the cracks become invisible at a load being much higher than that at which the concrete tensile stress

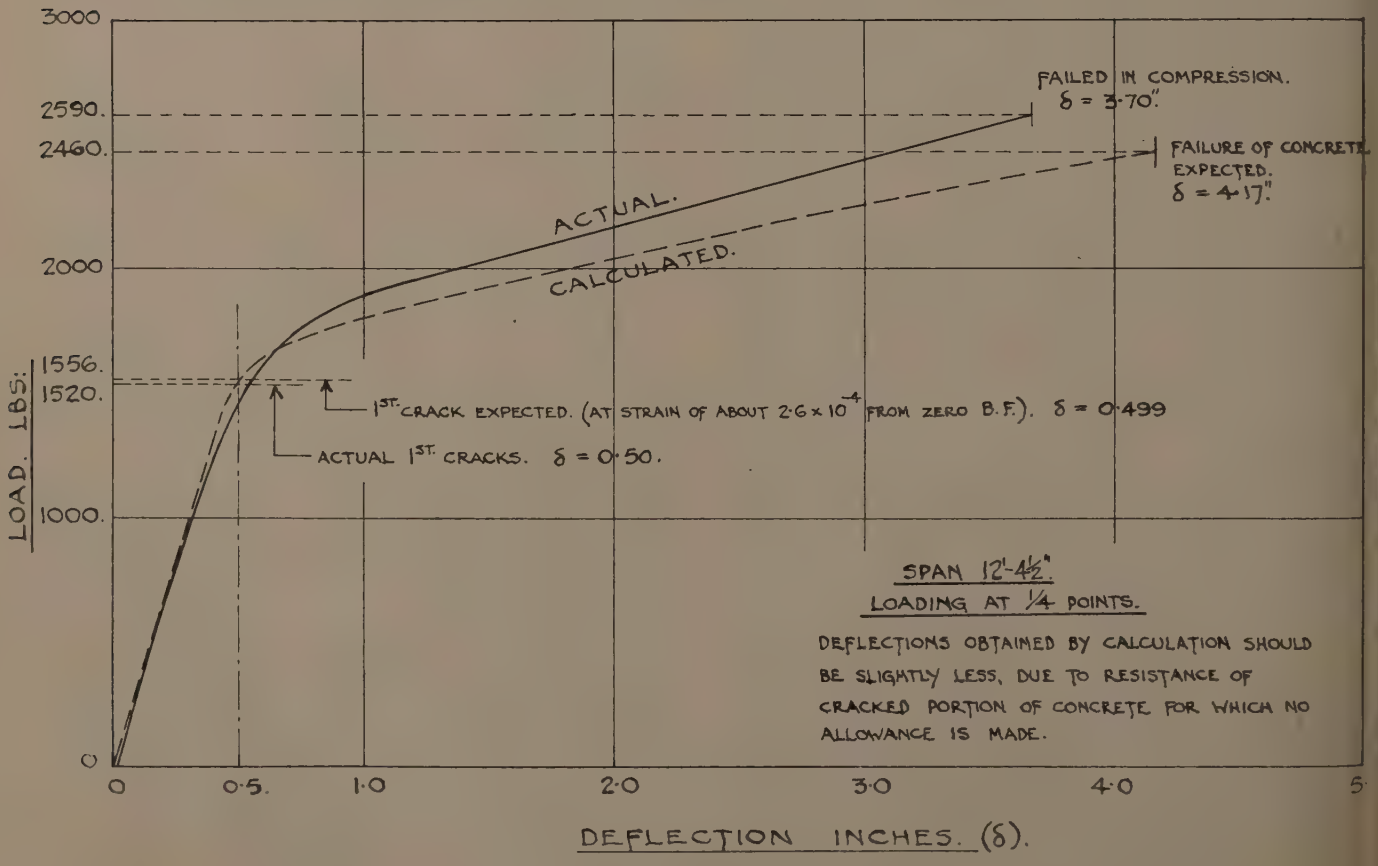

Fig. 35

reverses into compression (i.e. when the prestress which was interrupted at the cracks acts again over the full length). It is true the fact that the cracks become invisible does not mean that they do not exist, but they have become very narrow (say less than 1/100 mm. = 1/2,500 in.), and they are not visible to the unaided eye. The author has come to the conclusion that the bending tensile strength (also called modulus of rupture) amounts to only 624 lb. per sq. in., by his arbitrary assumption that the tensile stress is zero at the re-occurrence of visible cracks. The magnitude of this load depends on the skill of the observer to notice cracking, but has nothing whatsoever to do with the zero stress. Numerous tests in this country and abroad have shown that the modulus of rupture for high strength prestressed concrete amounts to approx. 1,000 lb. per sq. in., and is much higher than the value obtained from test specimens 16 in. × 4 in. × 4 in. of plain concrete. The bending tensile stress at first cracking can be obtained if the effective prestress is calculated ; this can be done fairly accurately, allowing for a suitable amount of losses of the initial prestress.

With regard to post-tensioned beams, unfortunately, no data are given to obtain the stresses at ultimate load. From the loads at failure given in Table G, the maximum bending moment cannot be calculated, and it would be appreciated if the author would publish the missing data (maximum bending moment, details of cross-section including position of steel, and concrete strength).

The author states on page 262 " that the use of wire 0.2 in. or larger diameter would not generally be recommended for bonded work unless it has been indented, deformed, or otherwise treated to improve its bonding qualities." While the writer agrees that a better bond is attained by using smaller diameter or indented or deformed bars, it must be said that plain 0.2 in. diameter wire without special treatment has proved very satisfactory in pre-tensioned construction, provided that sufficient bond length is available. The writer is connected with the construction of 17 highway bridges over railways, in which such wire has been used in beams of approximately 30 ft. length, and several full-size failure tests and numerous deflection tests have given complete satisfaction.

From : Mr. H. T. Horsfield

As the author has stated, the work at the Thatched Barn was primarily operational research, an important field of investigation hitherto neglected, and the main part of the paper deals with the valuable results obtained in tackling practical problems. The many tests carried out, however, afforded opportunities for investigating some of the fundamental properties of concrete, and on page 259 the author has derived a figure for that measure of safety of the quality of the concrete known as the bending tensile strength or modulus of rupture.

On page 253 in Fig. 18, the load-deflection curve is given for one of the tests on which the calculations were based. Under zero load, the effective prestress was the only stress acting in the bottom fibre of the joist. As the load was applied, this compressive stress was gradually reduced to zero, and further loading caused a tensile stress to develop. This tensile stress gradually increased until it exceeded the tensile strength of the concrete, and cracking was first observed at an applied load of 1,600 lb. Assuming a straight line stress distribution for the uncracked section, the change of stress in the bottom fibre can be easily calculated by dividing the applied bending moment by the section modulus. In this case, a tensile stress of 3,250 lb./sq. in. had been superimposed on the original compressive prestress. At the second loading, cracks were noted earlier, when the applied load was 1,300 lb., and the applied tensile stress in the bottom fibre was 2,650 lb./sq. in. The author takes this difference of 600 lb./sq. in. to represent the initial tensile strength of the concrete, which ceased to be effective when the section cracked under the first loading.

But a corollary of this reasoning is that when the cracks opened on the second loading, the stress in the bottom fibre had just become tensile, and that the applied tensile stress of 2,650 lb./sq. in. had just balanced the effective compressive prestress. On the data given for the section, 2,650 lb./sq. in. seems an impossibly high figure for the effective prestress.

Moreover, a reference to the similar load-deflection curves of Fig. 19 shows that as the first loading was reduced the cracks remained open at loads down to 80 lb./sq. ft., whereas the author's reasoning requires compressive stresses in the bottom fibre at all loads up to 95 lb./sq. ft., when the cracks re-open on the second loading. One cannot imagine the cracks remaining open if the stress really were compressive.

From the figures given in the paper, it appears that the effective compressive prestress in the bottom fibre of the joist at the time of testing was roughly 2,100 lb./sq. in. If the cracks first occurred when the applied tensile stress was 3,250 lb./sq. in., the concrete bending tensile strength would be about 1,150 lb./sq. in., but, as the author remarks, the first crack is difficult to define, and it is probable that unobserved cracking had occurred at a lower stress.

Reply to Written Communications

Mr. Masterman writes :

Dr. Abeles has raised questions concerning the stresses in the beams post-tensioned by the Lee-McCall method : he also challenges the author's statement that plain 0.2 in. diameter wire would not generally be recommended for bonded work, unless treated to improve the bonding qualities.

As mentioned in the paper, Professor Ross is publishing full particulars of the tests on the Lee-McCall beams, of which brief particulars were given in Table G. As regards bond with wire of 0.2 in. diameter, it is reassuring to know that Dr. Abeles has had satisfactory results from numerous bridge tests ; and it is well known that plain 0.2 in. diameter wire has so far been used successfully in large numbers of sleepers, where the anchorage length is short and the loading conditions are severe. There is no doubt that such wire does give satisfactory bond, provided the concrete is good, and the spacing and anchorage length is adequate. None the less, the author supports the present trend towards greater safety by improving the bond by some inexpensive surface treatment.

Considerable interest has been evinced in the author's suggestion that the bending tensile strength or rupture modulus of the concrete in a pre-tensioned unit may be assessed from the difference in load required to cause the first crack and to re-open the same crack on re-loading. In the paper, a rupture modulus of 624 lb. per sq. in. was deduced in this way, and was compared with a figure of 808, arrived at from laboratory tests on 4 in. × 4 in. beams made from the same concrete. The author believes that this figure of 624 lb. per sq. in. more nearly represents the actual bending tensile strength of the concrete in the joists concerned than does the figure of 808 obtained from specimens of substantially larger section, made and tested under laboratory conditions. It is considered a valuable advantage of the method suggested that the modulus

is obtained from the actual unit, and without having to make any assumptions as to the actual prestress in it.

However, both Dr. Abeles and Mr. Horsfield question the validity of the answer obtained, on the ground that the author has assumed that the tensile stress in the bottom flange is zero when the crack reappears. *No such assumption has in fact been made.* Nor is the author unaware of the uncertainties of crack observation especially in varying circumstances and by different observers. Such uncertainties are, however, largely eliminated in the proposed method, which requires a simple observation to be made twice by the same observer under identical conditions. He must say, on two loading occasions, when he first sees a crack appear. It does not much matter what is the size of the smallest crack he can see ; but there is some likelihood that he will see it a little earlier in its development on the second occasion, because he knows where to expect it ; to this extent, the modulus obtained will be a little on the high side. What *has* been assumed in putting forward this suggestion is that the load difference between the visible opening and re-opening of the first crack will not differ much from the difference between the load corresponding to no stress in the bottom flange, and the load at which the first crack really (though invisibly) does begin to form ; there are some grounds for this assumption.

The sequence of events on loading a prestressed beam are considered to be as follows : before loading begins, the bottom flange is in compression ; as load is applied, this compression is gradually reduced until a particular load exactly cancels it ; as loading is continued, the stress becomes tensile, *but no crack is started because the concrete is able to sustain a certain amount of tensile strain.* It is only when the load has increased to the point at which the limit of this strain has been reached that cracks begin to form (invisibly), and the load/strain or load/deflection curve turns away from the straight line. At a still later stage in the loading cycle the first crack becomes visible. This sequence is clearly illustrated in Fig. 36, which shows the actual strain readings recorded by Mr. Bate at the Building Research Station plotted against the load for one of the joists in question. The 1 in. Huggenberger extensometer actually bridged one of the first cracks.

Fig. 36 shows strain in the bottom flange plotted against applied load. A scale of calculated bottom flange stresses has been fitted on the left. The prestress of 1,720 lb. per sq. in. used as the base of this scale is a calculated figure of 1,900, reduced to 1,720 by the self-weight of the joist. This prestress of 1,900 is rather below what was intended, but is computed on the actual measurements of the joist, which was made in worn moulds, and was not strictly to the specified dimensions. At the top of the scale is the rupture modulus, obtained experimentally from 4 in. × 4 in. section test beams, and this is placed opposite the calculated cracking load.

On first loading, the prestress becomes cancelled at some point near to A, where compression in the bottom flange changes to tension (calculated load 940 lb.). Further loading is balanced by the tensile strain in the concrete, until at a point somewhere in the neighbourhood of B, the limit of this strain is reached, and cracks begin to form ; this is where the curve starts to leave the straight line. These cracks first become visible at C, where they are rather over half a thou. in width (observed load 1,520 lb.). On the unloading curve, the point at which the cracks close is not of interest because of the large hysteresis effect. On re-loading, the cracks will now begin to re-open near A, but are not seen until the point D is reached, where they are rather under half a thou. in width (observed load 1,200 lb.).

The author's modulus of 624 is marked in on the stress scale, and it will be seen that it corresponds to an applied load of about 1,286, represented by point E near where the curve is bending away from the straight line. It will also be noticed that the observed load difference between C and D is of about the same magnitude as that between A and B, which corresponds to the rupture modulus.

It would be interesting to know what method was used to derive the high rupture modulus figures quoted by Dr. Abeles.

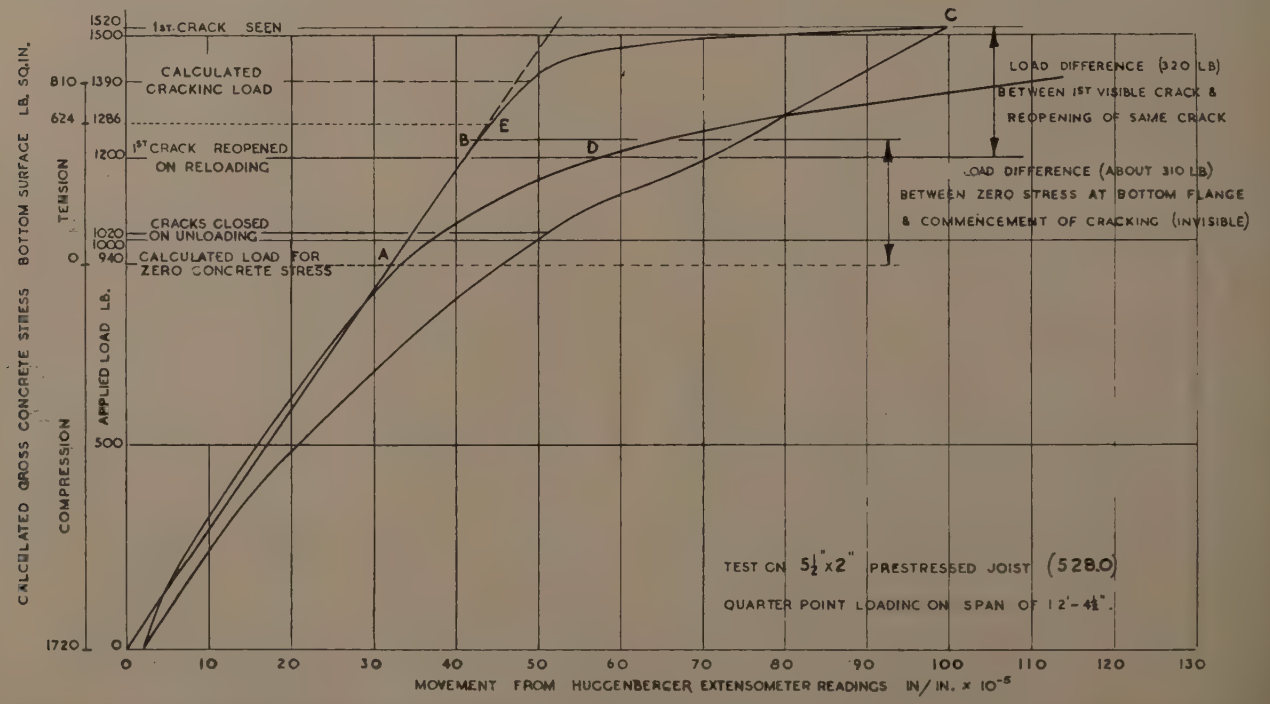

Fig. 36.—Huggenberger extensometer readings on the bottom concrete surface with gauge spanning one of the first cracks. Joist "O"

The Tuscan Boracic "Soffioni" and their Development at Larderello*

Discussion on Dr. Ing. Alfredo Mazzoni's Paper

Mr. R. C. S. WALTERS, B.Sc., M.I.C.E., President, Institution of Water Engineers, and President of the British Section of the Société, presided at the meeting. He was accompanied on the platform by Mr. J. E. Swindlehurst, M.A., M.I.C.E., the President of the Institution.

The CHAIRMAN introduced Dr. Mazzoni, and thanked him for having so kindly prepared the paper.

He said that in 1946 the Societa Larderello appointed Dr. Mazzoni as Director General to rebuild the installations destroyed during the war. Whilst that was carried out in record time, he proceeded to design the new Larderello III Generating Station, which was described in the paper. It was a very interesting and original piece of work.

(Mr. PETER GERARD, A.M.I.E.E., M.I.Struct.E., (Member of the Société), who had translated the paper, then presented it on behalf of Dr. Mazzoni and explained the large number of lantern slides which were shown. A film which had been produced at Larderello depicting the construction and the operation of the plant was also shown).

THE CHAIRMAN (Mr. R. C. S. Walters) congratulated Dr. Mazzoni on his paper, and paid tribute to Mr. Peter Gerard for having translated it and presented it to the meeting. He proposed formally the thanks of the meeting to Dr. Mazzoni and Mr. Gerard.

He said it was known that there was coal in Italy; apparently, the equivalent of 600,000 tons of coal per annum had been obtained through the old plant at Larderello, and of 900,000 tons per annum from the Larderello III plant. In the latter plant, apparently the steam was taken straight into the turbines, whereas in the old plant the steam had to go into boilers or heat exchangers on the way.

In Great Britain, the nearest phenomenon to the Italian source of underground heat was to be found at Bath, where the temperature of the Roman baths was 120°F., which was nowhere near boiling point; the water came up to the surface, as it had done since Roman times. There were one or two places in England also where we could get gas, and in one place, somewhere in Sussex, the gas was used for lighting the railway station. Dr. Mazzoni had set us wondering whether we could derive power from deep boreholes in this country. There were deep artesian boreholes at Slough (1,500 to 1,800 ft.), but if the water were allowed to overflow after it had been tapped, the boreholes became filled with mud in a matter of seconds. But apparently at Larderello the steam could be allowed to go into the air, or it could be corked down, which was very interesting as an engineering feat.

He asked if the origin of the steam was rainfall or water inherent in the molten rocks.

Mr. J. E. SWINDLEHURST, M.A., M.I.C.E., M.I.Struct.E. (President of the Institution of Structural Engineers), seconding the vote of thanks, said the meeting could not have listened to a more interesting paper, which indicated not only a magnificent feat of engineering, but a display of initiative which was very refreshing in these days. It was something of which Dr. Mazzoni and all associated with him would be extremely proud.

Mr. Swindlehurst, speaking Italian, warmly welcomed Dr. Mazzoni, and expressed the added pleasure it had afforded the meeting to welcome on that occasion Signora and Signorina Mazzoni.

(The vote of thanks was accorded with enthusiasm, and Dr. Mazzoni briefly responded).

Mr. W. HAWTHORNE, M.I.C.E., M.I.E.E. (Past President of the British Section of the Société), said that in the hall of the Institution of Structural Engineers, through the courtesy of that body, they had listened to many interesting papers, but none on so unusual a subject as that by Dr. Mazzoni. All were indebted to him for having prepared so lucid an account of the Larderello installations, and for having described and illustrated the work.

It had been explained that heat came from a reservoir of molten or very hot rocks, at varying depths down to 16,000 ft. below the surface of the ground. That was understandable; but where did the flow of water come from, and where was it converted into the steam, which formed 96 per cent. of the vapour emerging from the surface? The flow of $5\frac{1}{2}$ million lb./hr. amounted to about 25 cusecs, which was quite a sizeable stream, and that stream must be fed from a large perennial spring or a fresh water lake. Great courage must have been displayed by those who had installed tens of thousands of kilowatts of generating plant on the assumption that, with the increased utilisation of the vapour, the supply would not become exhausted in the near future. We were told that vapour had been coming out of the ground since the first century, but not at the rate at which it was coming out now, and one began to wonder how long the supply of water would last. No doubt there was plenty of heat.

The accessibility of subterranean heat had been exploited in Iceland, though on a much smaller scale than at Larderello. The city of Reykjavik drew hot water from wells about ten miles outside the city. There, some 40 of those wells or boreholes, of from 4 to 8 in. in diameter, which had been sunk to depths varying up to 2,000 ft., and they gave a flow of water of nearly 4,000 gallons per minute at a temperature of 190°F. The water was pumped to concrete tanks at the edge of the city, from which it was distributed to about two-thirds of the houses there for heating.

A matter on which Dr. Mazzoni and his co-workers deserved congratulations was the rapidity with which the war damage at Larderello was repaired and the

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**Read before a Joint Meeting of the British Section of the Societe des Ingenieurs Civils de France and the Institution of Structural Engineers at 11, Upper Belgrave Street, London, S.W.1., on Thursday, 5th April, 1951, at 6 p.m. Published in* THE STRUCTURAL ENGINEER, *Vol.* XXIX, *No.* 4, *pp.* 97-106. (*April*, 1951).

LARDERELLO II LARDERELLO III

Fig. A.—Generating stations

installations put into working order again. One rather doubted that it could have been done so quickly in this country. Larderello III was going to be a magnificent station, and obviously the machines were of first-class design. It had been stated that the power station would contain 30,000 kW. main sets, and would produce 800 to 900 million kWH. per annum, the consumption of vapour being 21 lb./kWH. That compared quite favourably with the performance of modern thermal stations in Britain, and it was evident that the plant at Larderello had been carefully designed for the local conditions.

The reference in the paper to sulphur and ammonia immediately raised the spectres of corrosion and embrittlement. Their effects might be retarded by the relatively low temperature and pressure of the vapour, but it would be interesting to know of what materials the pipes, valves, and turbine shafts and casings were made, and whether signs of corrosion had appeared where two different metals were in contact. Mr. Hawthorne also asked for how many hours the turbines were run before they were opened up and overhauled.

Dr. Mazzoni had not yet found a way of determining where boreholes should be sunk in order to secure a good flow of vapour, although he had tried geophysical, electrical and seismological methods. It was to be hoped that he would continue his investigations and that, when they had succeeded, he would come to this country again, and tell us where to put down bores in order to tap the unlimited supply of heat in the interior of the earth, whereby to supplement our diminishing supplies of coal. In any case, if Dr. Mazzoni did come back here, we should always be delighted to welcome him, and to hear more about the wonderful developments which were being made at Larderello under his guidance.

Mr. Bryan Donkin, M.I.E.E., discussing the power development side of work at Larderello, said the steam was generated in a "boiler" thousands of feet underground, and there was in effect a long pipeline between that boiler and the turbines. Something like one million pounds of steam would be extracted hourly. How did Dr. Mazzoni know at what pressure he would obtain it from the boreholes when drawing that quantity of steam? It seemed that he must have some very good data to work on in order to determine that pressure, because presumably he had to design the station and build the turbines before the full quantity of steam could be drawn.

In all the stations, surface jet condensers were used for condensing the natural vapour. Judging from what one had heard, and also from a reading of the Paper by Virgili on "The Application of Natural Steam to the Production of Electric Power," presented to the Fourth World Power Conference in 1950, it seemed that there was a great shortage of water at Larderello, and that the only way in which to make up the water required in the cooling towers was to add the cooling water from the steam turbine exhausts. It seemed also, that the circulating water from the turbines would gradually increase the quantity of salts in solution, before it came to the chemical works for the extraction of the boric acid. Mr. Donkin asked for a little more information about that.

The ordinary steam turbine designer would ask, why expand steam and vapour down to atmospheric pressure only, and why not take it right down to a much higher vacuum? One imagined the answer was that, because there was 6 per cent. of non-condensing gases, it would be necessary to compress the gases up to atmospheric pressure again. It seemed, however, that expanding only to atmospheric pressure meant throwing away a lot of heat to the cooling towers, whereas it would be valuable if it could be used; maybe Dr. Mazzoni used it later for the evaporation and concentration of the boric acid salts.

Mr. J. M. Burnett, A.M.I.E.E., raising a legal point, said that in this country until recent legislation was passed, if one bored for water there was nothing to stop anybody else boring next door and taking the output of one's boring. He wondered whether the company which operated at Larderello had a concession covering the whole of the area concerned, or whether there was the possibility of a pirate opening up very near to the company's boreholes.

In connection with the winning of oil and other liquid materials, a lot of time was spent in drilling wells which never came to anything. He asked whether all the drillings in the Larderello area were successful and had produced steam.

One would like to know also, whether any estimate had been made of the total amount of steam likely to be available from that area. The output was fairly large at the moment, but he would like to know what was the limit.

Again, he asked whether any effort had been made deliberately to increase the amount of steam by putting down water. In Alberta, Parsons had proposed putting water down a borehole and extracting steam from it.

Finally, if the output at Larderello were continually expanding and wells were being drilled continuously, there was a prospect, when a new well was being drilled

by the Larderello company, of reducing the pressure at one of its pre-existing wells. Mr. Burnett wondered whether that had in fact occurred, and whether the wells showed a general decline in output as the years passed.

Mr. F. S. Snow, M.I.C.E., M.I.Mech.E. (Past President, Institution of Structural Engineers), asked whether, as the result of the continual boring of wells, there was undue settlement of the ground surrounding the actual properties? Recalling also that the power houses were in many cases of tubular construction, he wondered

LARDERELLO III
Fig. B.—Tubular construction

whether, as the result of the continual taking of water from the subsoil, there was any settlement there, as happened frequently in this country.

Secondly, he asked whether any particular precautions were taken to protect the structural steel from the effects of the acid fumes?

Thirdly, what was the thickness of the water cooling tower walls, and were they affected by the acid and the fumes coming therefrom, or was it necessary to apply a solution to protect the concrete, or even on occasions to repair with gunite?

Mr. B. A. E. Hiley, M.I.C.E., M.I.Struct.E., asked what happened to the cooling tower which, as was shown by the pictures, was hit so centrally as to blow off the upper region. What reparations were necessary in the lower part of the tower? It had taken 12 months to reconstruct entirely, which he considered to be a record. He asked whether it had been necessary to strip it, or whether they just built on to what was left after the bombing of the structure?

Mr. Baliol Scott, Editor, THE MINING JOURNAL, asking exactly how deep were the boreholes at Larderello, said that oil boreholes had been driven to about 20,000 ft. in California or somewhere in the Western States. Were exploratory borings made at Larderello to test the conditions to the maximum depth that could be reached? Presumably, where they were boring deeply, they were definitely following the practice of the oil industry.

He also asked what was the output of gases in the course of a year?

Mr. Peter Gerard, replying on behalf of Dr. Mazzoni, said that in answer to Mr. Walter's question, the origin of the steam was not known with accuracy. According to the most modern theories, however, the steam arose from a magmatic mass in process of solidification (batolite), at a depth of about 20,000 ft.

It was already known that during the process of cooling and successive crystallisation of molten magma, remarkable quantities of superheated steam were released.

It was conceivable that this steam, mixed with small quantities of underground water vapour produced through infiltration of ground water by contact with hot rock formation. Such quantities were small, however, since heavy rainfalls did not seem to influence the rate of flow of the steam.

Dealing with the contribution by Mr. Hawthorne, he said that a method of estimation of the volume of this batolite would be given in the second edition of Dr. Mazzoni's new treatise: "The Tuscan Soffioni and the Installations at Larderello" now in the press.

It would show that the volume of batolite necessary to produce the actual quantity of steam during several thousand years would not appear excessive, but was well within the range of subterranean manifestations of steam.

No special corrosion had been observed in the steel water vapour pipes, which were well lagged against possible thermic losses.

Both the pure steam turbines and those fed from natural water vapour with exhaust to the atmosphere worked regularly, and required only short interruptions for maintenance purposes.

The direct injection natural water vapour turbines with condensers and compressors also worked regularly, but since they had been in commission only a short time, it was difficult to comment on the interruptions required for maintenance purposes.

Replying to Mr. Donkin, he said it was true that there was a very considerable pressure loss in the borings. At the bottom, the pressure was 25 to 30 atmospheres, and at the top it was 5 atmospheres, but even this low pressure was sufficient to generate electrical energy.

No increase in the quantity of salts in solution had been observed in the cooling tower ponds or in the open water channels.

Replying to Mr. Burnett, concessions had been obtained from the provinces of Pisa, Livorno, Grosseto and Firenze, covering territories which extended very widely around the centre of operations.

In the Larderello area, the majority of the borings gave excellent results. In other districts, however, a few borings gave disappointing results: some with very little steam and others with none at all. Electrical prospecting was now under way, and gave valuable information for the siting of new borings.

No assessment could be made at present concerning the availability of the water vapour in the subsoil, as they were not dealing with normal mineral deposits. However, whilst the Larderello district had been entirely developed, there were many other resources which still remained to be extracted within the extensive regions under concession.

A few borings carried out in the close vicinity of existing ones had slightly reduced their output; this would indicate that there might be subterranean contact between them. However, when the borings were sufficiently apart, say between 500 and 700 feet, there was no reduction in the output.

Replying to questions raised by Mr. Snow, Mr. Gerard said there were ground settlements, no doubt, but it had apparently not affected the ground structures, which were light structures designed to give a relatively small load on the ground. The Larderello III Power Station weighed only 300 tons and was designed so that, except for the 60 ton travelling crane, no heavy loads

were carried by the building structure. The roof had been designed for a superimposed load of 18.5 lb. per sq. ft., and the wind pressure had been taken at 16.4 lb. per sq. ft.

It was necessary to design the framework so that it could be transported over mountainous country, and the light sections adopted were admirable for facilitating transport and for effecting economy in materials.

Steel tubing was largely employed. The frames were designed so that practically all the material was used to its best advantage. The main tubes were placed to give the maximum moment of inertia to resist the bending moments, and the stiffening members were arranged to take the shear stresses.

To reduce the bending moments in the columns produced by the weight of the roof, ties were introduced. Under this loading, the columns acted largely as columns with hinges at the top. The bases of the columns were anchored to the foundation. With such a large light structure, the moments produced by wind pressure were exceptionally important. The designers had carried the crane girders on brackets so that the frames could be kept to a practically constant section, and consequently, made strong enough to resist the wind pressure and dynamic forces from the crane.

The work was carried out entirely by the firm of Antonio Badoni, of Lecco, who were specialists in this type of structure.

The only precautions taken to protect the structural steel from the effects of the acid fumes consisted of the usual red lead primer with one coat of aluminium finishing paint. The conductors for the transport of electrical energy were made of Aldrey cord, which was an aluminium alloy consisting of :—

98.7% aluminium ;
0.4% magnesium ;
0.6% silicum ;
0.3% iron ;

and possessing the following characteristics :

Specific gravity	2.7
Maximum tensile stress	30 to 35 kg./mm.² (43,000 to 50,000 lb. per sq. in.)
Elastic limit	21 to 25 kg./mm.² (30,000 to 35,000 lb./sq. in)
Modulus of Elasticity	6,300 to 6,500 kg./mm.² (9 millions lb./sq. in.)
Coefficient of lineal expansion	0.000023 per °C. (0.000013 per °F.)
Conductivity in relation to copper (equal section)	52 to 55%
Temperature coefficient	0.0036

The thickness of the hyperbolic shell of the water-cooling towers was 11 cm. (approximately 4½ in.), and it was true that the concrete was protected with gunite against the acid fumes in the atmosphere. The cooling tower ponds were protected with a ¾-in. rendering and two coats of bituminous paint.

In answer to Mr. Hiley's question, the second cooling tower of Larderello II Generating Station was rebuilt by Messrs. Siderocemento, who built the Larderello cooling towers. After examination of the damaged structure, it was decided to rebuild the tower shell on the same pond and foundations after damage had been made good. Tubular scaffolding was used throughout, and the shell was completed in four months.

Regarding Mr. Baliol Scott's remarks, the deepest boring ever carried out at Larderello was 5,300 ft., and the yearly output at Larderello was assessed at 20,000 million tons.

Dr. Mazzoni asked Mr. Gerard to say that a few days before leaving Italy, he was informed that a remarkable discovery was made by a mass spectographic on the gases from Larderello borings :

The two isotopes A^{40} and A^{36} in the rare gas Argon are in a ratio different from that usually found in the air. That is to say :

$$\frac{\frac{A^{40}}{A^{36}}\text{ Larderello gases}}{\frac{A^{40}}{A^{36}}\text{ Air}} = 1.26$$

This might lead to the belief that the origin of the gases at Larderello may be related to the probable deep strata of potash.

THE CHAIRMAN thanked Mr. Gerard again for the able manner in which he had dealt with the questions raised.

He also thanked the Institution of Structural Engineers, on behalf of the British Section of the Société des Ingénieurs Civils de France, for having published the paper in the Institution's Journal.

CORRIGENDA

The following amendments are noted :—

THE STRUCTURAL ENGINEER, April, 1951

Page 98, last paragraph, line 2 to be replaced by the following : " carbon dioxide and, in the following order : methane "

Page 104, line 27. For " Carborundum " read " Boron carbide."

Page 104. The caption under Fig. 8 should read " Larderello II rebuilt."

Institution Notices and Proceedings

ORDINARY GENERAL MEETING

An Ordinary General Meeting of the Institution of Structural Engineers was held at 11, Upper Belgrave Street, London, S.W.1, on Thursday, 28th June, 1951, at 5 p.m., Mr. J. E. Swindlehurst, O.B.E., M.A., M.I.C.E., M.I.Struct.E. (President) in the Chair.

The Minutes of the Ordinary General Meeting held on the 26th April, 1951, as published in the Journal in June, were taken as read, confirmed and signed.

The following members were elected in accordance with the Bye-Laws. Will members kindly note that the elections as tabulated below should be referred to when consulting the Year Book for evidence of membership.

STUDENTS

BARTLEY, Raymond, 111, Leggats Way, Watford, Herts.
BILLINGTON, Roy, 73, Hazel Road, Longview, Huyton, Nr. Liverpool.
BOOTH, Richard, 48, Holbrook Road, Belper, Derbyshire
BOSWELL, Peter, 3, Malvern Road, Leeds Road, Dewsbury, Yorks.
CALVER, Michael John, 50, Dover Road, Ipswich Suffolk.
CAMM, David Henry, c/o Mrs. Kirby, 265, Elland Road Beeston, Leeds, 11.

CLARK, Owen Gladstone, 45, Leo Street, Kenilworth, Johannesburg, South Africa.
CRAGG, Wilfred Shipham, 15, Banksia Road, Rosebank, Cape, South Africa.
DURLEY, John Edward Charles, 141, High Road, Loughton, Essex.
FERGUSSON, James Colin, c/o The Reinforcing Steel Co., Ltd., 94, Commissioner Street, Johannesburg, South Africa.
GIBSON, George, 48, First Avenue, Netherlee, Glasgow, S.4.
GLOYENS, Donald John, 69, Rose Valley, Brentwood, Essex.
HUNTER, James, 9, Laburnum Road, Liverpool, 7.
JOHNSON, William Morris, 16, Grove Park View, Harrogate.
JONES, William, 73, Elkington Road, Burry Port, Carmarthenshire.
JULIAN, Leonard William, 83, Llanover Road, North Wembley, Middlesex.
KEELY, Bryan Thomas Paul, 41, King Edward Drive, Grays, Essex.
LACK, William Brian, 29, Parkside Gardens, East Barnet, Herts.
LANE, Dennis John, 150, Links Road, Tooting Junction, London, S.W.17.
MIDDLETON, James Henry, 25, Derby Street, Moss Side, Manchester, 14.
NEAL, John, 50, Haslucks Croft, Shirley, Warwickshire.
O'CONNOR, Desmond Francis, 7, Wallasey Crescent, Ickenham, Middlesex.
OSORIA, John David Colaco, East Wall, Hedgerley Lane, Gerrards Cross, Bucks.
PEARSON, Matthew Thomas, 128, Albert Road, Jarrow, Co. Durham.
RAWNSLEY, Peter Michael, 75, Smith Avenue, Odsal Estate, Bradford, Yorks.
STINTON, Gordon John, 50, Charles Witts Avenue, Hunderton, Herts.
VAN ZYL, Dennis Christo, 6, Tempest Street, Airfield, Benoni, Transvaal, S. Africa.
WALTON, Jonathan, 22, Stamford Street, Sale, Manchester.
WHITE, Harold Hereward Dewe, 83, 7th Street, Orange Grove, Johannesburg, South Africa.
WILKIN, Brian John David, Sunny Side, Neath Road, Crynant, Nr. Neath, Glam.
WILSON, Russell, 1, Kirklee Quadrant, Glasgow, W.2.
WRIGHT, Frank, 23, Hopping Hill, Milford, Derby.

GRADUATES

ABRAMOWITZ, Alfred Isedore, B.Sc. (Eng.) Rand, 2, Aronby Court, Kenmere Road, Yeoville, Johannesburg, South Africa.
BARTLE, Peter Ronald, 49, Emmanuel Road, S.W.12.
BEAL, Raymond Alexander, 5, Heath Road, Timperley, Cheshire.
BECKWITH, Peter, 38, Ventnor Road, Linthorpe, Middlesbrough.
BIERNACKI, Wladyslaw Otton, 31, Avenue Gardens, W.3.
BOEUF, Frank, B.Sc. (Eng.) London, 37, Seaforth Drive, Waltham Cross, Herts.
BROOKSBANK, Fred, B.A. (Hons.) Cambridge, 86, Blackburn Avenue, Tettenhall, Staffs.
CAMPBELL, Gordon Arnison, M.Sc., Leeds, 6, Norfolk Walk, Chapel Allerton, Leeds, 7.
CAMPBELL, Peter, 1, Chapelfields, Marple, Nr. Stockport, Cheshire.
CARLING, Cecil John, "Mountsfoot," Childsbridge, Kemsing, Kent.
CAROLAN, Edward Joseph, B.E., N.U.I., 137, Tixall Road, Stafford.
CROSIER, Thomas Arnold, B.Sc.(Eng.), London, 9-11, Endsleigh Gardens, London, W.C.1.
DAWSON, John Derek, 30, Crown Street, Farington, Levland, Nr. Preston, Lancs.
DENNIS, Ronald Stanley, 11, Summerfield Gardens, Bramley, Leeds.
DEVARAJAN, Krishnaswamy, B.E., Madras, "Canal Head," Cwmbach Road, Aberdare, Glam.
ELLIS, Ryan, B.Sc. (Civil) Belfast, 30, Broadway, Falls Road, Belfast.
EYRES, Derek, 43, Church Street, Davenham, Nr. Northwich, Cheshire.
FOSTER, Donald, A.R.I.B.A., 7, Cambridge Avenue, Forest Hall, Newcastle-upon-Tyne.
GEDDES, James Douglas, B.Sc.(Civil), Durham, 20, Northway, Sheriff Hill, Gateshead, 9, Co. Durham.
GRAHAME, Russell Godfrey, A.M.I.C.E., 9, Clifford Court, Tanfield Avenue, Neasden, N.W.2.
HILL, Donald Gilbert, 89, Coldcotes Avenue, Harehills Lane, Leeds, 9.
HUBBERSTEY, Peter, Fairbrother, Hall & Hedges, Barclays Bank Chambers, Birley Street, Blackpool.
IYENGAR, Hemmige Vedantha Krishna, B.Sc., B.E. (Civil), Mysore, Messrs. Sir Lindsay Parkinson & Co., Ltd., Templenewsam, Leeds.
JAKEMAN, Leslie, 18, Alleny Road, Cadishead, Manchester.
JAYARAM, Thumula, B.E., Madras, 18, Ramanujam Street, T. Nagar, Madras, S. India.
LEWIS, Lawrence Henry, 22, River Way, Ewell, Surrey.
LONGBOTTOM, Derek Patterson, B.Sc. (Civil) Durham, 10, Lynton Road, Heaton Moor, Stockport, Cheshire.
MALLOWS, Dennis Lingwood, B.Sc. (Eng.) Cape Town, 3, 2nd Avenue, Fish Hoek, Cape Province, South Africa.
MALSHE, Prabhakar Govind, B.E.(Civil), Bombay, 759-46, Deccan Gymkhana Colony, Poona 4, India.
MAYOR, Harry Levi, 310, Mount Road, Levenshulme, Manchester.
RAYBOULD, Derek Leney, 56, Manor Drive, Wembley Park, Middx.
ROSE, Derrick Dudley, B.Eng. (Hons.) Liverpool, 8, Homestead Avenue, Old Roan, Netherton, Liverpool, 10.
SIMMS, Peter, 59, Greenhill Lane, Riddings, Derbyshire.
SMITH, Gordon Vivian, B.Sc. (Eng.) London, Hope Hill View, Esholt Hall, Nr. Shipley, Yorks.
STOKELD, Philip Gawthorpe, 10, Hans Place, London, S.W.1.
TRANFIELD, William, 53, Waverley Avenue, Wembley Hill, Middlesex.
TUCKER, William Duncan Alistair, B.A.(Eng.), Oxford, The Hall, Chancery Lane, Port of Spain, Trinidad, B.W.I.
VISVESVARAYA, Hosagrahar, Chandrasekharaiya, B.E., Mysore, The Concrete Association of India, 1-2, Queens Road, Fort, Bombay 1, India.
WATTIE, Neville, 44, Helen Street, Lower Broughton, Salford, 7, Lancs.
WILLMORE, Peter Gilbert, B.Sc.(Civil), Birmingham, 69, Pye Green Road, Cannock, Staffs.

ASSOCIATE

NIGHTINGALE, James, 81, Clifton Avenue, West Hartlepool.

MEMBERS

EDMONDSON, Harold Leonard, B.Sc.(Tech.), Manchester, A.M.I.Mech.E., 28, Beverley Road, Kettering, Northants.
GANDHI, Chhotalal Hirjivandas, B.Sc.(Eng.), London, Mehta Building, Meadows Street, Fort, Bombay, India.

Transfers

Students to Graduates

Campbell, Bernard Ernest, The Bungalow, " B " Power Station, Stourport-on-Severn, Worcs.

Collinson, Thomas Anthony, 415, Watery Lane, Sutton, St. Helens, Lancs.

Denson, Francis Ronald, 17, Burnham Avenue, Smithills, Bolton, Lancs.

Dilley, Eric Arthur, University Union, King's College, Newcastle-upon-Tyne, 1.

Higgins, Geoffrey Thomas, Flat 2, " St. Owens," Silverdale Road, Burgess Hill, Sussex.

Jory, Peter, B.Sc.(Eng.), Bristol, Public Works Department, Lagos, Nigeria, B.W.A.

Knight, Philip, " The Towers," Churwell, Nr. Leeds, Yorks.

Marsh, Peter John Barnes, Neville Lodge, 41, Woodlands Road, Isleworth, Middx.

Rajapakse, Rajapakse Kankanamalage Chandradasa, Water Works Branch, P.W.D., Colombo, Ceylon.

Topliss, Duncan Geoffrey, Meadow Cottage, Station Road, Rolleston, Burton-on-Trent.

Wright, James Stuart, Kongela, 52, Milton Road, Eastbourne, Sussex.

Graduates to Associate-Members

Allen, Arthur Charles, B.Sc.(Eng.), London, D.L.C. (Hons.), 8, Holdenhurst Avenue, North Finchley, London, N.12.

Armstrong, James Hodgson, B.Sc. (Civil) Glasgow, " Hainings Gate " Moorhouse, Carlisle, Cumberland.

Berry, Gordon William, 55, Masser Road, Holbrooks, Coventry.

Cooper, Stanley Maurice, " Arncliffe," 296, Bradford Road, Castle Bromwich, Nr. Birmingham.

Gould, Harry Bernard, 3, Boniface Gardens, Headstone, Harrow, Middx.

McEwan, Robert, B.Sc. (Civil) Glasgow, " Garfield," Abbotsgrange Road, Grangemouth.

Power, John Osborne, " Barnslea," 411, Langworthy Road, Pendleton, Salford 6, Lancs.

Westerman, Edwin, 8, Elmfield Avenue, Northenden, Manchester.

Associate-Members to Members

Bird, Kenneth Frederick, A.M.I.C.E., c/o Twisteel Reinforcement & Designs (Pty.) Ltd., P.O. Box 1189, Johannesburg.

Blades, William Durham, 7, Broxton Avenue, Prenton, Birkenhead, Cheshire.

Chaubal, Dwarkanth Ramchandra, B.A., B.Sc., B.E. (Civil) Bombay, Department of Civil Engineering, Government L.D. College of Engineering, Ahmedabad, Bombay Province, India.

Gepfert, George Albert, " Whyteleafe," 98, Furzehill Road, Boreham Wood, Herts.

Harris, John Seymour, A.R.I.B.A., A.M.I.C.E., A.M.T.P.I., 4, Greenfield Crescent, Edgbaston, Birmingham, 15.

Kulkarni, Govind Jivaj, B.Sc. (Civil) Bombay, A.M.I.C.E., M.R.San.I., A.M.I.E. (India), 1204/5, Shivajinagar, Poona 5, India.

Maxwell-Cook, John Charles, A.M.I.C.E., Ickworth, Bristol Gate, Brighton.

Mohile, Narayan Hari, B.E.(Civil), Bombay, M.I.E. (India), The Concrete Association of India, 1-2, Queens Road, Fort, Bombay 1, India.

Thomason, Reginald Harold, " Carleton," 48, Woodside Avenue, Esher, Surrey.

Tonge, Thomas Johnson, " Roseleigh," Stoke Orchard, Cheltenham, Glos.

HONOURS AND AWARDS

In offering their sincere congratulations to the following members on the distinctions recently conferred upon them, the Council feel that they are also expressing the good wishes of the Institution :

Order of the British Empire—C.B.E.

Mr. C. H. Aslin (Member).

Mr. G. F. Earle (Member).

Order of the British Empire—O.B.E.

Mr. W. Davies (Member).

REPRESENTATION

The Council have made the following nominations of members to represent the Institution :—

Imperial College of Science and Technology (Concrete Technology Advisory Committee)

Mr. E. Granter (Vice-President) — re-appointment.

Building Research Congress 1951

Mr. E. Granter (Vice-President).

Mr. S. Vaughan (Vice-President).

BUILDING RESEARCH CONGRESS, 1951

A copy of the preprint of the papers to be given in Division I of the Building Research Congress will be available in the Library early this month. A leaflet describing the scope of the above congress was distributed with the structural engineer, October, 1950.

ENGINEERING, MARINE AND WELDING EXHIBITION

The Directors of the Engineering, Marine and Welding Exhibition, which will be held at Olympia from 30th August to 13th September, 1951, have invited all members of the Institution to visit the Exhibition on Wednesday, 5th September, 1951. Tickets of admission, each available for two persons, which may be used for the official visit or on any other single day, may be obtained on application to the Secretary.

SUMMER SCHOOL IN RELAXATION METHODS

A Summer School in Relaxation Methods has been arranged for the three weeks September 3rd-21st, 1951, at the Imperial College of Science and Technology.

The course, which will cover the numerical solution of linear algebraic equations, framework problems, Laplace's and Poisson's equations, the biharmonic equation, eigenvalue problems, the heat-conduction equation, etc., will consist of daily lectures in the mornings, beginning at 10.15 a.m., with numerous examples to be solved under supervision in the afternoons.

All enquiries regarding the course should be addressed to Mr. D. N. de G. Allen, Imperial College, London, S.W.7.

PERSONAL

Dr. A. M. Ballantyne, B.Sc. (Eng.), A.M.I.C.E., (Associate-Member) has been appointed Secretary of the Royal Aeronautical Society.

Mr. F. R. Bullen, M.I.C.E. (Member of Council) has been joined in partnership by Mr. F. E. Fowle, A.M.I.C.E. (Member), and Mr. F. G. Etches, B.Sc., A.M.I.C.E. (Member), and the practice is now known as F. R. Bullen and Partners.

Dr. A. W. Hendry, B.Sc., A.M.I.C.E., (Associate-Member), has been appointed Professor of Civil Engineering at the Gordon Memorial College, Khartoum, Sudan.

Mr. L. E. Kent, B.Sc.,(Eng.), M.I.C.E., M.I.Struct.E. has been appointed the Institution's representative upon the Committee formed by Mr. Hugh Dalton.

Minister of Local Government and Planning, to revise model building By-Laws.

DRURY MEDAL AWARD

The third competition for the above award will take place in 1951, and the subject will be the design of a reinforced concrete garage with provision for erection above it of future superstructure, the garage being subjected to unbalanced earth pressure.

Graduates and Students of the Institution who wish to compete are invited to apply for full details to the Secretary; envelopes to be marked in the top left-hand corner "Drury Medal Award."

The closing date for the competition is October 1st, 1951.

The general conditions of the competition are as follows :—

1. The competition shall be for Graduates and Students of the Institution of not more than 25 years of age.

2. The subject of the competition shall be a design of structural character, that is to say, primarily structural design, not planning.

3. The subject of design and conditions shall be prepared and issued biennially by a group of five members appointed by the Council.

4. The Literature Committee shall appoint a jury of not less than five to examine the works submitted and to interview candidates, if found necessary.

5. In order to show that the work submitted is solely the work of the competitor, the documents submitted shall be countersigned by a corporate member of the Institution, or, failing this, shall be accompanied by a declaration on a prescribed form signed by the candidate in the presence of a Justice of the Peace or a Commissioner for Oaths.

RESEARCH AWARDS

The Council have instituted a Research Prize Fund, from which awards may be made annually to the author or joint authors of papers describing original research which they have carried out. Research awards may be made for papers read at Headquarters or in the Branches and published in the Journal, or for papers published in the Journal only without being read at an open meeting.

The assessment for such awards will be made annually, but awards will be made only to the contributors of such papers as reach a standard judged by the Literature Committee to be satisfactory.

Work submitted under this scheme must be original and may include any of the following :—

(a) investigations of an experimental or analytical character ;

(b) studies of historical or statistical records ;

(c) improvements in principles or methods of construction ;

(d) research into methods of structural engineering and building, the nature and use of plant and the organisation of engineering work ;

(e) any related or combined studies which are deemed by the Literature Committee to be of a research character.

In cases where the research work described in the paper was not the work of one individual, the names of all the collaborators should be given in the paper.

Awards may take any or all of the following forms :— a research medal ; a diploma ; a money prize.

Application for consideration for a research award must be made to the Secretary of the Institution, and in preparing papers for reproduction in the Journal, authors must comply with the conditions laid down for all such contributions. Particulars of these conditions may be obtained from the Secretary.

In judging research papers, the following factors will be considered :—

(a) the nature of the subject and its conclusions ;

(b) the value of the paper in advancing the science and art of structural engineering ;

(c) the standard of preparation and orderly arrangement of the subject matter.

Research papers will also be eligible for adjudication for the Institution Medals if they comply with the Regulations governing those awards.

The closing date for the receipt of applications in respect of papers published in the Journal between October, 1950, and September, 1951, is October 31st, 1951.

LONDON GRADUATES' AND STUDENTS' SECTION

A visit has been arranged to Bankside Power Station on the 25th August, to view the Station under construction — mainly steelwork. The party is to meet at 9.45 a.m. outside Southwark Cathedral. All members of the Section are encouraged to attend, as the visit should prove very instructive.

Hon. Secretary : D. B. Rogers, 4, Portland Rise, Finsbury Park, N.4.

BRANCH NOTICES

Lancashire and Cheshire Branch

The Branch Committee has been constituted as follows for the Session 1951-52 :—

Chairman : R. Gray (Member).

Vice-Chairman : W. Bates (Member).

Immediate Past-Chairman : A. V. Booth, B.Sc. (Member).

Hon. Auditors : F. Walkden (Associate-Member), J. E. Guest (Associate-Member).

Hon. Secretary : A. S. Sinclair (Associate-Member), 28 Kenwood Road, Stretford, Lancs.

Hon. Assistant Secretary : M. D. Woods (Graduate).

Committee : F. C. Brookhouse, A.M.I.C.E. (Past Chairman), P. Mather (Past Chairman), A. L. Grimshaw, F.R.I.C.S. (Member), F. J. Bown (Member), Professor J. A. L. Matheson, M.B.E., Ph.D., M.Sc., M.I.C.E. (Member), J. H. Morris (Member), M. K. Bryce (Associate-Member), G. A. Davis (Associate-Member), G. Greenlees (Associate-Member), J. Robinson (Associate-Member), A. E. Wright (Associate-Member).

Midland Counties Branch

Hon. Secretary : E. R. Deeley, A.M.I.Struct.E., Arranmoor, Adshead Road, Dudley, Worcs.

Graduates' and Students' Section

Hon. Secretary : M. H. Evans, B.Sc., 42, Church Hill Road, Handsworth, Birmingham, 20.

Northern Counties Branch

Hon. Secretary : Ian MacGregor, A.M.I.Struct.E., 9, Ellison Place, Newcastle-upon-Tyne, 1.

Northern Ireland Branch

Hon. Secretary : S. G. Duckworth, M.I.Struct.E., 13, Finaghy Road North, Belfast, Northern Ireland.

Scottish Branch

Hon. Secretary : D. G. Drummond, B.Sc., A.M.I.C.E., A.M.I.Struct.E., 11, Woodside Terrace, Glasgow, C.3.

South-Western Counties Branch

Hon. Secretary : E. W. Howells, M.I.Struct.E., 10-12, Market Street, Torquay, Devon.

WALES AND MONMOUTHSHIRE BRANCH

Hon. Secretary : E. R. Steward, A.M.I.Struct.E., Edrom, Ashleigh Road, Blackpill, Swansea.

WESTERN COUNTIES BRANCH

Hon. Secretary : C. E. Saunders, M.I.Struct.E., " Dunkery," Edward Road, Walton St. Mary, Clevedon, Somerset.

YORKSHIRE BRANCH

The Branch Committee has been constituted as follows for the Session 1951-52 :—

Chairman : A. Robb, B.Sc., A.M.I.C.E. (Member).

Senior Vice-Chairman : D. R. S. Wilson (Member).

Junior Vice-Chairman : J. Dossor, M.I.C.E., M.I.Mun.E., A.M.I.Mech.E. (Member).

Immediate-Past-Chairman : H. C. Husband, B.Eng., M.I.C.E., M.I.Mech.E., (Member).

Hon. Secretary : E. Wrigley (Associate-Member), City Engineer's Department, Civic Hall, Leeds, 1.

Committee : T. Barlow (Associate-Member), S. Champion, M.Sc., Ph.D., A.M.I.C.E. (Associate-Member), Professor R. H. Evans, D.Sc., Ph.D., M.I.C.E., M.I.Mech.E. (Member), G. McLean Gibson, O.B.E., M.I.Mech.E., A.M.I.C.E. (Member), A. J. Hodgkinson, A.I.E.E., A.M.I.Mech.E. (Associate-Member), Professor J. Husband, M.I.C.E. (Past President), J. Jenkins, M.Sc., M.I.C.E. (Member), Captain J. B. Lawson, M.B.E., A.R.I.B.A. (Member), S. Mackey, B.Sc., A.M.I.C.E.I. (Associate-Member), F. A. Maddock, B.Sc. (Member), L. Preston (Member), W. Hunter Rose, M.I.C.E.I., F.G.S. (Member), F. R. Rudkin (Member), L. C. Walker (Associate-Member), Captain F. A. Whitaker (Member), C. Wilkinson (Member).

UNION OF SOUTH AFRICA BRANCH

A meeting of the Branch was held in Johannesburg on Monday, April 9th, 1951, at 8 p.m., when Mr. Charles Rigby (Member) gave a paper entitled " European Visit." The paper, which was illustrated by films in colour, described the various methods of building construction which the author had studied on his visit to the Continent under the auspices of the National Building Research Institute.

On Monday, May 14th, 1951, a paper on " The Moment Distribution Method " was given by Mr. Charles Rigby. The paper was specially arranged for Students and Graduates, and a record number of 51 Members and visitors were present.

Branch Hon. Secretary : A. E. Tait, B.Sc., A.M.I.C.E., A.M.I.Struct.E., P.O. Box No. 3306, Johannesburg. During week-days Mr. Tait can be contacted in the City Engineer's Department, City Hall, Johannesburg, 'phone 34–1111 ext. 257.

Natal Hon. Secretary : E. G. Bennett, A.M.I.Struct.E., c/o Reinforcing Steel Co. Ltd., P.O. Box 478, Durban.

Cape Section Hon. Secretary : R. Stubbs, M.I.Struct.E., P.O. Box 1692, Cape Town.

OBITUARY

The Council regret to announce the deaths of Percival Martin COOPER, C.M.G., O.B.E., Ernest James THOMAS and Henry John Ernest WEBBE (Members), William Dudley BALE and William Cecil DUNTON (Associate-Members) and Frederick William CHARITY and Harold TOMLINSON (Associates).

RESIGNATION

Notification was given that the Council had accepted with regret the resignation of Lionel Cuthbert HALL (Retired Member).

JANUARY EXAMINATIONS

The Examinations of the Institution will next be held at centres in the United Kingdom and overseas on January 8th and 9th, 1952 (Graduateship), and on January 10th and 11th, 1952 (Associate-Membership).

Book Reviews

Building Construction and Drawing : Part I, Elementary Course, by G. A. Mitchell and A. M. Mitchell. (London, B. T. Batsford, 1950.) 648 plus viii pp., $7\frac{1}{2}$ in. × 5 in. 10s. 6d.

The improvements and additions to this twentieth edition of " Mitchell's Building Construction," have been so extensive that the entire work has been re-set and completely re-illustrated. Several new chapters have been added, including those on " Sequence of Trades in which a Building is Constructed," " Theory of Structures," and " Plumbing, Sanitary Fittings and Drainage." This new edition will be useful as a text-book for students and also as a handy general reference book for builders, architects and engineers.

Elementary Theory of Structures, by James C. Grassie. (London, Longmans. 1950.) 392 pp., $8\frac{1}{2}$ × $5\frac{1}{2}$ in. 25s.

There are many textbooks on the subject of Theory of Structures, but the author of this work has made a useful addition to the list.

The book is divided into nineteen chapters dealing with geometrical properties of sections ; forces ; graphical and analytical treatment of simple frames with axial forces ; shearing force, bending moment, and thrust due to lateral and combined loading on beams ; composite beam and frame structures ; travelling loads on beams : influence lines for simply supported beams, three-hinged arches, and framed structures ; strain energy due to direct forces, shear and bending ; deflection of beams and frames ; Williot displacement diagrams ; stability of struts under axial and lateral loading ; space frames and tension coefficients.

In all these matters the book deals with theoretical principles and with mathematical or graphical solutions and is not concerned with practical design, consequently such matters as masonry structures and reinforced concrete are omitted, whilst 90 pages are devoted to influence lines and 20 to space frames.

A comparison between these contents and the syllabuses of the Institution examinations will show that some of the needs of prospective candidates are met.

Owing to the large number of typical worked solutions together with problems to be worked by the reader, for which answers are given, the book will be specially useful in private study.

As is inevitable in every elementary treatise on theory of structures there is an overlap with strength of materials in the matter of beam and column theory, but these subjects are very well and concisely handled.

The book is illustrated with 275 neat and clear diagrams. A. A. F.

The Influence of Restraints on the Stability of Beams*

By A. R. Flint, B.Sc., Ph.D.

Introduction

The development of satisfactory design rules for beams of slender proportions, which are prone to lateral instability, has taken place gradually since the introduction of the first rolled sections, over a century ago. Until the presentation of rational data in the recently revised B.S. Code of Practice for the "Structural Use of Steel in Buildings", design formulæ have been of an empirical nature. These were based on the conception that lateral buckling was entirely due to collapse of the compression flange of a beam, as a strut under variable end load. In consequence the formulæ gave estimates of load carrying capacity which were often considerably in error, as a result of neglecting the torsional stiffness of the member.[1] The new design rules are based on the mathematical theory of stability[2] and will give reasonable estimates of the limiting stresses in beams whose slenderness ratios exceed a certain minimum value, in the same way that the Euler formula proves satisfactory in calculating the critical stresses in slender struts.

The majority of beams, however, are subject to considerable restraints under working conditions, either inherent in the construction of a framework or deliberately supplied as a precaution against buckling. These restraints have the effect of increasing the stability of the members, and in many cases it will be found that beams will fail due to yielding under plane bending stresses before collapsing laterally. Thus provided that the degree of restraint required to achieve stability may be determined, it will frequently be possible to design a slender beam on the basis of its material yield stress alone.

It is evident that increases in efficiency of the design of beams and gantries can be effected if designers allow for the influence of restraints, and realise how bracing members of low weight can afford considerable economies in material by raising the allowable stresses. With the introduction as structural materials of high strength steels, and also of aluminium alloys having low elastic moduli, there is need for a better understanding of the methods of preventing instability. This paper is intended to indicate the importance of restraints, and to illustrate the most effective methods of attachment of connecting members to deep beams.

The Stability of Unrestrained Beams

The maximum permissible compressive stress in structural beams under bending loads specified in the B.S. Code of Practice No. 113 is the lesser of either :—

$f_{bc} = 10$ tons/in.^{2} ; or

$$f_{bc} = \frac{920 r_1 k_1}{L} \text{ tons/in.}^2,$$

where L = the length between effective lateral restraints,

r_1 = the least radius of gyration of the beam section,

r_2 = the greatest radius of gyration of the beam section,

k_1 = a factor depending upon the ratio r_2/r_1 (approximately unity, except for small values of the ratio).

The second of these two values of f_{bc} is based upon the critical loading of a member, under uniform bending moment, with ends freely supported in bending, but rigidly prevented from twisting about its longitudinal axis at the supports[2]. In the case of a beam with no imperfections in shape and no eccentricities of loading, this represents one half of the maximum fibre stress occurring on the span immediately prior to buckling.

When the critical stress is exceeded, the unrestrained member becomes unstable, and if displaced by some slight disturbing force will buckle sideways and roll over, the large lateral displacement causing yielding in the flanges and consequent collapse.

Theoretical solutions to the problems of lateral instability of freely supported beams are available for a number of simple cases[3],[4] and it is found that, in general, the critical stress for a slender beam of uniform cross-section may be represented by the formula :—

$$f_{bc} = \frac{c\pi}{Z_2 L} \sqrt{\frac{EI_1 GJ}{\gamma}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

in which E, G are the moduli of elasticity and rigidity of the material,

I_1, I_2 are the minimum and maximum moments of inertia of the beam section respectively,

J is the torsion constant,

c is a numerical constant depending on the type of loading,

Z_2 is the modulus of section in the plane of loading,

$$\gamma = \frac{I_2 - I_1}{I_2}.$$

The values of the critical stress factor, c, for beams under two symmetrical point loads applied on the shear centre axis are shown in Fig. 1 for the whole range of load positions. It is seen that for loads near

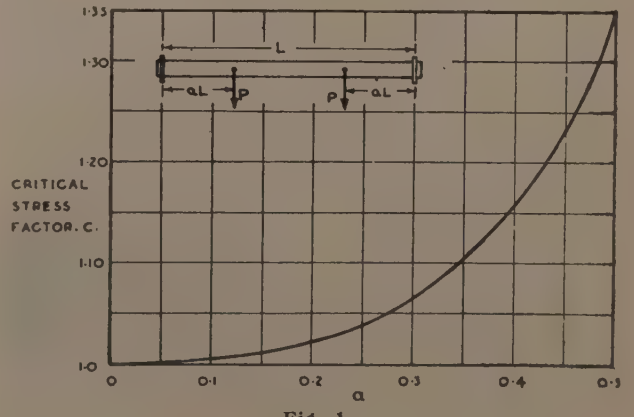

Fig. 1

*Paper read before the Western Counties Branch of the Institution of Structural Engineers at Bristol, on April 1st, 1949.

the ends of the span the critical stress tends towards that for a beam under uniform bending moment, in which $c = 1$, whereas the factor increases to a maximum of 1.35 for load at mid-span.

Further modifications to equation (1) are necessary if point loads are applied above or below the shear centre of the section, and when other load distributions are to be considered[4]. Similarly, the design of members subject to eccentricity of loading in the lateral direction, and those having initial imperfections, entails the application of theoretical analysis which gives limiting stresses lower than those estimated from equation (1)[5].

The Influence of End Restraints

Existing design rules for slender beams are based on theoretical results obtained for members with ideal conditions of support at their ends. The majority of end connections will, however, afford restraint to bending actions, as a result of which the buckling loads and allowable stresses will be modified. By basing the design of beams on the simple theories of bending, with an allowable stress governed by the yield stress of the material, it is possible to effect considerable economies in material if end fixity is considered. This advantage may be lost if the beams are slender and the design has to be based on the buckling loads under free ended conditions. It is consequently of value

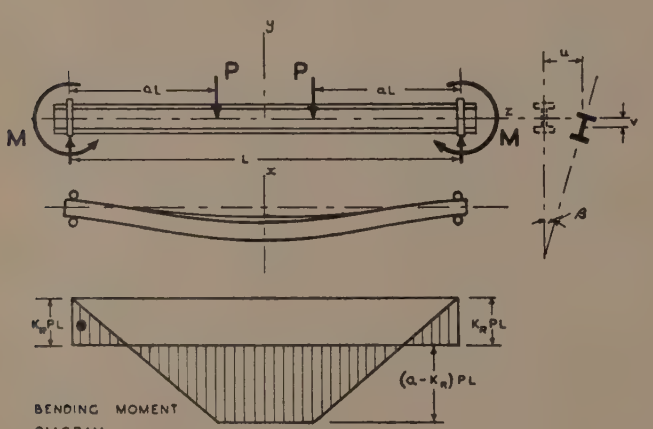

Fig. 2

to know the influence of end fixity on the stability of such members, and to have an indication of the safe working stresses.

As an example, let us consider the case of a beam of uniform cross section under the action of two equal loads, P, applied symmetrically at a distance aL from each support (Fig. 2). The loads are assumed to act on the shear centre axis, and for the purpose of theoretical analysis, the restraints afforded by the supports are considered to be elastic, having stiffnesses dependent on the rigidity of adjoining members. The ends of the beam are subject to equal restraining couples, M, in the plane of the web, which are proportional to the relative rigidities of the beam and its supports, and to the magnitude and position of the applied loads.

Defining the end restraint stiffness in terms of the moment necessary to cause unit rotation of the supports :—

$$R_2 = M/\Phi_2,$$

(where Φ_2 is the slope, $\frac{dv}{dz}$, of the beam at the supports), the corresponding end couples may be shown to be :—

$$M = \frac{PL\,a\,(1-a)}{\left(1 + \frac{2EI_2}{R_2 L}\right)} = K_R\,PL \quad \ldots \quad \ldots \quad (2)$$

When the loads reach their critical value, the beam will buckle sideways and twist if subject to any slight disturbing force. Provided that it is assumed that the end couples remain constant for small lateral deflections of the beam, the magnitude of the buckling load may be determined from theoretical analysis. The critical values of the loads, P, have been determined from the results of a strain energy solution* and may be expressed by the formula :—

$$P_{crit} = \frac{\pi \sqrt{EI_1 GJ}}{L^2 \sqrt{(2F_1 - 4F_2 K_R + K_R^2)}} \quad \ldots \quad (3)$$

in which $F_1 = \frac{1}{4\pi^3}\sin 2\pi a - \frac{a}{2\pi^2}\cos 2\pi a + \frac{a^2}{2} - \frac{(1-4a)}{3}$,

$$F_2 = -\frac{a}{2}(1-a) + \frac{1}{4\pi^2}(1-\cos 2\pi a).$$

In the simple case of a beam under point load at mid-span, with ends completely fixed in the plane of the web, ($K_R = \frac{1}{4}$), the critical load becomes :—

$$P_{crit} = \frac{43.5}{L^2}\sqrt{EI_1 GJ} = 2.53\left[P_{crit}\right]_{R_2 = 0}$$

The corresponding maximum bending stress, occurring at the load point, is :—

$$f_{crit} = \frac{P_{crit} L}{8Z_2} = \frac{5.45}{Z_2 L}\sqrt{EI_1 GJ}$$

$$= 1.29\left[f_{crit}\right]_{R_2 = 0}$$

It is, therefore, evident that a slender beam under the above loading conditions will carry a considerably greater load before buckling when the end connections afford a reasonable degree of fixity than if it were freely supported, and that the corresponding critical stress is also increased.

In the more general case shown in Fig. 2, the equation defining the critical stress may be derived from equation 3 in the form :—

$$f_{crit} = \frac{c\pi}{Z_2 L}\sqrt{EI_1 GJ}$$

in which the critical stress factor, c, depends on the values of a and R_2.

The variation of the critical stress with the stiffness of the end restraints is shown in Fig. 3 for three loading positions, a, as a relationship between the constant, c, and the non-dimensional restraint stiffness factor, $\frac{R_2 L}{EI_2}$.

The results apply to any slender beam, provided that the critical stress does not exceed the elastic limit of the material. The corresponding increase in load-carrying capacity due to end restraint is illustrated in Fig. 4. The end restraint stiffness is related to the ratio, n, between the buckling load of a restrained beam and that of a similar freely-supported member, and it is evident that considerably greater loads may be supported prior to collapse when the ends are partially fixed. For example, a beam under point load at mid-span has a buckling load of 2.5 times the "free" ended

*See Appendix 2.

critical load, if its ends are completely *encastre* in direction, and with point loads at the quarter points the buckling load may be increased to nearly 4 times the fundamental critical load.

The critical stress is always a minimum for freely supported conditions, and if this minimum value exceeds the yield stress it is unnecessary to consider instability in more detailed design. Hence, in deciding on a suitable section for a fixed-ended beam, it is necessary either to choose a member whose critical stress under free end conditions and similar loading is equal to or greater

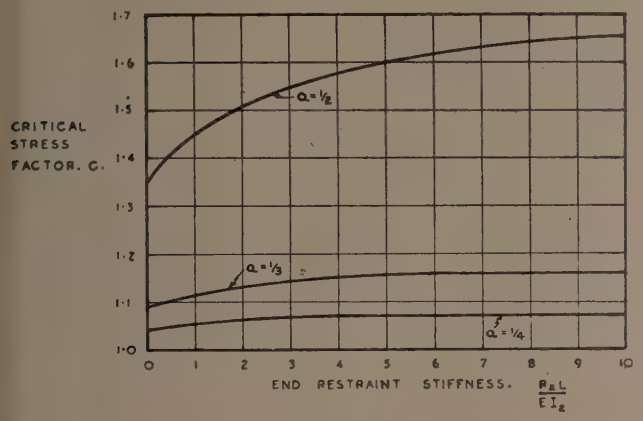

Fig. 3

than the yield stress, or to reduce the design stress for the fixed-ended beam to correspond to the buckling stress obtained for the free-ended case. This method will provide a safe design criterion for beams with any degree of end restraint.

End Restraint in Azimuth

End fixity in the plane of the web is generally accompanied by an additional restraint at the supports in the lateral plane. When a beam deflects laterally, restoring couples are induced at the ends which are proportional to the torsional stiffnesses of adjacent stanchions, and to the bending stiffnesses of any connecting beams. Consequently, the applied bending moment necessary to cause buckling is further increased.

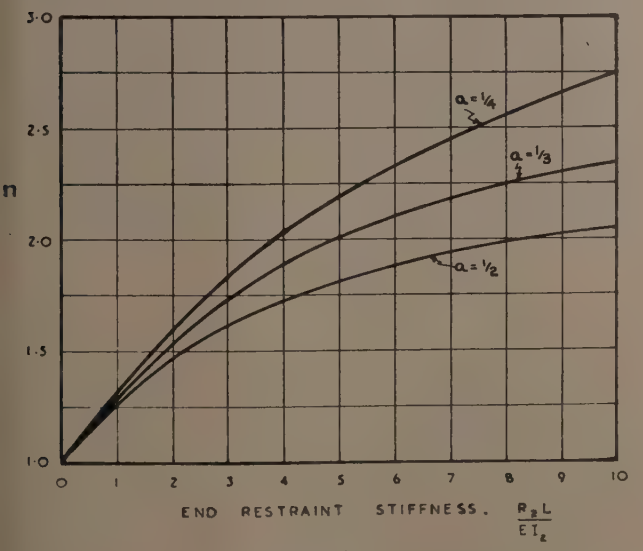

Fig. 4

As an example of the effect of this restraint, let us consider the simple case of a slender beam under uniform bending moment, and subject to elastic end restraint in azimuth only, as shown in Fig. 5. The end restraint stiffness will again be defined as :—

$$R_1 = m/\Phi_1,$$

where m is the couple, and Φ_1 the slope, at the supports in the plane xz. The beam is subject to no end couples in this plane until buckling takes place.

The theoretical analysis of this problem yields a relationship between the values of the critical stress factor, c, and the restraint stiffness given by* :—

$$\frac{R_1 L}{EI_1} = \frac{\pi c \sin \pi c}{1 - \cos \pi c} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

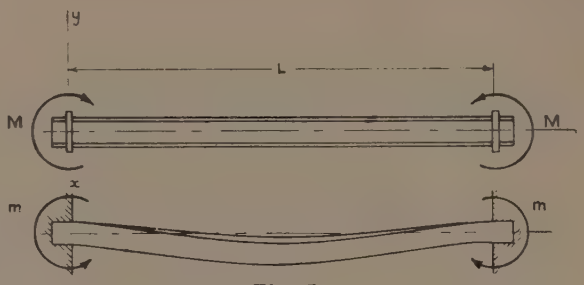

Fig. 5

from which the lower curve in Fig. 6 has been drawn. It is again apparent that appreciable increases in the buckling stress may be expected under working conditions in a majority of beams. In the extreme case of a member with ends completely fixed in the lateral plane, the buckling stress is found to be twice that for a free ended beam ($c = 2$). In practice, however, the restraint stiffness factor would seldom exceed 2, and it would consequently be inadvisable to assume complete end fixity in estimating design stresses.

The upper curve in Fig. 6 shows the influence of this type of end restraint on the critical stresses for beams under point load at mid-span. It is seen that the load-carrying capacity of a member is again considerably increased by quite a small degree of end fixity, and

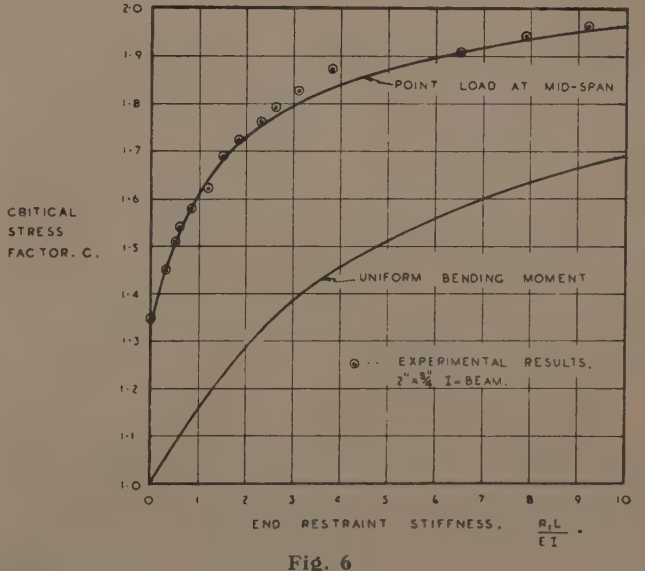

Fig. 6

when the ends are completely fixed in azimuth it is found that the critical stress factor becomes $c = 2.06$, giving an increase of 53 per cent. in buckling load above the free-ended value.

*See Appendix 3.

The experimental results shown in Fig. 6 were obtained from tests on a 2 in. × ¾ in. aluminium alloy I-beam, carried out in the Engineering Laboratories of the University of Bristol. The beam was supported in end fittings designed to provide "ideal" conditions of support by allowing free bending actions and rigidly preventing rotation at the ends. Load was applied to the shear centre at mid-span by means of iron weights hung on a load pillar. This pillar was suspended from a pulley, running on a loop of wire cable which passed over a large disc clamped to the test beam. By means of this system, the line of action of the loading remained vertical during buckling, and passed through the centre of the disc. The effective point of application

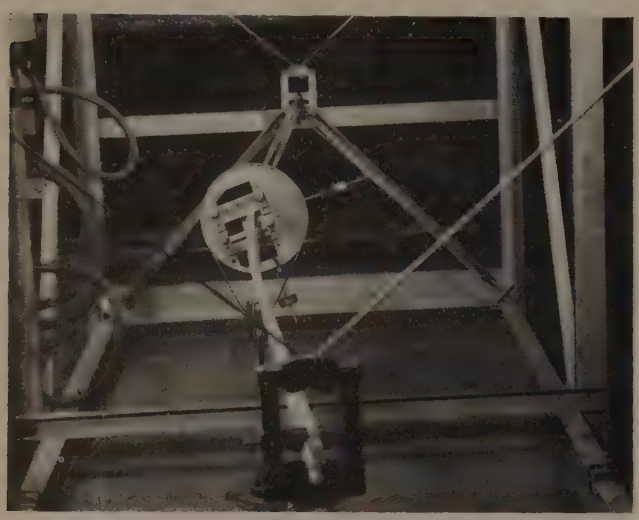

Fig. 7.—2 in. × ¾ in. I-beam buckling in fundamental mode

of a point load therefore coincided with the centre of the disc, and it was possible to load the beam at any point on its cross section. Fig. 7 shows the test beam buckling with free end conditions.

Having determined the fundamental buckling load, the beam was restrained at its ends in azimuth by clamping light cantilevers to the web at the supports, with their free ends stayed to the test frame. The stiffness of the restraint was varied by adjustment of the length of the cantilevers, and the collapse loads were recorded for a number of restraint stiffnesses. It is seen from Fig. 6 that good agreement was obtained with theoretical results, the error nowhere exceeding 2 per cent.

For the purpose of design, the curves in Fig. 6 may be applied directly to indicate the increase in safe working stress, or an effective span may be stipulated for members with partially-fixed ends, where the effective span, $L_e = \frac{L}{c}$

The Torsional Stiffness of Supports

The assumption has been made in all previous analysis that no rotation of the supports of a beam occurs during buckling. If these supports have only a limited elastic stiffness under torsion, however, the actual buckling load of the member will be lower than that estimated from idealised results. This rigidity will depend on the bending stiffnesses of adjoining stanchions, and, if these are under loads approaching the Euler value, the effective stiffness may be low.

It can be shown that for beams under single point loads, or under two symmetrical point loads, the critical load may be approximately related to the support stiffness under torsion by the formula* :—

$$n = 1 - \frac{4}{3R_T} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

in which n is the ratio between buckling loads for beams with finite and with infinite torsional support stiffnesses respectively.

$R_T = \frac{\theta GJ}{TL}$, where θ is the rotation of a support under a torque, T, and G, J and L refer to the beam.

The torsional stiffness of a member is usually considerably less than that of its supports, and the value of R_T will seldom exceed 0.02. It is consequently unlikely that a reduction in load-carrying capacity of more than 3 per cent. would occur in practice due to this behaviour. This reduction will usually be balanced by the increased stability afforded by the restraint of warping which occurs in most structural connections. It should be remarked, however, that the failure of many previous tests on beams to obtain results comparable with theory has been partially due to the fact that members have been supported on their lower flanges, with the upper flange free, which limited the effective torsional stiffness at the ends.

Model Portal Frames

As an illustration of the influence of end restraints, two model "H" portal frames were constructed, using a small brass I-section, and were loaded at mid-span, as shown in Fig. 8. The stanchions were of the same section as the beams, had the same length, and in one frame were connected to the beam with their webs in the plane of the frame, and in the other with their

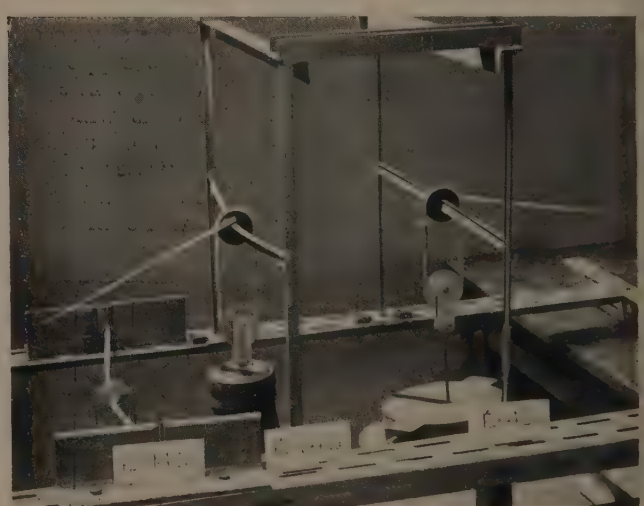

Fig. 8.—Buckled frame members

webs perpendicular to the beam. In this way two extremes of end restraint were obtained.

The critical loads were compared with that of a similar freely-supported beam of the same span, shown to the left of Fig. 8, and it was evident that the beams in the frames carried greatly increased loads prior to collapse. The experimental critical load ratios for the beam members were found to be 1 : 1.7 : 2.8, whereas the estimated ratios, allowing for bending restraints alone, were estimated to be 1 : 1.5 : 2.4. It was found that complete collapse of the portal

*See Appendix 4.

members did not occur, even if loaded considerably beyond the critical load, due to the influence of axial restraint[6], although the beams became bowed and twisted. Thus, in the stiffer frame it was difficult to determine the true critical load, and it is probable that the buckling load was less than that applied in Fig. 8.

Intermediate Restraints on the Span

Interconnection between loaded beams by means of bracing members is commonly employed in order to prevent failure due to lateral instability. The method of attachment of such stiffeners, and the rigidity of the member to which a beam is stayed, influence the buckling load of the system, and an indication of the most effective methods of connection may be obtained from consideration of certain simple cases.

Let us first take the case of a slender beam which is elastically restrained at mid-span against lateral bowing (Fig. 9), and is freely supported at its ends. When the beam buckles, a restoring force W is afforded by the restraint which is proportional to the lateral displacement at the point of connection.

i.e. $W = K_B(\delta + d.\beta_c)$,

where δ is the lateral deflection of the shear centre and β_c the rotation at mid-span respectively,

d is the distance of the point of connection above the shear centre,

K_B is the restraint stiffness (tons/in.).

The solution to the buckling loads may be obtained by means of a strain energy method, in terms of the restraint stiffness and the constants of the beam. If a simple cosine twisted form is assumed for the

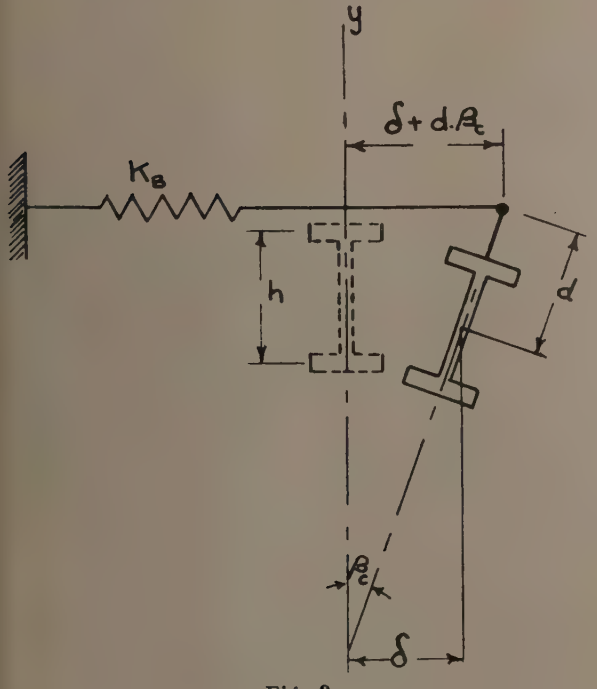

Fig. 9

buckled beam, the critical stress for a member stayed at the shear centre ($d = 0$) is given by*:—

$$c = \alpha\sqrt{(1+\lambda)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

where c is the critical stress factor,

and $\lambda = \dfrac{K_B L^3}{48EI_1}$ (the ratio between the restraint stiffness and the lateral bending stiffness of the beam).

α = 1 for beams under uniform bending moment,

α = 1.35 for beams under point load at mid-span.

This approximate solution may be appreciably in error when the applied loads approach the second critical values, and it is necessary to assume a twisted form containing at least two trigonometric terms in order to obtain a reasonable degree of accuracy. The results of this analysis may not be reduced to a simple formula, and are best presented graphically.

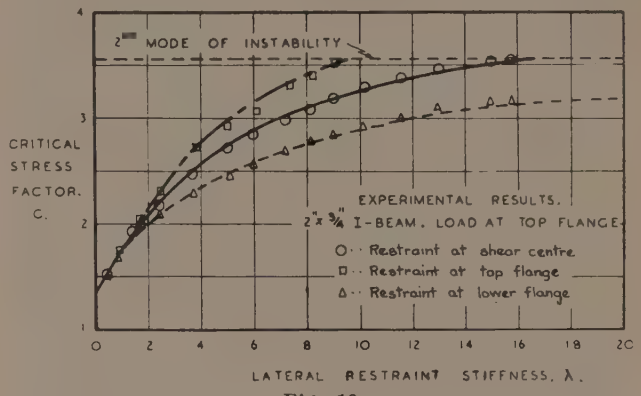
2nd MODE OF INSTABILITY
CRITICAL STRESS FACTOR. C.
EXPERIMENTAL RESULTS.
2"x ¾" I-BEAM. LOAD AT TOP FLANGE
O·· Restraint at shear centre
□·· Restraint at top flange
Δ·· Restraint at lower flange
LATERAL RESTRAINT STIFFNESS, λ.

Fig. 10

As an example, the influence of this type of elastic restraint on a slender 2 in. × ¾ in. I-beam has been studied both theoretically and experimentally. The variation of critical stress with the restraint stiffness is shown in Fig. 10 for load applied at the top flange at mid-span, with three positions of restraint on the loaded section. As would be expected, the optimum effect of staying the beam occurs when it is restrained at the top flange, due to the additional torsional restraint afforded. The early conception of lateral buckling assumed that the compression flange failed as a strut and that consequently, any restraint must be supplied to that flange in order to be effective. From these above results, however, it is obvious that by staying the web or tension flange the load carrying capacity may be greatly increased. In order to force the member to buckle in the second mode of instability, it is generally necessary to attach the stay above the shear centre of the section, and a restraint stiffness of the order of from 10 to 15 times the lateral bending stiffness of the beam is required. By staying the beam at a number of points along its span, the critical load may be increased to any desired value, the most effective points of application being at the points of maximum lateral displacement.

A comparison between the simplified results of equation (6) and the accurate solution shows that the former is in error on the unsafe side, for high values of critical load, but that a reasonable estimate of the effect of restraint may be derived from the formula for stiffnesses up to $\lambda = 1.5$.

The 2 in. × ¾ in. I-beam was tested under point load at mid-span with lateral elastic restraint at the loaded section, in order to verify the above conclusions. Fig. 11 illustrates the beam buckling under load, with restraint at the shear centre ($\lambda = 3$). The beam was stayed to a simple cantilever clamped to the test frame, and was supported with free ends. The critical loads were determined for three positions of restraint, and the results of these tests are shown in Fig. 10. The agreement between the theoretical curves and experimental points was satisfactory.

*See Appendix 5.

On reaching an applied load equal to the second critical load, the beam buckled in the mode shown in Fig. 12, beyond which any increase in the restraint stiffness was ineffective in raising the buckling load.

The Effect of Torsional Restraint

When two or more adjacent beams are loaded simultaneously, it is possible that they may buckle together in such a way that the lateral bending restraint, afforded

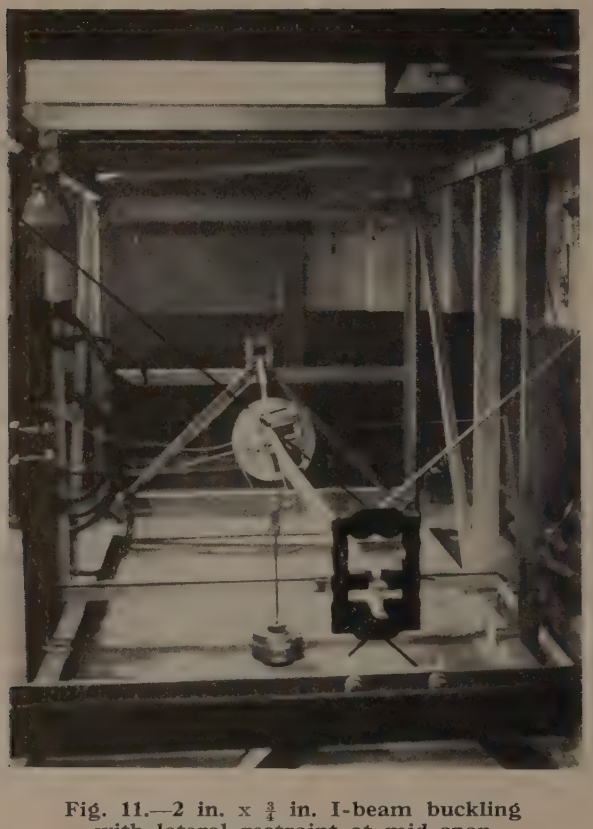

Fig. 11.—2 in. x $\frac{3}{4}$ in. I-beam buckling with lateral restraint at mid-span

by their interconnection, is zero. The presence of a connecting member will often afford torsional restraint, however, which may serve equally well as a means of stabilising the system.

Let us consider the simple case of two adjacent beams, both loaded under equal uniform bending moment, and connected by a filler joist rigidly attached to their webs at mid-span (Fig. 13). When the beams start to buckle, the secondary member affords restoring couples proportional to the rotation of the main beams, and the critical bending stress is consequently increased. By increasing the flexural stiffness of the connecting joist, it is possible to raise the buckling stress for the beams to the order of that for failure in the second mode of instability.

We may derive the expression for critical stress from the theory of stability, and for this type of loading the critical stress factor, c, may be related to the stiffness of the torsional restraint by the formula* :—

$$K_T = -2\pi c \cot \frac{\pi c}{2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

where the restraint stiffness is such that the restoring torque afforded at mid-span is given by $T = \frac{GJ}{L} K_T \beta_c$, in which K_T is a non-dimensional factor, representing the ratio between the torsional stiffnesses of the restrain and of the beams, and β_c is the angle of rotation a mid-span.

In the case where a connecting member is rigidl attached to the two beams, as shown in Fig. 13, i will have a point of inflexion in its bent form, and th above stiffness factor will be given by :—

$$K_T = \frac{6\,EI_cL}{GJ\,L_c},$$

where I_c and L_c refer to the secondary member.

The relationship between the same parameters ma be derived for members under point load at mid-span in terms of infinite series, which may be most con veniently represented graphically.

The variation of critical stress with restraint stiffnes is shown in Fig. 14 for members under uniform bendin and under point loads on the shear centre at mid-span these curves being applicable to any slender beam.

It is again evident that a considerable increase i the working stress may be obtained by the provisio of adequate connections of this type. Provided tha the junctions between the filler joists and main beam are very rigid, members may be made to suppor loads approaching the critical loads for buckling i the second mode. Fig. 15 shows two similar model beams with simple supports and a single bracing membe attached at mid-span, buckling under point load equal to 2.6 times the critical load for an unrestraine member. The buckled mode is a combination of single half wave and a first harmonic, with a node poin at mid-span, showing the members to be supportin loads almost as great as the second critical load.

In practice, the value of K_T will generally exceed 1,000 which should ensure the maximum increase in critica stress. By attaching a number of filler joists alon the span, it may usually be assured that lateral in stability is suppressed.

This form of restraint will frequently be found th most efficient in cases where there is no convenien means of providing lateral restraint, as in the case o

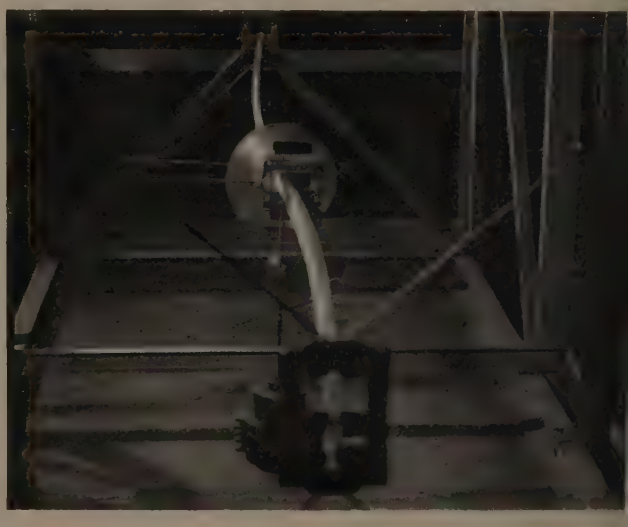

Fig. 12.—2 in. x $\frac{3}{4}$ in. I-beam buckling in second mode with stiff lateral restraint at mid-span

parallel crane gantries, in which connections of th type shown in Fig. 16 would have to be employe in order to provide clearance for operation of the hois

Conclusions

Owing to the paucity of data on the buckling beam members with elastic restraints, designers ha

*See Appendix 6.

reviously been forced to use the simplified results or slender beams, which are seen to be considerably a error in many practical cases. Although the conitions discussed above refer only to special cases, nough information may be derived from them to mphasise the necessity for raising the permissible ange stresses for restrained beams above the values erived for unrestrained members.

It is noticed that, with the exception of the effect f elastic torsional movement of the end supports,

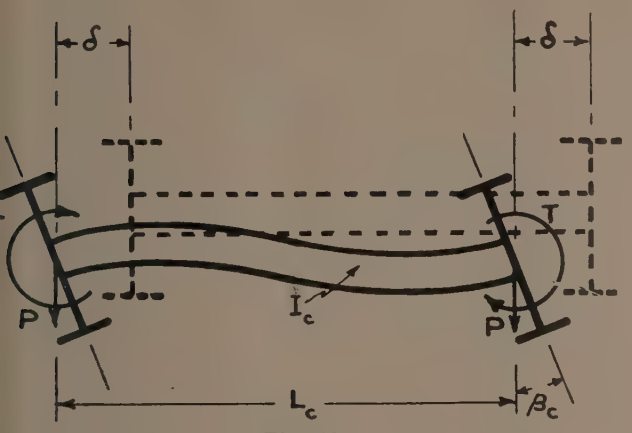

Fig. 13

ll the practical conditions, neglected in simplified heories, tend to increase the stability of members. he factors influencing the behaviour of beams may e summarised as follows :—

(1) *End Fixity.* The increases in buckling loads ue to end fixity are considerable, and allowance should e made for the influence of supporting connections n the design of slender beams. Provided that the ritical stress for a similar unrestrained member exceeds

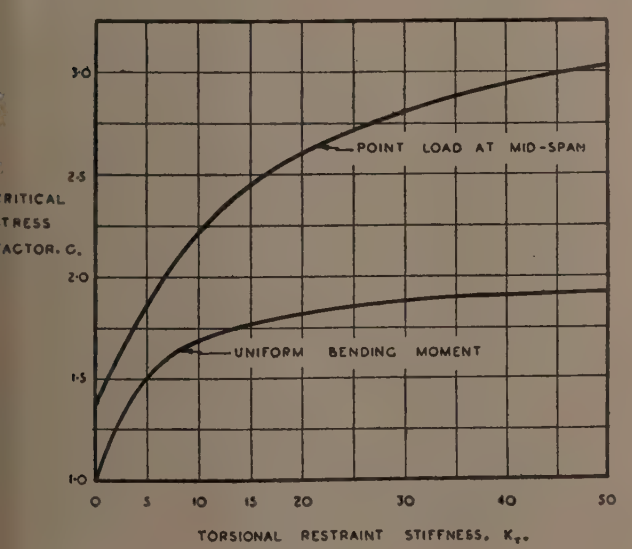

Fig. 14

he yield stress of the material, it is unnecessary urther to consider stability in the design of a beam with nd fixity. In cases where the permissible stress is overned by lateral stability, it will be necessary to se a working stress equal to the critical stress for ree ended conditions in the usual analysis for *encastre* nembers. The further increase in the buckling stress fforded by restraint in azimuth may be included in design by employing a reduced effective span in estimating the limiting stress.

(2) *Torsional Stiffness of Supports.* The stability of a beam is influenced by the stiffness of its supports under axial torque, and it is advisable to provide end connections giving a high combined torsional stiffness. Where these precautions are omitted, the load-carrying capacity of a beam may be reduced below that estimated from simplified theory. This effect will usually be balanced by an increase in stability caused by the restraint of warping at the supports.

(3) *Bending Restraints on the Span.* By provision of intermediate lateral stays attached to the beam, it is possible to increase the buckling stress to any required value. Connections should be made to the compression flange, at the points of maximum displacement in the buckled mode, for optimum efficiency.

Fig. 15.—Beams with torsional restraint

(4) *Torsional Restraints on the Span.* The use of filler joists between parallel beams may ensure stability of the main members provided that the connections are sufficiently rigid. This system of restraint will prove ideal in cases in which there are no suitable members to which the joists may be stayed, and will consequently be of value in the case of crane gantries.

The special problem of main beams loaded through secondary members has been discussed elsewhere[6], and it has been shown that such beams cannot buckle unless their initial shapes are considerably bowed.

Thus we see that in a majority of practical cases the existing design rules will give very conservative estimates of permissible stress. By judicious use of restraining members, it will frequently prove possible to increase the working stresses, and the influence

of end connections should be considered in efficient design.

APPENDIX 1

General Equations of Equilibrium

Let us consider a uniform slender beam, loaded by shearing forces and couples in the planes of maximum and minimum flexural stiffness. The ends are free to rotate with respect to the principal axes of inertia parallel to the x- and y- axes, whilst rotation with respect to the z- axis is rigidly restrained (Fig. 2). Let M_1 and M_2 be the bending moments at any point on the span in the planes xz- and yz- respectively, and let θ_1 and θ_2 be the slopes of the axis of the beam in these planes in the buckled state. The deformation of the axis is defined by the two displacements u and v in the x- and y- directions, and by the angle of rotation of the cross section β. The relation existing between the positive directions of the co-ordinate axes and the positive directions of rotations will be assumed to be the same as between translation and rotation of a right hand screw.

Considering the beam to have been displaced into a bowed and twisted form due to lateral buckling under

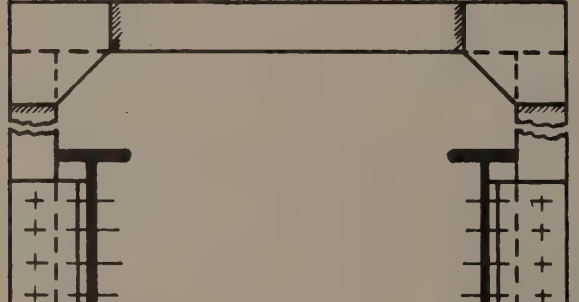

Fig. 16

the applied loads, we may determine the least moments necessary to maintain equilibrium. From the simple theories of bending and torsion :—

$$\left.\begin{aligned} EI_1 \frac{d\theta_1}{ds} &= M_1 \\ EI_2 \frac{d\theta_2}{ds} &= M_2 \\ GJ \frac{d\beta}{ds} &= T_z \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

where s is measured along the beam axis.

The third of these equations neglects the influence of warping on the torsional stiffness, the effect of which is slight in slender members.

Using the geometrical relations[3] :—

$$\left.\begin{aligned} \frac{d^2u}{dz^2} &= \frac{d\theta_1}{ds} + \frac{d\theta_2}{ds}\beta \\ \frac{d^2v}{dz^2} &= \frac{d\theta_2}{ds} + \frac{d\theta}{ds}\beta \\ \frac{d\beta}{ds} &= \frac{d\beta}{dz} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

the following differential equations of equilibrium, defining the deflections u and v and the angle of twist, β, are obtained by eliminating θ_1, θ_2 and s from equations (8) and (9) :—

$$\left.\begin{aligned} EI_1 \frac{d^2u}{dz^2} &= M_1 + \frac{I_1}{I_2} M_2\beta \\ EI_2 \frac{d^2v}{dz^2} &= -M_2 + I_2 M_1\beta \\ GJ \frac{d\beta}{dz} &= T_z \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

In the general case of a beam subject to uniformly varying bending moments, the moments at any section may be seen to be :—

$$\left.\begin{aligned} M_1 &= S_2 z\beta + Wz + m_2\beta - m_1 \\ M_2 &= -S_2 z - m_2 \\ T_z &= -S_2 \left(z\frac{du}{dz} - (u - u_0) \right) - m_2 \frac{du}{dz} - T \end{aligned}\right\} \quad (11)$$

where S_2 is a uniform shear in the direction of the y- axis, m_1 and m_2 are applied moments in the xz-and yz- planes respectively, W is a lateral shear in the direction of the x- axis, and T is an applied torque about the z- axis. The origin is taken at such a position that $M_2 = 0$ when $z = 0$, and all quantities of the second order of magnitude are neglected.

By substitution of these expressions for the moments in equations (10), the general equations of equilibrium for the determination of the magnitude of the least loads necessary to maintain the beam in its buckled form become :—

$$\left.\begin{aligned} EI_1 \frac{d^2u}{dz^2} &= \gamma (S_2 z + m_2)\beta + Wz - m_1 \\ EI_2 \frac{d^2v}{dz^2} &= S_2 z + m_2 \\ GJ \frac{d\beta}{dz} &= -S_2 \left(z\frac{du}{dz} - (u - u_0) \right) - m_2 \frac{du}{dz} - T \end{aligned}\right\} \quad (12)$$

where $\gamma = \dfrac{I_2 - I_1}{I_2}$.

These equations may be used to solve the majority of problems of lateral instability.

APPENDIX 2

Beams under Symmetrical Point Loads and End Couples

(Strain Energy Solution)

The critical loading of a slender beam under the conditions shown in Fig. 2 may be conveniently determined by application of a simple strain energy method[7], employing the principle of minimum total potential energy. When the member buckles sideways and twists under the action of applied loads, the intern

strain energy stored is increased since, in addition to bending in the plane of the web, we have bending in the lateral plane and torsion. The external loads do work in changing their potential energy, due to movement in their own lines of action. The critical loads are determined from the condition that the beam shall be in a state of neutral equilibrium, and the work done by the applied loads must equal the change in internal strain energy during buckling.

The lateral bending moment equations for the member shown in Fig. 2 may be derived from equations (12) as :—

$$\left.\begin{aligned} EI_1 \frac{d^2u}{dz^2} &= (Pz - M)\beta \quad \text{for } o < z < aL \\ EI_1 \frac{d^2u}{dz^2} &= (PaL - M)\beta \quad \text{for } aL < z < L/2 \end{aligned}\right\} \quad (13)$$

where z is measured from a support, and β and u are the angle of twist and lateral deflection, respectively, at any section. γ is assumed to be unity.

The change in potential energy of the external loads during buckling is given by :—

$$V = 2M\Phi_2 - 2P\Delta$$

where Φ_2 is the change in slope of the member, $\frac{dv}{dz}$, at the supports, and Δ is the change in vertical deflection at the load points due to slight displacement from the stable position.

$$\text{Hence } V = 2(M - PaL)\int_0^{L/2} \frac{d^2u}{dz^2}.\beta\, dz + 2P\int_0^{aL} \frac{d^2u}{dz^2}\beta\,(a_L - z)\, dz.$$

The increase in internal energy due to lateral bending and torsion is :—

$$U = EI_1\int_0^{L/2}\left(\frac{d^2u}{dz^2}\right)^2 dz + GJ\int_0^{L/2}\left(\frac{d\beta}{dz}\right)^2 dz.$$

By assuming a twisted form for the beam and by substitution from equation (13), the equation governing the critical conditions may be derived from the neutral equilibrium condition :—

$$U + V = 0.$$

As in other energy solutions to instability problems, it is found satisfactory to assume a simple trigonometric displaced form, and by putting $\beta = \beta_c \sin\frac{\pi z}{L}$ we may derive the critical relationship between P and M.

$$\text{Thus } P^2\frac{L^2}{2} - F_1 - PML\,F_2 + \frac{M^2}{4} = \pi^2\frac{EI_1\,GJ}{4L^2} \quad (14)$$

$$\text{in which } F_1 = \frac{1}{4\pi^2}\sin 2\pi a - \frac{a}{2\pi^2}\cos 2\pi a$$

$$\cos 2\pi a + \frac{a^2}{2}\left(1 - \frac{4a}{3}\right)$$

$$F_2 = \frac{a}{2}(1 - a) + \frac{1}{4\pi^2}(1 - \cos 2\pi a).$$

When the end couples, M, are due to elastic restraints at the supports in the plane of the web, we may assume that their magnitudes are not altered by the slight lateral displacement considered in deriving the above solution. Consequently, the end couples are those obtained from analysis of the beam in its stable position.

From the usual theories of bending of beams with restrained ends, the relationship between the applied loads and the end moments is found to be :—

$$M = \frac{PLa\,(1 - a)}{\left(1 + \frac{2EI_2}{R_2L}\right)} = K_R PL \quad \ldots \quad \ldots \quad (2)$$

where R_2 is the restraint stiffness previously defined.

Hence, by substitution in equation (14) the critical load may be evaluated in terms of K_R, a, and the beam constants :—

$$P_{crit} = \frac{\pi\sqrt{EI_1\,GJ}}{L^2\sqrt{\left(2F_1 - 4F_2K_R + K_R^2\right)}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

When the critical load is exceeded, the lateral deflection becomes large and additional vertical deflection of the beam occurs. The end moments are consequently increased, giving additional stability to the member and increasing the load necessary to maintain the buckled form. Thus it is possible for built-in members to support a load in excess of the initial buckling load without causing complete collapse.

APPENDIX 3

Beams with ends restrained in azimuth

Let us consider a slender member, under uniform bending moment, subject to end restraints in the lateral plane. Let the restraints have equal stiffnesses, R_1, as previously defined, and let there be no couples in the lateral plane prior to buckling.

Assuming that the applied moment has attained its critical value, and that the beam is slightly displaced from its stable position, the equations of equilibrium may be derived from equations (12).

$$\left.\begin{aligned} EI_1\frac{d^2u}{dz^2} &= \gamma M\beta - m_1 \\ GJ\frac{d\beta}{dz} &= -M\frac{du}{dz} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

where z is measured from one support.

By differentiation of the second of equations (15) with respect to z, and substituting in the first, the differential equation defining the twisted form becomes :—

$$\frac{d^2\beta}{dz^2} + k^2\beta - p = 0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

$$\text{where } k^2 = \frac{\gamma M^2}{EI_1\,GJ};\quad p = \frac{m_1 M}{EI_1\,GJ}.$$

The solution to this equation yields :—

$$\beta = A\sin kz + B\cos kz + p/k^2 \quad \ldots \quad \ldots \quad (17)$$

The arbitrary constants of integration are determined by consideration of the end conditions, $\beta = 0$ when $z = 0$; $\beta = 0$ when $z = L$,

$$\text{giving } A = -\frac{p}{k^2}\frac{(1 - \cos kL)}{\sin kL},\quad B = -p/k^2.$$

Also $\Phi_1 = m_1/R_1$ by definition.
Hence, from equation (15) :

$$\Phi_1 = -\left(\frac{du}{dz}\right)_{z=0} = \frac{GJ}{M}\left(\frac{d\beta}{dz}\right)_{z=0} = \frac{GJ}{M}Ak$$

$$= -\frac{GJp}{Mk}\,\frac{(1-\cos kL)}{\sin kL}$$

Therefore $\dfrac{R_1}{EI_1} = -k\,\dfrac{\sin kL}{(1-\cos kL)}$

But $k = \dfrac{\pi c}{L}$ where c is the critical stress factor.

Hence the value of the critical stress factor in equation (1) may be determined from the relation :—

$$\frac{R_1 L}{EI_1} = -\frac{\pi c \sin \pi c}{(1-\cos \pi c)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

APPENDIX 4

Beams with Elastic Torsional Supports

The critical loads of members whose end connections provide only a limited torsional stiffness may be estimated by modification of the simple analysis for slender beams. As an example, let us consider a beam loaded to instability by a point load P applied through the shear centre at mid-span. The ends will be supposed to be supported so that free bending actions may occur about the principal axes of inertia, and that rotation about the z axis is elastically restrained. This restraint will be assumed to be the same at each end and to be represented by the relation :—

$$\beta_e = T_e\, R_t\, \frac{L}{GJ},$$

where β_e, T_e, are the rotation and torque at the ends, respectively, and R_t is the end torsional restraint stiffness.

In this symmetrical case, a constant additional rotation will be given to the member when it buckles, due to bodily movement caused by elastic twisting of the supports. The modified equations of equilibrium may be written as :—

$$\left.\begin{aligned} EI_1\frac{d^2u}{dz^2} &= \frac{\gamma P}{2}\left(\frac{L}{2}-z\right)\left(\beta+\beta_e\right) \\ GJ\frac{d\beta}{dz} &= -\frac{P}{2}\left(\frac{L}{2}-z\right)\frac{du}{dz} \\ &\quad + \frac{P}{2}(\delta-u) \end{aligned}\right\} \quad (18)$$

where β is the rotation due to twisting of the member alone, δ is the lateral deflection at mid-span and z is measured from the load point.

The differential equation derived from equations (18) is thus :—

$$\frac{d^2\Phi}{dw^2} + K^2 w^2 \Phi = 0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

where $\Phi = \beta + \beta_e$, $w = \left(\dfrac{L}{2}-z\right)$, $K^2 = \dfrac{\gamma P^2}{4EI_1 GJ}$

The solution to this equation may be obtained by integration by the method of infinite series to give :—

$$\left.\begin{aligned} \Phi = A\Bigg\{1 &- \frac{K^2w^4}{3\cdot4} + \frac{K^4w^8}{3\cdot4\cdot7\cdot8} \\ &- \frac{K^6w^{12}}{3\cdot4\cdot7\cdot8\cdot11\cdot12} + \ldots\ldots\ldots\ldots\Bigg\} \\ + Bw\Bigg\{1 &- \frac{K^2w^4}{4\cdot5} + \frac{K^4w^8}{4\cdot5\cdot8\cdot9} \\ &- \frac{K^6w^{12}}{4\cdot5\cdot8\cdot9\cdot12\cdot13} + \ldots\ldots\ldots\ldots\Bigg\} \end{aligned}\right\} \quad (20)$$

The relevant end conditions may be substituted in this general solution to enable the arbitrary constants to be determined. It is seen that $\Phi = \beta_e$ at $w = 0$, and $\dfrac{d\Phi}{dw} = 0$ when $w = L/2$. Also the torque resisted by the end supports is seen from equation (18) to be $\dfrac{P\delta}{2}$, and thus by definition :—

$$\beta_e = \frac{R_T\, P\delta L}{2\,GJ}.$$

By using these conditions and evaluating δ by integration of the second of equations (18), the relationship between the end torsional stiffness and the buckling load may be expressed in the non-dimensional form:—

$$R_t = \frac{3\left(1 - \dfrac{q^2}{4} + \dfrac{q^4}{4\cdot5\cdot8} - \dfrac{q^6}{4\cdot5\cdot8\cdot9\cdot12} + \ldots\ldots\right)}{2q^2\left(1 - \dfrac{q^2}{4\cdot7} + \dfrac{q^4}{4\cdot7\cdot8\cdot11} - \dfrac{q^6}{4\cdot7\cdot8\cdot11\cdot12\cdot15} + \ldots\ldots\ldots\right)} \quad \ldots \quad (21)$$

where $q^2 = 4.48n^2$, and n is the ratio between the buckling load for the beam with the assumed end conditions and that for a member with rigid torsional support. This solution may be represented graphically, and it has been found that for a practical range of values of R_t the critical load may be approximately related to the support stiffness by the formula :—

$$n = 1 - \frac{4}{3}R_t \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (22)$$

This relation applies equally well to cases of loading by symmetrical point loads and by single point load at any position on the span.

APPENDIX 5

Intermediate Lateral Restraint

As an example of the effect of lateral restraint on the span, we will consider the case of a beam loaded at mid-span through the shear centre, and stayed at mid-span to an elastic support as shown in Fig. 9. The stay is attached at a distance d above the shear centre.

The strain energy method affords obvious advantages over the analytical solution in this problem. Considering the load, P, to have reached its critical value and the beam to have been slightly displaced, we may derive the equation of equilibrium from equation (12). If the lateral deflection at mid-span is δ, and the rotation of that section is β_c, then the restoring force afforded by the restraint is $K_B(\delta + d.\beta_c)$.

where K_B is the lateral restraint stiffness. Hence the bending equation of equilibrium is :—

$$EI_1 \frac{d^2u}{dz^2} = \frac{P}{2}\left(\frac{L}{2} - z\right)\beta - \frac{K_B}{2}\left(\delta + d.\beta_c\right)\left(\frac{L}{2} - z\right) \quad \ldots \quad \ldots \quad (23)$$

where z is measured from the load point.

The condition of neutral equilibrium is satisfied by equating the sum of changes in internal strain energy to the work done by external forces.

The work done by the external forces is seen to be :—

$$V = -P\int_0^{L/2} \beta\left(\frac{L}{2} - z\right)\frac{d^2u}{dz^2}\,dz + \frac{K_B}{2}(\delta + d.\beta_c)^2$$

The change in internal strain energy during buckling is :—

$$U_b + U_t = EI_1\int_0^{L/2}\left(\frac{d^2u}{dz^2}\right)^2 dz + GJ\int_0^{L/2}\left(\frac{d\beta}{dz}\right)^2 dz$$

We may assume a twisted mode such as :—

$$\beta = A\cos\frac{\pi z}{L} + B\cos\frac{3\pi z}{L} + \ldots\ldots$$

which satisfies all the end conditions. This assumed mode may be substituted into the above energy equations, from which the equation governing the critical load may be derived. Neglecting all but the first term in this expression, $A = \beta_c$. Then employing equation (23) the neutral equilibrium equation :—

$U_b + U_t + V = 0$

may be simplified to give :—

$$\frac{P^2L^3\beta_c^2}{192\,EI_1}\left(1 + \frac{6}{\pi^2}\right) + \frac{PK_BL^3}{\pi^3EI_1}\left(\delta + d.\beta_c\right)\left(\frac{\pi}{2} - 1\right)\beta_c - \frac{K_B}{2}\left(\delta + d.\beta_c\right)^2\left(1 + \frac{K_BL^3}{48\,EI_1}\right) - \frac{GJ\,\pi^2\beta_c^2}{4L} = 0 \quad \ldots \quad \ldots \quad \ldots \quad (24)$$

The relation between the lateral deflection and twist at mid-span may be obtained by considering a small element of the longitudinal axis of the beam of length dz. Due to curvature of this element, one end of which is considered fixed, the end of the beam will deflect through an arc of magnitude $\frac{d^2u}{dz^2}dz\left(\frac{L}{2} - z\right)$ in the xz- plane.

By integrating such increments of deflection over the half span, the lateral deflection at mid-span is seen to be :—

$$\delta = \int_0^{L/2}\frac{d^2u}{dz^2}\left(\frac{L}{2} - z\right)dz$$

which on substitution of the assumed mode yields :—

$$\delta = \frac{\beta_cL^3}{(48\,EI_1 + K_B\,L^3)}\left\{\frac{48P}{\pi^3}\left(\frac{\pi}{2} - 1\right) - K_B\,d.\right\}$$

Introducing this expression for δ into equation (24) and simplifying, the critical load may be seen to be :—

$$P_{crit} = \frac{17.2}{L^2}\sqrt{\left\{EI_1\,GJ\,(1 + \lambda)\left[1 + \frac{10\,\lambda}{(1 + \lambda)}\left(\frac{EI_1}{GJ}\right)\left(\frac{d}{L}\right)^2\right]\right\}} \quad (25)$$

where $\lambda = \frac{K_B\,L^3}{48\,EI_1}$, representing the ratio between the restraint stiffness and the lateral bending stiffness of the beam.

Hence the critical stress factor is :—

$$c = 1.35\sqrt{\left\{(1 + \lambda)\left[1 + \frac{10\,\lambda}{(1 + \lambda)}\left(\frac{EI_1}{GJ}\right)\left(\frac{d}{L}\right)^2\right]\right\}} \quad (26)$$

which reduces to $c = 1.35\sqrt{(1 + \lambda)}$ when the stay is attached at the shear centre (i.e. $d = 0$).

It is apparent that considerable increases in buckling load may be obtained by the attachment of stays at mid-span, and that it is more efficient to restrain the beam at points above the shear centre if possible. Where the stay is attached below the shear centre the final term is equations (25), and (26) is preceded by a negative sign.

A more accurate solution may be obtained by including two terms in the assumed form, and by applying the principles of minimum total potential energy. The solution is complex, and is best presented graphically as shown in Fig. 10 for the case of a 2 in. x $\frac{3}{4}$ in. I-beam.

An approximate solution has been obtained to the problem of a member under uniform bending moment with restraint at mid-span, and the critical stress factor has been found to be

$$c = \sqrt{\left\{(1 + \lambda)\left[1 + \frac{10\,\lambda}{(1 + \lambda)}\left(\frac{EI_1}{GJ}\right)\left(\frac{d}{L}\right)^2\right]\right\}} \quad (27)$$

which corresponds to that derived for the beam under point load with a numerical constant of unity.

APPENDIX 6

Intermediate Torsional Restraint on the Span

The influence of torsional restraint on the stability of a beam may be indicated by the investigation of the behaviour of a member under uniform bending moment, M, with an elastic torsional restraint at mid-span, as shown in Fig. 13.

We will define the torsional stiffness of the restraint, K_T, by the relation

$$T = \frac{GJ}{L}.K_T\beta_c,$$

where T is the restoring torque, and β_c the angle of rotation, at mid-span. The equations of equilibrium derived from equations (12) yield a general solution to the twisted form given by $\beta = A\sin kz + B\cos kz$,

where $k^2 = \frac{\gamma M^2}{EI_1\,GJ}$, and z is measured from the support.

The end conditions to be satisfied are :—

$\beta = 0$ when $z = 0$, hence $B = 0$.

Also $\beta = \beta_c$ when $z = L/2$,

therefore $\beta_c = A \sin \frac{kL}{2}$, or $A = \beta_c \operatorname{cosec} \frac{kL}{2}$,

and $\frac{d\beta}{dz} = -\frac{K_t \beta_c}{2L}$ when $z = L/2$.

Thus $\beta_c k \cot \frac{kL}{2} = -\frac{K_t \beta_c}{2L}$,

giving $K_T = -2kL \cot \frac{kL}{2}$.

But $kL = \frac{M L}{\sqrt{EI_1 GJ}} = \pi c$, where c is the critical stress factor.

Hence $K_T = -2\pi c \cot \frac{\pi c}{2}$ ... ... (7)

The same method of analysis may be used to solve the problem of the instability of beams under point load at mid-span with torsional restraint at the load point. The general solution to the twisted form may be derived from equations (12), as :—

$$\beta = A\left(1 - \frac{K^2 z^4}{3.4} + \frac{K^4 z^8}{3.4.7.8} - \frac{K^6 z^{12}}{3.4.7.8.11.12} + \ldots\ldots\right) + Bz\left(1 - \frac{K^2 z^4}{4.5} + \frac{K^4 z^8}{4.5.8.9} - \frac{K^6 z^{12}}{4.5.8.9.12.13} + \ldots\ldots\right) \quad \ldots \quad (28)$$

The end conditions to be satisfied are the same as those employed above, and by introducing these conditions into equation (28) and simplifying, the relationship between the critical load and the restraint stiffness may be obtained, as :—

$$K_t = 4\,\frac{\left\{1 - \frac{q^2}{4} + \frac{q^4}{4.5.8} - \ldots.\right\}}{\left\{1 - \frac{q^2}{4.5} + \frac{q^4}{4.5.8.9} - \ldots.\right\}} \quad \ldots \quad (29)$$

where $q^2 = 4.48n^2$, and n is the ratio between critical loads for the restrained beam and a similar unrestrained beam.

This series solution is represented graphically in Fig. 14, in which the critical stress factor is plotted against K_t.

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[3] "Elastic Stability of Long Beams under Transverse Forces." A. G. M. Mitchell, PHIL. MAG., Vol. 48, 1899.

[4] "The Buckling of Deep Beams." J. Prescott, PHIL. MAG., Vol. 36, p. 297 ; and PHIL. MAG., Vol. 39, p. 195.

[5] "The Stability and Strength of Slender Beams." A. R. Flint, ENGINEERING, Vol. 170, p. 545.

[6] "The Stability of Beams Loaded through Secondary Members." A. R. Flint, CIVIL ENGINEERING AND PUBLIC WORKS REVIEW, Vol. 46, Nos: 537-8, pp. 175 and 259.

[7] "Theory of Elastic Stability." S. Timoshenko. McGraw Hill.

Book Reviews

Elemental Theory and Design of Flexural Members, by J. Vawter and J. Clark. New York : Wiley. (London ; Chapman & Hall, 1950). 215 pp., 9 in. × 6 in., 32s.

The book, which is written for students, is fundamentally a text-book on the basic theory of flexure and its application to simple structural design.

Emphasis is laid on the principle that the same general physical laws are applicable to the three materials especially referred to in the book, namely: steel, concrete and timber—subject, of course, to certain peculiarities of each material.

Readers, not conversant with American Handbooks and Specifications, will welcome the explanatory paragraphs given in the introduction to the book.

The two important handbooks, referred to below, are made use of in the design calculations included in the book.

(i) The manual of the American Institute of Steel Construction (A.I.S.C.), STEEL CONSTRUCTION, 1949. This is very important.

(ii) Another important handbook is "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete."

The subject matter of the book deals with homogeneous and non-homogeneous beams, elementary design of steel beams and girders, elementary design of steel columns with bending, elementary design of reinforced concrete beams and columns, bending in unsymmetrical sections, bending in special beams (including those with thin webs). The appendix deals with the design of a floor in timber.

Details of welding technique and the standardised American welding symbols are given. There are numerous worked examples, accompanied by very clear diagrams, interspersed throughout the text. There are also some excellent photographs.

The book can be recommended to structural engineering students who desire an introduction into the American methods of calculation and American standards, in the sphere of structural members subjected to flexure.

Sets of problems are given at the ends of various chapters. It is a pity that answers to the problems are not provided. T.J.R.

A Course of Reinforced Concrete Design, by T. J. Bray. Second Edition. (London : Chapman & Hall, 1950). 228 pp., 10¼ in. × 7¼ in. 76 illus., 27 charts and graphs. 30s.

This book, based on notes prepared for the author's own use when lecturing, presented in amplified form, was first published in 1946 as a textbook for students and a reference book for young engineers. It has now been revised, but still refers to the 1934 Code of Practice, although an additional Appendix has been included which deals with the various recommendations and implications of the new B.S. Code of Practice, C.P. 114 (1948).

The book is well illustrated with diagrams, and design charts and graphs are placed at the end of the work for easy reference.

An Investigation of the Strength of Welded Portal Frame Connections

and

An Investigation of the Strength of Certain Welded Portal Frames in Relation to the Plastic Method of Design

Discussion on papers by Dr. Arnold W. Hendry *

THE CHAIRMAN (Mr. Walter C. Andrews, O.B.E., M.I.C.E., Vice-President), introducing Dr. Hendry, referred to his earlier paper on "An Investigation of the Stress Distribution in Steel Portal Frame Knees,"† in 1947, for which he was awarded the Sessional Medal and Research Prize. From that and his other contributions, listed in the papers now before the meeting, it seemed that Dr. Hendry had made a "corner" in portal frame knees and allied matters.

The Chairman proposed the hearty thanks of the Institution to Dr. Hendry for all the work which obviously he had put into his subject, for the way in which he had set out his experiments and drawn conclusions from them, and for the way in which he had presented the work. He had given much matter for thought and to which, perhaps, to give effect.

(The vote of thanks was warmly accorded).

Mr. JOHN KING (Associate-Member), asked for Dr. Hendry's observations on the rather vexed question of flange stability, purely from the practical angle. It was usually quite easy, he said, to give top flange restraint with members on the top of the portal, but architects had a nasty habit of objecting violently to members coming into the bottom flange of a portal, and it seemed from one of Dr. Hendry's illustrations that the bottom flange needed some stiffening near the knee.

Dr. HENDRY said that Mr. King had in mind the type of portal used for a building frame, but in the papers he had had in mind the bridge type of girder, in which the cross-girders usually framed into the beam and provided stability over the full depth. He was afraid there was nothing useful that he could add at the moment concerning the type of portal which obtained its stabilising force on the upper flange ; that problem was very wide and required full investigation.

Mr. D. J. DAVIES (Associate-Member) said he had had the pleasure some years ago of co-operating with Mr. Grisenthwaite in his work on portal-frame bridges, when it was realised that many problems were not readily solvable by the methods of analysis then available. Use was made, however, of information from tests carried out by the American Institute of Steel Construction, and from various papers on the subject of strains and stresses at frame knees. Rational methods of design for frame-knees were not fully developed by these earlier sources of information, and Dr. Hendry's pioneering work in this direction was likely to be very useful.

A number of points arose in design, for example the use of radial stiffeners against curved flanges. These had been frequently used on portal frames knees with inner curved flange thicknesses up to 2½ inches supports. The stresses in these flanges were reduced by putting in radial stiffener supports connected to the web, thereby achieving a certain saving in flange thickness. The question arose then of the flange forces arising from the curvature of the knee. The incidence of these radial forces on the web had to be allowed for in the design of the web to prevent buckling. These forces had been settled more or less empirically in the past, and checked by simple statical triangulation of forces, which seemed to offer a conservative solution for knee designs. With portal frames of 150—200 ft. span, the thickness of the web at the knee tended to increase to the stage where a more prodigal use of stiffening was required for economy in construction. In this connection the load in the stiffeners became an issue in design.

The Americans had carried out some very thorough tests on riveted and welded full-scale tapered-leg frames, but correlation of these results with design procedure of a rational nature by evaluation of principal stresses, became much too involved, and usually we had not the time to design by this method. The alternative had been to use previous experience of similar structures as a precedent for attaining sections that would satisfy the stress conditions.

One felt that engineers would agree that the work of Dr. Hendry might lead to the evaluation of design factors which could be incorporated in design procedure applied directly to many of the problems associated with the detail of portal frame knees. Problems on frame knees had been put to Mr. Davies by highway bridge engineers, and he felt re-assured that he could now refer them to the valuable research which Dr. Hendry has carried out, and which would add further to knowledge on this rather complex part of a portal frame bridge.

Dr. HENDRY said reference had been made to rather larger portal frames than he had had in mind, and to the possibility of going into the thinner types of

*Read before the Institution of Structural Engineers at 11, Upper Belgrave Street, London, S.W.1., on Thursday, December 14th, 1950. Published in THE STRUCTURAL ENGINEER, Vol. XXVIII, No. 10, p.265, and No.12, p.311.

†Published in THE STRUCTURAL ENGINEER, Vol. XXV, Nos. 3 - 4.

webs. That was a matter for further investigation. But from the work done so far, the conclusion might be drawn that even quite light stiffeners were very efficient in preventing premature failure ; he felt that in this particular matter, even if we had not full knowledge of the stresses in the various stiffeners in a system, probably anything which looked right would be satisfactory, and that was as far as we could go at the moment. There was a great deal in the experienced engineer's judgment. It was frequently found in this type of research work that the designer was somewhat ahead of theory and that, when theory caught up, the designer was usually found to have been essentially correct.

Lieut.-Col. R. F. GALBRAITH (Vice - President), congratulated Dr. Hendry and said it was to be noted that his tests were made on quite small members of small spans and quite light sections. But how far were we justified, he asked, in interpreting results on small scale tests and applying them to practical problems ? He asked whether larger scale tests had been made which would in any way support Dr. Hendry's theories.

Dr. HENDRY replied that experience, so far, had not revealed any serious differences between the behaviour of quite small sections and of very large ones. He had not been able to test anything very large, mainly for reasons of the space and equipment available ; but large scale full-size frames had been tested by Professor Baker and his colleagues at Cambridge, and he understood that the results obtained with the full-size frames fully confirmed the expectations based on small scale work. He believed also that comparisons with the very large portal frame connections tested by the Americans some years ago agreed with the conclusions arrived at on the basis of small scale work. It was very convenient to be able to do the basic work on small scale frames, to test theoretical methods and proposals, because it was so much cheaper, and the tests were easier to handle ; we simply could not afford to test a large number of frames costing perhaps £100, £150 or £200 each, and to do so was wasteful if we were not able to learn any more than we could learn by testing quite small frames. But, of course, it was necessary to check the results with those of large scale tests ; he believed that aspect was being handled by other people, and a certain amount of work on large scale frames had already been published.

Dr. J. W. RODERICK complimented Dr. Hendry on his most valuable contribution to the experimental verification of the simple theory upon which the plastic design method was based. The paper on connections was of particular interest, since it revealed the conditions under which knee type connections should be stiffened, and demonstrated the correct way in which this should be done.

One of the essential requirements of the plastic design method was, of course, that the connections should be capable of transmitting the full plastic moments of the members joined, and should do so with the minimum amount of deformation. Quite frequently one saw connections stiffened as shown in Fig. 1(a) ; while that might help in preventing web and flange buckling, it did very little to prevent excessive deformation. Under load, the web within the connection invariably began to fail in shear at an early stage, the square corner being deformed into a diamond shape as at (b). When the connection was provided with a diagonal stiffener such as Dr. Hendry had used, the deformation was considerably reduced.

Pointing out that care should be taken in applying the results of such tests to structures generally, Dr. Roderick said that ideally, we should like to examine the behaviour of connections in structures loaded to collapse. Unfortunately, the opportunities for tests of this kind were few, and we had to resort to the

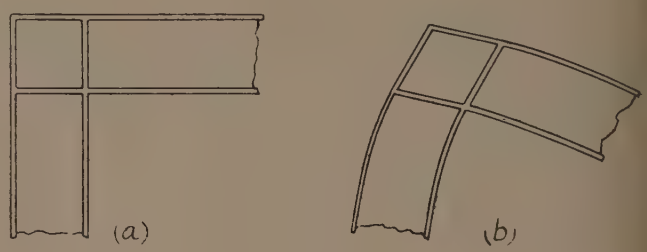


Fig. 1

testing machine as Dr. Hendry had been compelled to do. Since large tensile testing machines were rarely available, such tests had also to be confined to compressive loading. Illustrating a knee joint loaded as in Fig. 2, Dr. Roderick said that with this arrangement it was often difficult to obtain the desired ratio of bending moment and shearing force. Also, one had to contend with fairly large compressive components along the members contributing to the tendency to web and flange buckling in the vicinity of A, whereas for instance, in a tall portal frame the axial loads might be comparatively small. It was interesting to

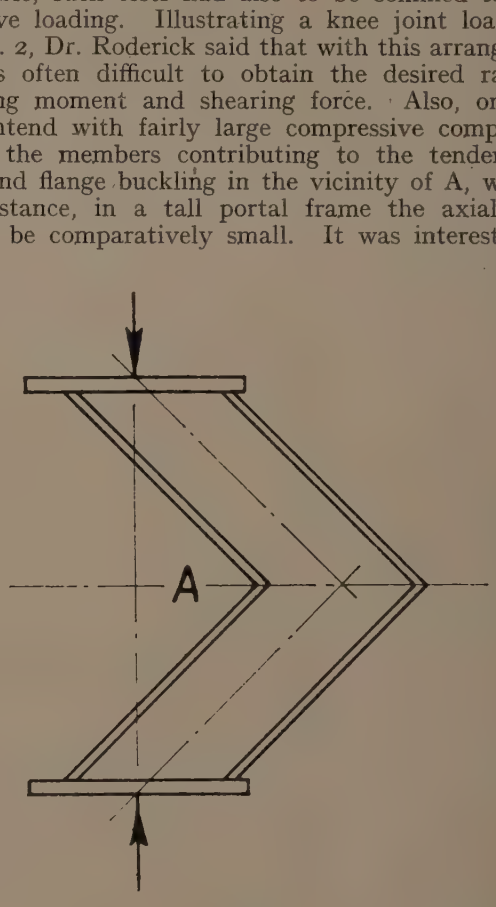


Fig. 2

note that in a series of tests by the German railway authorities*, carried out more than 20 years ago, some of these difficulties had been overcome by testing the connections in pairs and loading them as shown in Fig. 3.

The influence of shearing force on the full plastic moment had been referred to by Dr. Hendry. That problem had been investigated theoretically at

*Schmuckler, H. "Die Dresdener Versuche der Deutschen Reichsbahn - Gesellschaft und des Deutschen Stahlbau - Verbandes mi geschweissten Stahlkonstruktions - Verbindungen."
Der Stahlbau, p. 133 *June,* 1931.

Cambridge by Dr. Horne, who had produced an accurate solution for calculating the reduction in the moment.

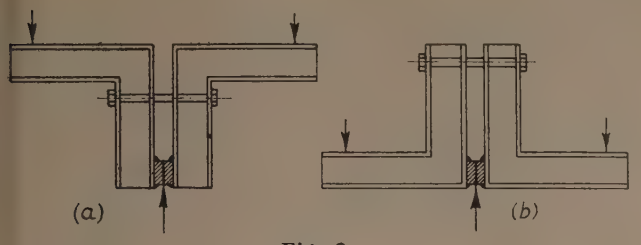


Fig. 3

For standard rolled steel sections, he had come to the same conclusion as had Dr. Hendry from his tests, namely that the reduction was usually small, and only became important under excessively high shearing forces.

Mr. J. C. H. Finlinson (Associate - Member), referring to the portal frames which were illustrated in Fig. 25 (p. 278), said it appeared that when they were tested, the strength was less in the case of A.1. than in the case of A.2. and A.3. He wondered whether that was due to any inherent merit in the pattern of the stiffeners in A.2. and A.3., or was it only due to the fact that the members in A.2. and A.3. had in effect been very substantially haunched. It appeared to him that the corner detail, especially in A.3, was very large relative to the frame. He asked Dr. Hendry whether A.2 and A.3 would have shown strengths usefully greater than A.1, had the proportion of span to size of corner detail been much greater.

Dr. Hendry first thanked Dr. Roderick for his interesting remarks, and for having called attention to the need for ensuring that one was really testing the effect that one was trying to represent.

In arriving at the proportions of the frames tested, a bridge type of portal having ratio of span to height of about 5 or 6 to 1 was envisaged. For this case, the ratio of bending moment to shearing force and axial load was correctly represented in the tests on L-frames, while a variety of other conditions were covered in the tests on complete portals.

In the L-frames he had tested, the axial compressive component had given rise to rather small compressive stresses, the effect of which on the strength of the frames would not be appreciable. Thus, he believed, the results would have been practically the same if one had tested the joint by putting it into bending and shear without a direct component. In practical portals, of course, there was a component in the beam whose magnitude equalled the horizontal thrust of the portals.

The first type of connection should be avoided wherever possible. But it was not always possible to avoid it, as was shown by some of the test frames having very deep beams, as illustrated in his second paper. If there were two beams of similar dimensions, that type of connection was weak.

Coming to Mr. Finlinson's remarks on the difference in strength and stiffness of the three frames having different knee stiffeners, as shown in Fig. 25, again he emphasised that he had in mind a bridge type portal in which one could have relatively deep brackets. The increase of stiffness and strength was entirely dependent on the size of the brackets in relation to the span of the portal.

For the bridge type girders which he had in mind, he did not think the brackets shown in Fig. 25 were very far away from what was feasible in practice.

Mr. P. L. Capper (Associate-Member of Council), discussing the reduction of the plastic modulus of a section by axial stress, called attention to Fig. 18 (p. 320), giving a series of lines connecting the ratio of the reduced modulus to the full plastic modulus of the member against the ratio of the axial stress to the yield stress ; and to Table IV, where some average factors were derived from those lines. The lines being nearly at 45° across the diagram, he suggested that a very simple approximation could be made by taking a line at 45°, which would then give the reduced plastic modulus/full plastic modulus, equal to the difference between the yield stress and the axial stress divided by the yield stress :—

$$\frac{\mu'}{\mu} = \frac{fy - fd}{fy}$$

Dr. Hendry said the suggestion appeared to be a very practical one, and the proposal might well be of value in design. One would over-correct by using that method to the extent that the straight line was under the curved line. The effect would not be serious in the case of squat members having the same order of depth as width, but it might be wasteful in the case of sections which were deeper in relation to their width. However, the correction might be suitable for practical purposes, and the suggestion was very interesting.

Mr. R. G. Braithwaite, M.I.C.E., referred to the unnecessary amount of detailed work at the knee-connections of many portal frames he had seen, which he attributed to the lack of information and knowledge of the stress distribution at this joint. He mentioned that he wished to underline and emphasise some of the facts that emerged in the paper, and referring to Figs. 1 and 14, said it was interesting to note the increased stiffness obtained by increasing the thickness of the web stiffener, without increasing the amount of workmanship or welding. He had himself carried out some similar tests on various types of joints (reported *Inst. Welding Transactions*, April, 1947), and had obtained similar results. Incidentally, it was an easy joint to make, as it was only necessary to make one bevel saw cut through the joint at the appropriate angle, and obtain the connection by reversing the edges.

The cost of making these joints was reflected in the details shown in Fig. 25 (p. 278). If one studied the graph in Fig. 27, and measured the quantity of weld required to make the joint, one found that Frame A.2 had nearly 5 times as much welding as Frame A.1, with not more than 12½ per cent. increase in strength at the yield point. Similarly, in Frame A.3 there was 7 times as much welding with only 35 per cent. increase in strength at the yield point.

The results of the tests on the curved flanges were worthy of emphasis, as they showed there was no particular advantage in stability between that and the straight sloping flange. This would form an important factor in the cost of fabricating heavy portal frames. Unless the fabricator had heavy presses to form the flanges cold, the cost of smithing and forging becomes extremely high, and there was also the added difficulty of fitting them to the web plate. Although from the point of view of aesthetics the curved flange might look better than the straight type.

Congratulating the author on the amount of detail he had put into the work, Mr. Braithwaite said it was obvious that he had devoted a number of years to his investigations, the results of which were a valuable contribution to the investigations carried out by

independent research workers, both in this country and on the Continent. Mr. Braithwaite remarked on the development of portal frame construction in Germany, and suggested that every opportunity should be sought to co-ordinate their research with our own.

Professor W. FISHER CASSIE (Hon. Treasurer) appealed to designers to take more note of the findings of research workers. The papers before the meeting, he said, put forward important points, not only regarding the problem of plastic failure, but also on fabrication, and a number of practical ideas had been introduced.

Looking at the history of any of the modern applications of science in engineering or elsewhere, we found that on the average, a period of about 30 years elapsed before an idea which was scientifically correct was applied in practice ; that was not an exaggeration. Papers such as those by Dr. Hendry could be produced and studied in the Institution, but how far were we progressing in the application of the ideas put forward ? Could Dr. Hendry or Dr. Roderick say how many portal frames of the kind discussed in the papers had been designed on the plastic theory, which, after all, had been known for some years to be workable ?

Dr. HENDRY, in thanking Mr. Braithwaite for his interesting remarks, said he was very gratified indeed to know that this work was of interest and value to the practical designer.

It was interesting to hear Mr. Braithwaite's comparison between the costs of the different frames which incorporated different amounts of welding. In actual fact, frame A.1 had cost about £9, and A.2 and A.3 £10 and £11 respectively. The problem of achieving a balance between the increase in strength and the increase in costs was one for the designer ; the research man had to indicate the possibilities, and the practical designer had to interpret them in terms of economy. At the same time, however, the research man should keep such matters well in mind.

The type of detail which Mr. Braithwaite had referred to, incorporating a diagonal stiffener, was undoubtedly most satisfactory. The question arose as to whether it was good practice to cut the beam in the manner described, and to have it welded to a piece of plate extending to the full breadth of the section. He had heard people criticise this detail on the grounds that if the stiffener plate contained laminations, the results would be serious ; but he did not know whether any cases of this kind had been observed in practice.

Dr. Hendry had no information concerning the number of portal frames which had been designed on the plastic theory, but he imagined that the number was around nil. He happened to know of one case in which a designer, whose ideas were well ahead, had designed a portal on the plastic theory ; when the calculations were submitted to a certain local authority for approval, the designer was asked to explain his method of design, which was regarded as incomprehensible. Some scope must be given to designers to apply new ideas, and one trusted that regulations would allow the plastic theory to be applied within the limits of acquired knowledge.

Replying to Professor Fisher Cassie, he said it was true that the time lag in applying a theoretical idea to practice was too great. The plastic theory had been well known for 10 or 12 years as a practicable method of design, but it was not yet used to any great extent ; one hoped that that state of affairs would be remedied in due course.

Dr. RODERICK said that, since Professor Fisher Cassie had mentioned his name when asking what progress had been made in the application of the plastic design method, perhaps he might be allowed to say that he thought Dr. Hendry had given a rather depressing account of the situation. One or two small buildings had been designed on that method and were erected, several more were contemplated, two of which would shortly be under construction.

It appeared that there was a belief that building regulations did not yet permit the use of the plastic design method. This was not so. The B.S. 449 (1948) dealt with design under three headings : simple design, semi-rigid design and fully-rigid design. For the last of these it is stated that " for the purpose of such design, accurate methods of structural analysis shall be employed leading to a load factor of 2, based on the calculated or otherwise ascertained failure load of the structure or any of its parts . . . . . " That is, the plastic design method was permitted, provided a value of 2 was used for the load factor.

In a handbook on the plastic design of simple frames, shortly to be issued by the British Constructional Steelwork Association, a load factor of 1.75 had been advocated. The reason for this was that a simply-supported beam designed for the working stress of 10 tons per sq. in. allowed under simple design in the British Standard, had a load factor on collapse of only 1.75. It was therefore hoped that the next stage in the progress of applying the plastic design method would be to get the regulating authorities to accept a load factor of this value.

Mr. B. L. CLARK (Associate-Member) said that in designing, he had always made a practice of using the stiffener to stabilise the bottom flange by connection back to some members which came on to the top flange, and in doing so he had used $2\frac{1}{2}$ per cent. of the stress in the bottom flange as the transverse force. Dr. Hendry had stated that the stabilising load on the bottom flange in his experiments was 3 per cent. of the flange load. Mr. Clark asked if Dr. Hendry could confirm the figure of $2\frac{1}{2}$ per cent. as one to which we could work in stabilising the bottom flange, because it was very important to designers, who could quite easily deal with the remainder of the portal frame, provided they were assured that the inner flange was quite stable where compression stresses occurred.

Dr. HENDRY said he could confirm the figure. That was the lateral load which was necessary in frame No. 5. In the paper (Fig. 9) the actual lateral load curve was given ; the lateral load was quite small in relation to the maximum load carried by the structure, but would be different as between one case and another. It depended, among other things, on the web thickness. For example, if the web were thin and deep, it was not so stiff and the lateral force necessary would be greater. It was difficult to put any particular figure on it, but he believed that, for sections of the proportions of rolled steel joists, the figure of 3 per cent. was probably quite reasonable.

THE CHAIRMAN commented on the great interest of the discussion, and thanked Dr. Hendry on behalf of the meeting for the able manner in which he had dealt with the questions raised.

The Construction of an Extension to a Factory at Silvertown*

Discussion on Mr. F. G. Etches' Paper

DISCUSSION

THE CHAIRMAN, proposing a vote of thanks to Mr. Etches for his excellent paper, said it was clear in the first place that the conception of the project as a whole, and also the development of the details gave evidence of much care and thought. The problems which had to be solved were considerable, because they had included not only the usual difficulties relating to the design of a new building, but they also included many particularly difficult problems associated with altering existing buildings and making them serve other purposes. He thought that Mr. Etches and all concerned with that project should be congratulated on the success which they had achieved in their solution to these many and varied problems. He had found it very interesting to compare the slide of one of the original buildings (roofed with Belfast trusses) with the slide of the new structure which had replaced it. The fuss and complication of the old structure had given way to the simple and finely proportioned portal framed structure, which he felt sure that everyone would agree was very satisfying to the engineering eye.

It was also clear that a great deal of ingenuity had been shown in the development of the details, for example, the specially shaped gutter, with its overflow weir just beyond the end wall—which would effectively prevent flooding inside the building in the event of particularly heavy rain or a blocked downpipe.

That was only one small example, but on all of the details pointed out, and in some others which they may have noticed on the slides (for example, the very simple and effective pipe bridge), there was evidence of clear and good engineering thought.

The paper was very comprehensive in scope, including not only piling and the many superstructure problems which were the direct concern of structural engineers, but also other matters such as heating and ventilation, electrical work, etc. The very fact that the paper was so comprehensive had made it impossible for Mr. Etches to enlarge on many of the interesting problems.

He therefore hoped that in the discussion, many questions would be asked which would enable Mr. Etches to amplify the information given in his paper.

Finally, he congratulated Mr. Etches on the very lucid way in which he had presented the paper, and for showing and explaining not only a very complete series of slides—but also a most interesting 'movie' show.

Following the hearty applause with which the Chairman's vote of thanks was received, the Chairman continued and said that there was one matter in particular on which he would like to ask for more information.

Concerning the piling, the Chairman said that Mr. Etches had referred to a test pile which was subject to a load of 112 tons, and which showed a total settlement after 24 hours of two inches. He would like to know whether a time-settlement curve was taken, and was it quite clear that settlement had finished at the end of the 24 hours? Two inches was a fairly high settlement, and it was important to know whether that represented the final settlement under the test load, or whether it was likely to go on. It would be interesting to know what was the nature of the soil through which the test pile penetrated, and also of the soil in which it finally took a bearing.

In connection with the piles also, he had found no mention in the paper of what load was put on them as a working load. That, he thought, would be a matter of considerable interest. In addition he would like to know the length and size of the piles, and any particulars as to the driving, the weight of the monkey, the height of the drop and the final set, and also what was the measured settlement (if any) of the piles under their actual load.

Mr. F. E. FOWLE (Member) said as a colleague of Mr. Etches, he would like to congratulate him on the excellence of his paper, and on the manner in which it had been produced.

Although he had taken no part in the work, yet he had watched with interest its progress both in the office and on the site, and to anyone who had seen the completed factory at Silvertown there could be no doubt that a very satisfactory result had been achieved.

The first point he wished to raise had already been dealt with by the Chairman, and concerned the load and settlement on the test pile.

His second point concerned the shape of the roof frames. On referring to the paper, it appeared that the shape of the roof frame had been dictated by the depth of the trough gutter, and this seemed to have been determined by the fact that no internal down pipes were permitted. In consequence, the gutter was 250 feet long, and discharged into an outlet at one end. apart from the desirability of discharging this length of gutter into a single outlet, he would like to ask if Mr. Etches would agree that had it been permissible to use internal down pipes, the depth of the gutter could have been considerably reduced and a complication in the shape of the frames avoided?

Mr. A. W. D. MARSHALL (Associate-Member) was interested in the actual covering of the roof, and the slope which was nominally 10 degrees, but appeared to be considerably flatter on the top portion. He asked whether there had been a practical test of the waterproofing.

With regard to the welding of the column legs, he had noticed from the photographs that the flange plates were welded to the web plates by continuous fillet welds. Could Mr. Etches say whether special precautions were taken at the works against distortion of the columns during the application of these welds?

His last point concerned the problem of choosing between portal frames and roof trusses. Architects

*Read before the Institution of Structural Engineers at 11, Upper Belgrave Street, London, S.W.1., on Thursday, 8th February, 1951, Mr. S. Vaughan, B.Sc., M.I.C.E., M.I.Struct.E. (Vice-President) in the chair. Published in "THE STRUCTURAL ENGINEER" Vol. XXIX, No. 2, p. 39. (February 1951)

and Engineers knew the improvement in appearance by avoiding the mass of tie members, and portal frames had the virtue of being a novelty, but had a lot of disadvantages. One disadvantage was that the works manager had nothing from which to hang apparatus. The first thing looked for was a main tie from which could hang bus-bars, electric leads for special machines, ventilators and load speakers. They had to balance a lack of usefulness against the novelty and cleanness of portal frames.

Mr. E. R. E. CATCHPOLE (Associate-Member) said his was a small question. In the method of erecting, where the stanchions and the haunch portions of the roof were erected first, and then the rafter section in each case was added, it seemed that each rafter section would be a closer, and would give rise to a problem in securing a good fit up for welding. He would be interested to know how this problem was overcome.

Mr. F. G. BOWEN (Associate-Member) said that a little more detail of the strengthening of the existing reinforced concrete columns would be of interest. Was any special treatment or strengthening of the column foundations necessary and, if so, how was it done ?

He thought that he could safely leave Mr. Etches to answer Mr. Marshall about the matter of roof frames *versus* trusses, but in regard to the contention that fittings could not be fixed conveniently on frames, he would suggest that the answer was for the Works Manager to make up his mind and specify his requirements in advance. Then the structural engineer would soon provide them for him. In fact, the factory described by Mr. Etches appeared to be a case in point because some of the frame fittings, judged from the illustrations, appeared to be very neat—he had in mind, in particular, the crane feeder brackets which, apparently, served their purpose admirably. Admittedly, from the structural engineer's point of view, it always seemed a pity to have to make a nice frame look like a Christmas tree, due to the addition of all the fittings that had to be put on to it !

Mr. H. RUTHERFORD asked if he might be allowed, as one on whom he thought lay the responsibility of finding the money for that building, to say how much he had enjoyed Mr. Etches' paper. He thought he had probably learned more about the building on that occasion than he ever knew before. He felt rather more inclined to talk about the two points the Chairman had mentioned. He would certainly have sleepless nights over those piles until he had heard the answer. He had been assured on more than one occasion that there was no chance of the whole building disappearing somewhere below the soil ! It was a very troublesome area. The references to the Belfast truss really had shaken him. That building was, in his opinion, quite a historic building. It was erected after the Silvertown explosion in 1917, and he thought it was put up in record time and as a temporary building, but it had been in existence now for very nearly 30 years, and they had a great affection for it.

To answer the various speakers who had been commenting about hanging things from the roof : just as infinite care had been given to the building itself, he could assure them that as much had been taken in the lay-out of the factory. If, in a few years' time it was necessary to start hanging things from the roof, then he thought it would be time to start thinking of a new building.

Mr. John FABER (Associate-Member) noted that the purlins used on the 25 ft. 3 in. span were of 8 in. × 4 in. rolled steel joists which appeared to be on the heavy side. As this was fundamentally a welded job, he had been interested to see that ordinary joists had been used for the purlins instead of one of the more fancy forms, such as castellated beams, or light girders formed by two angle- or T-flanges held apart by a wiggley rod welded at the intersections. He expected it was something to do with money, but he would like to have Mr. Etches' views on that.

The paper referred to the use of solvents and nitrocellulose, and it would be interesting to know whether any special waivers had been granted by the authorities for the use of these materials, and whether any special bund precautions had had to be taken. Had any separation traps been required in the drainage arrangements, and, if so, had there been any special linings, and how had these been formed ?

With reference to the ventilation, Mr. Faber observed that Mr. Etches had referred to special diffuser type inlets. In buildings of the kind described in the paper, it was quite usual to use nozzle inlets which introduce the air at high velocity, so that a certain amount of turbulence was set up, and the air worked its way into corners which might otherwise have become dead pockets. With these high velocity inlets, the atmosphere was sometimes more lively and stimulating than when slow velocity inlets were used. Could Mr. Etches say whether the ventilation had proved to be successful ?

Mr. A. J. SHEWAN, said, as one of the officials of the company owning the building, he would like to draw attention to the advantages of the portal type of frame. He was pleased that no one would have the opportunity of hanging weights from the bottom tie of a truss. With regard to the construction of the frames themselves, he was of the opinion that the superficial area was much less than of the equivalent type of truss, and therefore the corrosion to be expected in a period of years would be much less. It was also important in that particular factory that dust was kept to a minimum, and it was quite clear that dust on those comparatively small superficial areas of the portal frame was much less than any dust which could collect on the old common truss member.

Major R. M. H. NEAT (Associate-Member) said that while he was no protagonist for roof trusses, it just came to his mind that the American definition of an engineer was 'a man who could do with one dollar what it took another to do with many.'

He had one fundamental question : there was a problem to be solved ; a crane to be housed with a roof over the top of it. Could he ask Mr. Etches if he could give them an idea of the comparative cost between a more orthodox type of structure with a roof truss, and the structure using the portal frame ?

To this Mr. Reisser interjected to add : could they have at the same time the estimated cost of maintenance ? It was not only the initial cost, but the cost of maintenance afterwards that was important, and it would be interesting to have it.

Mr. ETCHES, replying to the discussion, said he valued very highly Mr. Vaughan's words of appreciation.

As far as the test pile was concerned, the settlement was admittedly rather high, but it was loaded with almost 100 per cent overload. The test continued for 24 hours, and the load settlement curve was plotted, and indicated that further settlement was extremely small.

The soil types that it passed through were first ordinary made-up ground, soft river deposits and so on,

as one got along the course of a tidal river, such as the Thames. The pile was driven down to, and came to rest in, the ballast which exists immediately below.

The test pile carried a maximum of 112 tons, but the average working load on the piles was of the order of 45 tons, although there were a few piles which carried rather more than that.

Mr. Fowle had made a point about the guttering. One of the considerations in the drainage of the factory was that the combined soil and surface water of the factory had been previously discharged into the main sewer which ran along North Woolwich Road. These sewers were very much overloaded, and it was the wish of all concerned that no extra load should be put on them, and if possible the original flow should be reduced. For that reason the whole of the surface water at the north end of the site was discharged into the Thames. The combined length of E and J Blocks was of the order of 450 feet. Had there been intermediate down pipes at the north end of E Block, at least one additional run of surface water drain right back to the river would have been required. Since also, it was highly desirable to discharge through a flood valve above the normal high water level, this would have been hardly possible with levels as they existed with this extended run of drain.

The capacity of the gutter was such that it could hold the equivalent of ½ in. of rain over the whole roof. That being so, it seemed quite reasonable to have down pipes at only one end of each shop, the depth of gutter being sufficient to give the necessary hydraulic gradient.

An additional reason for reducing the number of down pipes was to avoid cutting up the existing floors. In point of fact, many large areas were relaid, but in the original plan it was thought more areas could have been re-used, and therefore the less disturbance to the existing floors the better.

Mr. Marshall had made a point about the pitch of the roof which was 10 degrees throughout. There had been no leaks at all through the new roof, in spite of an extremely wet winter. As a matter of interest, the leaks which had occurred had been in the existing roof of F Block, which was a much steeper roof, although of course, there could be no direct comparison since this was a slate roof.

With regard to the distortion of the welding work, that had been anticipated by the fabricators. The flanges of the frames were set up to a curvature away from the webs. When the flange was welded to the web by a pair of continuous fillet welds, distortion caused the flange to straighten, and the finished job was entirely satisfactory.

They had heard a number of remarks about hanging apparatus from roof trusses and frames. He did not want to add anything more, except to say that the clients had provided the available information, and since all the services and the structures were handled by one organisation there was every opportunity of successful co-ordination. An illustration which showed that things which must hang from the roof did so reasonably and on well defined lines, was No. 8, which showed the fluorescent lighting fittings hanging from some of the purlins.

Mr. Catchpole had asked about the centre section of the roof. He thought perhaps there was a point of explanation needed there. The method chosen for the erection of the centre section of each frame was for the two halves to be bolted together at the ridge, and then dropped into place and welded. When it was designed, it was not anticipated that it would be erected in quite that fashion. Had the method used been decided earlier, there would have been no need to bolt the junction plate at the centre of the rafter ; it could have been a welded connection, and he was quite sure it would have been. As for the fit, there was no difficulty as far as he knew, and the joint came together without straining into position.

Mr. Bowen had asked about strengthening the columns in F Block. Fortunately, some information was available and some details were known. The surface of these was hacked back to the steel, shuttering was added, and suitable vertical rods and binders were taken round the column and the whole lot concreted up. There was no additional foundation work needed for the increased column loads, as the existing piles were quite capable of taking this, which as a matter of fact was not very considerable compared with the existing roof load which it was already carrying.

He thanked Mr. Rutherford for his remarks, and he hoped that the loading on the piles might be reasonably reassuring to him.

Mr. Faber had asked about the purlins, but the answer was in the price. It seemed to be nearly always cheaper to pay for material rather than workmanship. Certainly, a welded purlin could be made up using a less quantity of steel, but for such a span as 25 ft. 3 in. there was no doubt in his mind that a simple joist section was much cheaper than a welded section purlin, but this would probably be reversed if much longer spans had to be considered.

As for the protection given to the buildings, he could say that the nitro-cellulose store was given a 30 ft. clearance as a minimum from all other buildings. As for drainage, there was a complete floor below the steel storage tanks to catch any liquid, in the event of damage to any one of the tanks. A sprinkler system was installed throughout, and in special sections such as the sub-station, this was of the emulsifier type.

The diffuser outlets, or inlets, as the case might be, appeared to be quite satisfactory. He could say that it was quite comfortable to be in the shops at the moment with the plant working. There was no noticeable air movement, and yet it seemed quite fresh and pleasant.

He quite agreed with Mr. Shewan about the corrosion —it certainly must be less on the clean lines of a welded structure presenting less opportunity for corrosion to start and work unnoticed. It certainly did make the maintenance job much less formidable, particularly in any factory which was liable to more corrosion than some. The ledges for dust were certainly reduced to a minimum, which was particularly important in this case.

The question of cost had been raised by Mr. Neat. It was rather difficult at the moment, because the building was not quite complete and the final accounts had not been settled. The question of cost was one which he did not feel able to answer adequately at that moment.

There was one thing which he thought should have been said before—he did appreciate the help given by Mr. Reisser of Murex in producing the film, and the work of Miss Hickford in taking it. He thought all were particularly pleased to have a record of the steelwork erection and the early stages of the work on the site in that form.

The Chairman closing the meeting, thanked Mr. Etches on behalf of those present and on his own behalf for the very interesting evening he had provided and for the way in which he had answered the questions.

Institution Notices and Proceedings

FORTHCOMING MEETINGS

The following meetings will be held at 11, Upper Belgrave Street, London, S.W.1 :—

Thursday, October 11th, 1951

Presidential Address by Mr. Walter C. Andrews, O.B.E., M.I.C.E., M.I.Struct.E., 6 p.m.

Thursday, October 25th, 1951

Ordinary General Meeting for the election of members at 5.55 p.m., followed by an Ordinary Meeting at 6 p.m., when Dr. P. W. Abeles (Member) will give a paper on "Some New Developments in Prestressed Concrete."

Thursday, November 22nd, 1951

Ordinary General Meeting for the election of members at 5.55 p.m., followed by an Ordinary Meeting at 6 p.m., when Mr. G. B. R. Pimm, M.I.C.E. (Past President) will give a paper on "The Structural Engineer as Arbitrator, Expert Witness and Advocate."

Thursday, December 13th, 1951

Ordinary General Meeting for the election of members at 5.55 p.m., followed by a Joint Meeting with the British Section of the Societe des Ingenieurs Civils de France at 6 p.m., when Monsieur D. Laval will give a paper entitled "Reconstruction of the Port and Bridges at Rouen."

JANUARY EXAMINATIONS

The Examinations of the Institution will next be held at centres in the United Kingdom and overseas on January, 8th and 9th 1952 (Graduateship), and on January 10th and 11th (Associate-Member).

REPRESENTATION

The Council have made the following nominations of members to represent the Institution :—

British Standards Institution : Sub-Committee on Large Aluminium Alloy Structural Rivets
Mr. C. F. Pike (Member).

Joint Committee on Materials and their Testing
Mr. W. H. Woodcock (Honorary Editor).
Deputy : Mr. Ewart S. Andrews, B.Sc., M.I.C.E., (Past President).

OBITUARY

The Council regret to announce that Mr. L. F. Summerfield, M.C., Past Chairman of the Western Counties Branch, died suddenly at the end of June. Mr. Summerfield, who was 61, was elected to membership of the Institution in 1925. He served on the Branch Committee since 1932, holding the office of Chairman from 1948 until 1950.

PERSONAL

Mr. Arthur Burton (Member) has set up in practice as a Consulting Structural Engineer at 307, Acklam Road, Middlesbrough.

EXAMINATIONS

Preparation for the Examinations of the Institution by Attendance at Technical Colleges

A candidate for Graduateship or Associate-Membership may be able to attend a technical college ; these notes are intended to guide him in choosing the most suitable instruction.

Preparation for the Graduateship Examination

Technical colleges offer :

(a) Full-time courses for degrees or Higher National Diplomas in Building or Engineering.

(b) Part-time day or evening courses for Higher National Certificates in Building or Engineering.

If he obtains a Higher National Certificate or Diploma complying with Appendix II, Section V, of the Regulations Governing Admission to Membership, the candidate will be exempted from the Graduateship Examination.

Alternatively, he may study subjects selected from the available courses and sit the Graduateship Examination. At technical colleges courses are usually available in Building Science or Engineering Science, Strength of Materials, Theory of Structures and Surveying, but students are not normally allowed to select subjects from National Diploma or Certificate courses unless they can show evidence of sound training in more elementary studies. The advice of the College Authorities should be followed.

Preparation for the Associate-Membership Examination

At some technical colleges there are part-time courses in Structural Engineering which cover the syllabus of the Associate-Membership Examination. At other colleges the candidate must rely on Higher National Certificate courses or on advanced courses in Building, Civil Engineering or Municipal Engineering : these cover only part of the requirements for the Associate-Membership Examination.

Colleges in the first category provide at least two years of instruction in Theory of Structures and in Structural Engineering Design and Drawing up to Associate-Membership standard. They also give instruction in Structural Specifications, Quantities and Estimates.

The Colleges which have informed the Institution that courses in Structural Engineering are available are :—

Belfast College of Technology.
Birmingham Central Technical College.
Bolton Municipal Technical College.
Bradford Technical College.
Derby Technical College.
Dudley and Staffordshire Technical College.
Glasgow Royal Technical College.
City of Liverpool College of Building.
L.C.C. Brixton School of Building.
L.C.C. Hammersmith School of Building and Arts and Crafts.
Manchester College of Technology.
Middlesbrough Constantine Technical College.
Salford Royal Technical College.
South-West Essex Technical College, Walthamstow, E.17.

Colleges in the second category provide instruction in Theory of Structures from which the student may reach Associate-Membership standard, but instruction in Structural Engineering Design and Drawing and in Structural Specifications, Quantities and Estimates is not usually so complete. The colleges which have informed the Institution that such courses are available are :—

Brighton Technical College.
Cardiff Technical College.
Huddersfield Technical College.
Leeds College of Technology.
London Battersea Polytechnic.
London Northampton Polytechnic.
L.C.C. Westminster Technical College.
Plymouth and Devonport Technical College.
Preston Harris Institute.
Wigan Mining and Technical College.
Woolwich Polytechnic.

Students attending colleges in the first category are advised to take the organised courses in Structural Engineering. Students of Graduate Membership standard will usually be allowed to select subjects from courses provided by colleges in the second category.

DRURY MEDAL AWARD

The third competition for the above award will take place in 1951, and the subject will be the design of a reinforced concrete garage with provision for erection above it of future superstructure, the garage being subjected to unbalanced earth pressure.

Graduates and Students of the Institution who wish to compete are invited to apply for full details to the Secretary ; envelopes to be marked in the top left-hand corner "Drury Medal Award."

The closing date for the competition is October 1st, 1951.

The general conditions of the competition are as follows :—

1. The competition shall be for Graduates and Students of the Institution of not more than 25 years of age.

2. The subject of the competition shall be a design of a structural character, that is to say, primarily structural design, not planning.

3. The subject of design and conditions shall be prepared and issued biennially by a group of five members appointed by the Council.

4. The Literature Committee shall appoint a jury of not less than five to examine the works submitted and to interview candidates, if found necessary.

5. In order to show that the work submitted is solely the work of the competitor, the documents submitted shall be countersigned by a corporate member of the Institution, or, failing this, shall be accompanied by a declaration on a prescribed form signed by the candidate in the presence of a Justice of the Peace or a Commissioner for Oaths.

RESEARCH AWARDS

The Council have instituted a Research Prize Fund, from which awards may be made annually to the author or joint authors of papers describing original research which they have carried out. Research awards may be made for papers read at Headquarters or in the Branches and published in the Journal, or for papers published in the Journal only without being read at an open meeting.

The assessment for such awards will be made annually, but awards will be made only to the contributors of such papers as reach a standard judged by the Literature Committee to be satisfactory.

Work submitted under this scheme must be original and may include any of the following :—

(a) investigations of an experimental or analytical character ;

(b) studies of historical or statistical records ;

(c) improvements in principles or methods of construction ;

(d) research into methods of structural engineering and building, the nature and use of plant and the organisation of engineering work ;

(e) any related or combined studies which are deemed by the Literature Committee to be of a research character.

In cases where the research work described in the paper was not the work of one individual, the names of all the collaborators should be given in the paper.

Awards may take any or all of the following forms :— A research medal ; a diploma ; a money prize.

Application for consideration for a research award must be made to the Secretary of the Institution, and in preparing papers for reproduction in the Journal, authors must comply with the conditions laid down for all such contributions. Particulars of these conditions may be obtained from the Secretary.

In judging research papers, the following factors will be considered :—

(a) the nature of the subject and its conclusions ;

(b) the value of the paper in advancing the science and art of structural engineering ;

(c) the standard of preparation and orderly arrangement of the subject matter.

Research papers will also be eligible for adjudication for the Institution Medals if they comply with the Regulations governing those awards.

The closing date for the receipt of applications in respect of papers published in the Journal between October, 1950, and September, 1951, is October 31st, 1951.

THE MACLACHLAN LECTURE

General Conditions

Through the generosity of Mr. John MacLachlan (Retired Member), the Council was able in 1948 to institute an Annual Lecture to be competed for by Associate-Members. The conditions of the presentation are as follows :—

1. The Institution of Structural Engineers shall institute a written lecture to be known as the MacLachlan Lecture and to be held annually.

2. The subject of the Lecture may be on any aspect of Structural Engineering so long as in every second year the subject shall be confined to steel structures.

3. Entrance into the competition for the Lecture shall be confined to Associate-Members of the Institution, who are under the age of 32 years.

4. All papers entered for the competition shall be submitted to assessors to be appointed by the Council of the Institution, and all such papers (including the prize-winning lecture) shall be available for publication in the Journal of the Institution at the discretion of the Council.

5. No paper submitted shall have been published or read elsewhere.

6. The winner of the competition shall be required to present the Lecture to a meeting of the Institution at which he will be presented with the sum of £17 10s. 0d.

7. Should a competitor's paper be considered worthy of ranking second in merit he will receive a consolation award of £5.

8. In the event of there being no winner of the competition in any one or more years, whether because no lecture is submitted or because no lecture submitted is considered to be of sufficient merit to warrant an award, or for any other reason, the Institution shall transfer these sums to the Research Fund of the Institution.

Particulars of the Competition for 1952

1. The MacLachlan Lecture will be given at a meeting of the Institution to be arranged towards the end of 1952.

2. The subject of the Lecture shall be on any aspect of structural engineering.

3. The work should be submitted as the script of a lecture which the author, if successful in the competition, will deliver before an audience in the course of about

one hour. The development of mathematical formulæ and detailed calculations should be avoided as far as possible in the text ; if they are essential they should be embodied in appendices. Photographs, drawings, graphs, etc. which would appear as illustrations to the lecture in published form, should accompany the script. If additional illustrations would be shown as slides, a list of these should be included.

4. Six copies of each Lecture should be submitted and should be addressed to the Secretary of the Institution.

5. The closing date for the receipt of entries by the Institution is Monday, March 31st, 1952.

LONDON GRADUATES' AND STUDENTS' SECTION

A visit has been arranged to London Airport on Saturday morning, September 15th. Details will be circulated.

Mr. B. T. Hale has resigned from the Committee and Mr. J. Grace has been co-opted in his place.

Hon. Secretary : D. B. Rogers, 4, Portland Rise, Finsbury Park, N.4.

BRANCH NOTICES

LANCASHIRE AND CHESHIRE BRANCH

A visit to Woodhead to see the construction of the new double line tunnel, has been arranged for Wednesday, September 26th.

The opening meeting of the Session will be held at the Engineers' Club, Albert Square, Manchester, at 6.30 p.m., on Tuesday, October 16th, when the Chairman's Address will be given by Mr. R. Gray (Member). Films of the new Tyne Bridge and the Ford Works at Dagenham will be shown. The President and the Secretary of the Institution will attend the meeting.

Hon. Secretary : A. S. Sinclair, A.M.I.Struct.E., 28, Kenwood Road, Stretford, Lancs.

MIDLAND COUNTIES BRANCH

The Branch Annual Dinner will be held at the Botanical Gardens, Birmingham, on Saturday, October 6th.

A meeting of the Branch will be held at the James Watt Memorial Institute, Birmingham, at 6 p.m., on Friday, October 26th, when the Chairman's Address will be given by Mr. W. H. Veal, F.R.I.C.S. (Member).

Hon. Secretary : E. R. Deeley, A.M.I.Struct.E., Arranmoor, Adshead Road, Dudley, Worcs.

GRADUATES' AND STUDENTS' SECTION

The first meeting of the Session will be held at the James Watt Memorial Institute, Great Charles Street, Birmingham, on Monday, October 29th, at 7 p.m., when a paper on "The Dome of Discovery" will be given by Mr. J. S. Allen (Graduate).

Hon. Secretary : M. H. Evans, B.Sc., 42, Church Hill Road, Handsworth, Birmingham, 20.

NORTHERN COUNTIES BRANCH

The opening meeting of the Session will be held at the Cleveland Scientific and Technical Institution, Middlesbrough, on Tuesday, October 2nd, when the Chairman's Address will be given by Mr. J. Gerrard, M.I.Mech.E. (Member). A similar meeting will be held at the Neville Hall, Newcastle, on Wednesday, October 3rd. Both meetings will commence at 6.30 p.m., preceded by tea at 6 p.m., and will be attended by the President-Elect and the Secretary of the Institution.

Hon. Secretary : Ian MacGregor, M.I.Struct.E., 9, Ellison Place, Newcastle-upon-Tyne, 1.

NORTHERN IRELAND BRANCH

The opening meeting of the Session will be held at the College of Technology, Belfast, at 7.30 p.m., on Tuesday, October 2nd, when the Chairman's Address will be given by Mr. M. C. Gillies (Member).

Hon. Secretary : S. G. Duckworth, M.I.Struct.E., "Lisleen," 13, Finaghy Road North, Belfast.

SCOTTISH BRANCH

The opening meeting of the Session will be held at the Ca'doro Restaurant, Glasgow, at 6 p.m., on Wednesday, November 7th, when the Chairman's Address will be given by Mr. R. Summers (Member).

The Annual Dinner and a Buffet Dance will be held at the Central Hotel, Glasgow, on Thursday, November 8th.

Hon. Secretary : D. G. Drummond, B.Sc., A.M.I.C.E., A.M.I.Struct.E., 11, Woodside Terrace, Glasgow, C.3.

SOUTH-WESTERN COUNTIES BRANCH

Hon. Secretary : E. W. Howells, M.I.Struct.E., c/o Messrs. T. L. Harding & Sons, Ltd., 10-12, Market Street, Torquay.

WALES AND MONMOUTHSHIRE BRANCH

The opening meeting of the Session will be held at the South Wales Institute of Engineers, Cardiff, at 6.30 p.m., on Tuesday, October 30th, when the Chairman, Mr. G. H. Hodgson, M.I.C.E. (Member) will give an Address on "Problems in the Conveyance and Storage of Crushed Rock." The President and the Secretary of the Institution will attend the meeting.

A meeting will be held at the Mackworth Hotel, Swansea, on Wednesday, October 31st, at 6.30 p.m., when the Chairman's Address will be repeated and will be followed by discussion.

Hon. Secretary : E. R. Steward, A.M.I.Struct.E., "Edrom," Ashleigh Road, Blackpill, Swansea.

WESTERN COUNTIES BRANCH

Hon. Secretary : C. E. Saunders, M.I.Struct.E., Dunkery, Edward Road, Walton St. Mary, Clevedon, Somerset.

YORKSHIRE BRANCH

The opening meeting of the Session will be held on Wednesday, October 17th.

Hon. Secretary : E. Wrigley, A.M.I.Struct.E., City Engineer's Dept., Civic Hall, Leeds, 1.

UNION OF SOUTH AFRICA BRANCH

The Annual General Meeting and Dinner were held at the Langham Hotel, Johannesburg, on Wednesday, May 30th, when the following Committee was elected for the Session 1951-52 :—

Chairman : D. R. Ryder.

Vice-Chairman : Dr. A. J. Ockleston.

Committee Members : G. M. Frost, F. F. Binswanger, C. A. Pringle, C. A. Rigby, G. M. Anderson, J. G. Hay, D. D. Thorp, T. Breslin, A. E. Tait.

Branch Hon. Secretary : A. E. Tait, B.Sc., A.M.I.C.E., M.I.Struct.E., P.O. Box No. 3306, Johannesburg. During week-days, Mr. Tait can be contacted in the City Engineer's Department, City Hall, Johannesburg. 'Phone 34-1111, ext. 257.

Natal Section Hon. Secretary : E. G. Bennett, A.M.I.Struct.E., c/o Reinforcing Steel Co., Ltd., P.O. Box 478, Durban.

Cape Section Hon. Secretary : R. Stubbs, M.I.Struct.E., P.O. Box 1692, Cape Town.

The President 1951-52

Mr. Walter C. Andrews, who will take office on October 11th, as President of the Institution for the Session 1951-1952, represents the younger generation of engineers. He is of the twentieth century and is in fact the first President of the Institution to have been born after the close of the nineteenth century. At the time of his birth in London on March 1st, 1902, Queen Victoria had been dead just over a year and with the new reign had come a quickening in the development of new ideas in every walk of life, and in none more marked than in engineering. The age of steel and concrete had dawned, and six years later saw the foundation of the Concrete Institute, afterwards re-named the Institution of Structural Engineers.

Mr. Walter C. Andrews, O.B.E., M.I.C.E., M.I.Struct. E.

Too young to have taken part in the First World War, Walter Charles Andrews commenced his engineering career in 1918 as a pupil under agreement with Mr. R. W. Vawdrey (a Founder Member of the Institution), at the Indented Bar and Concrete Engineering Company. After three years' practical training he joined the staff of the Consulting Engineers, Walter Bridges & Co., as a structural assistant, and in 1927 he became junior partner in the practice of C. W. Glover & Partners.

During the Second World War, Mr. Andrews was appointed Director of Aircraft Production Factories at the Ministry of Aircraft Production. In the period 1941-1946, he was responsible for factories for incendiary bombs, aircraft parts, air frames, repair shops, an underground factory, aircraft hangars up to 180 ft. span, reinforced concrete runways, heavy duty hammer foundations (in connection with which he initiated research), and he acted as Adviser to the Ministry on foundation problems. The total value of these works was approximately £30,000,000, and for his services during this period, Mr. Andrews was appointed an Officer of the Most Excellent Order of the British Empire.

In 1946, he set up in practice as a Consulting Structural Engineer, engaging in the design and construction of all types of structures in steel, reinforced concrete and timber, and specialising in building and machinery foundations.

The new President's connection with the Institution of Structural Engineers began in 1925, when he was elected a Graduate. Two years later he became an Associate-Member, and his active participation in Institution affairs commenced in 1937, when he was appointed a member of the Panel for Hollow Tile Floors. Since then he has served on the Concrete Sectional Committee, the Science Committee, Literature Committee, Research Committee, Finance and General Purposes Committee, Membership Committee, Legislation Committee and Bye-Laws Committee. He has been Chairman of the following Committees :— Concrete Sectional, Science, Finance and General Purposes and Membership.

In 1940, Mr. Andrews transferred to full Membership of the Institution and in 1943 he was elected to the Council. He became Honorary Curator in 1946, Honorary Secretary in 1948, and Vice-President in 1949.

He has written a number of technical articles and papers, and in 1944 was awarded the Institution's London Prize for a paper on "Foundations in Combustible Material." He was also awarded the Gold Medal of the Institution of Engineers and Shipbuilders in Scotland with Mr. L. H. A. Crockett, for a paper on "Forging Hammers and their Foundations."

Mr. Andrews represents the Institution on the Code of Practice Council, the Joint Committee of the Four Institutions, the Organising Committee for Building Research Congress, and various technical Committees. He is Chairman of the Codes Committee for Suspended Floors and on a number of occasions he has served as a member of Committees of Assessors for Institution prizes and competitions.

Almost the whole of the President's professional life has been spent in Consulting practice. He is a Member of the Institution of Civil Engineers, a Member of Council of the British Section of the Societe des Ingenieurs Civils de France, and a member of the Association of Consulting Engineers. For a number of years prior to the war he lectured on building subjects at Evening Institutes. He is interested in local government and has been a member of a local council.

Gifted with quick perception and a lively wit, Mr. Andrews is a valued member of any Committee. These qualities, together with his exceptional grasp of detail, have made him an efficient chairman. To the office of President he brings a freshness of outlook and a likeable personality which cannot fail to win for him many new friends, or further to enhance the reputation of the Institution.

The Institution of Structural Engineers' Benevolent Fund

TO ALL MEMBERS OF THE INSTITUTION

The Council has been kind enough to permit me to use the columns of the Journal to communicate with you regarding the Institution's Benevolent Fund, and in this connection I approach you as Honorary Secretary of the Fund and not in my official capacity as Secretary of the Institution, for on the subject of the Fund I feel there can only be a personal approach.

The Benevolent Fund was instituted in 1936 when a great many of you were not as yet members of the Institution and I should like to take this opportunity so kindly provided by the Council to acquaint you with one or two facts about the Fund.

At the end of the first year a sum of £311 stood to the benefit of the Fund. This has gradually increased until by the end of the year 1950 we have accumulated funds of £7,240, of which £6,763 has been invested. Since 1936, we have distributed £1,344. It has taken fifteen years for the Fund to become established, although only in a small way, since the sum which we are enabled to dispose of annually in relief is derived from interest on investments and annual subscriptions only and now approximates £838 per annum.

I wish to take this opportunity of tendering to every member who has contributed, either by annual subscription or by donation from time to time, the very deep appreciation and thanks of the Committee of Management and myself, and I know that these supporters will continue to follow the dictates of their generous feelings.

There are many of you, however, who may not have thought of contributing, or to whom the Benevolent Fund has not become known. To you I would appeal to give a thought to those who, because of advanced age or infirmity may not be able to exist without help in the present difficult times. The vicissitudes of life affect us all and any of us may leave those dear to us in greatly restricted circumstances. I feel that it is only necessary for me to bring to your notice the fact that the Fund needs building up to a much higher figure for you to respond in a very generous manner.

Enclosed with this copy of the Journal are forms which provide for donations and annual subscriptions or for payments under a Deed of Covenant. The form of Banker's Order may be used if convenient. The undertaking to subscribe for seven years under a Deed of Covenant (applicable to those resident in the United Kingdom and paying the standard rate of income tax) is of particular value as the Fund benefits by the refunding of tax, which at the 1951 rate of 9s. 6d. amounts to approximately 90 per cent. of the subscription ; a subscription of £1, for example, realising a total sum of £1. 18s. 1d.

Thanks to the Council of the Institution the administrative costs of the Fund are negligible since most communications are enclosed with Institution matters and, of course, all services of the Committee of Management and officials are of an entirely honorary nature.

On behalf of all those who may benefit by your benevolence in the future I offer you my grateful thanks.

R. F. MAITLAND,
Honorary Secretary
Institution of Structural Engineers'
Benevolent Fund.

Some New Developments in Prestressed Concrete*

By P. W. Abeles, D.Sc., M.I.Struct.E.

Synopsis

Two ways of approach to prestressed concrete are considered, and results of research discussed. Recent tests carried out on composite partially prestressed bridge slabs by the Civil Engineer's Department, The Railway Executive, Eastern Region, are described and some results of fatigue tests mentioned. The essential features of prestressed concrete are investigated by distinguishing five different cases of under-reinforced prestressed concrete beams and three classes of prestressed concrete (monolithic members with bonded steel, monolithic members with non-bonded steel, and an assembly of blocks without adhesion at the joints and non-bonded steel).

Designs of the five original bridges erected in 1949 are discussed, and a later economic composite partially prestressed bridge deck design is described, which is being employed in the reconstruction of 14 railway overbridges, 10 of which have already been completed. The paper also contains a short report on the manufacture, supervision and erection of bridge decks. The application of partial prestressing to other designs is discussed.

In a second group, three designs are described, which relate to the application of prestressed concrete to structures affected by mining subsidence, including a novel turntable foundation.

Introduction

In a report on tests[1], published in the June, 1950, number of THE STRUCTURAL ENGINEER, it has been shown that prestressed concrete can be designed to act in any desired manner from an elastic material, similar to glass or cast iron with a brittle failure, to a plastic yielding material resembling reinforced concrete, but differing from the latter by its remarkable resilience. Because of these characteristics, and the possibilities of saving steel and expense, prestressed concrete has rapidly become of great interest to the structural engineer. Substantial evolution has taken place in recent years, although the basis for practical design was presented some 25 years ago. In the present paper some developments are discussed, with which the author is actively concerned. They fall into two groups, the main one relating to partial prestressing, and the other to constructions designed only for ultimate load conditions, as affected by mining subsidence. Partial prestressing was, until recently, a controversial and rather theoretical subject, but technical soundness has been proved by tests and its economic advantage by employment in decks of road bridges over railways, two of which have already been built.

Most of the tests, designs and structures shown in both groups relate to work carried out by the Civil Engineer's Department of The Railway Executive, Eastern Region (formerly L.N.E.R., Southern Area), during 1944-51. The author would like to state that the opinions and conclusions expressed in this paper are his own and should not be taken as the official views of the Railway Executive. The designs, however, by their application, are shown to be acceptable to the Railway Executive.

Paper to be read before the Institution of Structural Engineers at 11, Upper Belgrave Street, London, S.W.1, on Thursday, October 25th, 1951, at 6 p.m.

I. The Development of Partial Prestressing

(1) Two Different Ways of Prestressing Concrete

There are two ways of approach to prestressed concrete. One, called "full" prestressing, has been promoted by M. E. Freyssinet, the main point being that permanent compressive stresses should be introduced to counteract the tensile stresses under working load, thus creating a new homogeneous material under working load conditions, and allowing also a structural member to be assembled from individual blocks without any adhesion between them. If this idea is applied to a monolithic concrete member, the tensile resistance is ignored. As will be shown later, this neglect results in a difference between the actual behaviour of the material and that assumed. M. Freyssinet has presented in his paper[2], delivered in 1949, impressive examples of the practical application of prestressing for the promotion of which he has achieved so much. From his viewpoint, based on the idea of an assembly of blocks as, e.g., in an arch, he has emphasised the great importance of the "transformation" load, at which the tensile fibre of the structure ceases to be prestressed and becomes discontinuous, passing, as he states, from a zone of security to that of "insecurity." This is correct only for members assembled from individual blocks and held together by tensioned ties, but does not agree with the actual behaviour of a monolithic concrete member with well bonded tensioned wires.

The second method of approach considers prestressing as a development of ordinary reinforced concrete. This was first suggested in 1939 by the Austrian pioneer of reinforced concrete, the late F. V. Emperger, Hon. M.I.Struct.E.[3], the aim being to save steel and to reduce the extent of cracking when using work hardened steel, thus improving reinforced concrete by prestressing. All previous proposals, such as those by the Austrian Mandl, the German Koenen, and the Norwegian Lund, related to the creation of fully prestressed structures, but were unsuccessful since the tensioning stress in the steel was too low, and became soon ineffective. Emperger had in mind to have effectively prestressed wires added to the main reinforcement (in mild or work-hardened steel), these wires preferably being twin twisted, with transverse bars inserted, to increase their resistance to slipping.

In ordinary reinforced concrete the permissible steel stresses are limited to avoid wide cracks which might develop if the steel extended throughout its full length between the cracks. This may occur with mild steel of large diameter embedded in weak concrete, when the bond is entirely destroyed in the region of the greatest bending moment. Thus, under working load conditions, cracks of 0.01 in. width may develop in a design using the present permissible steel stresses, which width may be considered as the permissible

limit to avoid corrosion. However, with increased bond resistance, the conditions are considerably improved. This enhanced adhesion is obtained by using (a) thinner reinforcing bars, (b) reinforcement with special indentations, or by twisted bars, and (c) concrete of higher strength. In 1935/6, the author carried out comparative tests on rectangular beams [4], [5], reinforced with ordinary mild steel, and also with high strength steel having a yield point of approximately 42 tons per sq. in. (ultimate strength 63 tons per sq. in.), using weak and high strength concretes. These tests have proved that, with high steel stresses, the extent of cracking is greatly reduced if high strength concrete and bars of small diameter are employed.

On the results of his tests, the author suggested in 1940[6], a further development of Emperger's idea by using throughout high tensile steel, part of which being tensioned and part untensioned, or by a later alternative, the entire reinforcement being effectively tensioned to a lower degree. The main idea of this solution, called partial prestressing, was originally to make full use of the tensile resistance of the concrete and to permit the occurrence of fine hair cracks, but subsequently, it has become possible to adjust this principle in view of the actual behaviour of prestressed concrete, which has shown that such cracks need not necessarily occur, or become visible. The author's proposal to use the same high strength wire and the same tensioning stress as employed with full prestressing, but to reduce the prestressing force considerably (e.g., to half the magnitude), was immediately denounced as an inadequate solution[7], [8], since it was not realised that the extent of cracking is greatly reduced when a good bond is obtained. Statements made in[7] regarding the tensile stresses have since proved to be incorrect, but at that time it was impossible to prove otherwise. For example, it was stated in[7] that under a " heavy " concrete tensile stress of 923 lb./in.^{2} " cracks under load are certain " and that cracks occurring at a stress of 1,205 lb./in.^{2} " cannot be regarded as harmless." Again, in paper[8] it was even stated that at a concrete tensile bending stress of 771 lb./in.^{2} calculated in an example for partial prestressing " strong cracking existed," and also that the author's suggestions " did, in fact, eliminate the greater part of advantages derived from Freyssinet's processes, in exchange of imaginary other advantages." In the controversy[7] the author was told " that the provision of unstretched wires or rods in addition to the stretched wires or rods was proposed by M. Freyssinet as long ago as 1928, but not in the inadequate manner suggested by " the author. However, there would be no advantage in adding unstretched wires or rods in connection with full prestressing, with its relatively large tensioned steel reinforcement, the purpose of the untensioned wires according to the author's proposal being to ensure the required factor of safety against breaking, while the sectional area of the tensioned wires is reduced to the extent required for obtaining the desired effective prestress. The author concluded the controversy[7] with the words, " It will depend on tests to prove whether my ideas are adequate from a technical point of view." How far the author has succeeded in this is shown in the following.

(2) The Justification of Partial Prestressing

The author had to prove the suitability of his own suggestions by entirely new investigations. Emperger had discussed his ideas with the author, but the results of the tests reported in his book[3] were unknown to the author for some years. Preliminary comparative tests were carried out in London in 1942, and the results published in a paper[9]. These tests demonstrated the extraordinary resilience of prestressed concrete, and distinguished this material from ordinary reinforced concrete. This was supported by further tests, particularly those published recently[10], supplemented by tests presented in this paper.

The tests[1] and [10] and those presented in this paper have clearly proved that the author's basic idea proposed in 1940 was correct. Moreover, it has been shown that, in a prestressed high strength concrete, the stress at which fine cracks become just visible is much higher than that at which, according to paper[8] heavy cracking would have already occurred. In the criticism[7] the author's proposal was called " a compromise between ordinary construction and prestressing." This idea is apparently held also by M. Freyssinet, who recently said " There is no half-way house between reinforced and prestressed concrete ; any intermediate systems are equally bad as reinforced structures or as prestressed structures and are of no interest." This conception of partial prestressing as intermediate case between ordinary reinforced and fully prestressed concrete is erroneous. Either a structure is effectively prestressed, then it *is* prestressed whether fully or partially, or the prestress has become ineffective, then it *is not* prestressed. Apparently partial prestressing is even yet not considered by M. Freyssinet as a " correct " kind of prestressing, since he says that the " misapprehension " of the discontinuity " forms the origin of a number of erroneous ideas capable of doing great harm to the correct development of the idea of prestressing." However, these sweeping statements are entirely contradicted by the facts which he quite correctly expressed in the following statement, also taken from[2] : " In a prestressed structure, a load in excess of the transformation load is very clearly indicated by an exaggerated deflection, which, however, being completely reversible, brings no unfavourable consequences in its train."

The author must express his obligation to the criticisms[7] and [8], in which his proposal was condemned as " inadequate," thus being forced to prove the validity of his assumptions, which required extensive work and much trouble, but included interesting research. Partial prestressing is much too complex a problem to be solved on an entirely fundamental basis. While the new methods of physics offer a very interesting fresh approach, special prolonged studies and knowledge of the modern development of theories concerning matter are required. The elementary theory of elasticity is insufficient to explain the behaviour of material frequently showing inelastic properties. It is stated in [11] " Too often in engineering practice an attempt is made to interpret the mechanical behaviour of real materials solely in terms of elastic theory rather than to consider the nature of the material and its actual response to forces as the manifestation of its heterogeneous internal structure." Mr. Freudenthal suggests this more fundamental approach in the research of the behaviour of materials based on the laws governing the formation of matter from particles and larger structural elements at different levels of aggregation. This approach derived from the recent developments of physics, considers the individual crystals formed from atoms, ions or molecules, and is certainly most interesting but still requires much study and seems particularly difficult to apply to a combination of two heterogeneous materials each of which would have to be studied separately before their combined action under specific conditions could be investigated. In these circumstances the actual test, preferably on full size specimens, seems to be more convincing, leading more quickly to results by

giving " ocular proof " of particular properties. It will be seen in the following that these tests have established the technical adequacy and economic superiority of partial prestressing.

The author would like to acknowledge with gratitude the understanding which he received from the Railway Executive[12], and particularly from the Civil Engineer of the Eastern Region[10]. The author appreciates highly the encouragement he received from Professor R. H. Evans[13], Professor G. Magnel[14], [15], and Dr. F. G. Thomas[16]. The first publication particularly recognises the adequacy and economy of partial prestressing. Thus it has been possible to incorporate the essential ideas of partial prestressing in the " First Report on Prestressed Concrete " prepared by an *ad-hoc* Committee, of which Committee the author was a member. It may be stated that the development of partial prestressing according to the author's proposals, using high strength wire, is so far limited to Great Britain and will have to be introduced into other countries.

Before discussing the essential features of full and partial prestressing in detail, various investigations and tests are explained, which lead up to the full interpretation of prestressing. After dealing with these more theoretical considerations, the development of an economic bridge design is shown. Some bridges are described, including manufacture and erection. Some other designs are then discussed.

(3) The Importance of High Bond Resistance

The magnitude of the bond resistance is of great influence upon the extent of cracking. In 1933/35, the author had an opportunity of carrying out extensive tests upon spun concrete members[17]. They were reinforced with high carbon steel of a strength of 63-70 tons per sq. in., and the concrete had a high degree of density and strength due to centrifugal moulding. A very good distribution of cracks with limitation of their width occurred, which indicated that increased adhesion was of great importance. The purpose of these tests on rectangular beams[4], [5], mentioned before, was to investigate whether this high strength steel could be employed in reinforced concrete. Three kinds of high strength concrete were tested with three different percentages of reinforcement, and weak concrete with two percentages, as indicated in Table I. For each of the 11 types of beams, four specimens were tested. In the table, the mean values of the results are shown for each beam by the ratios of $r_1 = \frac{t_{max}}{t_y}$ and $r_2 = \frac{t_{max}}{t_{ult}}$, whereby t_{max} is the calculated steel stress obtained in the conventional manner, for failure load, t_y is the yield point stress, and t_{ult} is the ultimate steel strength.

It is seen that with weak concrete[1], the calculated steel stress t_{max} exceeds the yield point only for the lower percentage, whereas with the high strength concrete, t_{max} is in all cases greater than t_y, and with a low percentage and a small diameter bar, even approaches the ultimate strength, although the steel has a definite yield point stress, the ultimate extension of steel being up to 20 per cent. From these results it is seen that with bars of 5/16 in. and even 15/32 in. diameter, apparently a high degree of adhesion is obtained, provided these bars are well embedded in high strength concrete. Instead of a few wide cracks, a greater number of fine cracks developed. The average strain in the steel, measured over lengths of approximately 4 in., was considerably less than the calculated value, and thus the bond between steel and concrete must have remained intact except for the immediate vicinity of each crack which was in good agreement with the limited extent of cracking. Similar effects were obtained with work-hardened steel, as shown in the papers[18] and [19]. In these tests, not only did the calculated steel stresses exceed the ultimate steel strength in many cases, but failure occurred in individual cases by fracture of the steel, which proved the high extent of adhesion. In the discussion of paper[18] the author has given an interpretation for the excess of the calculated steel stress over ultimate strength by comparison with the stress concentration in notches of steel bars.

Recent tests may be mentioned, which were reported in the lecture[18]. It was possible to load beams up to the fracture of the tensile reinforcement of high strength steel comprising wires gauge 12 of a strength of 120 tons per sq. in., the ratio $r_2 = \frac{t_{max}}{t_{ult}}$ indicating the efficiency being 1.09.

(4) Preliminary Comparative Tests on Prestressed Concrete

Hoyer[20] published results of comparative tests on four sets of beams of the same cross section, using piano wire of an ultimate strength of 165 tons per sq. in. In each case, the ultimate load was nearly the same, although the initial prestress varied in these four series between 6.35, 25.3, 50.6 and 76 tons per sq. in.

Emperger published in [3] test results on four sets of beams of rectangular and T-shape of low strength concrete (3,200 to 3,550 lb. per sq. in.). Some of these beams were reinforced with mild steel having a yield point of 17.1 tons per sq. in. and some with work-hardened Tor-steel of an equivalent yield point of 25.4 tons per sq. in. Emperger combined in one set 95 per cent. of mild steel with 5 per cent. of wire reinforcement, and in the other three sets, 81, 64 and 58 per cent. work-hardened Tor-steel with 19, 36 and 42 per cent. wire reinforcement respectively. The wire had an ultimate strength of 95 tons per sq. in., and a yield point of 76 tons per sq. in., but the initial tensioning stress was as low as 25.4 tons per sq. in. Failure occurred when approximately the yield point in the mild steel or Tor-steel and the ultimate strength in the wires were reached; in two of the series the wires really fractured at failure.

Professor Evans carried out other comparative tests[21] whereby two sets of beams were compared, one having wire reinforcement, and the other high strength steel rods. Five different cases were established for each set, by providing four prestressing forces of different magnitude, the fifth set being non-prestressed. In the four cases the entire reinforcement was tensioned uniformly. In one alternative, with rod-reinforcement, having the highest prestress, the bond was intentionally destroyed beforehand, and thus the different behaviour compared with the bonded solution. The non-bonded beam had a much greater deflection, since the rods could extend over the entire length, again proving the beneficial influence of bond. In addition, these tests showed that it is possible to find any convenient intermediate case where the deformation and cracking is limited to a definite extent. This is, in fact, in accordance with the author's original proposal of partial prestressing. It can be done either by tensioning the whole reinforcement to a lesser degree than necessary for full prestressing, as in the tests by Prof. Evans, or by tensioning some of the wires to a high initial prestress, the other wires remaining untensioned and being pre-compressed at the release of

the tensioning force on the concrete, which alternative, in the author's opinion, is preferable.

In view of the high bond resistance between high strength concrete and thin wires, as pointed out at the end of Section 3, the author expected that the co-operation of tensioned and untensioned wires would be satisfactory, and would even restrain the elongation of non-bonded ties. Reference may be made to comparative small-scale tests on fully, partially and non-prestressed beams of the same cross section and wire reinforcement, in the partially prestressed beams

Professor Magnel carried out a test on a beam, in which, very probably for the first time, this idea was applied to post-tensioning[14] and [15]. Half the wires were untensioned and bonded, whilst the other half were post-tensioned and not bonded.

(5) L.N.E.R. Beam Tests on Prestressed Concrete Sleepers

In 1946 extensive beam tests were carried out by the former L.N.E.R. under its Chief Engineer, Mr. J. C. L. Train, M.I.C.E., on prestressed concrete sleepers, some

TABLE 1.—Ratios $r_1 = \frac{t_{max}}{t_y}$ and $r_2 = \frac{t_{max}}{t_{ult}}$

Tests on Rectangular Beams 1935/36 : t_{ult} = 63.5 — 70.3 tons per sq. in.

All beams were under-reinforced except that marked thus *.

Percentage of steel p.	Diameter of bars	t_y	Concrete (1) c_u = 2100		Concrete (2) c_u = 6300**		Concrete (3) c_u = 6300**		Concrete (4) c_u = 8400	
%	in.	tons per sq. in.	c_u = cube strength in lb. per sq. in.							
			r_1	r_2	r_1	r_2	r_1	r_2	r_1	r_2
0.38	5/16	42.5	1.15	0.70	1.51	0.91	1.54	0.93	1.53	0.92
0.85	15/32	38.6	0.83*	0.48*	1.26	0.73	1.43	0.83	1.42	0.81
1.50	5/8	40.5	—	—	1.20	0.73	1.15	0.71	1.20	0.73

**The tensile strength of concrete (3) was much higher than that of concrete (2).

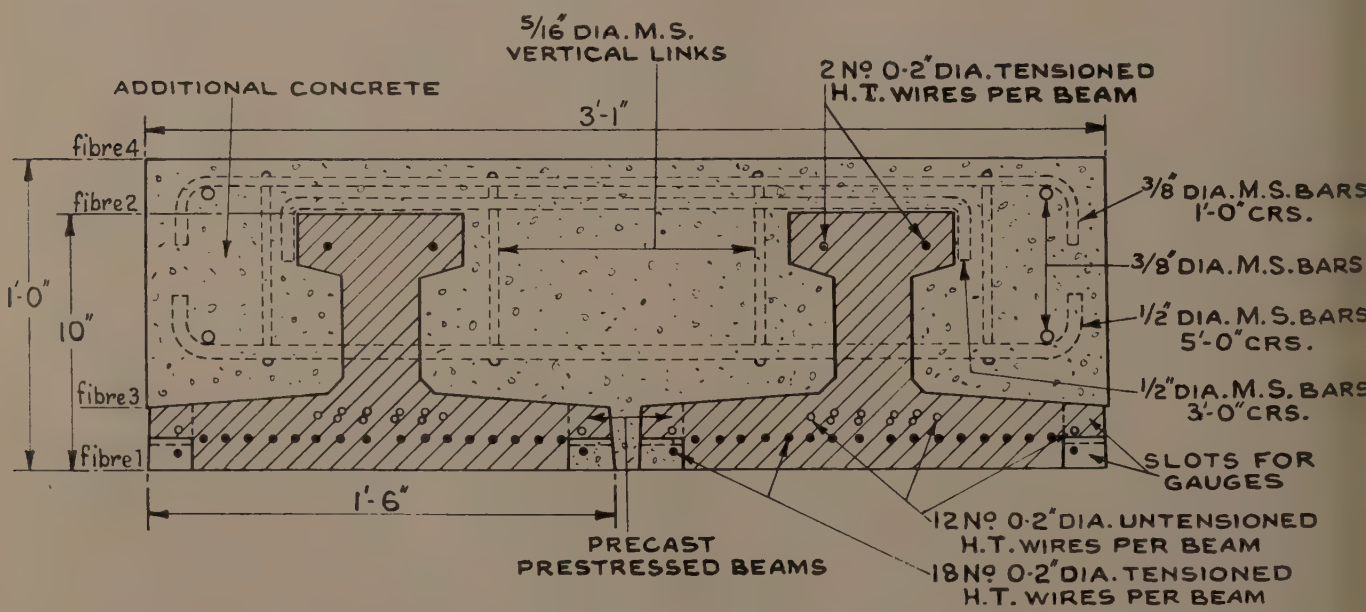


Fig. 1.—Cross Section of Test Slabs S1-S4

40 per cent. of the wires being tensioned. These tests were conducted in collaboration with Dr. K. Hajnal-Konyi at R. H. Stanger's Testing Institute, London, in 1942[9]. Although the concrete strengths were not the same in the various beams, and thus an exact comparison was impossible, the tests proved that the author's suggestions were sound, and they showed, in particular, the high degree of resilience, not known to the author previously. However, before practical application of this idea, still further tests were necessary to investigate the behaviour of partially prestressed concrete, both before and after cracking.

of which had been in the track for 2½ years[22].

The sleepers were loaded as simply supported beams, two point loads acting upwards, as indicated in Fig. 8 of [23]. The cycle of loading was to increase the load to somewhat above half of the ultimate load, then to reduce it nearly to zero, increase it again to 70 to 90 per cent. of the ultimate load, to reduce it again nearly to zero and finally to increase it until the sleeper failed.

From the prestressed concrete sleeper tests[22], the conclusions were drawn :

that (1) in constructions in which repetition of the maximum load occurs including impact (such as railway

underbridges, sleepers, and certain factory floors), the development of any cracks ought to be avoided and only compressive stresses should be allowed at present under working load, and that (2) in constructions in which the maximum load occurs only occasionally (such as road bridges, poles, roofs, or certain floors), nominal tensile bending stresses of 600 to 1,200 lb. per sq. in., and even more, may be considered as permissible provided that, under dead weight, concrete tensile stresses do not occur.

(6) Loading and Breaking Tests on Full Size Bridge Beams, 1949

Based on the tests in Section 5, the first beams were designed and employed as part of a composite construction, which will be discussed in more detail later. These beams were designed for permissible tensile stresses of 500-600 lb. per sq. in. under the maximum working load, and no cracking is expected under this loading.

Tests on three full size beams have been described in detail in paper[10]. The purpose of these tests was to give the basis for supervision of manufacture and to obtain special information for proposed developments. The relatively small cost of these tests was only a fraction of the immediate saving obtained by application of the results to the design of the deck construction of 19 railway overbridges being built on the Eastern Region of British Railways as partially prestressed composite slabs.

Checking tests were carried out as a routine control on all bridge contracts. One beam of each batch manufactured on the same bed was tested to a load corresponding to a concrete tensile bending stress of 750-800 lb. per sq. in. At this loading cracking should not occur and the deflection should not exceed a value calculated for $E = 3.75 \times 10^6$ lb. per sq. in. Although this might be considered as a very severe test, all beams tested in four contracts passed the conditions ; in fact, the measured deflection corresponded in most cases to E-values between 5×10^6 and 6×10^6 lb. per sq. in. By these routine tests good quality of the concrete and the required safety against cracking has been ensured. In one case the load was maintained for 30 days without any cracks developing under this tensile stress. During this time the total deflection increased from 0.768 in. to 0.856 in. after 5 hours, to 0.884 in. after one day, to 0.958 in. after six days, and to 1.327 in. after 30 days, the measurements being taken daily, and the nominal E values, corresponding to the total deflection, varied between 5×10^6 and 2.8×10^6 lb. per sq. in.

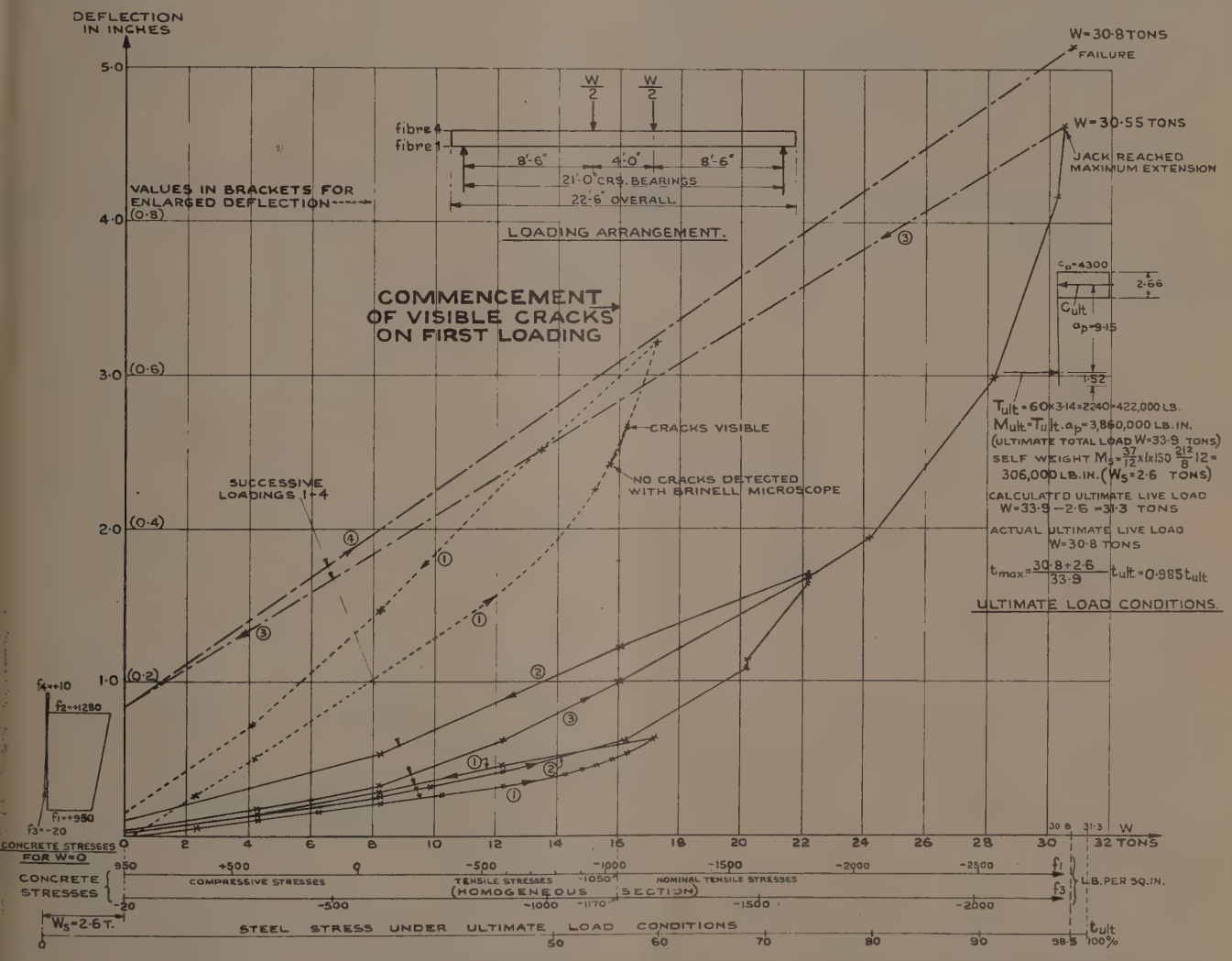


Fig. 2.—Test on partially prestressed composite slab S3 at Shorne on June 6th, 1951; load deflection curves

(7) Breaking and Fatigue Tests on Partially Prestressed Composite Slabs S1-S4 of 21 ft. Span, 1951

In the tests for composite beams T1 and T2[10] separation took place between the prestressed and the added concrete at a stage when steel and concrete were apparently still capable of carrying higher loads. It was,

therefore, decided to test slabs formed from two bridge beams, as shown in Fig. 1. The additional concrete was in these cases placed not only around the prestressed beams as with beams T1 and T2, but also between the prestressed beams, as is the case in the actual bridge deck construction. Two of such slabs (S3 and S4) were tested statically by Mr. R. H. Stanger at the factory of Atlas Stone Co., Ltd., at Shorne, near Gravesend, as part of a bridge contract let by the Civil Engineer, Eastern Region, B.R., and two other slabs

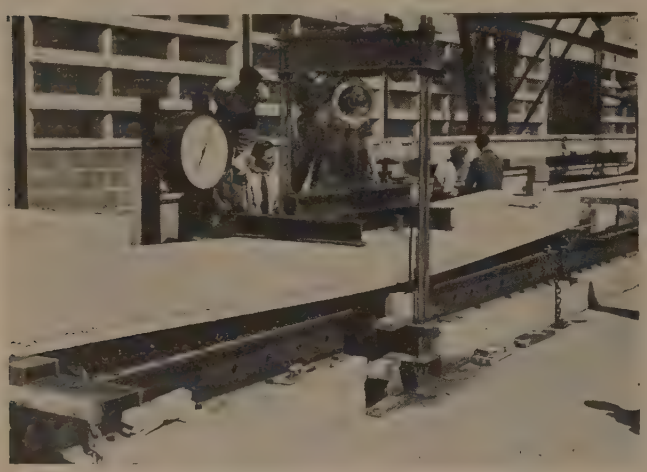


Fig. 3.—Slab S3 at loading of 30.55 tons

(S1 and S2) were tested dynamically by Professor F. Campus' Laboratory (Université de Liège—Institut du Génie Civil) at Liège, Belgium, on behalf of the Railway Executive and their Research Department.

The beams Nos. 1-4 for the slabs S1 and S2 were cast on March 29th, 1951, at Shorne, by the Atlas Stone Co., Ltd., and made in rapid hardening Portland cement. The beams 5-8 for the slabs S3 and S4 were manufactured on April 16th, 1951, with sulphate and acid-resisting "Sealithor" cement, which is now being used for the bridge deck constructions. The mix of the concrete in all beams was 1 : 1.25 : 2.5 by weight with a water-cement ratio of 0.38-0.40. The aggregates obtained from local pits, washed and crushed, were graded from BS 100 to $\frac{1}{2}$ in. The composition of the added concrete for slabs S1-S3 was the same as that for beams Nos. 1-4, and cast for S1 on April 19th, S2 on April 20th, and S3 on May 10th, 1951, but the added concrete in slab S4 was made of a much weaker mix in order to obtain a smaller strength, the vibrated concrete being of a mix of 1 : 3.4 : 5.1 by weight (water-cement ratio 0.88), and cast on May 16th, 1951. The tensile reinforcement of these slabs comprised 60 cold-drawn high tensile wires of 0.2 in. diameter, 36 of these wires were initially tensioned to 150,000 lb. per sq. in. (2.1 tons per wire) after a force of 2.2 tons per wire had been temporarily applied to offset creep of the steel. The remaining 24 wires were untensioned and placed in pairs, except for the outer wires, which were arranged singly to suit slots as shown in Fig. 1, provided for fixing strain gauges. The strength of the wire was reputed to be of a guaranteed minimum of 100 tons per sq. in., but tests carried out on specimens indicated a strength of only 90 to 95 tons per sq. in. for the wire used for the dynamic tests, while the strength of the wire used for the static tests varied between 95 and 107 tons per sq. in. In addition to the high tensile wires, ordinary steel bars were placed near the top similar to the mesh which is usually provided in bridge deck constructions to ensure a good load distribution ; similarly transverse bars were placed through holes of the web and connected with the upper mesh, as indicated in Fig. 1.

As with the beams T1 and T2, described in paper [10], the prestressed beams were simply supported when the non-prestressed concrete was added, resulting in effective concrete stresses at the fibres 1-4, as indicated in Fig. 2, for slab S3, the prism strength being 4,300 lb. per sq. in. In the load-deflection diagram, Fig. 2, are plotted the nominal concrete stresses f_1 and f_3, calculated for a homogeneous section and the average steel stress t calculated for a cracked section under failure conditions, similar to Figs. 7 and 10 in paper[10]. Although this stress t is obtained for a rectangular concrete stress distribution under failure conditions, the lever arm will hardly differ from that occurring for other stress distributions. Thus, this stress t approximates the steel stress in a crack also at a much lower loading. The exact stress distribution within a crack of a prestressed concrete construction is not known and depends greatly on the bond efficiency of thin wires. Consequently it is, at present, only possible to illustrate it on the one hand by the steel stress t in a cracked section calculated for ultimate load conditions and, on the other hand, by the magnitude of the nominal concrete tensile stress f_1, i.e., the stress which would occur in a straight line stress distribution assuming the section to be still homogeneous.

The calculated cracking load, at which a tensile bending stress of 1,000 lb. per sq. in. is obtained for f_1 amounts to 15.75 tons, but the first cracks were not noticed before the load of 16.25 tons, after visible cracks were not noticed at a load of 15.75 tons. As with tests T1 and T2, no cracks in the added concrete were observed even with a Brinell microscope, although the tensile stresses f_3 are higher than f_1 at that stage. However, from the enlarged load deflection curve which is shown in dotted line in Fig. 2 it is seen that just before visible cracking a considerable increase in deflection occurred, which indicates that invisible cracks may have developed, as stated by Professor Evans[13], [24], and already mentioned in paper [10]. It can be taken from Fig. 2 that the load at which visible cracking commenced, was approx.

Fig. 4.—Slab S4 on removal of load of 24 tons

16 tons, corresponding to respective concrete tensile stresses $f_1 = -1{,}050$ and $f_3 = -1{,}170$ lb. per sq. in. With a high powered microscope cracks might have been detected earlier ; but this is of no practical significance whatsoever, since even haircracks of limited width are harmless.

For a maximum mean tensile stress of 100 tons per sq. in. corresponding to an average tensile strength of the wire, and a maximum concrete stress of 4,300 lb.

per sq. in. obtained from prisms, the calculated failure load amounts to 31.3 tons, while the actual value was 30 tons. At a loading of 30.55 tons, shown in Fig. 3 the maximum extension of the jack was reached and the load had to be reduced to underpack the jack. Failure occurred after considerable yielding of the slab, the immediate cause being crushing of the concrete compression zone. After the commencement of crushing of the concrete, a separation took place between the prestressed and added concrete in the fibre 3 at one side of the slab.

failure crack. The bending moment at the point of failure was only approx. 93 per cent. of the maximum value between the loading points. Thus there must have been a weaker part in the added concrete with the greatly reduced cement contents. When a second loading was applied after failure similar to beam P1 of paper [10], a maximum load of 25.75 tons only was reached, when a heavy inclined shear crack developed adjacent to the site of the previous compressive failure, which crack continued horizontally in the bottom portion and went over a certain length right through

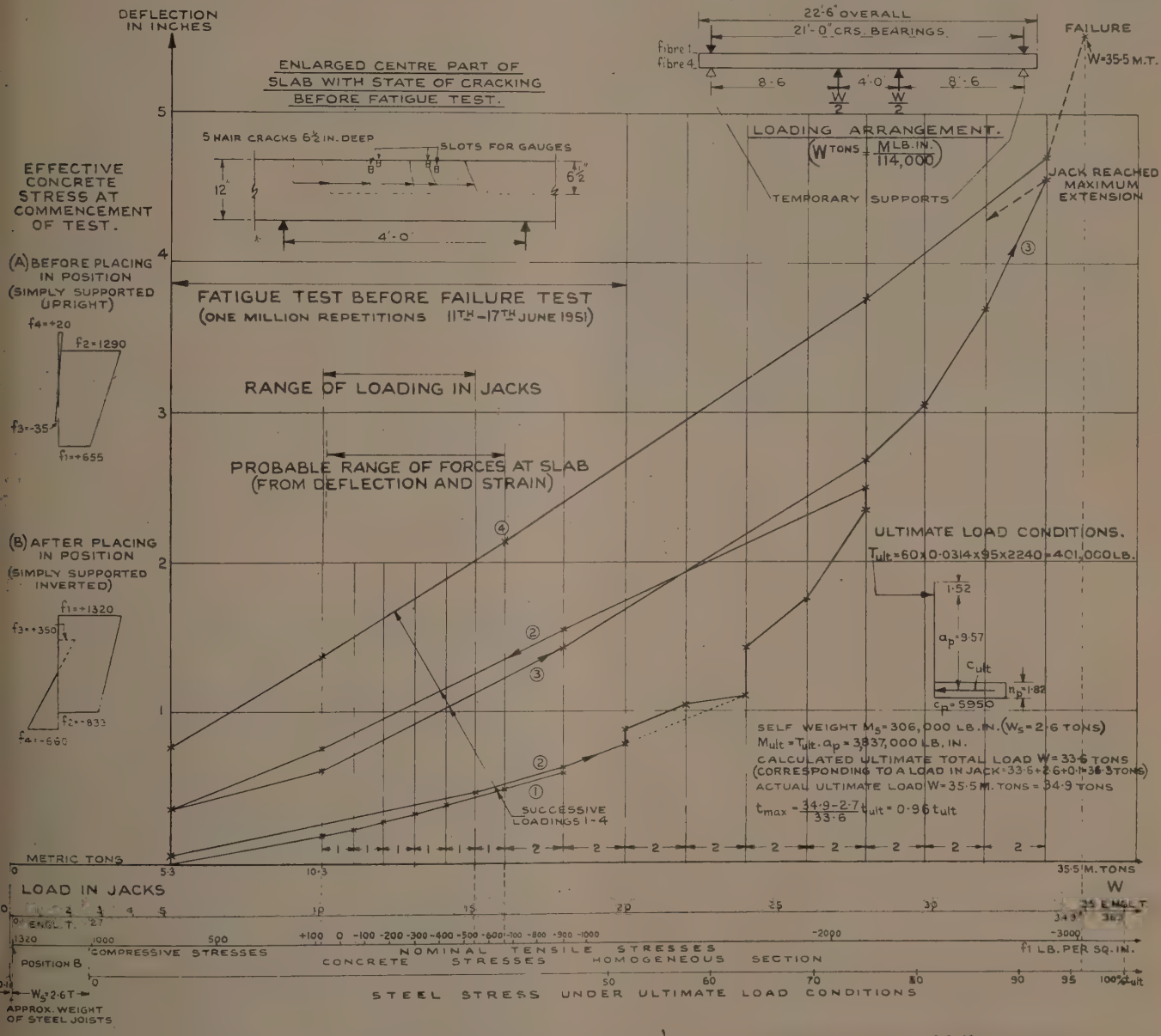

DEFLECTION IN INCHES
ENLARGED CENTRE PART OF SLAB WITH STATE OF CRACKING BEFORE FATIGUE TEST.
5 HAIR CRACKS 6½ IN. DEEP
SLOTS FOR GAUGES
22'-6" OVERALL
21'-0" CRS. BEARINGS
FAILURE
W=35·5 M.T.
LOADING ARRANGEMENT.
(W TONS = M LB. IN. / 114,000)
TEMPORARY SUPPORTS
JACK REACHED MAXIMUM EXTENSION
EFFECTIVE CONCRETE STRESS AT COMMENCEMENT OF TEST.
(A) BEFORE PLACING IN POSITION (SIMPLY SUPPORTED UPRIGHT)
FATIGUE TEST BEFORE FAILURE TEST
(ONE MILLION REPETITIONS 11TH–17TH JUNE 1951)
RANGE OF LOADING IN JACKS
PROBABLE RANGE OF FORCES AT SLAB (FROM DEFLECTION AND STRAIN)
(B) AFTER PLACING IN POSITION (SIMPLY SUPPORTED INVERTED)
ULTIMATE LOAD CONDITIONS.
T ult = 60 x 0·0314 x 95 x 2240 = 401,000 LB.
SELF WEIGHT Ms = 306,000 LB. IN. (Ws = 2·6 TONS)
Mult = Tult . ap = 3837,000 LB. IN.
CALCULATED ULTIMATE TOTAL LOAD W = 33·6 TONS
(CORRESPONDING TO A LOAD IN JACK = 33·6 + 2·6 + 0·1 = 36·3 TONS)
ACTUAL ULTIMATE LOAD W = 35·5 M. TONS = 34·9 TONS
SUCCESSIVE LOADINGS 1–4
METRIC TONS
LOAD IN JACKS
35·5 M. TONS
W
POSITION B
COMPRESSIVE STRESSES
NOMINAL TENSILE STRESSES
CONCRETE STRESSES
HOMOGENEOUS SECTION
f1 LB. PER SQ. IN.
Ws = 2·6 T
APPROX. WEIGHT OF STEEL JOISTS
STEEL STRESS UNDER ULTIMATE LOAD CONDITIONS

Fig. 5.—Test on partially prestressed composite slab S1 at Liège; load-deflection curves of failure test on June 18th, 1951

In the second slab S4 of the static test with the weaker concrete visible cracks were noticed at a load of 15·75, while the value calculated for a modulus of rupture varies between 14.58 and 15.75 tons, computed for a ratio of 1.25 to 1.0 of the moduli of elasticity of the two types of concrete employed. The failure load calculated for a mean steel strength of 100 tons per sq. in. and a concrete stress of 3,000 lb. per sq. in. amounts to 31.8 tons, while at a load of 27 tons a slight destruction of the compressive zone outside one of the loading points occurred, together with a limited separation between prestressed and added concrete in fibre 3 next to the

the prestressed flange at one side of the slab, where a separation between prestressed and added concrete had not taken place. Fig. 4 shows slab S4 after a load of W = 24 tons, previously applied, had been removed.

Fig. 5 is the load deflection diagram of a failure test carried out on slab S1 tested for fatigue resistance at Liège. The purpose of these dynamic tests was to investigate the behaviour of a partially prestressed bridge deck construction designed for concrete bending tensile stresses of 500 to 600 lb. per sq. in. under working load. Although no cracks ought to occur, even if the maximum working load were applied under fatigue

conditions, it was considered possible that excessive loads might occur occasionally, resulting in the development of cracks. It was argued in some quarters that such cracks would be detrimental if the maximum working load were applied subsequently under fatigue conditions to a cracked section designed as homogeneous for a tensile stress of the order mentioned.

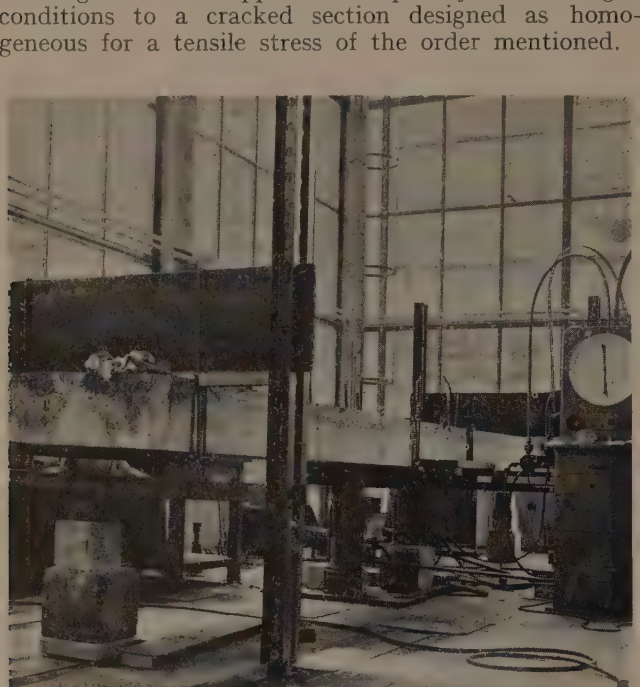


Fig. 6.—Slab S2 after 3 million repetitions before static failure test

The prestressed concrete beams Nos. 1-4, for slabs S1 and S2 were cast on March 29th, and the initial prestress was less since at tensioning a total prestressing force of approx. $18 \times 2.1 = 37.8$ tons only was applied instead of $20 \times 2.1 = 42$ tons, and a stress f_1 of approx. 1,320 lb. per sq. in. was effective, when the slab was in inverted position B, indicated in Fig. 5. At the failure test the prism strength of the added concrete cast on April 19th for slab S1 was approx. 5,950 lb. per sq. in. when tested on June 18th, and that added on April 20th for slab S2 was 6,150 lb. per sq. in. on July 3rd. The slabs were first cracked by a static loading and afterwards a fatigue test was carried out in accordance with a prearranged sequence. The slabs were tested in inverted position with upward acting loads, as seen from the photograph, Fig. 6, showing slab S2 in the testing machine before final failure.

Slab S1 was tested to failure on June 18th, 1951, after one million repetitions of loadings had been completed. The ultimate load calculated in this case for a maximum steel stress of 95 tons per sq. in., in accordance with the actual strength, amounted to 36.3 tons, while the actual maximum load was 34.9 t. (35.5 m. t.), corresponding to 96 per cent. of the maximum steel stress, the bending moment due self-weight being taken into account.

The slab S2 was broken on July 3rd, 1951, after three million repetitions of fatigue loading had been completed, the third million being carried out in a range between nominal tensile stresses in the beams of 300 and 900 lb. per sq. in. based on forces in the jacks. Thus during this last million of repetitions there was a continuous tension in the concrete, the cracks widening four times per second, but not completely closing. During this period two outer tensioned wires fractured where exposed for $\frac{1}{2}$ in. length in slots provided for affixing gauges. Nevertheless a slightly higher failure load (36.1 m. tons) was obtained than with slab S1 reaching 101 per cent. of the resistance calculated for the prism strength and 58 wires, the deflection being over 6 in., as seen in Fig. 7. This proves that fatigue does not affect well bonded wires in prestressed concrete up to 0.2 in., whether they are tensioned or untensioned, the full strength of both types being reached at failure, though the same wire failed, under fatigue conditions at approx. half the load where not bonded. Thus the ultimate resistance of under-reinforced prestressed concrete with bonded wires is not affected by fatigue.

These severe tests have also proved that if cracks have occurred in prestressed concrete and open under subsequent loading, they will close completely on removal of the load, even if the loading is repeated one million times. Thus the claim of the opposers of partial prestressing, who state that even the occasional occurrence of fine cracks is dangerous, is shown to be wrong. A complete report is expected in due course from the Research Department of the Railway Executive.

(8) The Essential Features of Prestressed Concrete

From the tests discussed in the previous sections, it is seen that the effective prestressing of concrete, independent of the magnitude of the prestressing force, produces a material which behaves quite differently from ordinary reinforced concrete, since, even after it has been damaged by cracking, it retains complete elasticity and resilience, up to a loading approaching failure. This phenomenon distinguishes prestressed concrete from any other material.

Fig. 8 shows a series of load-deflection curves plotted in an idealised form for a simply supported beam having bonded tensioned wires. The concrete bending tensile stresses, computed for a homogeneous section, are

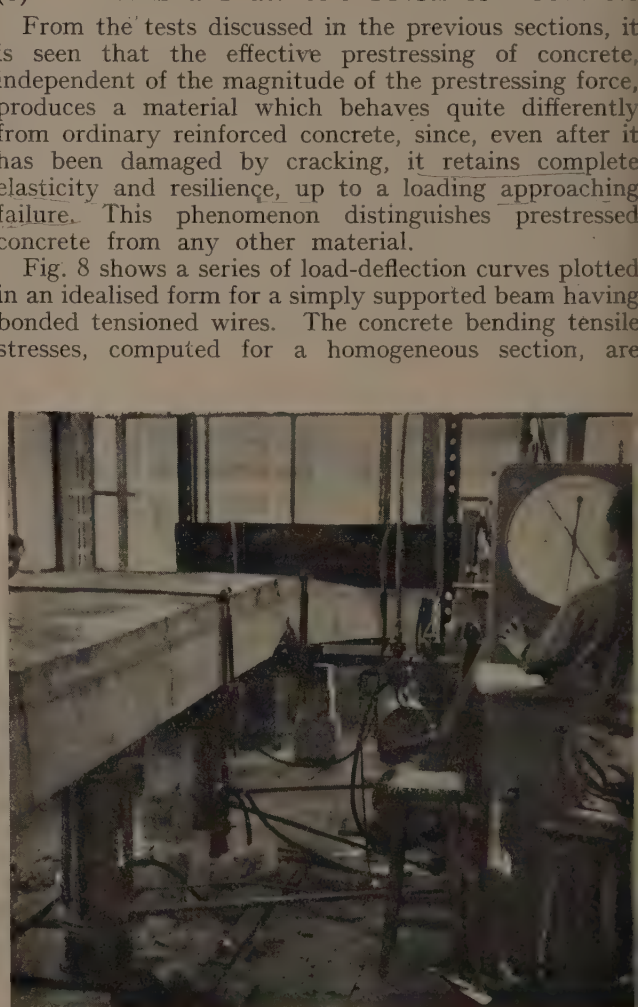


Fig. 7.—Slab S2 after failure

plotted in addition to the loads. The curves show that at the load W_0 the initially compressive fibre stress becomes tensile, and at the load W_r (suffix r for rupture) first cracking occurs. Up to this load W_r, the deflection curve is in agreement with calculated values for a homogeneous section, computed with $E_c = 4 \times 10^6$ to 6×10^6 lb. per sq. in., dependent upon concrete quality and shape. If the load exceeds W_r, the deflection lin-

approximates, in principle, to that of an ordinary reinforced concrete beam calculated for a neglected tensile zone and $m = 15$. As long as the load only slightly and rarely exceeds W_r, the rather fine and short cracks become invisible on reduction of loading below that giving stresses equivalent to the modulus of rupture of a homogeneous section in spite of the previous disrupture of homogeneity. However, after repeated or sustained loading the deflections increase, but still as a straight line corresponding to a lower modulus of elasticity of, say, $E_c = 2.5 \times 10^6$ to 4×10^6 lb. per sq. in. Exact intermediate measurements result in hysteresis curves between the two limiting lines. When the load exceeds W_r to a greater extent, the cracks become more distinct, opening and closing below W_r and, finally, only at W_o, a loading at which the tension in the concrete bottom fibre reverses into compression. At this condition, the pre-compression which was previously interrupted in the cracks, reasserts itself along the tensile zone of the beam. The phenomenon of cracks becoming invisible at loadings between W_o and W_r, as indicated in Fig. 8, may be explained by the minuteness of the cracks. Obviously, this applies only if the tensioned wires are well bonded to the concrete. Where the member consists of blocks with tensioned ties without mortar joints, the latter will open as soon as the load exceeds W_o, and close at this loading on reduction of the loading, but only if permanent elongation does not remain in the steel, which in this case, is allowed to extend during loading along its entire length.

From Fig. 8, the importance of bond can be recognised. To avoid cracks in an assembly of blocks without mortar joints it is essential that the load W_o is not exceeded, whereas with well bonded wires an occasional load exceeding the much higher value W_r does not prevent full freedom from cracks as soon as this loading has been reduced.

The modulus of rupture, or flexural strength c_t requires clarification. The expression "Strength" is not quite appropriate since c_t is only a nominal and quite arbitrary stress, calculated for a homogeneous section. The modulus of rupture varies for different concrete strengths and plasticities and depends also on the shape and size of the cross-sections. It amounts to about 1.5 to 2.25 times the tensile strength c_t.

The value of c_t is itself difficult to ascertain. In extensive tests[25] the tensile strengths of 200 to 530 lb. per sq. in. have been obtained at various ages for concretes with cylinder strengths of 1,500 to 8,000 lb. per sq. in. respectively. The highest values were rather infrequent and it may be assumed that the tensile strength exceeding 500 lb. per sq. in. is unusual, even with concretes of the highest strength properties. On the basis of the test results in paper [25] the following approximation for the tensile strength c_t may be given :

$$c_t = 530 - \left(\frac{10{,}000 - c_u}{8{,}000} \right)^2 \times 330$$

This formula gives results for a 28 days' cube strength c_u of 7,500 lb. per sq. in. of $c_t = 427$ lb. per sq. in.

As already stated, it is difficult to measure the tensile strength. Tests on briquettes of a cross section 1.5 × 1.5 have shown great uniformity, but too high values are obtained due to the stress concentration. With regard to the modulus of rupture, a value of 1,000 to 1,050 lb. per sq. in. has been proved to be appropriate according to the tests discussed in Sections 4 and 5 for inverted T-sections in high strength concrete with cube strengths of approximately 7,000 lb. per sq. in. The appropriate value for rectangular sections may be taken as 950 to 1,000 lb. per sq. in. The value for an I-section lies between these two values, and that for an upright T-section is smaller than that for a rectangle.

With regard to sustained loading, the prolonged test mentioned in Section 6 has proved that cracking did not occur when a load was maintained for 30 days, which corresponded to a tensile stress approximately 75 per cent. of the modulus of rupture, and at this stress cracks should not develop if the duration of loading is extended indefinitely. From research data, it is known that the tensile resistance to fatigue loading of plain concrete amounts to 50 to 60 per cent. of the tensile strength at static loading. With prestressed

Fig. 8.—Typical load-deflection curves of a prestressed beam with bonded wire

concrete the resistance is greater than with plain concrete, the ratio of cracking load in dynamic tests to that in static tests being 85 to 90 per cent., as stated in paper [16]. Thus, repeated loadings up to the maximum working load are expected not to cause cracks when the maximum bending tensile stress does not exceed 600 to 700 lb. per sq. in., whereas for sustained maximum working load the appropriate stress would be 750 lb. per sq. in. for a modulus of rupture of 1,000 lb. per sq. in.

When considering failure, two cases must be distinguished with bonded steel, viz., under-reinforced and

over-reinforced beams. In the first case, failure is primarily caused by steel after considerable deflection has occurred, either by fracture of the tensile reinforcement or by excessive yielding of the beam, which leads to reduction of the depth of the concrete compressive zone, and to consequent crushing. With an over-reinforced beam, failure always occurs through limited compressive resistance at a stage when the deflection

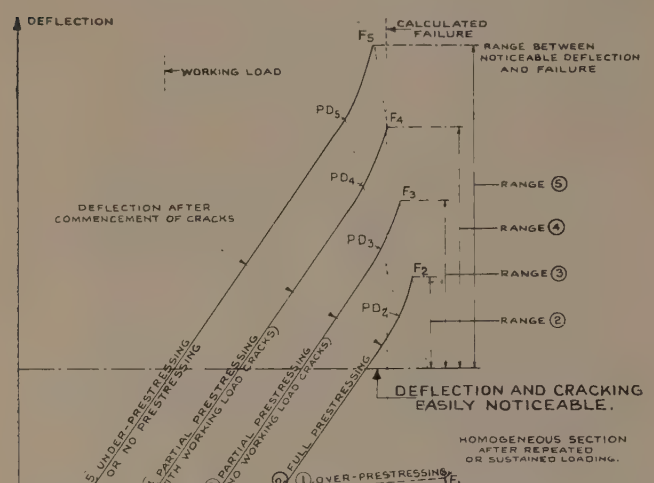

DEFLECTION
CALCULATED FAILURE
RANGE BETWEEN NOTICEABLE DEFLECTION AND FAILURE
WORKING LOAD
DEFLECTION AFTER COMMENCEMENT OF CRACKS
RANGE ⑤
RANGE ④
RANGE ③
RANGE ②
DEFLECTION AND CRACKING EASILY NOTICEABLE.
HOMOGENEOUS SECTION AFTER REPEATED OR SUSTAINED LOADING.
HOMOGENEOUS SECTION FIRST LOADING.
⑤ UNDER-PRESTRESSING OR NO PRESTRESSING
④ PARTIAL PRESTRESSING (WITH WORKING LOAD CRACKS)
③ PARTIAL PRESTRESSING (NO WORKING LOAD CRACKS)
② FULL PRESTRESSING
① OVER-PRESTRESSING
LOAD
M.O.R. = MODULUS OF RUPTURE. W.L. = WORKING LOAD.
C.C. = COMMENCEMENT OF CRACKING. F = FAILURE.
P.D = PERMANENT DEFLECTION. Wult. = CALCULATED ULTIMATE LOAD.
(COMMENCEMENT)

Fig. 9.—Load-deflection curves for under-reinforced prestressed concrete beams with bonded wires

and state of cracking do not give warning of danger. The optimum condition would be a balanced design with equal tensile and compressive resistance. However, it is hardly possible to calculate the balanced design without knowing the exact properties (strength and plasticity) of the concrete in relation to the shape selected. Thus, it is advisable to use, wherever possible, under-reinforced sections, and make safe assumptions for the stress distribution of a balanced design, so as to avoid over-reinforced designs. The tests described in [1] have proved that the approximate design for failure conditions suggested by the author in [7] and [14], is appropriate for under-reinforced beams.

In the idealised diagram Fig. 9, five cases of under-reinforced prestressed concrete beams are compared. In case 1, which may be called over-prestressed, cracking and failure take place simultaneously, as with a brittle material, such as glass or cast iron. With case 2, which is a fully prestressed beam, a small range is afforded between the state of noticeable cracking and deflection and that of failure. When cracking takes place, a considerable change in the slope of deflection occurs, but this "transformation" does not become noticeable to the observer before both deflection and cracking have reached an appreciable amount, as indicated in the graph. Case 3 is a partially prestressed beam in which full freedom from cracking is ensured under working load, provided the concrete tensile bending stress is below the modulus of rupture. In this instance, the warning range between noticeable deflection and failure is larger than in case 2. This warning is further increased with case 4, representing a partially prestressed beam in which temporary fine cracks may develop under the rarely occurring maximum working load. These cracks, as previously stated, would close completely under normal loading, provided that only compressive stresses occur at that stage even if the loads did not occur occasionally but repeatedly. Case 5 of an under-prestressed or non-prestressed beam may be considered as undesirable. With this type the deflection and amount of cracking could easily become excessive under working load, although in such a case, heavy cracking could be avoided by partial prestressing. In the graph points C.C. represent the commencement of cracking. At a load PD permanent deflections occur, whereas failure is indicated by the loads F_1 to F_5. The slope of the line F_1-F_2-F_3-F_4-F_5 depends on the relative strength of steel and concrete, and particularly on the bond resistance. With low percentages, this line may become vertical. However, in case 5, the steel stress at failure may be less than f_{ult}.

Fig. 9 relates to both prestressed and composite members, in which only a part is prestressed. These five cases correspond to the boundaries between the individual types mentioned in the "First Report on Prestressed Concrete."

In Table II the conditions for these five cases are summarised. Cases 1 and 2 are seen to be the boundaries of the solution which originates from M. Freysinnet's idea of creating a new homogeneous material which can also be built up from blocks, whereas cases 3 to 5 originate from the approach via reinforced concrete. When considering prestressing, three different classes of material should be distinguished and each class treated differently, viz., (a) monolithic material with pre-tensioned wires or post-tensioned effectively bonded wires; (b) monolithic material with post-tensioned non-

TABLE II.—Summary of Five Cases of Fig. 13

No.	Condition at W.L. (Working Load)	Bottom Fibre Stress at W.L. in a nominal Homogeneous Section		Commencement of Cracks	Kind of Failure	Range between Noticeable Deflection and Failure
1	No Cracks	Compressive Stress	Considerable	At Failure	Sudden and Brittle	None
2	No Cracks	Compressive Stress	Approaching Zero	Much Above	Working Load	Rather Small
3	No Cracks	Tensile Stress	Below M.o.R†	Above	Gradual and Yielding	Large
4	Fine* Cracks	Tensile Stress	Above M.o.R†	Below	Gradual and Yielding	Larger
5	Heavy Cracks	Tensile Stress	Much Above M.o.R†	Much Below	Gradual and Yielding	Very Large

*They close up entirely on removal of the load. †M.o.R. = Modulus of Rupture

bonded wires ; and (c) non-monolithic material (assembly of blocks without mortar joints), and post-tensioned wires. Class (c), designed as fully prestressed, is based on the same principles as case 3, partially prestressed, which is censured as being bad design by the advocates of full prestressing. In fact, there is no justification whatever for ignoring the established tensile strength of concrete which is available in class (a) because of the fact that there is no tensile resistance in class (c).

The graph of Fig. 9 relates to bonded wire ; in the case of non-bonded wires there is, firstly, the main difference, that instead of many well-distributed cracks, a much smaller number of isolated cracks occur, which become much wider. Secondly, dependent on the stresses, the steel extends over the entire length, with the consequence that failure occurs earlier and suddenly. Particularly, if the tensioning stress in the steel is low, failure occurs at a relatively low steel stress. The essential differences between prestressed concrete with bonded and non-bonded steel was, for the first time, clearly indicated by Prof. Evans in [21].

(9) The Design of Prestressed Concrete based on Test Results

In view of the previous consideration, it is seen that a straight line stress distribution for a homogeneous material is applicable, even after cracking has occurred, whereby " nominal " tensile stresses may be taken into account, for full homogeneity is again ensured on reduction of the load. The magnitude of stresses for a homogeneous section has to be investigated for three loading stages, i.e., transfer, transport, which includes handling, and working load. To these, in certain cases, a fourth loading ought to be added, viz., dead weight, or frequently occurring load.

Quite apart from these loadings, conditions at failure have to be considered, based on a designated factor of safety. M. Freyssinet says in [2], " the usual expression of the factor of safety has literally no meaning whatsoever," and further, " it is thus insufficient to say that the classic expression of the notion of safety loses its significance ; the very idea itself is completely changed ; the condition for the passage from the zone of security to that of insecurity are quite novel." These statements may very well apply to members of class (b) and particularly class (c) mentioned in Section 6, for which it seems difficult to compute the failure load, but they do not agree with the actual behaviour of class (a), which will be the more usual case of prestressed concrete. In the previous sections, it has been clearly shown that, with under-reinforced beams, it is possible to determine exactly a minimum failure load based on the ultimate tensile resistance and the lever arm in a cracked section.

The author has derived design formulæ in his book[23], of which the following formula (1a) may be quoted :—

$$Z_{1\ \min} = \frac{M_w - R_o.M_s}{R_o.f_{ct} + f_{tw}}$$

whereby f_{ct} is the permissible concrete compressive stress at transfer, and f_{tw} the tensile stress under working load; Z_1 is the section modulus for the bottom fibre, M_w and M_s are the bending moments for working load and self-weight of beam respectively, whereas R_o is the ratio of the minimum effective prestress to the stress at transfer (which has been denoted by η). It is seen that the denominator contains the tensile stress f_{tw}. For a member type (3) of Fig. 9, the value may be assumed as 600, 750 or 950 lb. per sq. in. respectively, to ensure full freedom from cracks either for repeated, sustained or rarely occurring loading. If $R_o.f_{ct}$ is taken as 2,000 lb. per sq. in. (e.g., 0.85 × 2350), the following reductions of $Z_{1\ \min}$ are obtained for the three cases mentioned before : $\frac{2000}{2600} = 0.77$, $\frac{2000}{2750} = 0.73$ and $\frac{2000}{2950} = 0.685$ respectively. These are considerable reductions, and there is really no justification for ignoring the concrete tensile resistance. In all these cases full freedom from cracks is obtained. Moreover, such structures would show a better performance under loads above the working load than a fully prestressed member made up from blocks (without adhesion at the joints), designed for the same working load conditions. There is no need for a large factor of safety against cracking, since a small margin is sufficient. This has been proved by the numerous successful checking tests based on tensile stresses of 750 to 775 lb. per sq. in.

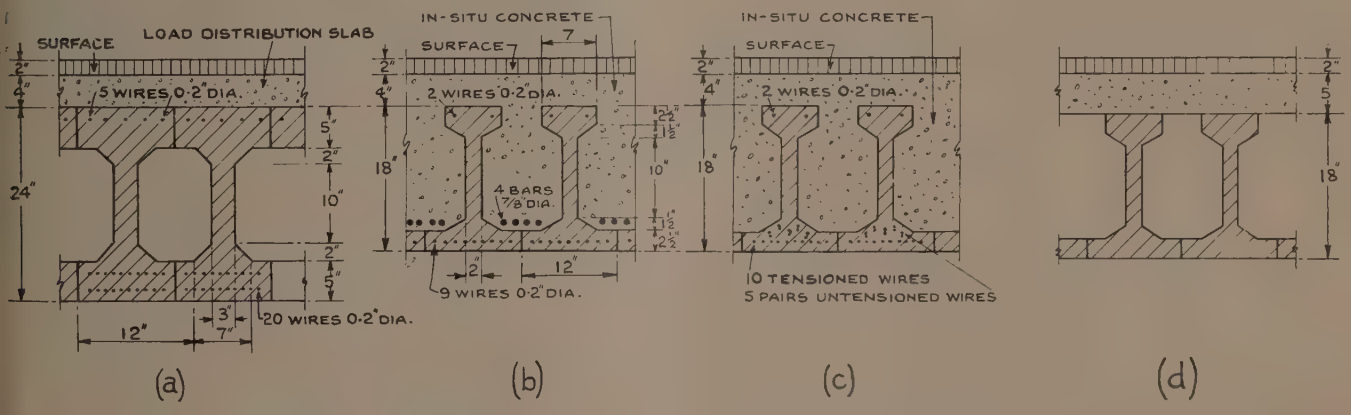


Fig. 10.—A comparison of four designs of bridge deck construction 1945

The superiority of partial prestressing is further enhanced in case (4) of Fig. 9. Let f_{tw} be 1,300 lb. per sq. in. for a rarely occurring load on the assumption that under dead weight only compressive stresses occur, then the section modulus is reduced to $\frac{2000}{3300} = 0.60$ of the value which would have been taken had partial prestressing not been considered.

(10) Development of a Composite Bridge Design

In the discussion[26] to the paper [9], Dr. K. Hajnal-Konyi suggested the combination of precast prestressed and *in-situ* concrete in view of the satisfactory behaviour of partially prestressed members. This proposal was investigated by the author in his closure[26] for a bridge

deck construction. As a basis of comparison, a construction was selected as shown in Fig. 10 (a) resembling an established road fully prestressed bridge deck design over railways[27], comprising I-shaped precast prestressed beams and a load distribution slab of 4 in. depth. The design was made for rather low concrete stresses, e.g., $f_{ct} = 1{,}200$ lb. per sq. in., since at that time it was not yet considered prudent to permit high concrete stresses. Designs shown in Fig. 10 (b) and (c) represent alternative solutions which are both partially prestressed, and comprise composite slabs according to Dr. Hajnal-Konyi's proposal. This arrangement allows for a considerable reduction of depth in the precast beams, since they are designed to be fully prestressed for the dead weight only. The precast prestressed beams in all three designs have, in addition to the bottom reinforcement, some tensioned wire reinforcement in the top flange. This was provided to allow for easy handling by slinging at the centre without danger of cracking. In previous constructions in which such upper wires were omitted, placing had proved rather cumbersome[27], since the beams had to be suspended at both ends, the self weight counteracting the prestress.

Designs Fig. 10 (b) and (c) differ from each other by the arrangement of the untensioned steel. In the first case mild steel bars are placed in the *in-situ* concrete, whereas in the second case, the entire reinforcement required is of high strength wire placed within the precast prestressed concrete section. The untensioned wires are arranged preferably in pairs, twisted to increase the bond.

(11) The Prototype: Gilroyd Lane Bridge, near Barnsley, Built 1949

In 1945, the first designs were evolved on the lines of Fig. 10 (b), based on higher permissible concrete stresses, and the economy of the design was improved in accordance with the particulars shown in the upper part (design 1) of Fig. 12, paper [10]. The beams shown in

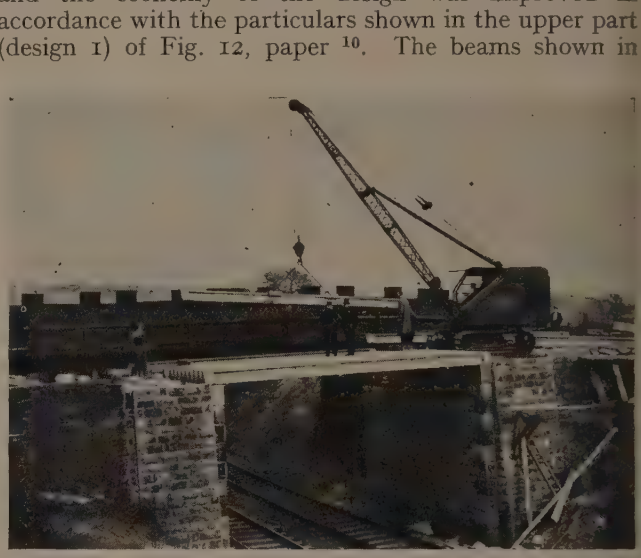


Fig. 12.—Gilroyd Lane Bridge; placing of bridge beams 1949

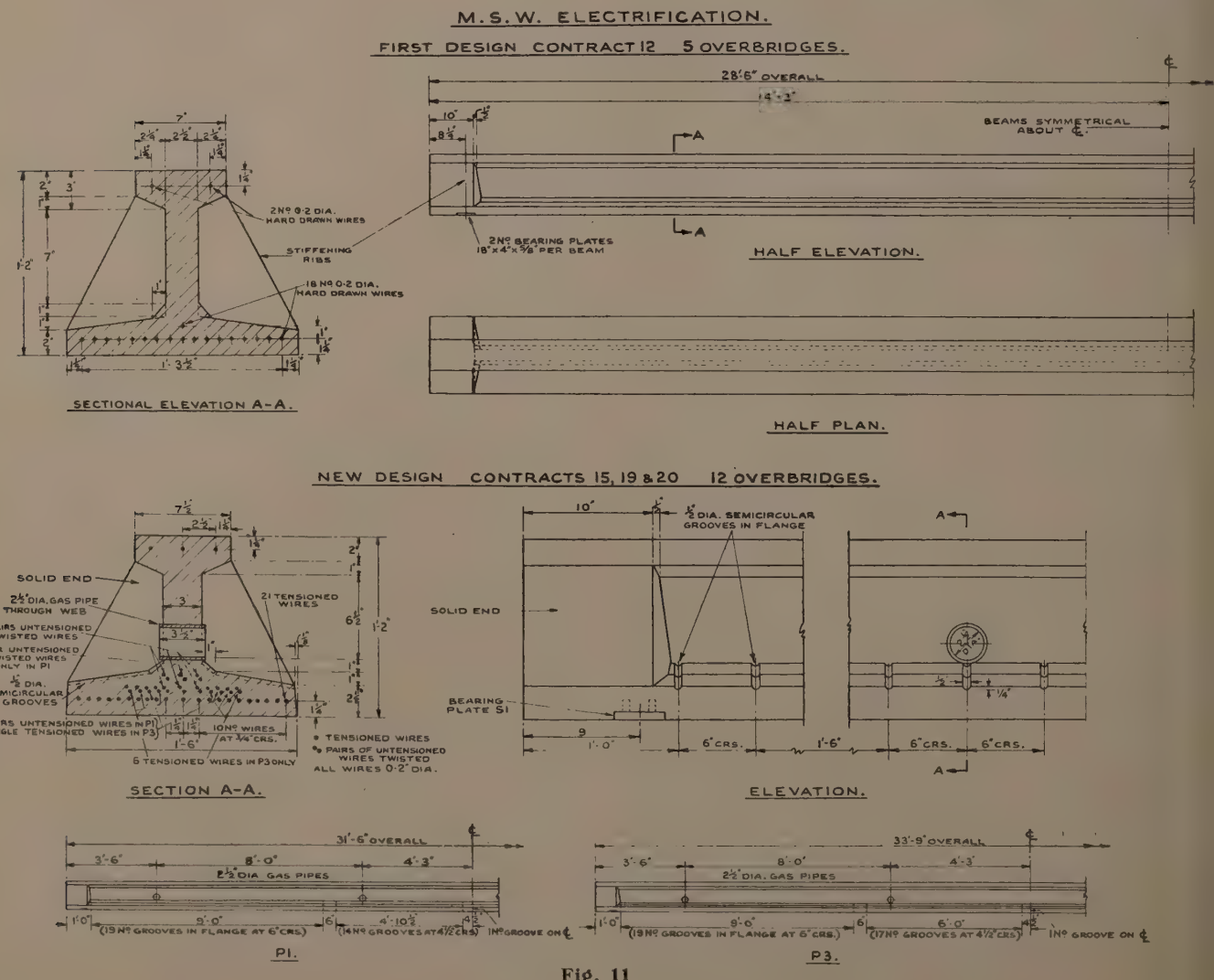

M.S.W. ELECTRIFICATION.
FIRST DESIGN CONTRACT 12 5 OVERBRIDGES.
28'-6" OVERALL
BEAMS SYMMETRICAL ABOUT ℄
2Nº BEARING PLATES 18"x4"x⅜" PER BEAM
HALF ELEVATION.
2Nº 0·2 DIA. HARD DRAWN WIRES
STIFFENING RIBS
18 Nº 0·2 DIA. HARD DRAWN WIRES
SECTIONAL ELEVATION A-A.
HALF PLAN.
NEW DESIGN CONTRACTS 15, 19 & 20 12 OVERBRIDGES.
SOLID END
2½" DIA. GAS PIPE THROUGH WEB
5 PAIRS UNTENSIONED TWISTED WIRES
1 PAIR UNTENSIONED TWISTED WIRES ONLY IN P1
½" DIA. SEMICIRCULAR GROOVES
21 TENSIONED WIRES
2 PAIRS UNTENSIONED WIRES IN P1
2 SINGLE TENSIONED WIRES IN P3
10 Nº WIRES AT ¾" CRS.
6 TENSIONED WIRES IN P3 ONLY
• TENSIONED WIRES
% PAIRS OF UNTENSIONED WIRES TWISTED
ALL WIRES 0·2" DIA.
SECTION A-A.
½" DIA. SEMICIRCULAR GROOVES IN FLANGE
SOLID END
BEARING PLATE S1
6" CRS.
1'-6"
ELEVATION.
31'-6" OVERALL
2½" DIA. GAS PIPES
(19 Nº GROOVES IN FLANGE AT 6" CRS.)
(14 Nº GROOVES AT 4½" CRS.)
1 Nº GROOVE ON ℄
P1.
33'-9" OVERALL
(19 Nº GROOVES IN FLANGE AT 6" CRS.)
(17 Nº GROOVES AT 4½" CRS.)
P3.

Fig. 11

detail in the upper part of Fig. 11 are of the same cross section as the test beam P1, discussed in paper [10]. As indicated in the figure, a few links $\frac{1}{4}$ and $\frac{3}{8}$ in. dia. were provided in the stiffener ribs at the ends of some of the beams to avoid horizontal cracks at release of the prestress, but tests proved that they were unnecessary and most of the beams (also test beam T1) of this first type were cast without them. In this design 1, mild steel

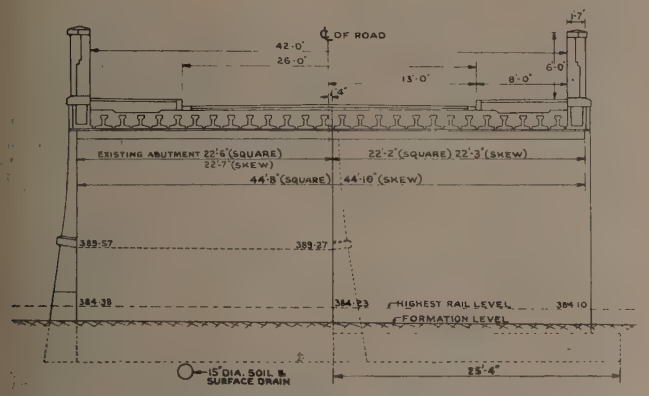


Fig. 13.—Sectional elevation of Lodge Lane Bridge; reconstruction and widening 1950

reinforcement has been added in the *in-situ* concrete to carry, in conjunction with the prestressed beams, the full design load, i.e., M.o.T. loading, or 15 units B.S.S. + 50 per cent. impact. The first contract, for five overbridges, was let during the autumn of 1948 after comparing an alternative quotation for a similar bridge deck construction, with encased steel joints of the same limited depth. This proved the great economy of a composite partially prestressed design ; in fact, the alternative construction would have been dearer by some 65 per cent. than the prestressed equivalent, and the steel consumption would have been approximately $5\frac{1}{4}$ times as great.

Manufacture of the prestressed bridge beams for the first five overbridges was carried out by Dow-Mac, Ltd., at Tallington, Lincs., in the open air during the mild winter of 1948/49. Side parts of moulds were placed after the wires were tensioned and remained until just before release. Six beams at a time were manufactured upside down on one 200 ft. bed. A number of test specimens were cast and cube strengths of over 5,000 lb. per sq. in. obtained, before the prestress was released, i.e., after five days. Immediately after casting, the concrete was protected against frost by placing straw around the moulds.

Fig. 12 shows the placing of the prestressed beams for the deck of Gilroyd Lane Bridge, which was carried out in an afternoon in the intervals between trains, without a special possession, using a road crane, the weight per beam being approximately 25 cwt. The provision of the tensioned wires in the top flange allowed lifting up the beams at any point without the risk of cracks.

Precast parapet concrete slabs were employed to reduce the weight, but in subsequent designs, local stone or suitable brick were used in order to meet the wishes of the local planning authorities.

In 1949, in addition to a second public road bridge of the same type, three deck constructions for private road bridges were built on the lines of the design shown in Fig. 10(d). However, this latter design has proved less economical. Precast slabs were employed and homogeneity secured by concreting the inter-spaces between them above the prestressed beams, and by links protruding from the slabs and beam into this *in-situ* concrete jointing. The cost of this type of deck construction was higher than that of the solid slab and the carrying capacity less ; in addition, transport of beams with protruding links was more difficult. Consequently, in later bridge decks, whether for public or private roads, the same composite slab construction has been employed embodying beams with untensioned wires. This arrangement not only saves steel, but reduces labour at the site.

(12) New Design—14 Bridges, Built 1950-51

The Shaw Lane Bridge, Mouselow, is the first of these 14 bridges built in accordance with the revised design. Since untensioned wires were also embedded in the prestressed beams, the cross section was modified, as shown in the lower part of Fig. 11, so as to give sufficient space for the wires. The beams are longer than those for Gilroyd Bridge, being 33 ft. 9 in. long, as against 28 ft. 6 in. In both cases steel bearing plates are provided. A steel plate 4 in. by $\frac{5}{8}$ in. is cast into the underside of each beam, to which intermediate bearing plates are fixed, bearing plates being provided on the abutments. The Lodge Lane Bridge, at Hyde, near Manchester, comprising a replacement and widening of the previous bridge, was the first public road bridge containing untensioned wires incorporated in the beams. As is seen from Fig. 13, a cross section through the bridge, channel-shaped composite parapet beams were provided to carry the heavy brick parapet.

The prestressed beams were manufactured by the Atlas Stone Co., Ltd., at Shorne, near Gravesend. In this case the beams were manufactured in an upright position, three beams being cast on the same day, and the side parts of the moulds were removed after one day. In the second contract for four bridges, ferrocrete cement was used, whereas for the remaining bridges, super-sulphated acid resisting Sealithor cement was employed. In the latter case smoke boards provided in previous designs are dispensed with. Surface carbonation of the super-sulphated cement was countered by the application of a solution of lime water shortly after casting.

Fig. 14.—Reconstruction of Mouselow Bridge, 1950. Placing of bridge beams

Fig. 14 illustrates the placing of the beams at Mouselow Bridge. In this case a temporary bridge was

provided above the former bridge, after the main arch was removed, and the beams were placed by a road crane from the temporary bridge. This work took place on Sunday, March 12th, 1950, during intervals between trains, when the road traffic was interrupted.

The next sites where the new design was employed were near Gorton, Manchester. In this case, there was no possibility of direct access to both bridges as only

Fig. 15.—Reconstruction of bridges near Gorton, 1950. Lifting of parapet beam from railway wagon

a temporary footbridge was provided. Consequently, the bridge and parapet beams were brought to the site by rail and lifted by a 10-ton railway crane from the wagons, as shown in Fig. 15. During temporary possession of one railway track on Sunday, April 16th, 1950, all beams, viz., 18 ordinary beams and two parapet channels, were placed during two intervals of 1½ hours and one hour respectively, In Fig. 16 the beams are seen being placed at the widening of Lodge Lane Bridge ; four transverse rods are already threaded through the

Fig. 16.—Widening of Lodge Lane Bridge, 1950. Placing of beams

holes of the beams. Fig. 17 shows concreting of the deck, the distribution bars being seen in the left part of the photograph. All bridges referred to in Sections (11) and (12) were erected by Messrs. Wellerman Bros., Ltd., Sheffield.

(13) The Design of Composite Bridge Deck Construction

As already mentioned, the prestressed beams form the shuttering for the *in-situ* concrete and are designed to carry the weight during placing (see Fig. 18 (a)). The combined section of precast and *in-situ* concrete (Fig. 18 (b)) carries the road surfacing and the live load without the occurrence of any concrete tensile stresses under dead weight. Table III shows a practical example. In this case it is assumed that the moduli of elasticity E_0 and E_c^1 of prestressed and *in-situ* concrete respectively are equal. In some cases the precast concrete will have

Fig. 17.—Widening of Lodge Lane Bridge, 1950. Placing of additional concrete

a higher strength and modulus of elasticity than the *in-situ* concrete. However, the ratio of these moduli will only slightly exceed unity and be hardly ever greater than 1.25. Although at low stresses the modulus of elasticity of the precast concrete may be much higher than that of the *in-situ* concrete, under higher stresses and greater strain the modulus of the precast concrete will be reduced, while the stresses in the *in-situ* concrete are much less than those in the prestressed concrete so that the two moduli will be nearly equal under working load.

(14) Economy of the Bridge Deck Construction

The cost of precast prestressed concrete placed in position is rather high, depending mainly on the depreciation of moulds. If larger numbers of units are

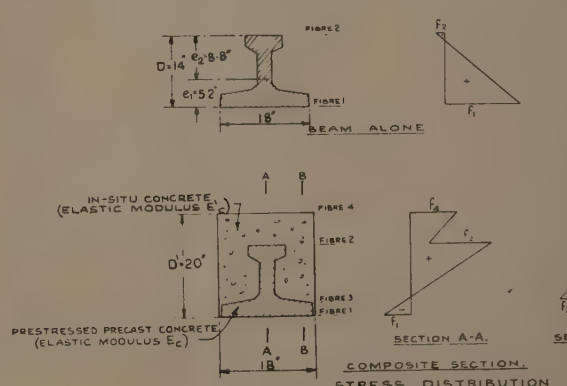


Fig. 18

required, the costs are obviously reduced. Precast units of the kind described in this paper, which were easily transported and placed in position, cost approximately 30s. per cu. ft., including the cost of high strength wire. Ordinary concrete placed in position costs about 3s. to 4s. per cu. ft. Thus, a composite slab consisting of ¼ precast and ¾ *in-situ* concrete may total approxi-

mately 11s. to 12s. per cu. ft. There is no need for special shuttering or propping, since the precast part forms the shuttering and is capable of supporting the *in-situ* concrete. The cost of the mesh placed in the *in-situ* concrete is of little influence.

From the above data, it is seen that the economic superiority of such a composite slab depends mainly on the reduction in volume of the precast concrete component. Obviously, any construction made entirely in precast prestressed concrete is more expensive. It is the particular feature of partial prestressing that the cross section can be decreased, and thus the part of the precast component reduced. This feature applies also to non-composite prestressed members.

When partial prestressing is applied to post-tensioning, not only is the cross section reduced, but also the unit price per cu. ft. concrete will be decreased, since the costs of anchorages and tensioning are correspondingly less when some of the wires are placed untensioned in the concrete.

(15) Other Designs in Partial Prestressing

Prestressed I-beams and precast concrete slabs form the roof of a water treatment plant at Woodhead Tunnel. This construction has proved to be cheaper than another design using steel joints. In this case wires gauge 14 have been used for the beams of 22-ft. length and 12-in. depth. These beams, which were manufactured by Concrete Developments, Ltd., Iver, Bucks., in 1949, have passed the checking test with a load corresponding to a tensile stress of 750 lb. per sq. in.

TABLE III.—Example for Composite Bridge Deck Construction

Concrete stresses in homogeneous section in lb. per sq. in. ($E_c = E'_c$) (+compression, — tension)

	Fibre	Precast concrete		*In-situ* concrete		Z values
		1	2	3	4	in.³
1	Prestress at transfer ...	2,324	— 113	—	—	
2	Effective prestress ...	1,825	— 89	—	—	$Z_1 = 404$
3	Self weight of slab alone (precast concrete and wet *in-situ* concrete) ...	—1,190	+2,015	—	—	$Z_2 = 239$ (Fig. 18)
	Resultant ...	+ 635	+1,926	*	*	
4	Additional dead weight alone...	— 152	+ 61	—114	+152	$Z'_1 = 1250$
	Resultant at complete dead weight ...	+ 483	+1,987	—114	+152	$Z'_2 = 3125$
5	Live load alone	—1,070	+ 428	—804	+1070	$Z'_3 = 1008$
	Resultant at working load	— 587	+2,415	—918 **	+1222	$Z'_4 = 1250$ (Fig. 18)

* Due to creep and shrinkage small stresses are imparted to the *in-situ* concrete after it has hardened ; however, they are neglected here.

**This stress in fibre 3 is of no influence upon visible cracking, since in all investigations cracks have first been observed at the bottom fibres.

Other designs in preparation employing partial prestressing relate to purlins and roof girders for sheds, such as the design for girders of 76 ft. at a proposed new goods shed at Bury St. Edmunds according to Fig. 19, employing post-tensioned cables. In this case a preliminary estimate has shown substantial savings as compared with steel construction. The roof cover is asbestos cement troughing requiring a fall of at least 4°, the dead weight being approximately 5 lb./sq. ft. Fig. 20 shows a design for large railway sheds. Preferably a cantilever construction is used if the length exceeds 100 ft. Also in this case the light asbestos cement roofing is employed. The cantilever beams in this design are 30 ft. apart.

Another instance in which partial prestressing can be suitably applied is poles. Tensile stresses of 1,000 lb. per sq. in., or more, may be taken as permissible. A factor of safety of 2 would ensure complete serviceability, even at a loading of 1.8 times the maximum live load. This is a higher value than obtained with steel masts, for which a factor of 1.5 applies to the serviceability, depending on the yield point. In cases where loading occurs in two opposite directions, partial prestressing is particularly advantageous, because, the precompression is reduced, which influences the loading capacity, since the resistance is often governed by the compressive stress in the first case. A design for masts has been prepared for consideration with planned future railway electrification using overhead conductors. Fig. 21 shows an anchor mast in prestressed concrete superimposed over the elevation of a steel mast. The prestressed concrete structure having I-shaped columns and box-shaped horizontal members, is in this case designed to be free from cracks, a concrete tensile bending stress of 700-800 lb. per sq. in. being permitted for the maximum load which would occur only either at fracture of a cable or temporarily at fixing the cables at one of the anchor masts.

II. Constructions Suitable to Mining Subsidence

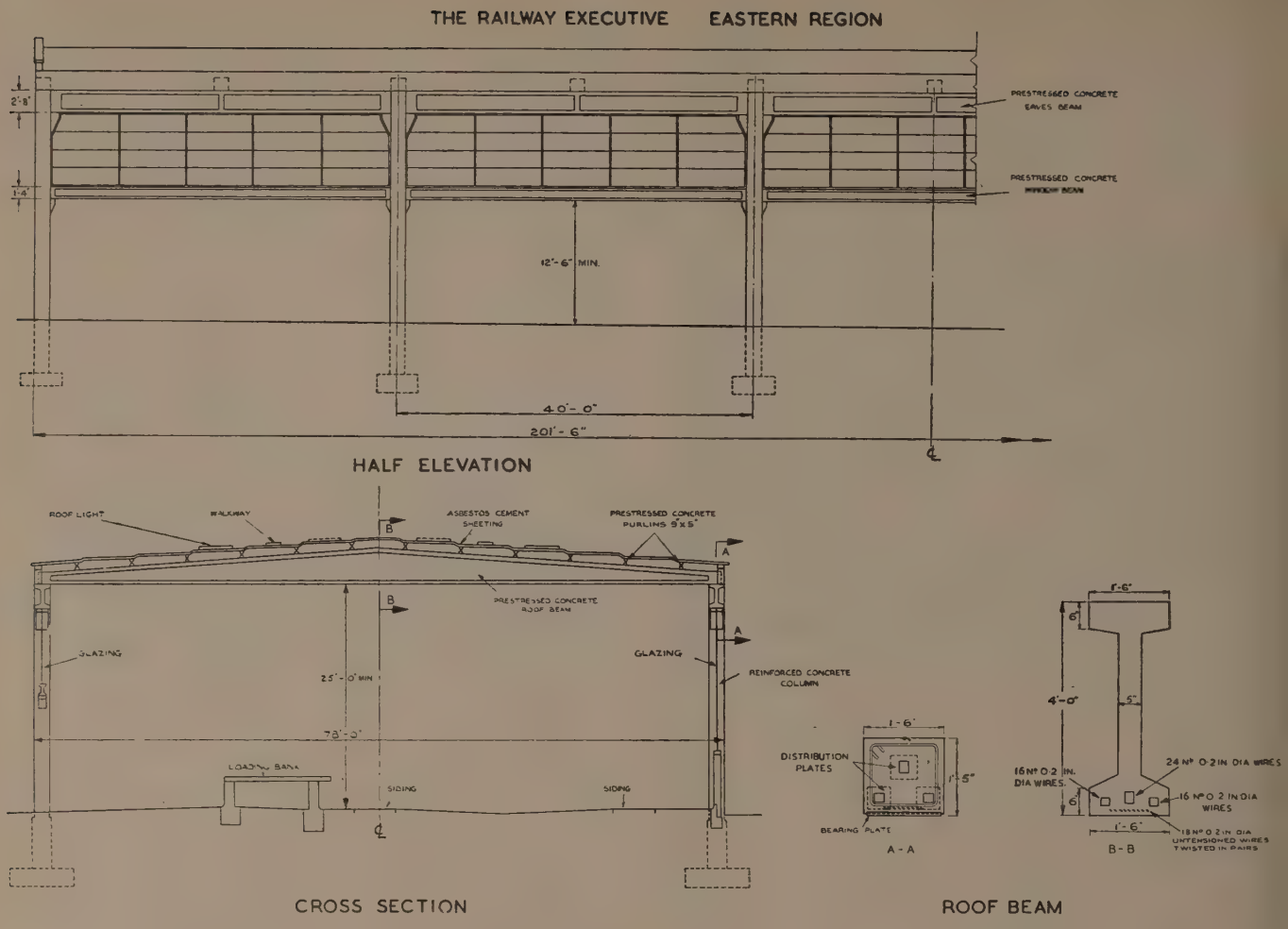
(Designed only for ultimate load conditions ensuring serviceability)

1. General Considerations

Prestressed concrete is particularly suitable for use in such constructions, since it is resilient almost to failure load. It is difficult to know what loading will apply at subsidence ; it is, therefore, necessary to make the construction as strong as possible. The ultimate resistance, therefore, becomes nearly identical with that for which the structure has to be designed. It is obviously of great importance to make full use of all possibilities ; thus, the co-operation of the tensile resistance is essential and, where an assembly of blocks is employed, mortar joints should be used. Prestressed concrete has an advantage, as compared with other materials, in that it remains serviceable after a loading approximating the maximum load has taken place.

(2) Strengthening a Church Tower at Silverdale Staffordshire

Strengthening of the tower at St. Luke's Church was necessary in 1948, because tilting had taken place, and serious cracks had developed owing to mining subsidence, further subsidence being expected. The underlying idea of the design, as described more in detail in [23], is illustrated in Fig. 22. Chases " a " were cut on both



BURY ST EDMUNDS NEW GOODS SHED

Fig. 19

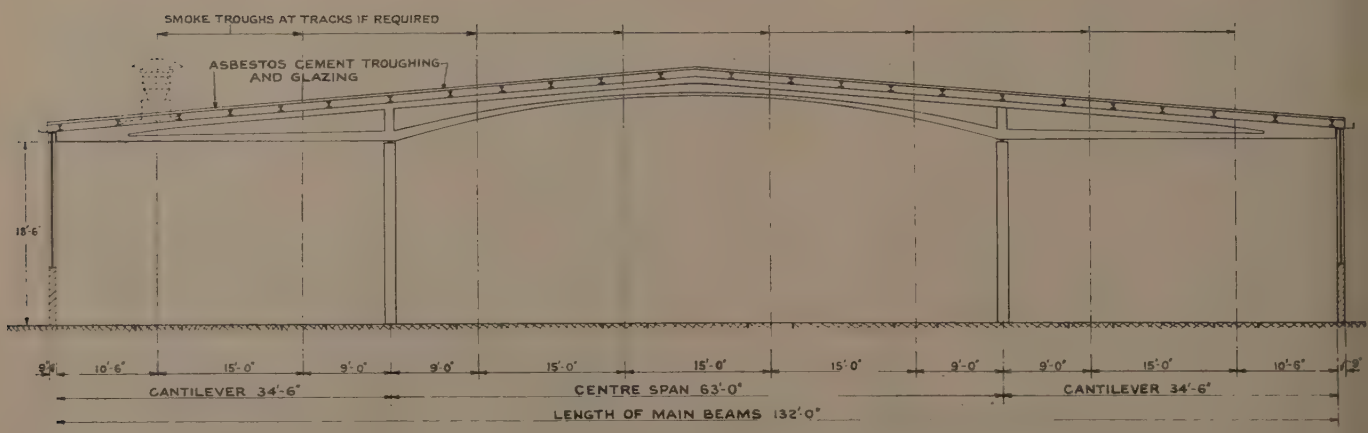


Fig. 20.—Prestressed roof construction for large railway sheds. Example, shed with eight tracks

sides of the wall with recesses " b " at the two ends. Wires " c " placed in these chases were tensioned and secured by anchorages " d " to R.S. Joists " e," the prestress being transmitted to the wall by means of distribution plates " f." The chases and recesses were afterwards filled with cement mortar.

Thus, a construction was created in the wall capable of acting, in the event of further subsidence, either as a simply supported beam or as a cantilever ; the respective compression zones in the masonry at the centre above the chases, and at the ends below them, were previously consolidated by inserting cement mortar through holes, as indicated in Fig. 22 by shading.

(3) Cill Beams for Existing Road Bridges

Twelve cill beams have been designed in the Civil Engineer's Office of the Railway Executive, Eastern Region, at King's Cross, for use in connection with the raising of bridges at Wath-upon-Dearne, which became necessary owing to mining subsidence. In view of the high soil pressure at the existing foundation, these cill beams had to be as light as possible. It was

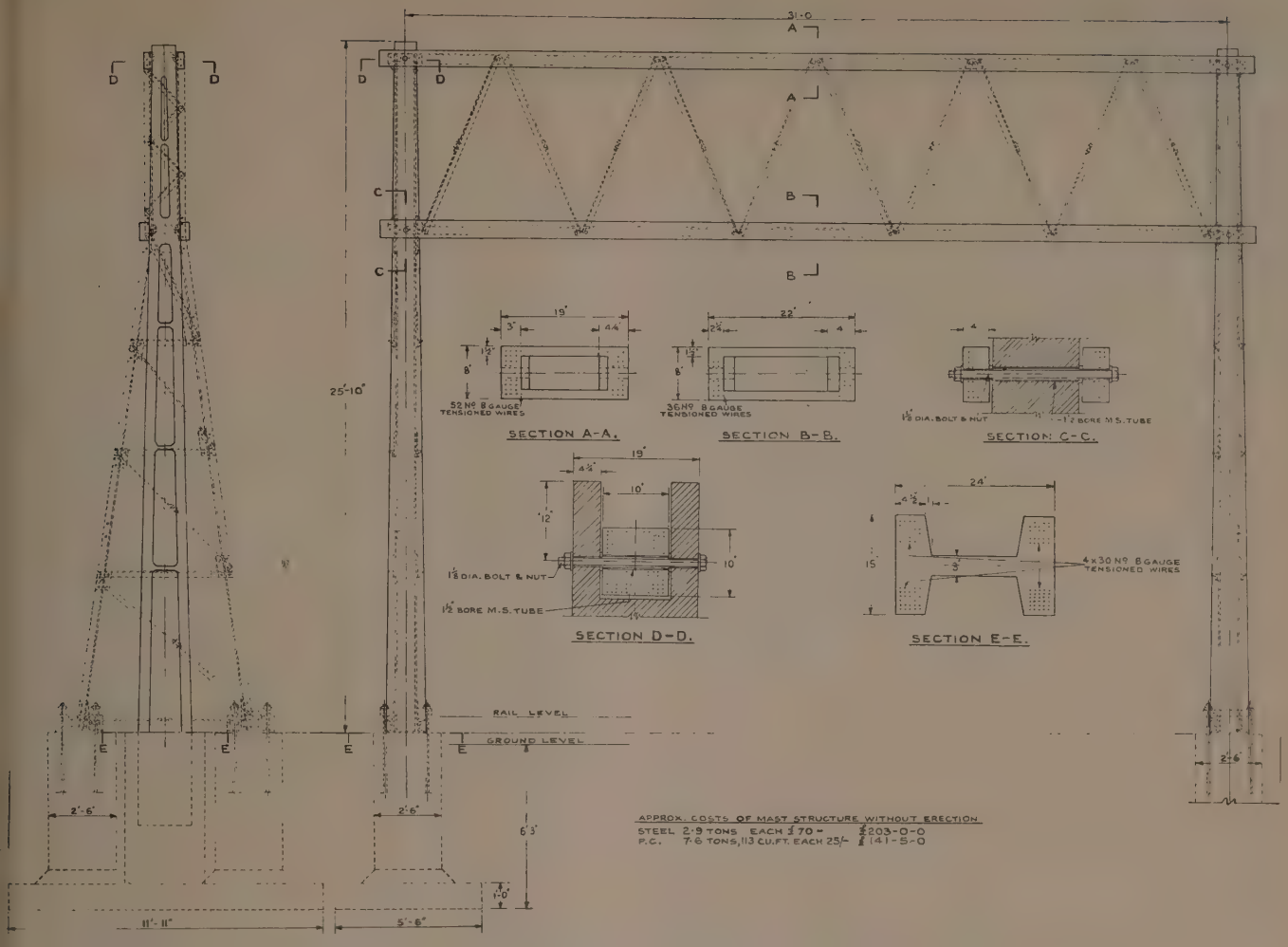
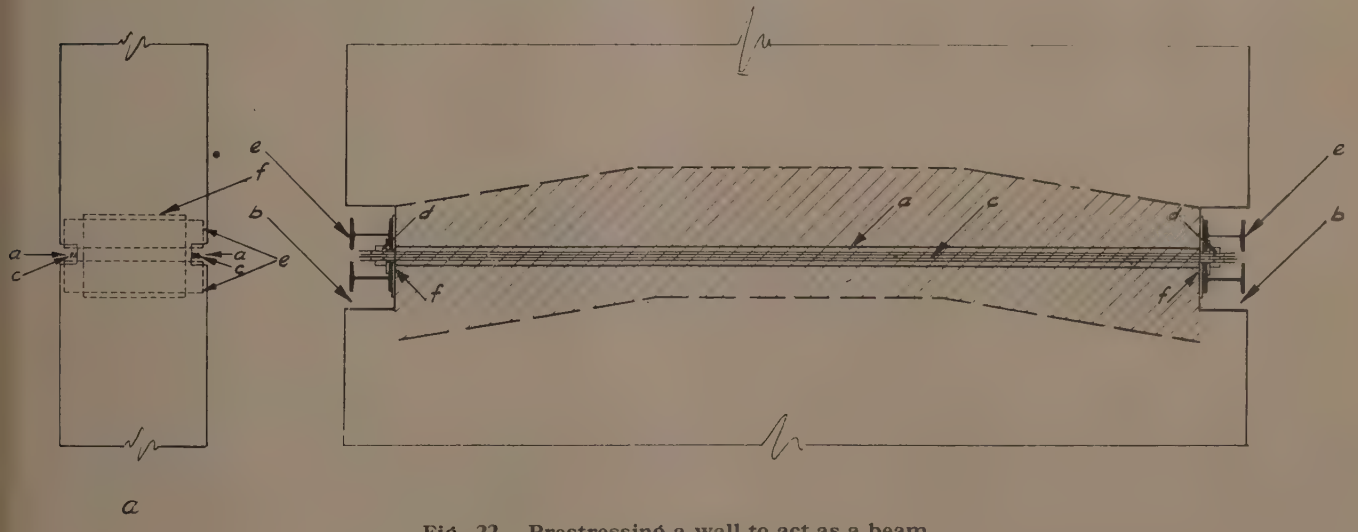


Fig. 21.—Anchor structure. Comparison between prestressed concrete and steel structures



a

Fig. 22.—Prestressing a wall to act as a beam

therefore, decided to insert a number of hollow blocks in the piers, and abutments, and to prestress this assembly of blocks by means of post-tensioned cables after the joints had been grouted, whereupon the wires were to be grouted in. Owing to difficulties peculiar to railway operation, the method of erection was changed, and it was not possible to adhere to the original design. However, the beams were manufactured in accordance with the original design, in order to test the application of this type of construction.

Considerable difficulties occurred in making sufficiently strong cement mortar joints between the blocks. It would certainly have been a more satisfactory and economic solution to have a monolithic hollow beam produced, either with pre-tensioned or post-tensioned wires. Such beams built from blocks do not appear generally suitable unless the blocks are singly inserted, as originally intended, and as would be the case in underpinning.

For the manufacture of these 12 cill beams, Magnel-Blaton cables were used, and the tensioned wires embedded in grout. In order to increase the ultimate resistance to the maximum possible, efficient grouting is essential. The beams were designed to act as simply supported beams or cantilevers, as the case may be, at subsidence. Consequently, top and bottom reinforcement were approximately equal in the beams.

Fig. 23.—Cill beams built from hollow blocks with post-tensioned cables, 1949; grouting

During grouting, shown in Fig. 23, various mixes were used and tested, in order to determine the most satisfactory solution. Some difficulties occurred in providing continuity of the apertures at the joints between blocks. This made it more difficult to insert a stiff grout from the end of the beams. Table IV contains briquette and cube strengths of several mixes of grout. It is seen that the very liquid mix of 1 : 1 : 1, often recommended, has relatively low strength properties. A mix 1 : 2, with a water-cement ratio 0.58, has been found to be more suitable from the point of view of easy grouting and obtaining a higher strength. However, in a future case with straight-through holes, a grout, having a lower water-cement factor, might be preferable. These cill beams were made by The Costain Concrete Co., Ltd., Childerditch, Essex, in 1949, and inserted in the bridge in 1950.

TABLE IV.—Compressive and Tensile Strengths of Various Grout Compositions
(Strengths in lb. per sq. in.)

Mix	sand : cement	3 : 1	2 : 1	1 : 1	1 : 1
	Water/cement ratio	0.8	0.58	0.49	1.0
Cube strength (2 in.) at 28 days		1600	3800	2000	1350*
Briquette strength (1 in.) at 28 days		240	300	250	220†

*7 days' strength.
†5 weeks' strength ; the 7 days' briquette strength was 175.

(4) Turntable Foundation

Fig. 24 shows details of the design of a foundation for a turntable built at Wath-upon-Dearne, in 1950, where mining subsidence notices have been received to a total of 5 ft. 1 in. The construction is designed to permit the table to continue in use after considerable settlements have taken place, and consists of a wheel with a number of spokes ; the basic idea of this construction originated from Mr. R. E. Sadler. The rim and spokes are designed to act as beams or cantilevers. Thus, in the event of a minor local settlement, the construction should be capable, without breaking, of carrying an engine and, after removal of the locomotive, the resilience of the construction should permit a simple restoration to level with jacking and under-packing, if necessary. This is a construction built *in-situ*, involving 110 cu. yd.

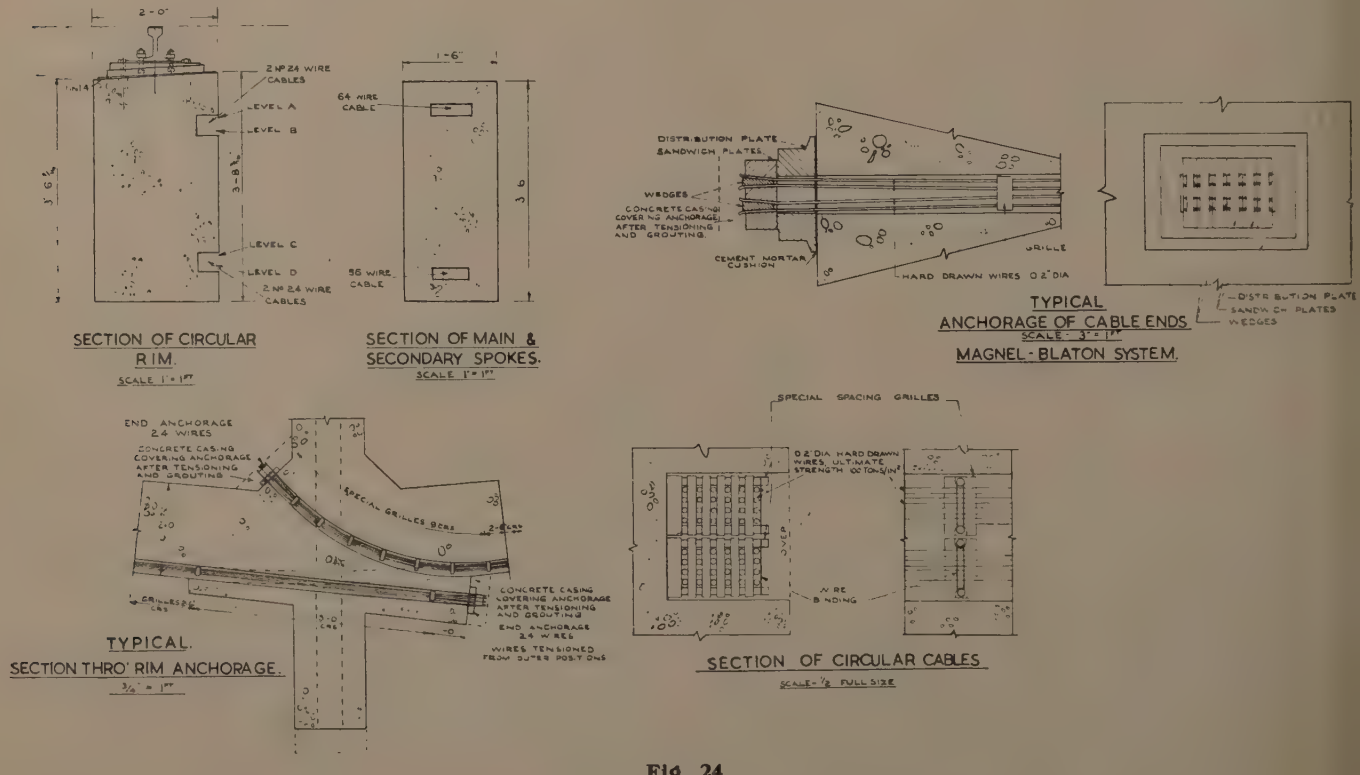

SECTION OF CIRCULAR RIM.
SECTION OF MAIN & SECONDARY SPOKES.
TYPICAL ANCHORAGE OF CABLE ENDS
MAGNEL-BLATON SYSTEM.
TYPICAL SECTION THRO' RIM ANCHORAGE.
SECTION OF CIRCULAR CABLES

Fig. 24

concrete, and using Sealithor acid-resisting sulphated cement, and Magnel-Blaton cables were used. Some of these were threaded through holes and some placed in grooves provided at the circumference of the wheel. The foundation construction seen in Fig. 25 was built in 1950 by Messrs. C. R. Price, Doncaster.

III. Summary. Some Advantages

A. Prestressing Generally

(1) The structure remains flexible up to 90 to 99 per cent. of the failure load, resulting in enhanced service-

Fig. 25.—Prestressed Turntable Foundation at Wath on Dearne 1951

ability. (Steel and reinforced concrete become unserviceable due to permanent deformation much below the failure load.)

(2) Freedom from cracks under working load or under frequently occurring load, as desired. (Impossible with ordinary reinforced concrete.)

(3) Increased life, no maintenance. (As compared with steel and timber.)

(4) Reduction in construction depth for simply supported beams (as compared with ordinary reinforced concrete).

(5) Saving steel.

(6) High degree of resistance to fatigue.

B. Partial Prestressing

(1) Improvement in economy.

(2) Further reduction of construction depth.

(3) Permanent freedom from cracks under working load in spite of concrete tensile stresses :—

Up to 600 lb. per sq. in.	where repeated loading occurs	for modulus of rupture.
,, ,, 750 ,, ,,	for sustained loading	
,, ,, 950 ,, ,,	for rare loading	of 1,000 lb. per sq. in.

(4) Cracks temporarily visible under load close and become invisible on reduction of the load even if the maximum load occurs with many repetitions.

(5) Very high resistance to failure even after fatigue with constant opening and closing of cracks.

C. Composite Members

(1) Greatly increased economy, if the rather expensive prestressed component represents only part of the entire quantity.

(2) Simplification of work at the site.

D. Prestressing of Walls to form a Carrying Structure

By providing chases in walls and housing tensioned wires, the wall can be employed to act as a beam.

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[26]" Fully and Partly Prestressed Concrete," correspondence and closure, JOURNAL AM. CONCRETE INST., Vol 42 (Supplement), November, 1945.

[27]" The Use of Pre-cast Prestressed Beams in Bridge Deck-construction," by A. A. Paul, JOURNAL INST. CIVIL ENG., No. 1, 1943-44, November, 1943.

Book Reviews

Winds, Waves and Maritime Structures, by R. R. Minikin. (London : Griffin, 1950.) 216 pp., 9 in. × 6 in, Illustrated, 25s.

English literature is deficient in books on maritime construction, and Mr. Minikin's little book is doubly welcome for that reason. We wish the book were three times as large, so that many of the subjects briefly touched upon could be more fully developed.

Mr. Minikin has presented a full and clear discussion on the difficult subjects of wind, weather, wave generation, wave form and wave pressure, and has brought forward some of the newest researches on wave pressure, including some original methods of his own for arriving at results without difficult mathematics. Even more stress might have been laid on the difference between the moderate pressures exerted by reflected waves, about which relatively little is known.

The principles of harbour shape are dealt with and there is full treatment of breakwater design, though more might have been said about the mound type of breakwater. The long chapter on design of jetties and dolphins is approached from an up-to-date angle, and brings in several recent developments. Sea walls, slipways and dredging are only lightly touched upon.

When a second edition is published, and we are sure it will be soon, we hope that it will be a larger book, and that some re-grouping of the information will be possible, so that all the matter on breakwaters comes together. Captions under the excellent diagrams would help in reading, and a bibliography would be appreciated by those wanting more information.

M. N.

Reinforced Concrete Design, by John Berry, B.Sc. (London : Hutchinsons Scientific and Technical Publications, 1945. Second impression). 108 pp., 7½ in. × 5 in., 10s. 6d.

This little book is divided into five sections. The first deals with the physical properties of concrete and its constituents, while the second introduces the elementary principles of design. The third and fourth develop the design of concrete members, while part five gives two short examples of design. This section might have been extended with some advantage.

Only elementary algebra and mensuration are used, but the mathematical basis of design is fully developed from first principles, which may possibly make a stronger appeal to young Engineers than to Architects or Surveyors.

A particular feature of this book is the consideration given to the ultimate strength of concrete members. The alternative principles of design, using allowable stresses, or using a load factor based on ultimate strength is discussed, and an introduction to the Whitney theory is given.

It is felt that the stresses used should now be revised to agree with C.P.114, but this does not seriously affect the book's usefulness.

It can be recommended to young Engineers starting the subject, and those with some experience are likely to find it of interest.

Both the type used and the illustrations are very clear, and the quality of the paper good. F. G. E.

City and Guild Services. Sketching for Craftsmen by Ernest Hoyle (London : English University Press, 1950). 80 pp., 8½ x 5½ ; illus., 5/-.

This book deals in a simple and clear manner with the elementary principles of perspective in a series of carefully graded exercises explained on the adjacent pages by notes in easy and colloquial style. It is primarily for beginners, but experienced draughtsman may well find some points of interest in the ingenuity of the diagrams. It will be of particular use to those who are not natural draughtsmen and who find difficulty in pressing their ideas in a workmanlike manner on paper.

The author's technique shows itself best in the workshop sketches and the machine perspectives ; the necessity of keeping good proportion while sketching is emphasised. Hints on sketching simple parts later to be built up into a more intricate picture, show how essential it is to have a fore-knowledge of the general shape of all individual parts in order to make a clear picture of the whole. The notes at the end of the book and the very sound advice on dimensioning, are points worthwhile following.

This is a thoroughly useful book for the examinee and as such it is very competently fulfilling its purpose. Should it fulfil its author's hopes and cause students to sketch for a hobby as well as a career, the sound grounding in basic perspectives it will have given them, will make their pens and pencils, a source of profit and interest in their serious work and their leisure moments.

S. J. C.

Special Treatments for Portland Cement Concrete

Discussion on Mr. C. L. a'Court's Paper*

Before presenting this paper, Mr. a'Court mentioned that it was written about a year ago. As it dealt mainly with ideas and was not a record of work done, it might be found that some remarks which he was about to make did not agree exactly with what was said in the paper as printed. He felt that this was inevitable with ideas, which must necessarily develop and change.

Thanking the lecturer and opening the meeting for discussion the Chairman said they had had a most interesting lecture. Mr. a'Court had thrown out a wealth of ideas and the most intriguing one was that as concrete was such a readily used material we were stretching it to the limit to which it could be used. He wondered if that was so, for when they could produce a light weight concrete which was strong there was a whole range of other uses which had not yet come into being.

They had had a most interesting lecture and he proposed a formal vote of thanks.

Mr. DONOVAN H. LEE (Member) said he had greatly enjoyed Mr. a'Court's lecture. It had opened up great opportunities for questions and for example, he would have liked to hear more about air entraining. On the one hand we had spent a long time trying to get all the air out of concrete, for example by using vibrators, even getting vacuum concrete which Mr. a'Court had not mentioned, and now the Americans had found that the only way they could make their roads stand up to the salt treatment in the winter was to put in air!

With regard to pozzolanas, he thought they would all like to know if they were available in England. In view of the number of reinforced concrete structures built from fifteen to forty years ago which were now in a shocking state of deterioration it seemed the use of concrete of a poor mix, or uncompacted or with an excessively high water-cement ratio did lead to serious results in the course of time, particularly in the industrial areas where the action of the atmosphere on the concrete was greatest. He assumed the pozzolana did combine with the lime of the concrete, and he assumed wider use of pozzolanas was desirable although not a way of avoiding the need for a good mix, proper compaction and so on.

With regard to linseed oil as a surface treatment, was it true that it could be used? Mr. a'Court had mentioned oxygenating and evaporating as the way linseed oil dried. He understood that linseed oil could not evaporate and that there was no extractable water in it. It could only oxygenate and the common practice of warming paint to dry it was wrong because all that was needed was oxygen, and a lot of it. In other words a fan was better than warmth.

He had noticed Mr. a'Court had shown a graph of the effect of wear or abrasion of concrete surfaces, but it was generally understood that the reason for its use was to prevent dusting and this it did do. His experience had been generally that the second treatment merely created a gluey substance on the surface and sometimes even the first treatment seemed as though it would not get into the pores. He was not clear whether with very good concrete anything did penetrate. Did sodium silicate start a chemical reaction with the cement next the surface of the concrete, and did it have a similar effect like case hardening of a metal? Did it do that, or did it merely fill the pores if any existed?

Dr. T. W. PARKER remarked that the subject, as Mr. a'Court had told them, had many sides and subdivisions. At the Building Research Station they had found it helpful to divide additions to cement into two groups—those in which there was a large percentage addition and those in which the addition was only a few per cent. (and often less than one per cent.). In the first case one got such substances as pozzolanas or granulated slags, or inert additions, in one group. It was an advantage that most of these were materials which could with most advantage be admixed by the manufacturer rather than on the site. The user was freed from the need to adopt controlled mixing; they were fully controlled, obtainable by bag.

The second group contained the minor constituents which influenced the working properties, etc. of the cement, such as calcium chloride or wetting agents. These could not be conveniently admixed by the manufacturer but required to be added by the user. A subdivision of this sort was essential for any systematic treatment of the subject.

On the question of the additions of small amounts of calcium chloride, or air entraining agent, or wetting agent, one of the important effects of the lecture was to give some authorisation guide to the order of results to be obtained. There was need for this, for otherwise there was no basis of comparison with the claims of various proprietary products, whose composition need not necessarily be disclosed to the user.

The lecture emphasised the point that certain minor additions effectively modified the properties of cement other than that of strength. The extension of the use of Portland cement by the engineer, architect and builder had brought the material into fields where strength was not the only important property. Workability, heat generation, and shrinkage might enter in ways which were not normally matters of great importance formerly, but were becoming so now. The more important they became the more important became the minor additions which could affect the properties other than strength of the Portland cement.

He had one query relating to Fig. 5 of Mr. a'Court's paper, in which he showed the effect on the abrasion resistance of sodium silicate. Had there been any scientific theory to explain the phenomena? It did occur to him that it might be related to the degree of carbonation of the cement surface. The action of sodium silicate on a concrete surface in which the free lime had carbonated might be different from the action on a

Read before the Institution of Structural Engineers at 11, Upper Belgrave Street, London, S.W.1., on Thursday, 8th March, 1951, at 6 p.m. Lt.-Colonel R. F. Galbraith, M.C., B.Sc.(Eng.), M.I.Struct.E., A.M.I.C.E. (Vice-President) in the Chair. Published in THE STRUCTURAL ENGINEER, *Vol. XIX, No. 3. pp. 71-77. (March 1951).*

surface in which the free lime was still uncarbonated. In other words, if the lime was still present as free lime, the sodium silicate might produce a lime silicate gel, whereas if the lime was carbonated, the silicate treatment might produce silica gel, this difference between the end products possibly accounting for the difference in resistance. It might be possible to explore this by dyeing the surface with napthalene green B which might be used to indicate the depth of carbonation.

Mr. J. SINGLETON-GREEN (Member of Council) said he had noticed with interest what Mr. a'Court had said at the beginning of his talk about ideas, not jobs, and wondered what contractors would think if the implication was that jobs did stand still!

On the last slide shown there was no vertical scale and no reference to zero. A second point about the curve was that people did not usually know after 14 days whether the floor was going to be satisfactory or not.

He was glad that Mr. a'Court had amplified his earlier conclusion by saying that perhaps almost any floor, even a good floor, could be improved by the use of silicate of soda. He had perhaps seen more bad floors than anyone in the country and had still to find one that could not be improved by the *intelligent* use of silicate of soda. It might need one, two, or three coats, or perhaps even six, but if used properly silicate of soda would improve the floor; the exception occurred if the floor was very old, and in such a case perhaps some oil treatment was the only thing which could be expected to keep down the dust.

He did not go all the way with the speaker when he condemned jumps in grading. He thought we were reaching the stage where we in fact knew less about grading than we thought we did ten years ago. There were people who liked the flat part of the curve and they could not ignore the fact that they were getting some very good results.

A case that came to mind was a job in U.S.A. where they used $\frac{3}{8}$" limestone and very fine sand. They obtained a cylinder strength of 18,000 lb. per sq. in., which would probably be 24,000 lb. per sq. in. on a cube.

As Mr. a'Court had indicated and Mr. Donovan Lee had mentioned, there was concrete and concrete. He thought it a great pity that we could not have different names for the different kinds of concrete. The average man had not seen really good concrete and did not know what it looked like.

The main thing about air entraining was that the air was not in the same form as that which they tried to take out. The air taken out was where they did not want it. The air put in was in minute bubbles and did a job of work.

He did not know whether Mr. a'Court would consider linseed oil under paint technology or not, but he had seen surfaces treated with linseed oil and cement mixtures and they were perfect after 40 years.

They had always regarded strength as the thing that mattered. It was important, obviously, but it was not the most important thing. Durability was far more important and if they could look upon concrete in terms of durability some problems would assume a different aspect. Unfortunately there was no convenient "yard stick" for durability, so they would continue to talk in terms of strength. If Mr. a'Court could suggest some simple "yard stick" for measuring durability, he thought they would get something which was badly needed.

Mr. T. WHITAKER said he would like to ask Mr. a'Court a question and if the reply were that it was difficult to answer he would agree with him. How could one differentiate between workability caused directly by a workability additive which was put in to the concrete, and the workability caused by the air which that aid might entrain? Let them look at the question against the background of American experience. Up to about 1941 the Americans spoke a lot about treated cements and about workability aids. From about this time one heard of air-entrainment, and the Americans now considered that the workability and beneficial results which had accrued from treated cement, or aids put in to the concrete, had been the result of the entrained air rather than the aid itself. He added that if a builder bought ready mixed concrete in America, and did not carefully specify that he wanted non-air entrained concrete, he would invariably be supplied with air entrained concrete, such was the extent of air entraining in America.

Mr. J. M. FISHER referred to gap graded concrete and said there had been issued by the Ministry of Works a memorandum on the saving of cement in which its use was strongly recommended in certain circumstances. He had not had any experience of that type of concrete, but in view of the fact that it had been commented on so favourably by a Government Department, he would like some more information on it.

With regard to workability agents, he agreed with Mr. a'Court that probably the greatest curse of concrete was the fact that water was such a cheap workability agent, it was delivered to the site at 6d. per ton.

The use of an air entraining agent in America as Mr. Whitaker had mentioned, had become almost universal. Whether the adoption of an air entrained concrete was technically justified or not, it was used largely because it was such an aid to increased workability and concrete could be placed with very little effort

A full scale experiment had just been carried out in this country with an air entraining agent in connection with the laying of a concrete pavement and the conclusions reached might be interesting.

The addition of an agent which entrains air, reduces the compressive strength of concrete and at the same time increases workability to a considerable extent. Its use also allows of a reduction in the sand content to the extent of the amount of entrained air. Both reduction in sand content and increased workability allow of a reduction in the amount of water in the mix since the workability of the concrete must remain at a fixed figure in pavement work. The reduced water/cement ratio results in a concrete which has a strength approximately equal to the non air-entrained concrete.

Regarding pozzolanas, he had seen a reference in recent American literature to their use in the Hungry Horse Dam. He believed that pulverised fuel ash had been used extensively and had proved extremely satisfactory. Another example of their use was in the construction of a flood control dam. The concrete was one part cement to eighteen parts aggregate by weight and approximately one third of the cement was a natural pozzolana. An air entraining agent was also used. That type of concrete was something of an experiment, but not a single crack had been revealed in a total length of about 1,600 ft. It would seem that such a development might be very useful in the construction of hydro-electric schemes in the North of Scotland.

Mr. H. C. ERNTROY referred to Fig. 2 and said that he understood that Mr. a'Court attributed the reason for "a whole series of concrete failures" to the poor grading of the combined aggregate (as shown in the figure) which would result in the use of a too high water/cement ratio. The grading shown is not much finer than No. 3 of Road Research Technical Paper No. 5 over the fine aggregate range and this would not, in the ordinary way, be expected to require an excessive amount of water. He

was wondering whether the real cause of the trouble experienced was due to imperfect compaction as a result of the segregation of the relatively fine mortar in the mix.

His other question related to Table 2. In it the author quoted the C3S content of three types of cement and the corresponding three day strength. He would like to know whether Mr. a'Court considered that these factors were related and whether the quoted values of 45, 50 and 66 per cent for the C3S content were typical for these types of cement or referred solely to the samples used for those tests.

Mr. D. E. Hennessey (Associate-Member) asked whether Mr. a'Court could say what the special ingredients were of the sulphate resisting and low-heat types of cement now being marketed and whether these ingredients had any effect on the strength or behaviour of the finished concrete.

If low-heat cement was a cement made with a base of pozzolana, could he tell them why it was more costly than ordinary Portland Cement.

The lecturer had referred to quick-setting concrete. Mr. Hennessey found a method of obtaining this—by accident—it was by mixing aluminous with ordinary Portland cement. He would not recommend the use of a mixer in these circumstances.

Major H. A. Holt (Member) said he was interested in the aggregate grading curve which Mr. a'Court had shown, particularly as he was inclined to attribute some failures to it. He noticed there was about 33 per cent of sand passing the No. 7 sieve and no aggregate between that and the 3/8" sieve. He had been wondering whether if that proportion of sand had been reduced to perhaps 20 per cent the disadvantages which accrued from the use of that aggregate might not have been apparent? He had also been interested in the remarks about replacing some of the coarser particles of the cement with fine stone. In connection with that, his father, who was a manufacturer of cement in a small way fifty years ago, used to add a small proportion of crushed rock, finely ground, to the cement and he claimed most remarkable properties for the cement. Unfortunately, the Standards association of those days put a stop to it. Nevertheless he did get remarkable increases of strength by so doing.

He rather felt we were on the threshold of new discoveries in using air entrainment. He understood it was a fact that air was one of the most efficient lubricants it was possible to use and he would very much like to have Mr. a'Court's view as to whether the reduction in strength amounting as he said to about 5 per cent for every one per cent of air, was not in some measure tied up with the particle size of the entrained air bubble. If, for example, the normal air in freshly mixed concrete were to be abstracted and then, either by chemical or mechanical means, entrained air of almost micron size introduced, was it beyond the bounds of possibility that one might be able to use air to give workability and water purely to hydrate the cement, thereby obtaining, in as far as the water was concerned, the absolute optimum strength from that cement?

Reply to the Discussion

Mr. a'Court, replying to the discussion, referred to Mr. Donovan H. Lee's enquiries about pozzolanas and their availability in this country. As far as he knew, such materials were not generally available. There had been attempts or attempted projects to manufacture them from time to time but these did not seem to have become commercial propositions. He referred to development now going on in the Western Highlands where it was claimed that there was a material with pozzolanic properties; whether this was the natural material or whether it was necessary to heat-treat it he did not know.

As regards linseed oil, he was not aware that he had used the word "evaporated"; he thought he had said "dried". He understood from his chemist friends that linseed oil dried by oxidation, thus forming an impermeable skin. Mr. Donovan Lee had referred to difficulties in the application of sodium silicate which resulted in a gluey residue on the surface. In connection with any recommendations for sodium silicate treatment, it was necessary to say first that the correct brand should be used—there were many types of sodium silicate, depending on the soda/silica ratio. The correct type having been obtained, this should first be applied at a dilution of 4 parts of water to 1 of silicate. Further coats should then be applied; generally, these could be progressively richer, going from 4 to 1, to 3 to 1, to 2 to 1 dilutions. If, however, the concrete was only slightly porous it might be unwise to use the richer concentrations for the second and third coats, as the "gluey mess" to which Mr. Lee referred might result. Invariably, however, the recommendation was to wash off the floors between coats, after it appeared that maximum penetration had taken place; with even the best laid concrete there were slight inequalities to be observed in which shallow pools of liquid could form.

He had been asked what sodium silicate did and to what thickness it penetrated. The main effect was produced by the breakdown of the silicate and the deposition of silica gel in the pores of the concrete. It was likely that there was, in addition, reaction with the free lime present which resulted in the formation of calcium silicate. Linked up with this was Dr. Parker's question on the abrasion graph which had been shown. The answer to this question was that no explanation of the phenomenon, based on scientific data, had yet been attempted. He agreed that the optimum period coincided with the time when there should be the maximum amount of lime present as such, and it was a likely hypothesis which might well prove to be the explanation. Such was his own present view but it had not yet been substantiated.

Mr. Singleton-Green had also referred to this diagram and to the lack of scale. The author had deliberately not inserted a scale as, in the first place, the units would not convey very much, and, in the second case, the actual shape of the curve depended upon the quality of the concrete. He indicated that there was evidence to show that the depth of the trough was negligible (i.e. that the curve reached a straight line) on the one hand when the concrete was of great density and hardness and, at the other extreme, when it was very poor indeed. In between these limits the depth of the trough probably varied from nothing to a maximum. As to the depth of penetration it seemed that this was effectively about 1/64th or 1/32nd of an inch depending on the quality of the concrete.

Mr. Singleton-Green had also asked about floor conditions and had pointed out that it would not be known what the floor was like until it had been tried, and consequently advantage could not be taken of the optimum period of application. The conditions to which the floor would be subjected were known. If these were going to be rigorous, sodium silicate should be specified when the floor was being put down and advantage could then be taken of the factor under discussion. Almost any floor at any time, however, would benefit from such an application. The present point was to show that there was an optimum of which advantage could be taken.

Gap grading was a subject on which he had to admit that he had no experience ; it was one upon which he, himself, looked forward to enlightenment. As regards the various questions relating to the Fig. 2, he would like to repeat that this was diagrammatic and not related to any specific job. It was typical of a whole number of jobs and that was why it had been introduced. In examining causes of quite a number of troubles of which he had had full record, it had been found that although the trouble might be reported as slow setting, slow hardening, low cube strength, or in various other ways, there was always one constant factor which was an aggregate grading of the type indicated. There were, of course, always other factors ; these varied, but the one quoted occurred in every case.

As concerned Mr. Singleton-Green's point about the non-static nature of jobs, he would point out that that was not the meaning which he had intended : what he had meant was that a job, once completed, was a matter of history and a true record of it remained the same.

He agreed with Mr. Singleton-Green that it would be very desirable to have a measure of quality distinct from that of strength, and added that another very desirable introduction would be a means of determining the eventual quality of the concrete while it was still in its fluid state.

He had been very interested in Mr. Whitaker's remarks on his experiences in America and the use of air entrainment in that country. In this respect also he appreciated the contribution made by Mr. Fisher who had given us the benefit of his own experiences. As regards American practice, the author had no personal experience but had been told that it was necessary to take into account the fact that concrete in that country was not normally of the quality which we should expect here, and that, as a result, additives offered an easy way of cutting down the water and so improving the quality.

As concerns the figures quoted in Table 2, he referred firstly to the correction which should be made in the last figure of Column 1 which should read '1760'. He then added that these related to American cements and he could not say that they were necessarily typical of cement bought in the open market in this country. The point was, however, that they all showed the great increase of strength for a small addition of calcium chloride and, in addition, gave three points on a curve which showed the relationship of the effect to the C_3S content.

He was afraid he could not answer Mr. Hennessey's question as to the precise composition of sulphate-resisting and heat-resisting cement produced in this country ; by sulphate-resisting he took it that Mr. Hennessey meant the modified Portland type cement, as no others were manufactured commercially here. The ingredients, so far as he knew, were substantially the same as for ordinary cements, but with a modification of the final compounds formed. The American specification, for example, required a lower tricalcium aluminate content in sulphate-resisting cements.

The mixing of Portland cements and high alumina cements was a very dangerous thing to do and generally to be avoided at all costs.

Reverting to Mr. Donovan Lee's question on the size of the air bubble in concrete, it was necessary to remember that the air bubble left in by accident was very different from that introduced in air entrainment. Artificially induced air consisted of discrete bubbles dispersed through the mass whereas the air ordinarily entrained during mixing resulted in bubbles of greatly varying sizes scattered indiscriminately, and liable to be connected by capillary paths. He found Major Holt's suggestion of the reintroduction of air a most interesting one, but it seemed to him that it might be a little expensive.

Written Communication

In a written communication, Professor A. BADIAN (Member) points out the danger of using sugar as an additive to concrete. The author replies :-

I am entirely in agreement with what is said. Sugar is a highly dangerous additive to concrete. In the paragraph in question in the paper, I have simply indicated some of the commoner materials which affect the setting time of concrete and I have expressly warned the general practitioner against experimenting on his own account as all treatments which affect the setting time weaken the resultant concrete to a sign ifica nt degree

Book Reviews

Elementary Structural Design, by C. O. Harris. Chicago : American Technical Society ; London : Technical Press, 1951. 163 pp., 11 in. × 8¼ in., Illustrated, 40s.

Professor Charles Harris of Michigan State College, states that his aim has been to produce an introductory course for college students and for those wishing to study the subject at home.

The clear text, profusely illustrated by first-class diagrams and drawings, is presented in such a manner that the reader can master the material without assistance.

It is assumed that the reader possesses no knowledge of statics or elementary elasticity, but it is unfortunate that formulæ relating to these two subjects are presented without proofs of any kind.

The chapters on steel beams and reinforced concrete floor slabs quote working stresses and elastic constants which do not accord with British Standard recommendations or London bye-laws, and symbols used are not those normally adopted in this country.

The book will be most useful to those studying for the Intermediate and Final Examinations of the R.I.B.A., and to first year students of the ordinary National Certificate in Building.

First year University men may regard it as a useful second textbook, and a good illustration of American design methods. D. W. C.

A Code for Sewerage Practice, by L. B. Escritt. (Contractors Record and Municipal Engineering, 1950.) 73 pp., 8¾ in. × 5½ in. 7s. 6d.

This concise handbook on sewerage practice is divided into three sections, sewerage, drainage building and sewage disposal works. Data are given for both towns and for isolated buildings, and recommendations are included in many instances. The book is illustrated with line drawings and tables.

Hydraulic Diagrams. No. 3. Design of Surface-Water Sewers. Frequency of Rainstorms. (Contractors' Record and Municipal Engineering, 1950.) 7½ in. × 5 in. 4s. (3s. 6d. per copy for not less than six copies).

This is one of a series of hydraulic diagrams framed to assist water and drainage engineers in their calculations. By a short system of trial and error the requirements for the design of surface-water sewers are found, according to the Lloyd-Davies formula. The diagram is a thin, pocket-size, convenient for carrying about.

The Construction of the Royal Festival Hall

Discussion on paper by Mr. E. Roland Hole and Mr. F. C. Eales*

The CHAIRMAN, proposing a vote of thanks to the Authors, said all were grateful to Mr. Hole and Mr. Eales for having put on to paper an almost epic story of achievement, an achievement against odds of weather, shortages of materials, shortages of labour and the thousand and one other factors which could so easily have set back their schedule and prevented the completion of the work on time. All would wish to join in congratulating them heartily on a magnificent job well done.

(A cinematograph film was then exhibited, showing the erection of the roof trusses on the Royal Festival Hall).

Mr. GUTHLAC WILSON (Member) recalling Mr. Hole's remark, when presenting the paper, that he had considered reinforced concrete was the ideal material for the construction of the concert hall, said that when his firm had first come to the job they had thought so and they had thought so all through, although they had found a definite pre-conceived opinion that it ought to be done in steel, in the interests of speed. Looking back, however, he did not think it could have been built faster in any material than it was built by Messrs. Holland & Hannen and Cubitts, Ltd., who had achieved a remarkable record.

Some people had asked why the roof was not made o reinforced concrete, as a shell roof. The answer was plain when one saw how the ceiling was suspended from it, and the mass of pipes and ducts that passed through the girders. He did not think a shell roof would have been quite so helpful in meeting all the conditions which had arisen, nor could the work have been done so quickly, as would be appreciated after having seen the film showing the erection of the roof trusses.

The Royal Festival Hall, said Mr. Wilson, was the last major work in which his late partner, Mr. W. L. Scott, was engaged, and in many parts it reflected the results of his many years of experience in reinforced concrete.

Mr. A. C. VIVIAN (Member of Council), in order to give an idea of the size of the job, in addition to that given by the pictures the authors had shown, asked what was the number of man hours devoted to the work. Did the contractors run the whole job on a man hour basis, planning on the basis of so many man hours for each particular trade?

Raising the controversial matter of incentives, Mr. Vivian said he appreciated very thoroughly the point made by the authors that the job was essentially one for planning from start to finish, there being presumably a central Planning Department, aided by the material chasers and progress men on the site. But in addition to planning, and in addition to the very admirable system described for ensuring that the whole of the people engaged on the work were kept in touch with all that was going on, was it also found possible to offer an incentive bonus? If so, were the contractors able to introduce an incentive bonus based on certain trades, or did they consider, even if they did not adopt it, the possibility of producing a scheme whereby they could pay an incentive bonus assessed on a percentage completion of the work as a whole.

Mr. M. M. KHAN (Member), referring to the excavations, said there were four fairly large machines for the excavation work, and their output might be assessed at something of the order of 2,000 cu. yards per day; the actual output from similar machines in use before had been greater than 2,000 cu. yards per day. But he had noticed that that capacity on the site had actually produced an output of the order of only 900 cu. yards daily, and he wondered why there was such a difference between what might have been expected and what was attained. It might have been due to lack of lorries, or to weather conditions, or other matters.

Presumably the hard excavations on the site were the foundations of the earlier buildings. They were fairly old, and therefore it might reasonably be assumed, in the absence of more information, that they were made with lime concrete, or might consist of bricks or stonework built in lime. He would have thought that an easier method than the use of the pneumatic hammer might have been considered.

Referring to the rooters pulled by D.8. tractors, probably they were turned down because the hard excavation was not widespread.

The well point system was a very interesting method of lowering ground water, but it was also very expensive, as the authors had stated in the paper.

The sheet piling presumably had quite a big bearing on the de-watering of the site. To avoid undermining by suction of fine material, it had been considered desirable to put sheet piles on one side of the site, and later on, piles were extended to the other side, i.e., the river side. So that nearly half the periphery of the site was sheet-piled, the piles going right down into good clay. The problem was that if the sheet piling had been continued right round the site it would have entailed something like another 300 tons of steel in piling, the cost of driving them and the cost of extracting them. If the salvage value of the piles after extraction were taken into account, it represented half the extra cost entailed in continuing the sheet piling right round the site. Mr. Khan wondered whether the net cost of that additional sheet piling had been considered against the cost of the well point de-watering. The comparison might throw some light on why the well point system was used. But the driving of sheet piling right round the site might have given quite good security. After all, there were four diesel pumps in operation all the time, the cost of which must have been fairly considerable.

Coming to the very bold method of constructing foundations in mass concrete, Mr. Khan referred to the process of depositing stone rejects in position and subsequently grouting. It was a very speedy method

*Read before the Institution of Structural Engineers at 11, Upper Belgrave Street, London, S.W.1, on Thursday, 29th March, 1951, (The President, Mr. J. E. Swindlehurst, M.A., M.I.C.E., in the Chair). Published in the STRUCTURAL ENGINEER, Vol. XXIX, No. 3, p. 78, March, 1951.

to use, but he supposed many engineers would be horrified to think of putting much load on that concrete. Obviously it was difficult to estimate its strength ; but normally foundations of that type did not carry very heavy forces, and where the stresses were low he was heartily in favour of using that method.

Finally, Mr. Khan drew attention to a printers' error in the paper, in the spelling of the name of the very famous McKiernan Terry hammer (p. 82, second column).

Mr. H. G. HUCKLE, speaking as an architect, said that recently in one or two journals there had been some comments on the methods by which jobs were being carried out, following the report of the Building Productivity Team which had visited America. One of the criticisms, which he could not help feeling was against the architect, was that drawings were not completed before the job commenced, so that the Quantity Surveyor did not prepare a full Bill of Quantities, but had to re-measure. It was said that this was bad and cost money.

It appeared from the very last paragraph of the paper, where it was stated that a copy of the key plan was posted in the architect's office as a solemn warning that it was " now or never " for the details of holes, fixings, etc., required, that that part of the job had been done with full collaboration between the contractors, the architect and the engineer, and that the drawings had been produced as the building work had proceeded. Leaving out the all-important question of time, he asked if Mr. Hole could say whether in his opinion the construction of the Royal Festival Hall had been a good and economical job—in other words, whether in that particular case we were teaching the Americans something.

Reply to the Discussion

Mr. EALES, replying to Mr. Khan's question on excavation, concerning the relationship of the output capacity of the plant on the site to the actual output, said the only answer was that the lower actual output was due to the high proportion of hard material encountered in the excavations ; there was something of the order of 11,000 cu. yards of hard material out of the 63,000 cu. yards of total excavation, and the reduced output of the plant was due largely to that.

Dealing with the question as to whether some other method of dealing with the hard excavation might have been preferable to the use of the pneumatic hammer, he said the hard excavation did not consist of relatively weak lime concrete, but it was extremely hard concrete. The question of dealing with it in another fashion was considered, of course, but the conditions governing the excavation—namely, having to excavate a trench around the whole site down to the water level, and to permit of the installation being carried out at the same time as the excavation proceeded—meant that they could not deal with the hard material over a large area, but had to nibble at it in small quantities. He agreed that if they could have had access to the whole area and could have carried out the excavation and breaking up as apart of the one process, in all probability another method would have been selected.

The next point was whether it might have been more economical to drive sheet piling all round the site and to deal with the water by other means than by installing the well point system. The main consideration there was that the sheet piling of both the railway side and the river side did not arise until the excavation was well under way, at least down to the water level, and it was then in effect too late. The relative economics of the two systems could not really be considered, because one would not entirely replace the other.

With regard to the suction of fine material from the viaduct side, he said there should not have been any danger of the suction of the fine material there. But, of course, the ideas of the Railway Executive and their engineers had to be considered, and he believed it was true to say that it was not entirely in the hands of the contractors to decide on the matter.

As to the placing and grouting of reject stones in the basement he said the concrete there was not in any sense structural ; it was required there by reason of its weight. He believed—although the matter was out of his province—that stone rejects alone would have been sufficient, but grouting was used to make a more perfect job.

Mr. HOLE, replying to Mr. Guthlac Wilson, said it was very pleasing that so fine a structure would remain as one of the last pieces of work on which Mr. Wilson's late partner, Mr. Scott, was engaged ; one was quite sure that his great experience was a very great asset to the whole job.

The question of incentives and man hours and of how such matters were arranged, involved quite a long story. An incentive system was in operation throughout the whole job, and the man hours were calculated very carefully at the outset of the work ; they were related to small units of work and were used as targets, agreed upon by the representatives of the men as being fair targets, having regard to the conditions under which the work was carried out. There were men engaged on the site to check the amount of work done against the hours worked, and the bonus was paid on the agreed basis from those calculations. The method had proved of very great value throughout the job. The bonus was related to all the trades and applied with the trades throughout. The targets fixed, of course, were related to the planning, and they were adjusted as the conditions were varied, as obviously they would vary by reason of the changes brought about by the adjustment of planning.

Commenting on Mr. Huckle's question arising from the findings of the Building Productivity Team which had visited America, to find out why in America things were done so well, Mr. Hole said it could not be doubted that if the drawings were completely available before the work started they would help everyone concerned. But he could not speak at first hand on what the economy would have been as the result of so doing ; he was afraid he had not experienced that situation ! He could state quite definitely, of course, that the building of the Royal Festival Hall was a very economical job ! Further, he did feel that, bearing in mind the task of the architect—and, incidentally, the task of the engineers, because it was upon them that the contractors had relied for such a long time—the bringing together of the information on the site as required to meet the very fast time schedule was a great achievement. The work was helped by very careful planning ; and the value of the planning was that time was not wasted in the drawing office. Everyone knew what was wanted, and even if they could not have it just when they wanted it, at least it was known, and when they did get it they were all very grateful.

Institution Notices and Proceedings

PRESIDENTIAL ADDRESS—SESSION 1951-52

A General Meeting of the Institution of Structural Engineers will be held at 11, Upper Belgrave Street, London, S.W.1, on Thursday, October 11th, 1951, at 6 p.m., when Mr. Walter C. Andrews, O.B.E., M.I.C.E., M.I.Struct.E., will be installed as President for the Session 1951-1952, and will give the Presidential Address.

FORTHCOMING MEETINGS

The following meetings will be held at 11, Upper Belgrave Street, London, S.W.1 :—

Thursday, October 25th, 1951

Ordinary General Meeting for the election of members at 5.55 p.m., followed by an Ordinary Meeting at 6 p.m., when Dr. P. W. Abeles (Member) will give a paper on "Some New Developments in Prestressed Concrete."

Thursday, November 22nd, 1951

Ordinary General Meeting for the election of members at 5.55 p.m., followed by an Ordinary Meeting at 6 p.m., when Mr. G. B. R. Pimm, M.I.C.E. (Past-President), will give a paper on "The Structural Engineer as Arbitrator, Expert Witness and Advocate."

Thursday, December 13th, 1951

Ordinary General Meeting for the election of members at 5.55 p.m., followed by a Joint Meeting with the British Section of the Société des Ingénieurs Civils de France at 6 p.m., when Monsieur L. P. Brice will give a paper entitled "Construction of the Temple Barrage on the River Lot."

Members wishing to bring guests to the Ordinary Meetings announced above are requested to apply to the Secretary for tickets of admission.

JANUARY EXAMINATIONS

The Examinations of the Institution will next be held at centres in the United Kingdom and overseas on January 8th and 9th, 1952 (Graduateship), and on January 10th and 11th (Associate-Membership).

INSTITUTION PUBLICATION

The Institution has issued the following publication, copies of which are now available:

32/1951. **First Report on Prestressed Concrete**
Price 3/6

Corporate members of the Institution may obtain one copy free upon application. The report is available to non-corporate members and others at the published price, and additional copies may be purchased by corporate members. Copies may be obtained from the Institution's offices, 11, Upper Belgrave Street, London, S.W.1.

THE BUILDING EXHIBITION

The Directors of the Building Exhibition, which is to be held at Olympia from November 14th to 28th, 1951, have extended an invitation to all members of the Institution to attend the Exhibition on Friday, November 23rd, 1951. Admission tickets, for use on the day of the official visit, or on any other single day, may be obtained on application to the Secretary.

THE INSTITUTION OF ROYAL ENGINEERS

The Council of the Institution of Royal Engineers have asked that it be brought to the notice of members of this Institution who may have served as commissioned officers in the Corps that they are eligible to become either Full or Associate-Members of the Institution of Royal Engineers, Chatham. The Secretary is Brigadier C. C. Phipps, C.B.E., M.C., to whom all enquiries should be addressed.

RESEARCH AWARDS

The Council have instituted a Research Prize Fund, from which awards may be made annually to the author or joint authors of papers describing original research which they have carried out. Research awards may be made for papers read at Headquarters or in the Branches and published in the Journal, or for papers published in the Journal only without being read at an open meeting.

The assessment for such awards will be made annually, but awards will be made only to the contributors of such papers as reach a standard judged by the Literature Committee to be satisfactory.

Work submitted under this scheme must be original and may include any of the following :—

(a) investigations of an experimental or analytical character ;

(b) studies of historical or statistical records ;

(c) improvements in principles or methods of construction ;

(d) research into methods of structural engineering and building, the nature and use of plant and the organisation of engineering work ;

(e) any related or combined studies which are deemed by the Literature Committee to be of a research character.

In cases where the research work described in the paper was not the work of one individual, the names of all the collaborators should be given in the paper.

Awards may take any or all of the following forms :— A research medal ; a diploma ; a money prize.

Application for consideration for a research award must be made to the Secretary of the Institution, and in preparing papers for reproduction in the Journal, authors must comply with the conditions laid down for all such contributions. Particulars of these conditions may be obtained from the Secretary.

In judging research papers, the following factors will be considered :—

(a) the nature of the subject and its conclusions ;

(b) the value of the paper in advancing the science and art of structural engineering ;

(c) the standard of preparation and orderly arrangement of the subject matter.

Research papers will also be eligible for adjudication for the Institution Medals if they comply with the Regulations governing those awards.

The closing date for the receipt of applications in respect of papers published in the Journal between October, 1950, and September, 1951, is October 31st, 1951.

THE MACLACHLAN LECTURE

General Conditions

Through the generosity of Mr. John MacLachlan (Retired Member), the Council was able in 1948 to institute an Annual Lecture to be competed for by Associate-Members. The conditions of the presentation are as follows :—

1. The Institution of Structural Engineers shall institute a written lecture to be known as the MacLachlan Lecture and to be held annually.

2. The subject of the Lecture may be on any aspect of Structural Engineering so long as in every second year the subject shall be confined to steel structures.

3. Entrance into the competition for the Lecture shall be confined to Associate-Members of the Institution, who are under the age of 32 years.

4. All papers entered for the competition shall be submitted to assessors to be appointed by the Council of the Institution, and all such papers (including the prize-winning lecture) shall be available for publication in the Journal of the Institution at the discretion of the Council.

5. No paper submitted shall have been published or read elsewhere.

6. The winner of the competition shall be required to present the Lecture to a meeting of the Institution at which he will be presented with the sum of £17 10s. 0d.

7. Should a competitor's paper be considered worthy of ranking second in merit he will receive a consolation award of £5.

8. In the event of there being no winner of the competition in any one or more years, whether because no lecture is submitted or because no lecture submitted is considered to be of sufficient merit to warrant an award, or for any other reason, the Institution shall transfer these sums to the Research Fund of the Institution.

Particulars of the Competition for 1952

1. The MacLachlan Lecture will be given at a meeting of the Institution to be arranged towards the end of 1952.

2. The subject of the Lecture shall be on any aspect of structural engineering.

3. The work should be submitted as the script of a lecture which the author, if successful in the competition, will deliver before an audience in the course of about one hour. The development of mathematical formulæ and detailed calculations should be avoided as far as possible in the text ; if they are essential they should be embodied in appendices. Photographs, drawings, graphs, etc., which would appear as illustrations to the lecture in published form, should accompany the script. If additional illustrations would be shown as slides, a list of these should be included.

4. Six copies of each Lecture should be submitted and should be addressed to the Secretary of the Institution.

5. The closing date for the receipt of entries by the Institution is Monday, March 31st, 1952.

LONDON GRADUATES' AND STUDENTS' SECTION

A meeting of the Section will be held at 11, Upper Belgrave Street, London, S.W.1, on Tuesday, November 13th, 1951, when an informal discussion will be presided over by Mr. Leslie Turner, B.Sc., M.I.C.E. (Past-President), Mr. S. M. Reisser, B.Sc., A.M.I.C.E. (Member) and Mr. C. H. Hockley (Associate-Member).

Hon. Secretary : D. B. Rogers, 4, Portland Rise, Finsbury Park, N.4.

BRANCH NOTICES

Lancashire and Cheshire Branch

The opening meeting of the Session will be held at the Engineers' Club, Albert Square, Manchester, at 6.30 p.m., on Tuesday, October 16th, when the Chairman's Address will be given by Mr. R. Gray (Member). Films of the new Tyne Bridge and the Ford Works at Dagenham will be shown. The President and the Secretary of the Institution will attend the meeting.

Hon. Secretary : A. S. Sinclair, A.M.I.Struct.E., 28, Kenwood Road, Stretford, Lancs.

Midland Counties Branch

The Branch Annual Dinner will be held at the Botanical Gardens, Birmingham, on Saturday, October 6th.

A meeting will be held at the James Watt Memorial Institute, Birmingham, at 6 p.m., on Friday, October 26th, when the Chairman's Address will be given by Mr. W. H. Veal, F.R.I.C.S. (Member).

On Monday, November 5th, a meeting will be held at Derby at 7 p.m., when the following films will be shown : " San Francisco Bay Bridge," " Collapse of the Remagen Bridge," " Loch Sloy," " Plan into Action."

On Friday, November 23rd, a meeting will be held at the James Watt Memorial Institute, Birmingham, at 6 p.m., when Mr. F. Brooksbank, B.A., (Graduate) will give a paper on " Designing for Welding."

Hon. Secretary : E. R. Deeley, A.M.I.Struct.E., Arranmoor, Adshead Road, Dudley, Worcs.

Graduates' and Students' Section

The following meetings will be held at the James Watt Memorial Institute, Birmingham :—

Monday, October 29th, 1951, *7 p.m.*

Paper by Mr. J. S. Allen (Graduate), on " The Dome of Discovery."

Thursday, November 29th, 1951

" Some Recent Midland Structures."

Hon. Secretary : M. H. Evans, B.Sc., 42, Church Hill Road, Handsworth, Birmingham, 20.

Northern Counties Branch

The opening meeting of the Session will be held at the Cleveland Scientific and Technical Institution, Middlesbrough, on Tuesday, October 2nd, when the Chairman's Address will be given by Mr. J. Gerrard, M.I.Mech.E. (Member). The meeting will be repeated at the Neville Hall, Newcastle, on Wednesday, October 3rd. Both meetings will commence at 6.30 p.m., preceded by tea at 6 p.m., and will be attended by the President-Elect and the Secretary of the Institution.

A meeting will be held at Middlesbrough on Tuesday, November 6th, at 6.30 p.m., when Mr. G. M. Boyd (Associate-Member) will give a paper entitled " Contrast and Similarities between Ship and Land Structures." The meeting will be repeated at Newcastle on Wednesday, November 7th.

Hon. Secretary : Ian MacGregor, M.I.Struct.E., 9, Ellison Place, Newcastle-upon-Tyne, 1.

Northern Ireland Branch

The opening meeting of the Session will be held at the College of Technology, Belfast, at 7.30 p.m., on Tuesday, October 2nd, when the Chairman's Address will be given by Mr. M. C. Gillies (Member).

A meeting will be held at the College of Technology, Belfast, at 7.30 p.m., on Tuesday, November 6th, when Dr. R. M. Kenedi, A.R.T.C., will give a paper on " The Use of Equal Strength Principles in the Design of Structures for Minimum Weight."

Hon. Secretary : S. G. Duckworth, M.I.Struct.E., " Lisleen," 13, Finaghy Road North, Belfast.

Scottish Branch

The opening meeting of the Session will be held at the Ca'doro Restaurant, Glasgow, at 6 p.m., on Wednesday,

November 7th, when the Chairman's Address will be given by Mr. R. Summers (Member).

The Annual Dinner and a Buffet Dance will be held at the Central Hotel, Glasgow, on Thursday, November 8th.

Hon. Secretary : D. G. Drummond, B.Sc., A.M.I.C.E., M.I.Struct.E., 11, Woodside Terrace, Glasgow, C.3.

SOUTH-WESTERN COUNTIES BRANCH

A Meeting of the Branch will be held at Plymouth on Friday, November 2nd, when the Chairman's Address will be given by Mr. L. F. Vanstone, L.R.I.B.A. (Member). The President and the Secretary of the Institution will be present.

Hon. Secretary : E. W. Howells, M.I.Struct.E., c/o Messrs. T. L. Harding & Sons, Ltd., 10/12, Market Street, Torquay.

WALES AND MONMOUTHSHIRE BRANCH

The opening meeting of the Session will be held at the South Wales Institute of Engineers, Cardiff, at 6.30 p.m., on Tuesday, October 30th, when the Chairman, Mr. G. H. Hodgson, M.I.C.E. (Member) will give an Address on "Problems in the Conveyance and Storage of Crushed Rock." The President and the Secretary of the Institution will attend the meeting.

The Chairman's Address will be repeated at the Mackworth Hotel, Swansea, on Wednesday, October 31st, at 6.30 p.m., when it will be followed by discussion, and at the County Buildings, Colwyn Bay, on Saturday, November 10th, at 6 p.m.

On Wednesday, November 28th, a meeting will be held at the Mackworth Hotel, Swansea, at 6.30 p.m., under the title "Structural Engineering Questions Answered." The meeting will be repeated at Cardiff on Tuesday, December 4th.

Hon. Secretary : E. R. Steward, A.M.I.Struct.E., Edrom, Ashleigh Road, Blackpill, Swansea.

WESTERN COUNTIES BRANCH

The opening meeting will be held on Friday, October 5th, at 6 p.m., when the Chairman, Professor A. G. Pugsley, O.B.E., D.Sc., M.I.C.E., F.R.Ae.S. (Member), will give an Address entitled "Structural Engineering and the Universities." The President and the Secretary of the Institution will be present.

A Joint Meeting with the Institution of Civil Engineers will be held at 5.30 p.m. on Thursday, November 1st, when Mr. E. Bateson, M.I.C.E. (Member) will give a paper on "Some Interesting Bridge Reconstruction Schemes."

Hon. Secretary : C. E. Saunders, M.I.Struct.E., Dunkery, Edward Road, Walton St. Mary, Clevedon, Somerset.

YORKSHIRE BRANCH

The opening meeting of the Branch will be held at the Great Northern Hotel, Leeds, on Wednesday, October 17th, at 6.30 p.m., when the Chairman's Address will be given by Mr. A. Robb, B.Sc., A.M.I.C.E. (Member).

A meeting will be held on Wednesday, November 21st, when Mr. T. E. S. White, B.Sc., M.I.C.E., will give a paper on "The Baitings Reservoir."

Hon. Secretary : E. Wrigley, A.M.I.Struct.E., City Engineer's Department, Civic Hall, Leeds, 1.

UNION OF SOUTH AFRICA BRANCH

Branch Hon. Secretary : A. E. Tait, B.Sc., A.M.I.C.E., A.M.I.Struct.E., P.O. Box No. 3306, Johannesburg. During week-days, Mr. Tait can be contacted in the City Engineer's Department, City Hall, Johannesburg. 'Phone 34-1111, ext. 257.

Natal Section Hon. Secretary : E. G. Bennett, A.M.I.Struct.E., c/o Reinforcing Steel Co., Ltd., P.O. Box 49, Merebank, Durban.

Cape Section Hon. Secretary : R. Stubbs, M.I.Struct.E., P.O. Box 1692, Cape Town.

ADDITIONS TO THE LIBRARY

ARCHITECTS' YEARBOOK, 3. London, 1949. Presented by Mr. G. Crossley.

AMERICAN SOCIETY FOR TESTING MATERIALS, Proceedings, Vol. 49, 1949 ; BOOK OF STANDARDS, 1949, and 1950 SUPPLEMENT, Pt. 2. Philadelphia, 1950.

BARKAS, W. W. (D.S.I.R. Forest Products Research. The Swelling of Wood under Stress. London, 1949.

BIRD, E. L. and DOCKING, S. J. Fire in Buildings. London, 1949. Presented by Mr. C. E. Cannons.

Central Board of Irrigation, India, Annual Report (Technical), 1948, Pts. I and II. Simla ; 1949.

CHALMERS, Bruce, Editor. Progress in Metal Physics. London, 1949. Presented by Professor A. G. Pugsley.

Colston Papers. Engineering Structures. London, 1949. Presented by Professor W. Fisher Cassie.

CRIMP and BRUGES. Tables and Diagrams for use in designing Sewers and Water Mains. 2nd Edition. London, 1949.

ESCRITT, L. B., and RICH, S. F. The Work of the Sanitary Engineer. London, 1949.

FAREY, C. A., and EDWARDS, A. T. Architectural Drawing, Perspective and Rendering. London, 1949. Presented by Mr. E. G. Warland.

GREGORY, E., and Others. Engineering Materials. London, 1949. Presented by Dr. C. M. Moir.

Iron and Steel Directory and Handbook, 6th Edition. London, 1950. Presented by the Publishers.

LOW, B. B. Mathematics : A Textbook for Technical Students. London, 1948. Presented by Mr. E. J. Gouk.

MANSON, J. L., and DRURY, F. E. Experimental Building Science, Vol. II, Pt. I. Mechanics of Structures. 2nd Edition, Cambridge, 1950. Presented by Mr. H. H. Clapham.

REINER, Markus. Deformation and Flow : An Elementary Introduction to Theoretical Rheology. London, 1949. Presented by Professor W. T. Marshall.

Tees-side Chamber of Commerce and Tees-Side Industrial Development Board. Tees-Side Enterprise. Derby, 1950.

VAN LANGENDONCK, T. Calculo de Concreto Armado, Sâo Paulo, 1950.

WYNN, A. E. Estimating and Cost-Keeping for Concrete Structures. 2nd Edition. London, 1949. Presented by Mr. W. Barber.

WYNN, A. E. Design and Construction of Formwork for Concrete Structures. 4th Revised Edition. London, 1947. Presented by Mr. W. Barber.

YOUNG, C. R., and MORRISON, C. F. Elementary Structural Problems in Steel and Timber. 3rd Edition. New York, 1949. Presented by Mr. W. Fitton.

Book Reviews

Welded Deck Highway Bridges (Edited by James G. Clark). Cleveland, Ohio : James F. Lincoln Arc Welding Foundation, 1950 ; 247 pp., $8\frac{3}{4}$ in. × $5\frac{3}{4}$ in, Illustrated $2.50.

Many structural engineers will be familiar with the competitions organised by the James F. Lincoln Arc Welding Foundation to encourage the development of the design of welded structures. This recently published book describes and illustrates very clearly selected designs from the 1949 Award Programme "Welded Bridges of the Future," in which the specific task was to design a two-lane deck highway bridge, of welded steel construction, supported on two end piers 120 feet apart.

Designs were submitted by bridge engineers of state highway departments, consulting engineers, designers of steel fabricating companies, structural engineers of research organisations, and University professors of structural engineering, and entries were received from twenty countries. The varied character of the designs will be evident from the following brief descriptions of those awarded the first three prizes :—

1. One space truss of triangular cross section with two top chords and one bottom chord, employing new steel shapes in both chords for easier assembly and economy of metal.
2. Two vertical girders with steel deck of $\frac{5}{8}$ in. floor plate welded to Y-shaped stringers, composite action between steel deck and girders.
3. Two vertical trusses using 12 in. and 10 in. pipe as members, and new shapes to facilitate making the welded connections.

Many other types of main frame were employed, including—to quote from the introduction—"a tied arch, a two-hinged arch, a stiffened arch, an inverted arch, a modified Vierendeel truss, a girder with large holes, a truss and girder combination, a tensed cable, a rigid frame, a forked frame, and a cellular box."

It must not be thought, however, that the entrants sought only the unusual or spectacular ; ingenious constructional features were evolved, but generally they were not introduced at the expense of sound practical design. Details of quantities and estimated costs were required to be submitted, and some discussion on this subject is included in a final chapter.

Professor James G. Clark, who edited the book, was himself a member of the Awards Panel, under the chairmanship of W. M. Wilson, Research Professor of Structural Engineering, University of Illinois. The Lincoln Arc Welding Foundation hope that "the publication of some of the excellent ideas presented in this programme will benefit the field of bridge engineering"—and it need only be said that most bridge engineers will find something of interest and value in these ideas. H. A. C.

Theory of Structures, by H. W. Coultas, 3rd Edition (London : Pitman, 1950)) 538 pp., $8\frac{1}{2}$ x $5\frac{1}{2}$; 20s.

The third edition of this book, planned to cover the syllabuses of the university and professional examinations in the theory of structures, has been revised and brought up-to-date. A chapter has been added dealing with methods of analysis which have been developed since the publication of the second edition, the method of "slope deflection" and the "method of successive approximations" being fully stated and discussed. Frames, both with and without lateral restraint, are considered, and it is shown how the equations for calculating redundant moments can be written. Many examples are given to illustrate the solution of various problems by these methods. In addition, a chapter has been included in which the laws of Maxwell and Betti have been developed, and the application of these laws to the mechanical solution of statically indeterminate structures has been stated.

Mathematical Solution of Engineering Problems, by J. Jennings. (London : Spon, 1951.) 200 pp. 9 in. by $5\frac{1}{2}$ in. 25s.

The mention of the word "mathematics" sends a cold shiver down the spine of many engineers, and although some might be frightened at the title of this book, no professional engineer should be unable to understand its contents, and to find them useful.

The book assumes a knowledge of algebra and trigonometry up to Ordinary National Certificate standard and the elementary principles of the differential and integral calculus, and the mathematics in it does not go much beyond this.

The title is perhaps a little misleading, for the solutions of the engineering problems contained in the book are already known, but the author has endeavoured to show how certain fundamental mathematical principles can be of assistance to the designer in his arithmetical computations, for ultimately every design problem reduces to one of arithmetic.

The chapter headings are briefly as follows :—Analysis of experimental data ; approximations, areas, volumes, centroids ; nomograms ; dimensional analysis ; linear simultaneous equations ; periodic phenomena ; statistical methods ; problems involving fractions and integral numbers. These give a good outline of its contents.

The student class rather than the experienced designer will profit most from the book. The latter generally has his own methods (often the result of much "blood, sweat and tears") for his arithmetical computations.

It is to be regretted that on the cover sheet the book is described as being planned for "technicians." The reviewer would have preferred the term "technologists," for the book would appear to be primarily for the use of designers and research workers, neither of whom can really be classified as technicians.

W. T. M.

Civil Engineering Handbook. 3rd Edition. Editor-in-Chief, L. C. Urquhart. (New York and London : McGraw-Hill, 1950.) 1,002 plus x pages, 9 in. by 6 in. 72s. 6d.

This valuable handbook, compiled by leading United States specialists, has been revised and enlarged. All the main branches of civil engineering are treated in ten sections, on surveying, railway, highway and airport engineering, mechanics of materials, hydraulics, stresses in framed structures, steel design, cement and concrete, soil mechanics and foundations, sewerage and sewage disposal, and water supply and purification.

In the section on surveying, new material has been added on photogrammetric surveying and on the legal authority and liability of the surveyor. New information is also included on hydraulics, concrete, soil mechanics, on the design of pavements for highways and airports, and on recommended loadings and practice in steel design.

The Structural Engineer as Arbitrator, Expert Witness and Advocate*

By Gower Pimm, M.I.C.E., M.I.Struct.E., F.I.Arb.

Arbitration

Arbitration is a method of settling disputes by referring them to one or more specially appointed persons, instead of in the Courts. A great variety of matters can be referred to arbitration, but its advantages are greatest when the issues are of a technical nature. It has the more general advantage that the time and place of the hearing can be fixed to suit the convenience of the parties and others engaged in the proceedings. The text-books on the subject do not stress this point.

In confining himself, of necessity, to certain aspects of such a large subject, the author felt that he could not do better than direct his attention mainly to this question :—" To what extent is the arbitrator at liberty to decide the issues solely in the light of his own technical knowledge, administrative experience and sense of equity, and to what extent must he be prepared for the intervention of the Courts ? " The need for an answer to this question is apparent as soon as we open any text-book on Arbitration, for the first thing we see is a long Table of Cases. In " Russell on Arbitration," for example, the Table runs to 38 pages, and comprises some 1,500 cases to which reference is made in the book. As all ordinary arbitrations are private, there is no record of the proceedings except in the papers of those concerned, and at first sight it may seem remarkable that there are so many cases of which published reports are available. The explanation is that the cases referred to are those which, for one reason or another, have not been settled within the four walls of the arbitration room, but have reached the Courts. The number is so large that they cannot be dismissed as abnormal, and that the position causes some concern is evident from the following observations of one of the greatest authorities on arbitration in this country :—

" If England were to come into line with international commercial law *there should be an alteration to make it possible for two parties to agree that an award should be final. That law applied throughout the British Dominions and most countries of the world,* and an alteration in our system was of vital importance for our international commercial relations."

Although this quotation refers particularly to international relations, the words in italics apply equally in the domestic sense. It is true that Section 16 of the Arbitration Act, 1950, like the earlier Acts, states that unless the arbitration agreement expresses the contrary intention the award of the arbitrator is final, but like many other provisions of the Act this must be read in conjunction with other Sections, and it is quite certain that the finality of an award can be challenged on various grounds.

Many of the cases in this long list relate to the authority of the arbitrator—whether he has been properly appointed, or whether the matter is one that should go to arbitration at all. Others result from the arbitrator himself having decided to " state a case " for the decision of the Court on a legal issue. These will be discussed later, but considering for the moment only those in which the question involved is the finality of the arbitrator's award, when made, this question can only arise on one or other of the following grounds :—

(1) That the arbitrator has misconducted himself.

(2) That he has misconducted " the proceedings."

(3) That he has made an error of law or of fact, and has allowed the error to appear on the face of his award.

The first need not be discussed at length, as it implies culpable misconduct, of which of course no member of the Institution would be guilty. If he were, it is proper that there should be the right of appeal to the Courts.

Misconduct of " the proceedings " applies to cases where, although there has been no moral turpitude on the part of the arbitrator, there has been some technical irregularity, such as admitting evidence which would not be admitted in the Courts, or making an award that is not definite, or one which does not deal with all the matters referred and with those matters only.

(1) above was in the 1889 Act, and (2) was introduced in the 1934 Act, both (1) and (2) being re-enacted in the 1950 Act. No agreement between the parties can deprive either of the parties of the right of appeal to the Courts which the provisions give. They can of course enter into a gentleman's agreement not to exercise that right, but if either party decides to be ungentlemanly, the other party cannot prevent him from breaking it. On the other hand, if both parties are resolved to accept the decision of the arbitrator for better or worse, there are ways, which are quite legitimate, of making it very difficult for either party to depart from the undertaking ; for instance, they may decide that neither party shall be legally represented. However, an engineer of the standing that would justify his appointment as arbitrator should have no difficulty in steering clear of serious irregularities in procedure, and if he is unintentionally guilty of any technical irregularity, he should not hesitate to rectify it if his attention is called to it at the time. For instance, he should not insist upon accepting evidence in the face of a valid protest.

It would not appear at first sight that (3) above should be a pitfall, as it should be the easiest thing in the world for the arbitrator to avoid giving any reasons for his decision, and especially any legal reasons, on the face of his award. In that connection, however, the following case is perhaps one of the most remarkable ever decided :—

Questions of law arose, and the arbitrator sought for and obtained from a Divisional Court answers to a special case as to how those questions should be decided. He then published his award, from which it appeared that it was based upon the answers of the Divisional Court. Upon an appeal to the Divisional Court and later to the Court of Appeal, both Courts refused to set the award aside, but on an appeal to the House of Lords, that House did set the award aside, on the grounds that the answers given by the Divisional Court were

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erroneous, and that this error appeared on the face of the award.

While the powers of the Courts in relation to arbitration have been increased rather than diminished by successive enactments, it by no means follows that the swing toward the Courts is thereby accelerated. The following examples show that the result may be quite the reverse.

Prior to the passing of the Arbitration Act, 1934, the "better or worse" principle applied to cases in which a named arbitrator was not or might not be impartial, e.g., an engineer appointed as arbitrator in a dispute between his client and a contractor. In a great many cases of this kind the Courts refused to revoke the authority of the arbitrator, on the grounds that "the aggrieved party knew, or ought to have known, that the arbitrator, by reason of his relation towards any other party to the agreement, or of his connection with the subject referred, might not be capable of impartiality." Section 14 (1) of the Arbitration Act, 1934, which is re-enacted in Section 24 (1) of the Arbitration Act, 1950, reversed this, and laid it down that the considerations quoted above do not constitute grounds for refusing to revoke the authority of the arbitrator. In other words, since 1934 it has been possible to challenge the authority of the arbitrator even though the party knew, when agreeing to his appointment, that he might not be impartial. The result of this enactment, making it clear that in such circumstances an application for leave to revoke would probably be successful, has not been to increase the number of applications, but to remove the cause, which after all is the essence of good law-making.

To take another example. It was formerly quite usual for the reference to be to three arbitrators, one to be appointed by each party, and the third by the two so appointed. The 1889 Act gave no guidance as to the functions of the third arbitrator. Was he on the same footing as the other two, in which case he would have to attend the hearing and the three would make a joint award, or was he in the capacity of an umpire, to settle any differences between the other two? The 1934 Act cleared up this point, and laid it down that in such a case the agreement should have effect as if it provided for the appointment of an umpire, and not of a third arbitrator. It also laid it down that when the agreement provides for the appointment of three arbitrators otherwise than as above, the award of any two of them shall be binding. These provisions of the 1934 Act are also re-enacted in the 1950 Act, and the effect has been to over-rule many earlier decisions of the Courts. Here again, by making it clear that, notwithstanding anything in the agreement, the third arbitrator is not an arbitrator in the strict sense, but an umpire, the law now removes the need for an application to the Court to clarify the position.

One other example will be given of the way in which clarification, while emphasising the right of appeal to the Courts, has tended to reduce the number of such applications. As it emerges from the Arbitration Act, 1950, it may be appropriate here to give a brief outline of the effect of that Act, before proceeding to the example, which is perhaps the most interesting of all.

The Arbitration Act, 1950, which came into operation on September 1st, 1950, consolidates the law contained in the Arbitration Acts, 1889 to 1934, which are repealed by this Act with the exception of certain provisions relating to Merchant Shipping. So far as Structural Engineers are concerned, therefore, it is now "the Arbitration Act." The material provisions of the repealed Acts are re-enacted in substance, and in the main in identical terms, but the arrangement of the Sections is different. It may be observed in passing, that while this makes for simplification of the law relating to arbitration, there has been the inevitable result that all existing text-books on the subject immediately became out-of-date to the extent that the references to Sections that they contain are no longer appropriate. This was particularly unfortunate in the case of "Russell on Arbitration," as the 14th Edition of that standard work, published as recently as 1949, was the first edition since 1935.

As in military operations, the "consolidation" of Acts of Parliament gives the opportunity for the abandonment of any untenable position, and the following extracts from the three Acts provide a remarkable example of that strategical principle :—

(1) Arbitration Act, 1889.

1. A submission, unless a contrary intention is expressed therein, shall be irrevocable except by leave of the Court or a judge. . . . . "

27. In this Act, unless the contrary intention appears—

"Submission" means a written agreement to submit present or future differences to arbitration, whether an arbitrator is named therein or not. . . . . "

(2) Arbitration Act, 1934

21 (2) In this Act unless the context otherwise requires—

The expression "arbitration agreement" means a written agreement to submit present or future differences to arbitration, whether an arbitrator is named threin or not. . . . . "

(Note that this is identical with the definition of "submission" in the 1889 Act).

(3) Arbitration Act, 1950

1. The authority of an arbitrator or umpire appointed by or by virtue of an arbitration agreement shall, unless a contrary intention is expressed in the agreement, be irrevocable except by leave of a High Court or a judge thereof.

Section 27 of the 1889 Act was very loose, and failed to distinguish between "submission" and "agreement to submit." It is true that the two can be embodied in the same document, as when it is desired to submit to arbitration a matter in respect of which there is no previous agreement to do so, but this is seldom the case in building or engineering arbitrations. In these the Arbitration Clause is the arbitration agreement, but there is no submission until a dispute arises and is referred to arbitration.

Section 1 of the 1889 Act would have been quite correct if the dictionary meaning of "submission" could have been read into it, but Section 27 confused the issue, and the 1934 Act avoided it. The Arbitration Act, 1950, straightens things out by making it clear that "the authority of the arbitrator" is irrevocable, and not merely the arbitration agreement, which was always irrevocable. It is not surprising that between 1889 and 1950 there were many legal battles upon this issue.

Thanks partly to the clarification of the law and partly to an increasing appreciation of the nature of arbitration it would be true to say that many cases which reached the Courts in the past would not do so to-day. One important factor in this is that the Courts have repeatedly made it clear that they will not interfere except for the strongest reasons. It might almost be said that the only cases which ought to reach the Courts are those

in which the arbitrator " states a case " for the decision of the Court. In the course of an arbitration a question of law arises, and the arbitrator can then do one of three things. He can ask the parties if they are willing to accept his decision on the point, and if they agree he can proceed accordingly, being careful, however, not to give his conception of the legal aspects in his award. Secondly, he can himself decide to state a case for the decision of the Court. Thirdly, if one of the parties presses for this course he will be wise to adopt it, but if he feels that the party is being " awkward " he can drop a hint that this may be taken into account in awarding the costs, in so far as they relate to the application to the Court. This may have the effect of reducing the pressure, without doing any injustice to the party who believes himself to be aggrieved. Another matter for reflection is that the setting aside of an award, or its remission to the arbitrator, does not necessarily mean that the ultimate decision will be materially different.

In 1947 Mr. E. J. Rimmer, M.Eng., B.Sc., A.M.I.C.E., Barrister-at-law, presented a very erudite and helpful Paper on " The Civil Engineer as Arbitrator," to the Institution of Civil Engineers. In the discussion of this Paper, Mr. Stewart Brown made the following comforting observations :—

" In a long experience of the author (Mr. Rimmer), in arbitrations, Mr. Stewart Brown had never heard him ask an arbitrator to state a special case, but he believed that that was due not to Mr. Rimmer's confidence in arbitrators, but in his own ability to persuade an arbitrator to decide in his favour, whatever the law might be ! "

This really sums up the matter very neatly. If the arbitrator has confidence in himself, and if the parties and their legal representatives have confidence either in the arbitrator or in their own persuasive powers, the arbitrator can with perfect safety make his own decision on a point of law. If he is wrong, his decision will still be binding, always provided he does not allow the error to appear on the face of his award. He need have no scruples about this, for he may be wrong in law but right in equity. As the parties have submitted their differences to arbitration, it may be assumed that they prefer the latter.

There is one rather curious exception to this rule that an award may be invalidated if an error in law appears on the face of it. The rule does not apply if the error relates to a point specifically referred to the arbitrator.

In the matter of the appointment of an arbitrator, the arbitration clause in most engineering contracts provides that disputes shall be determined by a single arbitrator to be agreed upon between the parties, or failing such agreement, to be appointed by the President of one of the (named) Professional Institutions. In the latter event, it will be observed that the word used is " appointed," and it is the function of the President to appoint, and not merely nominate or suggest an arbitrator. In practice the arbitrator is usually informed that his appointment is acceptable to both parties, and of course they have the right, for a valid reason, to object to a particular arbitrator, but apart from that the mere absence of a notification that the arbitrator is acceptable to the parties, does not invalidate the appointment. As in most things in arbitration, a right that is not exercised at the appropriate time is deemed to have been waived.

Perhaps one of the most difficult things with which the arbitrator has to deal is the question of " costs." Unless a contrary intention is expressed in the arbitration agreement, the costs are in the discretion of the arbitrator. On one point the Act is very definite, however. Any provision in the agreement to the effect that the parties shall in any event pay their own costs, or any part thereof, is void, except that it is permissible if the agreement was made after the dispute arose. This only restricts the right of the parties themselves to agree to pay their own costs, and does not preclude the arbitrator from apportioning the costs in that way, although it is more usual for the costs to " follow the event." The former offers an easy way out of the difficulty, but the arbitrator will not of course be tempted to make an award in that form merely because it is the easy way. He will probably adopt the more usual course of directing that the costs shall be paid by one of the parties. It will then be his duty to settle the scale of the costs, and either tax them himself or direct that they shall be taxed, and unless he has had considerable experience he will be wise to take legal advice as to the wording of this part of his award, upon his own decision as to the principle. In certain exceptional circumstances the arbitrator may decide to form his own opinion as to what the costs of the successful party should be, to add that to the amount of his award, and then direct that the respective parties shall pay their own costs.

It has long been a custom for the arbitrator to direct that his own fees and expenses are to be paid by the party taking up the award, the party in question being entitled to recover the amount from the other party if the costs are awarded against that party. For several reasons this is unsatisfactory. The parties may settle their dispute before it comes to a hearing, or even after the hearing. In the former event there will be no award, and in the latter neither party will be disposed to take up the award. The arbitrator will then be in a weak position, although it is right to say that if the parties have acted upon legal advice their solicitors will see that the arbitrator's charges are paid. But they may not have acted upon legal advice. In the second place, if the parties feel that the arbitrator's charges are excessive, they have the right to apply to the Court to have them taxed. By far the better way is for the arbitrator, when accepting the appointment, to state what his charges will be, and get the agreement of both parties. The agreement should provide that if the reference is determined by the act of the parties before the award is published, the arbitrator's fees shall be paid by the parties in equal shares, or jointly and severally.

In conclusion, the author would make a plea for the setting up of a Court of Engineering Arbitration, on the lines of the London Court of Arbitration established by the London Chamber of Commerce. The arbitrator's duties are sufficiently onerous in dealing with the issues involved, without having to be, in addition to " Judge," Clerk of the Court, usher, and all the other officers of the Court. All such matters can be arranged much more conveniently by a Court of Arbitration.

Expert Evidence

The Structural Engineer may be called upon to give expert evidence before tribunals of many kinds. He may be Engineer or Consulting Engineer for a project which is the subject of a Private Bill in Parliament, or of a Departmental Inquiry, and in such cases he will be advocating his own proposals, or endorsing the proposals of another engineer employed by his clients. In that connection it is necessary to comment upon the dictum that has been laid down by a very high authority, that the expert witness is not an advocate. In cases of the kind referred to he certainly is, and all that can be read

into the dictum is that before agreeing to support a proposal he must satisfy himself that he can do so conscientiously.

In arbitrations and Court cases the position of the independent expert witness is somewhat different, because here he is giving evidence upon issues which are not, if one may so express it, of his own making. There will be a matter in dispute, and the engineer will have been asked by one of the parties to the dispute, after it has arisen, to give evidence on their behalf. How does the dictum now apply? The author suggests that the same principle is involved, viz., that the engineer must first consider whether he can support the submissions of the party who has approached him, and if he decides that he can, he is again entitled to advocate their acceptance. If on the other hand they do not coincide with his own views, it is his duty to decline. It is also quite within his province to "sift" his client's contentions, and to say that some can be supported and others cannot.

One of the greatest misconceptions about the expert witness is that his participation in the case begins and ends with his appearance in the witness-box. His client's legal advisers will see that it does not, and actually his real contribution is not in giving evidence, but in preparing evidence. A further important part of his duties is to advise on the questions that should be put to the other side. Perhaps the best way to illustrate his functions is to outline an actual case, from the time the engineer is first approached until the end of the hearing.

The first communication will come from the party who wishes to retain him, or from that party's solicitors, and will give an idea of the nature of the dispute, asking if he is willing to be retained, and may or may not ask him to say what his fees would be. If it appears to be a matter in which he could assist, he will ask for further particulars, e.g., a sight of the relevant documents, and at this stage he should state his fees, whether asked to do so or not. The most convenient way of doing this is to send a copy of the Institution's Scale of Charges, indicating the items which apply, and stating the actual fee where the Scale gives only a minimum. It is the author's practice to require, in addition to the fees for attendance and perusal of the documents, a nominal "refresher" of, say, £1 1s. per day for every day of the period commencing with the date from which he is told he must hold himself in readiness until the date upon which he is actually called. He may be told, for example, that the case may come on at any time in the week commencing January 15th, but he may not be called until January 22nd. During that week he would be unable to make any other appointments except from day to day, and the daily "refresher" is in the nature of a small compensation for that inconvenience. It is seldom necessary to make this charge, but the desired object is achieved if the obligation to pay it results in the client keeping the engineer closely advised as to the date of the hearing.

Occasionally the engineer may be asked to say whether he would prefer to attend under "subpœna." It is unnecessary to go into the reasons for this rather curious question, but in the author's opinion the reply should be a most emphatic negative. In some cases all the witnesses are called under "subpœna," but that is a different matter. The point the author wishes to make is that the engineer should not be persuaded to express a wish to be so called. If he does he may find himself in a false position.

Assuming now that he has made himself conversant with the matters at issue, and has decided to accept, the engineer will next prepare a report upon which his Proof of Evidence will be prepared. In practice, if he has drafted his report in suitable terms, it will be adopted, with little alteration, as his proof. He will probably have access to the proofs of other witnesses on questions of fact, and he will of course include in his report his observations upon any other evidence so placed before him.

All the "proofs" will go to counsel, if counsel are engaged in the case, and subsequently the engineer will be required to attend a conference with counsel. If the case is an important one, there may be many such conferences both before and during the hearing. On these occasions the engineer must be prepared to submit to a little good-natured "third degree" questioning, but having looked at both sides of the case he will be able to get his own back to some extent, by suggesting similarly awkward questions that might be put to the other side when the time comes. Reference will be made later to the importance of grasping similar opportunities as the case proceeds.

We now come to the actual hearing. As in cricket, one side must go in first, but it is not decided in the same way. The rule is that the case for the plaintiff in a Court case, or the claimant in an arbitration, is first presented. The witnesses for that side then give their evidence-in-chief, following which they are usually cross-examined by the opposing counsel, and may be re-examined by their own counsel. Questions put in cross-examination are confined to matters arising out of the evidence-in-chief of the witness, and those put in re-examination to those arising out of his cross-examination. The case for the defendant (in a Court case) or the respondent (in an arbitration) is then presented, and the witnesses for that side are examined, cross-examined and re-examined in the same way. The role of the expert witness therefore depends to some extent upon the side for which he is acting, as this determines the "timing" of his appearance in the witness-box.

It is of the utmost importance that he should be present when the witnesses for the other side are in the box, and this applies both ways. He will thus hear the evidence of his "opposite number," and will be able to prompt counsel with any questions arising out of the evidence, that might be put to that witness in cross-examination. If he is for the defendant, or respondent, he will have the additional advantage of being able to suggest to counsel any questions arising out of the opposing evidence he has heard, that might be put to him in his own evidence-in-chief later. If he is for the plaintiff, or claimant, it is his "opposite number" who will have this opportunity of offering rebutting evidence.

It may not be out of place to add a few words on the difficult problem of "prompting counsel." If the examination of an opposing witness is completed, or in progress, when the Court rises, it affords an opportunity of putting any suggestions to counsel, either directly or through the solicitor, at leisure, but otherwise more prompt action is necessary. The usual method is to pass a note to the solicitor, or to junior counsel, who, if he thinks the point a good one, will pass it to the examining counsel at the opportune moment. Unless there is a very complete understanding between the expert witness and the examining counsel, he would be ill-advised to interrupt the latter, even if he were near enough to do so. In favourable circumstances the author has sometimes found that he has been able to anticipate the points he may wish to bring before counsel as they arise, and to

have the necessary " chits " typed out in readiness, but to suggest that this can always be done would be misleading.

One thing should be mentioned in conclusion. It is a strict unwritten rule that if the Court adjourns while a witness is under examination, he should have no contact whatever with those on his side during the adjournment. This hint may remove any uneasiness that may be occasioned by the very proper aloofness of those with whom he is associated in the case. If he is not aware of this rule he may receive the impression that he has incurred their displeasure by some imperfection in his performance. The author, before he was aware of the rule, once spent a very unhappy week-end from this cause.

The Structural Engineer as Advocate

In the second part of this Paper some brief observations were made on the question of whether the engineer, in the capacity of expert witness, should be an advocate —with a small " a." An even larger question is whether, in an arbitration, he is entitled to be, and qualified to be an Advocate—with a capital " A." There is no doubt whatever that it is permissible. The parties can entrust the conduct of the case to anyone they choose, and the only question is the ability of the person charged with this responsibility. It has been the author's privilege to be associated with many distinguished corporate members of the Institution who have discharged this duty with the utmost brilliance and success. No one who has been examined and cross-examined by a barrister will be disposed to withhold tribute to the members of the legal profession for the way in which they grasp the technicalities, and their skill in presentation ; but engineers too are skilled in the art of convincing presentation, and of course they enjoy the advantage of being already conversant with the technicalities. On purely technical grounds, therefore, the engineer is in every way qualified as an Advocate. There remains the question of law, which has loomed so large in this Paper. Here again, in the course of the engineer's normal work he has to deal with matters which contain at least an element of law, and often to exercise judicial and semi-judicial powers. He is therefore not unfamiliar with the law, although it will be conceded that arguing the legal niceties is outside his province. The author's view is that in arbitration the legal niceties can be easily overdone, and may even obscure the principles of equity. Added to this, there are the uncertainties of the law, and examples of both are given in the Paper.

If the party against whom an award is given, or seems likely to be given, could be certain that an over-ruling decision of the Court would be in his favour, no one could blame him for seeking that decision, but there can be no such certainty. The decision of the Court, from the point of view of the parties, may be quite academic. For example, it may be to the effect that the arbitrator has done something that he ought not to have done, and that the arbitration, wholly or in part, must begin all over again, with the same or a different arbitrator, as the case may be. It may be for such reasons it is sometimes said that arbitration is costly. The author submits that it is not arbitration that is costly, but the disavowal, by the parties in certain cases, of the principles of arbitration to which they have subscribed.

The question for the parties to decide, therefore, is whether they wish to avail themselves of every right the law allows them, regardless of cost, or to accept a decision based solely upon the technical and practical arguments which an engineer is well qualified to present in the capacity of Advocate.

A Method of Analysis of the Continuous Vierendeel Girder

By Edgar Lightfoot, M.Sc.(Eng.), A.M.I.C.E., A.M.I.Struct.E.

Introduction

The " Vierendeel," or " open-frame," girder, has been extensively employed on the Continent and elsewhere, mostly in bridge construction. Few *continuous* girders of this type have been built, but many uses may yet be found for them. A paper recently published in Japan[1] deals with a method of analysis of this highly-redundant continuous structure, and is probably the first in the field. The method propounded there is based on the well-known Slope-Deflection Method, and involves the systematic writing of a large number of simultaneous equations and their subsequent solution by iteration.

This particular structure can be analysed in a straightforward way as explained below, and any method which is suitable for the solution of the simply-supported Vierendeel girder may be used.

Outline of the Method

(a) With one Redundant Reaction

Consider a loaded Vierendeel girder simply-supported at its ends and continuous over a single roller support at the same level. (Fig. 1.)

Let the loading and the redundant support C be removed and replaced by a vertical unit load at C. The girder is now simply-supported and may be solved by any of the numerous methods of analysis for this type of

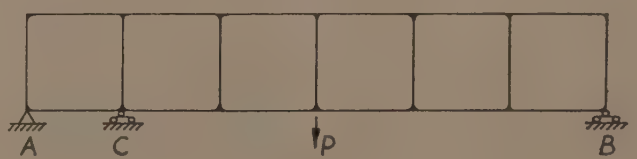


Fig. 1

girder. The bottom chord AB takes up a deflection line which may be determined from the M/EI diagram along the chord AB (the effects of direct and shearing forces being neglected). From Müller-Breslau's Principle it is observed that this deflection line is, to scale, the influence line for the reaction at C. This reaction may now be calculated for the applied external loading, and the reactions at A and B then follow simply from Statics. The girder may then be solved completely

by any method suitable for the simply-supported Vierendeel girder.

(b) With more than one Redundant Reaction

Remove the redundant reactions and apply unit loads in turn as before to obtain the respective deflection lines. These deflection lines must now be incorporated to give the true influence lines for the various reactions. This is easily done by writing and solving simultaneous equations, which express the fact that the actual deflection at a redundant support is zero. The number of unknowns—the multiplication ratios—is one less than the number of redundant reactions. With the redundant reactions obtained from the influence lines for reaction the solution follows as before.

Assumptions

The proposed method can be generalised for flexural effects, but several simplifying assumptions are usual (at least, in a first analysis). These are :

(1) the usual Elastic Theory assumptions ;

(2) that the redundant supports are all of the roller type, except one, which is pinned ; and that these supports are all in line, unyielding, and capable of supplying reactions in either sense.

(3) that the secondary effects due to direct and shearing forces, and to joint shape, can be neglected, and

(4) that the loading is applied at the panel points only, and at right-angles to the line of the supports.

Examples

The examples chosen for solution are the two solved by Takabeya and Uchida in their recent paper[1].

The method of analysis to be used for solving the simply-supported girder will be the Maugh Panel Method[2*], which is one of the easiest methods for girders with chords equally stiff in each panel. In such girders the moments in the chords are identical in any vertical section, and points of inflection occur at the mid-points of the posts.

The deflection lines of the girders will be obtained by use of the Conjugate Beam Method, which may easily be deduced from the Moment-Area Theorems.

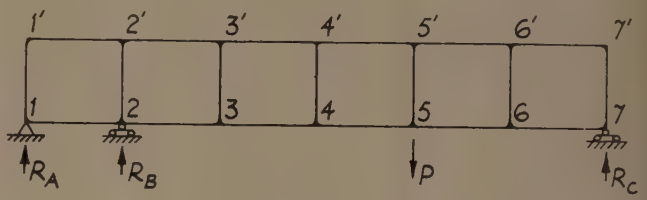


Fig. 2

In Fig. 2 all members are of equal stiffness K, and Young's Modulus E. Panels are of equal length and the chords are parallel.

Remove the applied loading, P, and the redundant reaction, say B, and then apply any convenient known load at B. Using Maugh's Panel method and nomenclature, without explanation.

$r = s = 1$ and $\alpha = 0$, for each panel.

$D = 6 + r + s + \alpha(2\alpha + \alpha s + 2s + 6) = 6 + 1 + 1 + (0) = 8$ for each panel.

Primary Moments

$$M'ad = \frac{\alpha M - V.l}{2D}(3 + s + \alpha(2 + s)) = -\tfrac{1}{4}.V.l$$

$$M'da = \frac{\alpha M - V.l}{2D}(3 + r + \alpha) = -\tfrac{1}{4}.V.l$$

Panel Carry-Over Factors

$(r/D) = \frac{1}{8}$; $(r/D)(1 + \alpha) = \frac{1}{8}$; $(s/D)(1 + \alpha) = \frac{1}{8}$; $(s/D)(1 + \alpha)^2 = \frac{1}{8}$.

In the table below the primary moments are written in the first line and distribution follows, corresponding

**It is interesting to note here that an almost identical method of analysis has recently appeared. In "Side Sway in Symmetrical Building Frames." which was published in the* STRUCTURAL ENGINEER *for April, 1950, Mr. N. Naylor presented a development of the well-known Moment Distribution Method which will apply to the Vierendeel girder with equal parallel chords. His method may be regarded as a particular case of the Maugh Panel Method, and is slightly easier to remember and to use. The Maugh Panel Method, however, will also apply to Vierendeel girders and towers with sloping chords.*

Distribution for Terminal Moments in the Top and Bottom Chord Members

PANEL	1		2		3		4		5		6	
CARRY-OVER FACTORS	0	0	⅛ to end →		→		→		→		→	
	←	←	←	←	←	←	←	←	⅛ to end		0	0
PRIMARY MOMENTS	-1600	-1600	+320	+320	+320	+320	+320	+320	+320	+320	+320	+320
			+200	-200	- 15	+ 15	-42	+ 42	- 45	+ 45	- 46	+ 46
	+ 70	- 70	+ 43	- 43	+ 40	- 40	+39	-39	+ 34	- 34		
			+ 9	- 9	+ 6	- 6	+ 8	- 8	+ 8	- 8	+ 5	- 5
	+ 1	- 1	+ 1	- 1	+ 1	- 1	+ 1	- 1	+ 1	- 1		
SUMMATION	-1529	-1671	+573	+67	+352	+288	+326	+314	+318	+322	+279	+361

Table 1

to panel interaction. The primary moments are chosen to correspond to an arbitrary panel length.

Deflection Line by the Conjugate Beam Method

The final chord moments obtained from Table 1 are drawn in Fig. 3.

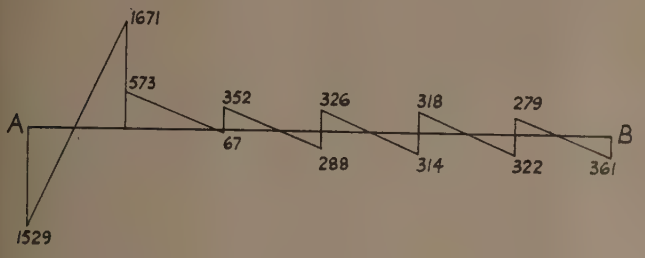


Fig. 3

EI is constant for the bottom chord of the girder and so is omitted. If θ_a is the reaction at A to the M loading along the bottom chord, and each panel is taken to be three units of length,

$$18\,\theta_a = 3/2\,(1671 \times 16 + 573 \times 14 + 353 \times 11 + 326 \times 8 + 318 \times 5 + 279 \times 2) - 3/2\,(1529 \times 17 + 67 \times 13 + 288 \times 10 + 314 \times 7 + 322 \times 4 + 361 \times 1)$$

whence $\theta_a = 817^1/_6$

Now $\theta_a + \theta_b = 3/2\,(1671 + 573 + 353 + 326 + 318 + 279) - 3/2\,(1529 + 67 + 288 + 414 + 322 + 361) = 958\frac{1}{2}$

Thus $\theta_b = 958\frac{1}{2} - 817^1/_6 = 141^1/_3$

The deflection at panel point 2, d_2, is given by the bending moment at point 2 on the conjugate beam.

Thus, $d_2 = 817^1/_6 \times 3 + 3/2\,(1529 \times 2 - 1671 \times 1) = 4532$

And, $d_5 = 141^1/_3 \times 6 + 3/2\,(361 \times 5 + 322 \times 2) - 279 \times 4 - 318 \times 1) = 2370\frac{1}{2}$

Thus for P at point 5,

$R_B = (2370\frac{1}{2}/4532) \times P = 0.524.P$

Taking moments,

$6.R_A - 5 \times 0.524.P = 2 \times P$

whence $R_A = -0.103.P$, and from

$R_A + R_B + R_C = P$, $R_C = 0.579.P$

(Takabeya and Uchida obtained values :

$R_B = 0.513.P$, $R_A = -0.094.P$, $R_C = 0.581.P$).

All the moments in Table 2 should be multiplied by $P.L/4000$ to give correct values, where L is a panel length. The final moments multiplied by $P.L/1000$ are as follows, with the values obtained by Takabeya and Uchida immediately beneath :

Panel (1)		Panel (2)		Panel (3)	
—10	—41	—123	—87	—103	—108
— 8	—39	—122	—87	—103	—107
Panel (4)		**Panel (5)**		**Panel (6)**	
—69	—141	—178	—111	—131	—159
—69	—141	—179	—111	—131	—159

The moments in the posts may be obtained at each panel point from the chord moments there, since $\Sigma M = 0$ at each joint. Direct and shearing forces may be found from Statics once the moments are known.

Example 2

This example will not be worked in full ; only the main results at each step are given below :

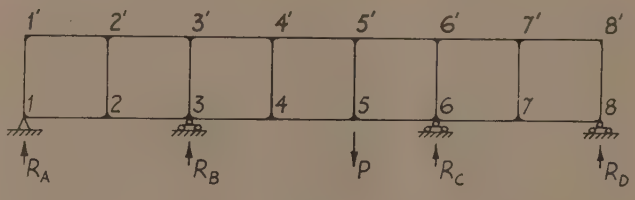


Fig. 4

Distribution for Final Terminal Moments in the Top and Bottom Chord Members

PANEL	1		2		3		4		5		6	
CARRY-OVER FACTORS	0	0	1/8 to end						1/8 to end		0	0
PRIMARY MOMENTS	+103	+103	-421	-421	-421	-421	-421	-421	+579	+579	+579	+579
			- 13	+ 13	+ 51	- 51	+ 59	- 59	+ 60	- 60	- 65	+ 65
	-60	+60	-50	+50	-34	+34	+88	-88	+64	-64		
			- 7	+ 7	- 7	+ 7	- 5	+ 5	+ 10	- 10	+ 9	- 9
	- 1	+ 1	- 1	+ 1	0	0	+ 1	- 1	+ 1	- 1		
SUMMATION	+42	+164	-493	-350	-411	-431	-278	-564	+714	+444	+532	-635

Table 2

In Fig. 4 all chord members are of equal stiffness K and all posts are of equal stiffness $2/3\ K$. Young's Modulus, E, is constant throughout. The panels are of equal length and the chords are parallel.

With R_B, R_C and P removed, unit load is applied at B and the chord moments are found to be as follows :

(where $\frac{(5/7)L}{4} = 810$).

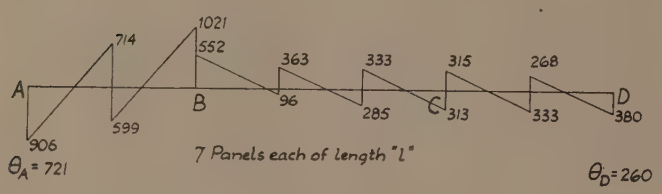


Fig. 5

The deflection line for unit load at B with R_B, R_C and P removed is as shown in Fig. 6. The deflection line for unit load at C with R_B, R_C and P removed is the same shape, laterally inverted, and is shown dotted.

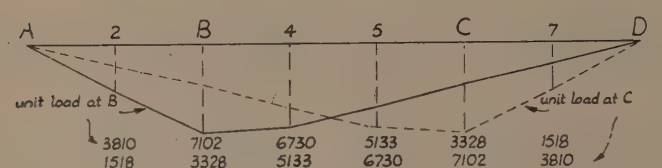


Fig. 6

The actual deflection line for unit load at B, with R_C operative and such that the deflection at C is zero, is obtained by applying an upward reaction at C to remove the deflection there. This reaction must clearly be 3328/7102 of the unit load giving the deflection lines above, but it could otherwise be found by writing an equation for the deflection at C to be zero :

$3328 - 7102.R_C = 0$, and thus, solving, $R_C = 3328/7102$.

The actual deflection line for unit load at B with R_B and the loading removed is given in Fig. 7, and this line is also, to scale, the influence line for R_B.

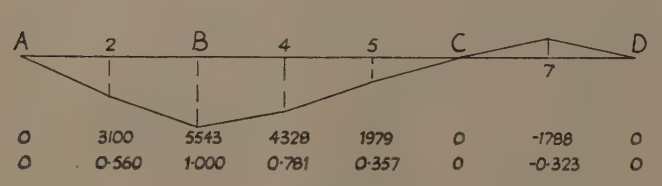


Fig. 7

From Fig. 7 it is seen that for P at point 5, $R_B = 0.357.P$, and since the influence line for R_C is the same shape, but laterally inverted, $R_C = 0.781.P$.

(Values obtained by Takabeya and Uchida were $0.357.P$ and $0.779.P$.)

The reactions at A and D may now be obtained from Statics, and the final terminal chord moments are then found by a final distribution, as before.

Approximate Methods of Solution

(*a*) *The Method of Static Analysis*

The Vierendeel girder is often solved on the assumption of points of inflection existing at the mid-points of all members. This assumption is nearly correct for panels distant from the loading (see Figs. 3 and 5, above), but may lead to large inaccuracies in the moments in some members. The inaccuracies in maximum stresses, however, may not be so great, since direct forces contribute. The method is very useful for preliminary designs and for cases where the open-frame action is secondary (e.g., wind loading on an open-frame trestle).

This method may easily be extended to the girder with redundant supports. In the examples above, the primary moments are, in fact, equal to the (final) terminal moments in this approximate method. Thus, for example, the (final) chord moment values in Fig. 3 by this method would be 1600, 1600, 320, 320, etc. It is found that this method gives the slope of the chords to be zero at each joint, and the joints to deflect to fall in line from support to load on each side, and thus Fig. 7 becomes :

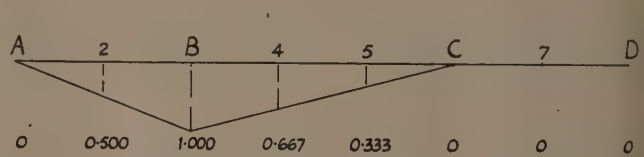


Fig. 8

From Fig. 8 it is seen that $R_B = 0.333$, $R_C = 0.667$, and $R_A = R_D = 0$.

(*b*) *Use of an Equivalent Beam*

A common device, which gives approximate values of the redundant reactions on continuous trusses, is to replace the truss by an equivalent beam. The EI value at any section of the beam is made proportional to the EI value of the chord sections taken as acting together at the corresponding section of the truss. If this approximation is applied to the continuous Vierendeel girder the results obtained vary with the particular case considered, but may be quite inaccurate. The method might be useful for a quick preliminary design.

Conclusion

The continuous Vierendeel girder is amenable to solution by various methods. The approach given above breaks the problem down into the calculation of the redundant reactions and then the final calculation of moments, and allows any method already familiar in the analysis of the simply-supported Vierendeel girder to be applied. If modern methods of analysis are used an accurate solution may be obtained with little difficulty, using the slide rule. Certain approximate methods may be used for a preliminary design.

References

[1]F. Takabeya and I. Uchida. "Calcul d'un Pont a Poutre Vierendeel Continue." Memoirs of the Faculty of Engineering, Kyushu University, Augut, 1949.

[2]L. C. Maugh. "Statically Indeterminate Structures." Wiley, 1946.

Building on Fill with Special Reference to the Settlement of a Large Factory

Discussion on Dr. G. G. Meyerhof's Paper*

Discussion

The CHAIRMAN, whose pleasurable duty it was to propose formally a vote of thanks to Dr. Meyerhof for his paper, said there was no need to emphasise the importance of the subject discussed therein ; its importance was indicated by the large number present at the meeting. In the paper Dr. Meyerhof had given a very nice blend of the theoretical and the practical ; in other words, he had given the results of research work and had analysed them, and an indication of their practical application. The meeting wished to thank him heartily for his paper and for his excellent presentation of it.

Lieut.-Colonel G. W. KIRKLAND, M.B.E., M.I.Struct.E. expressed the pleasure it had afforded him to listen to Dr. Meyerhof's presentation of the paper, and said that his firm had been very glad of Dr. Meyerhof's collaboration and interest in the problem. As the engineer responsible for the remedial measures, Colonel Kirkland had found the confirmatory opinions gratifying.

At Dr. Meyerhof's suggestion, Colonel Kirkland showed a number of slides to illustrate one of the major problems arising from the settlement which had occurred. The owners of the premises, he said, were manufacturers of a well-known building product.

Slide 1 showed the machine which was the main production unit of the factory ; had anything happened to it, production would have ceased. It was of heavy construction, with heavy cast-iron frame sides. The production flow was from the left-hand end towards the right, and settlements varying between $\frac{3}{8}$ in. and 3 in. had occurred. As a result the bearings ran hot, much power was wasted in friction, and various components had broken. One of the processes involved the use of a blanket to convey unbonded raw materials through rollers and the life of the blankets had been short. The machine was the most vulnerable unit and settlement was jeopardising the cast-iron frame, repair of which, immediately following the war, was well-nigh impossible.

Slides 2 and 3 (reproduced) illustrated various stages of the work. In Stage A bored piles were placed on the right-hand side of the machine.

In Stage B an excavation was opened up in readiness to receive cantilever beams, and a heading was cut under the frame sides to receive the cantilevers which were to take the load. Simultaneously, on the left of the machine, which was very congested, further piles were placed.

Stage C showed the cantilever beams in position, wedged to the underside of the foundation and tailed down on to the tailing down piles. Up to this point the plant had remained operative.

At Stage D the plant became inoperative. Fortunately, the annual holiday period enabled the owners to allow 21 days in which to do the job, and work was carried out with both day and night shifts. The whole of the drive mechanism was taken out on the left of the frame side, and cantilevers were inserted there.

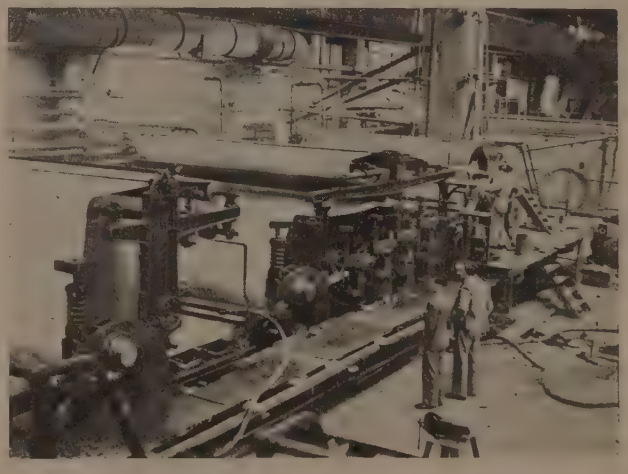
The next three stages covered the repetition of that work on the other side of the machine. The volume of work was considerable. In between the frame sides there, a concrete slab had cracked badly, and was removed.

Stage H showed the machine re-assembled. Within the 21 days the work was completed, apart from the floor finishes.

Colonel Kirkland said he had been pleased to hear only that evening that the machine was still working perfectly, having been in operation for $2\frac{1}{2}$ years since the work was completed. A level check had shown that there had been no subsequent movement.

Slide 4 showed details of the various components used.

Slide 5 (reproduced) was a photograph of the machine after it had become operative again. On the left of the centre the original foundation had been exposed where the load had already been taken by the under-pinning beams.



Slide 5, shown by Lt.-Colonel G. W. Kirkland

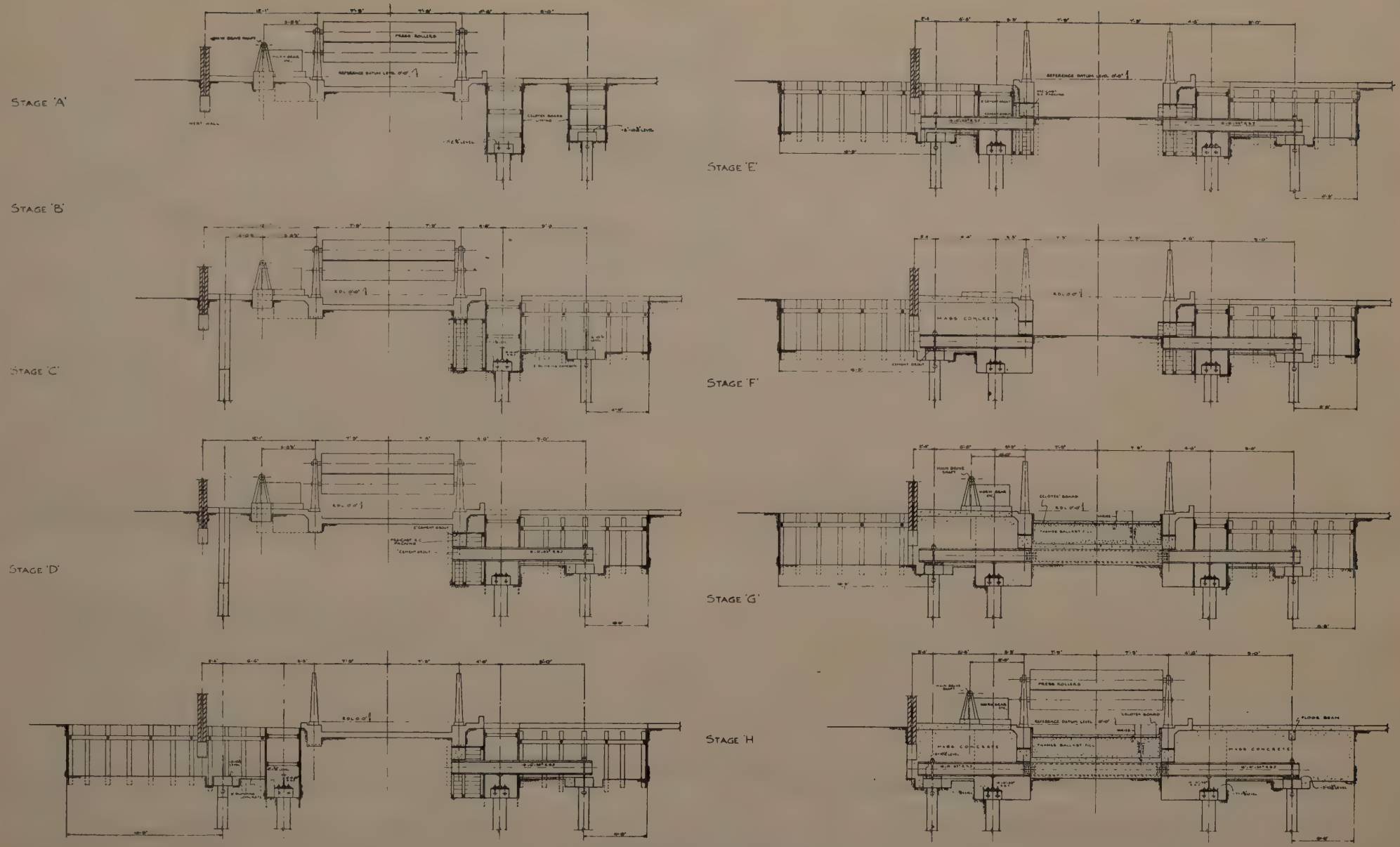
Slide 6 showed the driving side of the machine, where the mechanism had made conditions very restricted for piling operations. Quite apart from the low headroom and the overhead driving shafts, particular care had to be exercised in locating the load-bearing piles.

Slide 7 illustrated the operations at the time the machine was re-levelled.

Slide 8 was another view of the finished machine back in operation. Colonel Kirkland said he believed that at that time the production had increased considerably.

Measures were still being taken, concluded Colonel Kirkland, to remedy defects in the structure. His firm had been busy there for four years, and it looked as though they would be there another three years making good defects.

Read before the Institution of Structural Engineers at 11, Upper Belgrave Street, London, S.W.1, on Thursday, February 22nd, 1951, Mr. Walter C. Andrews, O.B.E., M.I.C.E. (Vice-President), in the Chair. Published in THE STRUCTURAL ENGINEER, *Vol.* XXIX, *No.* 2, *pp.* 46-57.



CELOTEX FACTORY—WEMBLEY

SCHEME I FOR UNDERPINNING SOFT BOARD PRESS UNIT

Slides 2 and 3 shown by Lt.-Col. G. W. Kirkland

The paper gave a very good lesson of the results of being penny wise and pound foolish, illustrating the wisdom of ensuring that the structure of a building would do the job required of it.

Mr. A. H. Toms, after thanking the Secretary of the Institution for having invited him to attend the meeting, referred to some tests made by British Railways (Southern Region) with compaction machines for the purpose of minimising settlement of railway tracks on newly laid filling over clay formations.

Discussing some tests made on crushed Meldon stone, he said its specific gravity was 2.74 or 170 lb. per cubic ft. The material used for the filling was the residue from the crushing of this stone which passed ¼ in. sieve. When dumped uncompacted its dry density was about 100 lb. per cu. ft. After loaded dumpers had passed over it several times, their weight being equivalent to about a 3-ton pneumatic-tyred roller, the dry density of the material was about 125 lb. per cu. ft., and the vertical density gradient was well over 1 lb. per cu. ft. per inch depth. A 4½-cwt. vibrating roller was tried, and though very handy and having a good output of some 500 sq. yards per hour, it was very sensitive to changes in moisture content of the fill and required very wet material, which involved extra wetting in most cases. The maximum dry density of the material was about the same as before at 125 lb. per cu. ft., but the gradient was higher, being about 2 lb. per cu. ft. per inch depth.

The machine now employed for compacting the filling in excavations was a 10-cwt. or 1-ton frog-rammer. It was easy to handle, because it pulled apart for handling, but had a lower output in sq. yards per hour than the roller to give maximum density, but this maximum density was about 130 lb. per cu. ft. after three or four passes, and the vertical density gradient was 1 lb. per cu. ft. per inch of depth. Moreover, that machine would tackle 20 in. depth of the loose material and would compact down to 15 in. in one layer, which was very valuable.

There had been considerable scatter in the results obtained, but the figures quoted gave a general idea.

A Vibrosoil compactor, weighing about 5,500 lb., lent by one firm, had a base plate turned up at the edges and supporting a casing in which there were out-of-balance rotating weights. Normally its vibratory motion propelled it, but when in difficulty a winch on the machine was brought into use. It was found to give densities as high, or higher than the frog-rammer, and a vertical density gradient not greater than that produced by the latter. A disadvantage was its much greater weight than the frog-rammer ; but on open sites it might have a very good field of use.

The Vibromax Soil Consolidator, which he had seen tested at the Road Research Laboratory, was smaller than the Vibrosoil compactor but was very similar in principle ; it was reversible under its own power, whereas the other was not, and its performance appeared to be about the same.

For anyone who had to compact considerable amounts of filling, to the maximum density practicable, the frog-rammer or the vibrating plate compactors would be valuable.

Finally, Mr. Toms mentioned that a certain firm, when asked by British Railways to put down boreholes to ascertain the combustion area in an ash tip (on which a locomotive was erected) and which had caught fire, had had the idea of forming a cut-off wall by putting down boreholes, overlapping, and filled with sand. So far as could be judged, that procedure had proved satisfactory.

Mr. P. T. French (Associate Member), said that at the large factory discussed in the paper there had been observed a certain amount of horizontal movement of manholes and drainpipes. He asked whether that had any bearing on the subject and if it were of interest it might be considered further.

Commenting on the author's statement that the main compaction of the fill took place in the first two years after it had been deposited, Mr. French said he believed the fill had started about the middle of the last century. The factory had been built approximately 14 years, and he wondered how to account for the excessive or large settlement which had occurred since then.

Did this mean the loads had not been allowed for in the calculations or had the piles been overloaded since ? Could the brown clay be used if calculations proved it offered sufficient resistance or would it be advisable in the future to penetrate to the blue clay ?

Mr. D. J. Maclean (Road Research Laboratory), discussing soil compaction in relation to the paper, said that a vertical movement might occur in a fill as a result of either a change in the air content or in the moisture content of the soil.

The occurrence of vertical movement in fill due to a change in moisture content had an important bearing on the choice of moisture content at which soils should be compacted. In the technical literature it was stated commonly that soils should be compacted at an optimum moisture content as determined by an arbitrary laboratory test. Recent research at the Road Research Laboratory suggested that that concept was wrong so far as this country was concerned. The view held at the Laboratory was that ideally soil should be compacted at a moisture content, which they described as an "equilibrium" moisture content, that would subsequently remain constant under any superimposed loading.

Taking a practical case, that of London clay, he said that the optimum moisture content, as determined by the standard laboratory method, was 21 per cent. ; but the " equilibrium " moisture content for the same soil under a building would be about 25 per cent. If this soil were compacted at the optimum moisture content of 21 per cent. a considerable uplift of the surface would subsequently occur as a result of the swelling of the clay as it increased in moisture content from 21 to 25 per cent.

On the other hand, in this country it was much more likely under the prevailing weather conditions that the clay would often be placed at a higher moisture content than the " equilibrium " value. If this were the case, settlement would subsequently occur as a result of the consolidation of the soil.

It was also important to compact sands and other fine granular materials at a moisture content which would subsequently remain constant. He was aware of several cases in road and airfield work where sand sub-grades had settled from ½ to 1 in. as a result of a decrease of moisture content subsequent to the compaction of the sand. Although the sub-grades had been well compacted during construction the subsequent loss of water from the sand resulted in an increase in the air content, which was equivalent to bringing the sand to a poorly compacted condition where stresses arising from traffic could produce further compaction. Sands were particularly susceptible to compaction by vibration, and that factor might be important in connection with buildings constructed on sand.

Coming to the practical use of compaction plant, Mr. Maclean emphasised first that their performance depended very much on the moisture content of the soil. He illustrated this for London clay by a graph

(Fig. 1) which showed the effect of moisture content on the dry density (state of compaction) that could be obtained with a sheepsfoot roller, a smooth-wheeled roller, a pneumatic-tyred roller and a medium-sized track-laying tractor. The moisture content at which the greatest compaction could be obtained with these

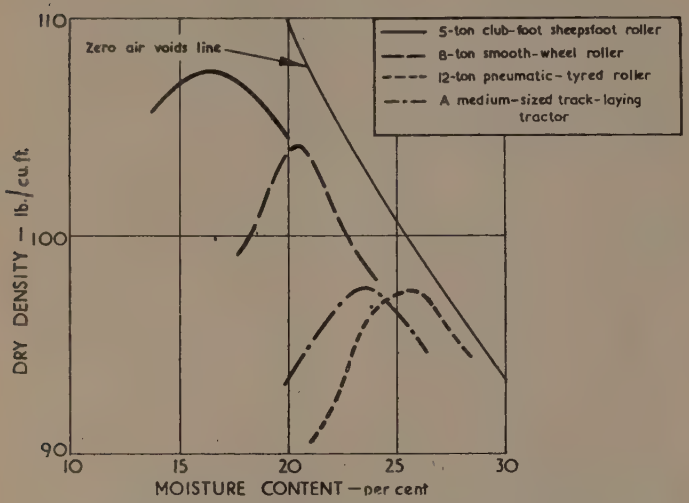


Fig. 1.—Dry density/moisture content relationships for a clay soil when compacted by different types of plant

(Crown copyright reserved)

types of plant ranged from 16 per cent. for the sheepsfoot roller to 26 per cent. for the pneumatic-tyred roller. It was thus necessary to employ that type of plant which would give the best performance at the particular moisture content at which the soil was to be compacted. With the London clay this would be about 25 per cent., and therefore the pneumatic-tyred roller would probably be the best machine to use in this instance.

In order to obtain good compaction it was also necessary to limit the thickness of the layer of soil compacted in one operation. A graph (Fig. 2) showed

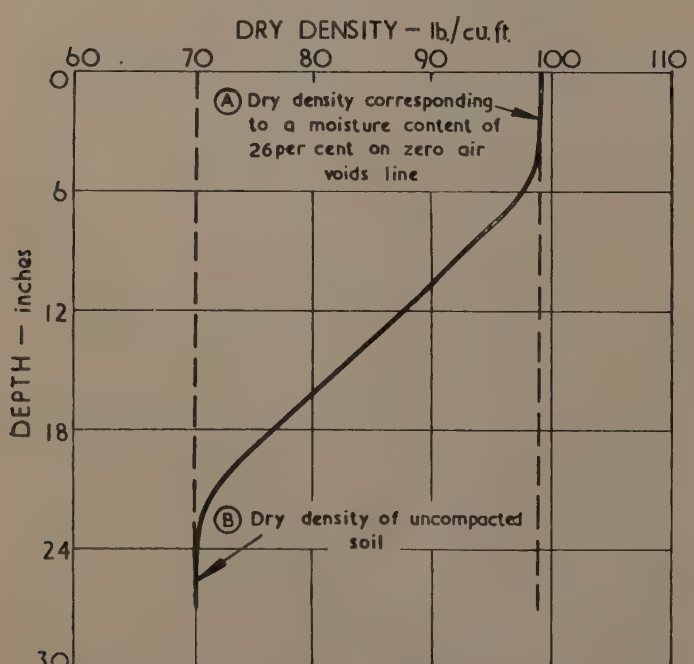


Fig. 2.—General form of relationship between dry density and depth through a compacted layer of soil

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a general relationship between the dry density and the depth through a layer of compacted soil. Near the top of the layer the soil was in a high state of compaction, all the air having been driven from the soil, but at a certain depth the dry density began to decrease approximately linearly with depth, until finally a depth was reached at which no increase in compaction occurred, the soil there being at the same dry density as placed. Since the presence of air in the soil would permit subsequent settlement of the fill, Mr. Maclean suggested that it was necessary to limit the thickness of each layer of soil to that at which the air content began to increase linearly with depth. With rollers the thickness of soil placed should not be more than 9 in., but with frog-rammers and heavy vibrating plates, thicknesses as much as 2 ft. might be used.

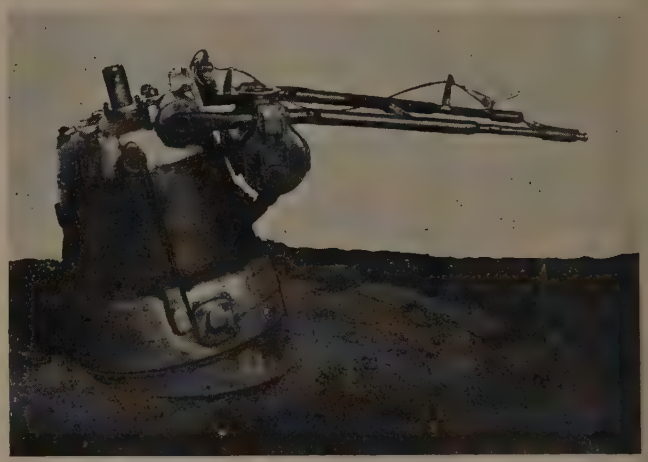

Fig. 3

(Crown copyright reserved)

As Mr. Toms had mentioned, the frog-rammer seemed to be a very suitable plant for compacting small areas of fill. A machine of this type weighing ½ ton (Fig. 3) had been tested at the Road Research Laboratory. Some results obtained with this plant showing the effect of moisture content on the compaction of soils ranging from a gravel to a heavy clay were given in Fig. 4. At the higher moisture content the soils contained only a small air content after compaction.

A vibrating plate machine tested at the Laboratory was illustrated in Fig. 5. This had a heavy base plate on which was mounted a motor operating at about 1,000 r.p.m., the vibration being obtained by means of a set of out-of-balance weights. Results obtained with this machine (Fig. 6) showed that a very satisfactory performance was obtained with a gravel-sand clay, sand and sandy clay but on a heavy clay the performance was less satisfactory.

Mr. Maclean said there was little information on the relationship between the settlement of fill and its initial state of compaction, but the information obtained from data published in an American paper (Fig. 7) might be of interest. Over the range of compaction indicated, there were settlements ranging from slightly under 0.1 per cent. to over 0.6 per cent. of the height of the fill. Finally, Mr. Maclean suggested that it might be possible to obtain some idea of the probable settlement of fill by carrying out repeated plate-bearing tests and obtaining a relationship between load and settlement as shown in Fig. 8. He suggested that the permanent deformation after the application of a load, say 10 times, might be related to the probable settlement of the fill.

Mr. Maclean congratulated Dr. Meyerhof on his very interesting paper and thanked the Institution for inviting him to the meeting.

Mr. John Faber, B.Sc., A.M.I.C.E. (Associate-Member), commenting on the suggestion in the paper that, on made ground, the use of bored piles had an

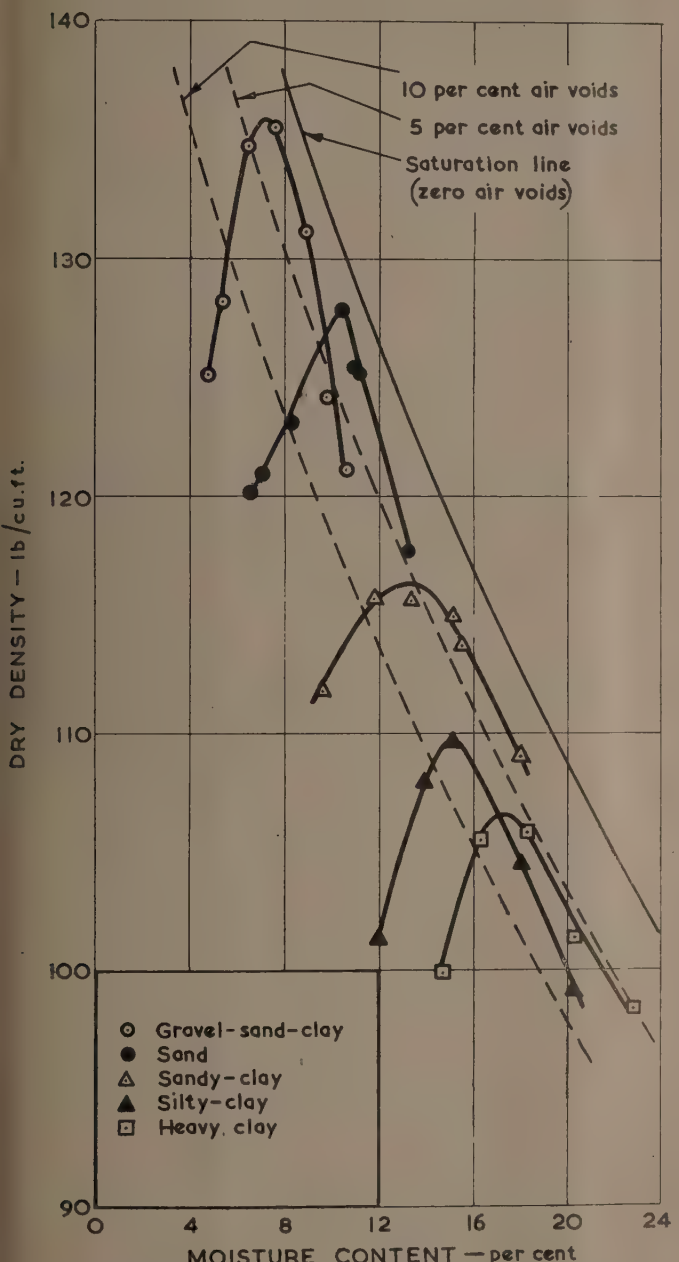

Fig. 4.—Relationship between dry density and moisture content for 5 coverages of a ½-ton Frog-Rammer on five different soils for a 9-inch uncompacted layer

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advantage over driven piles because of the settlement caused by the driving of the piles, said there could be no doubt that ground would settle when it was disturbed by the driving of piles ; but many engineers preferred driven piles to bored piles, and it should be understood that there was a difference between the *immediate* settlement caused at the time of the piling and the *ultimate* settlement which might occur after some considerable lapse of time after the piling.

The graph Fig. 9 showed the settlement of an unpiled building in relation to time. It covered a period of about three years, and the total settlement was about 8 in. There were six curves, each representing the average for about 20 columns. The columns were in rows progressively further away from a site where pile driving was at one time in progress, and the reason for the differences of the six curves was that the ground formation under those particular rows of columns was different in the different cases. It could be seen clearly that the shaking of the ground due to the piling had broken down the internal frictional forces and had accelerated the rate of settlement. After the piling had ceased there was no further disturbance, during which time the internal frictional forces of the soil were slowly built up again ; and it was not until those frictional forces had built themselves up again over a period of time and were eventually overcome so as to cause internal failure of the soil that the building had started to settle again.

The principle, Mr. Faber suggested, was very much like that of the ordinary aneroid barometer, which never really indicates the true pressure ; it was always a little behind the truth, and it was not until one tapped the barometer and overcame the internal friction forces

Fig. 5

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in the mechanism that the needle would jump to the position where it wanted to be. Then the needle would remain in that position until the forces due to a change of external conditions were sufficient to exceed the friction forces in the barometer mechanism which were trying to prevent the needle moving. The same sort of thing happened in soils ; there were internal friction forces tending to stop movement, but when those friction forces were overcome, the ground settled.

It was thus the *ultimate* settlement which very often mattered more than the *immediate* settlement. The building referred to was erected about ten years ago ; the maximum settlement after five years was about 12 in., and the building was still settling. The *ultimate* settlement was causing trouble with sliding doors, drains and so on ; but the little kinks in the curves which had been produced by driving piles near to the columns had been of no consequence.

Mr. S. J. Crispin (Member) asked whether the author could say if any great differences were found in compacting various soils in depressions or pits where they were kept in position at the sides, and on free embankments or slopes of which the edges were free.

Mr. H. M. Gell said the paper did serve to emphasise the results of the penny wise and pound foolish policy, as Colonel Kirkland had mentioned, and he suggested that in this country we did not spend enough on the initial investigations. On the other hand, in the

United States people would spend very often something like 5-7 per cent. of the total value of the project on those investigations. Consultants and architects would say, of course, that it was difficult to persuade the building owner to spend that amount ; an owner always thought that the money should come out of their fees. That was wrong ; the Consulting Engineer or architect ought to be strong enough to convince the building owner that it was an excellent investment against

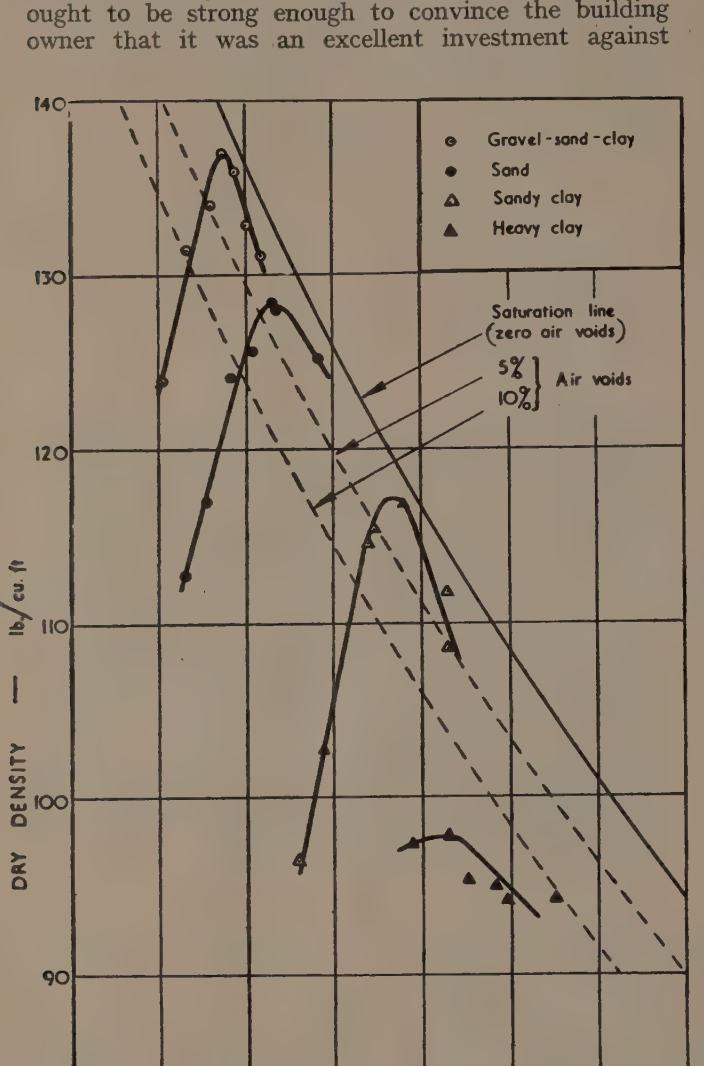

Fig. 6.—Relationships between dry density and moisture content for four different soils when compacted in 9-in. loose layers by six passes of the 2-ton Vibrosoil compactor

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failure and probably a safeguard on the economy of the whole project. The proper policy when we did not know was to find out. His preaching to young men was always to recall the activities of Rothschild in connection with the Battle of Waterloo. He had wanted to know who was going to win the battle ; England and the share market were depressed. He had persuaded someone to sail him over there, he went to the battle area, came back to the Stock Exchange and instructed his agents to buy and to continue to buy. That was one of the many reasons for the firm foundation of his fortune. Thus Mr. Gell always advised young men, " When you do not know, go to the Battle of Waterloo and find out."

In Mr. Gell's view the author had made a most valuable contribution in stressing the essential and vital need of a thorough site investigation on all such projects, and the paper on this aspect alone was of considerable value.

Mr. M. J. TOMLINSON referred to a settlement of a light structure on fill which his company had recently investigated, which served to emphasise how rapidly settlement could take place even in clay fill. The site was on a chalk hillside ; a cut had been made into the chalk to form a level base for a single-storey timber hut and fill was deposited to extend the levelled area as in Fig. 10.

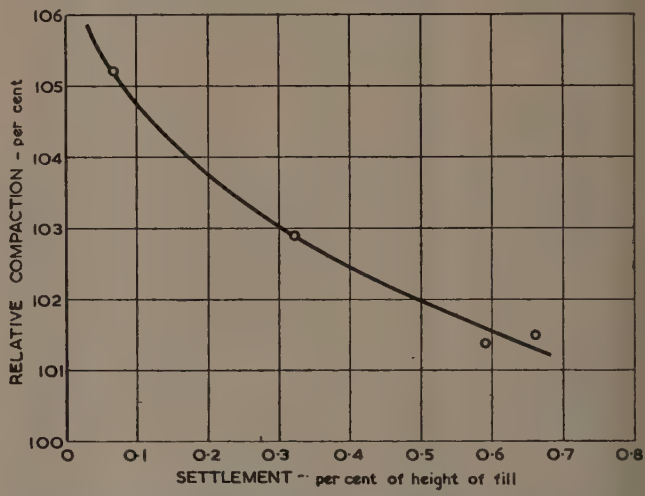

Fig. 7.—Relationship between relative compaction of the fill and percentage settlement 3 years after construction of silty clay embankments in Ohio, U.S.A.

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The fill was mainly of clay, but it contained some chalk fragments. After it had been in place for a few months, the hut was built with its foundations partly on the original chalk subsoil and partly over the fill. The hut was erected on a shallow brick sleeper wall resting on a concrete foundation. Below the front edge of the hut there was 9 ft. of fill, but at a distance of 20 ft. from the front there was no fill at all. After six months there was settlement of about 4 in. at the front end of the hut and a crack in the foundations 20 ft. towards the rear at the junction of the cut and fill, and the structure of the hut was distorted. The trouble must have been due to the settlement of the fill itself under its own weight rather than of the relatively incompressible chalk subsoil.

Again, at a factory in Aberdeen they had investigated a bad case of settlement. There was 10 ft. of very poor fill on 20 ft. of reclaimed ground (sand and silt), and below that there was 10 ft. of gravel and then granite bed rock.

The filling was deposited about 100 years ago, and after it had been in place for 50 years a two-storey factory of brick construction was built on shallow strip foundations supported directly by the filling. This building had settled about 15 in. in the centre, and it was considered that the greater proportion of this settlement had taken place in the upper 10 ft. of poor fill which consisted of ash, gravel, clay, and pieces of timber. It was estimated that the reclaimed ground (mainly of sand but with some silt) was relatively incompressible.

Reply to the Discussion

Dr. G. G. MEYERHOF, expressing his appreciation of the interest that had been shown in the paper, thanked

the many contributors to the discussion ; the subject was one on which, unfortunately, there was very little quantitative information. He hoped that when the discussion was printed, it might be possible to include some of the interesting photographs and diagrams which had been exhibited, because it was only by such case records that we learned how to deal with that difficult problem.

In thanking Colonel Kirkland for having illustrated in detail the method used for under-pinning the plant in the factory dealt with in the paper, he said it was a difficult job, particularly as the plant had been kept

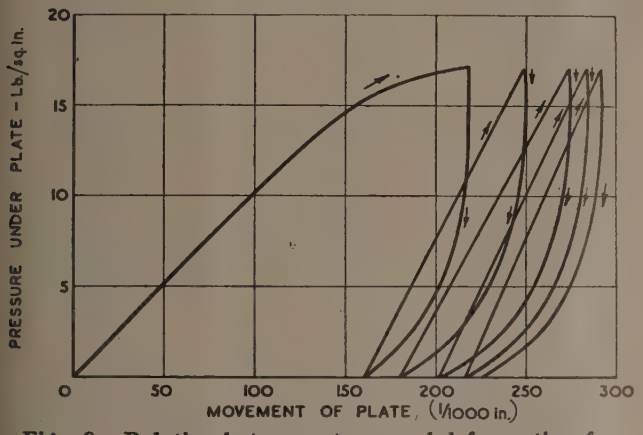

Fig. 8.—Relation between stress and deformation for plate bearing tests with repeated loads

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running for quite a long time and remedial measures had to be fitted in while it was in operation. It would be agreed that the methods proposed were interesting, especially the idea of jacking steel beams horizontally through the fill ; if the resistance had not been so great, that might have been a very useful procedure.

Coming to Mr. Toms' reference to power rammers and vibrating compactors for consolidating fill, Dr. Meyerhof said that there were many points in favour of that method where fairly small quantities of materials were involved. It might also be used behind retaining walls and in other cases where a fairly narrow width of material had to be compacted. When compacting earth against a structure, however, we must be careful not to compact too much ; it had been observed in several cases that over-compaction might be detrimental to the structure, particularly retaining walls where, apart from causing large pressures, expansion after the release of stress might produce some movement.

In reply to Mr. French, he could not say whether the observed horizontal movement of manholes and drain-pipes had anything to do with the movement of the fill. On the whole the latter movement was vertical, and any horizontal component must be due to some exterior cause.

On the question of the settlement of the factory in relation to the amount of previous consolidation of the fill, he had mentioned in the paper that a period of two years might be considered as a minimum before structures were erected on fills consolidating under their own weight, because the observations available at present showed that subsequent movements were relatively small, and because in any case structural precautions and special foundations were usually required. Of course, one could wait considerably longer than two years, and in some cases it might be preferable to do so. The reasons why the settlement of the particular factory was so detrimental were, first, that the structural precautions and special foundations were not effective, and, secondly, that the floor loads were exceptionally high. However long we waited for the consolidation of fill under its own weight, we must expect further settlement if additional heavy loads were to be applied.

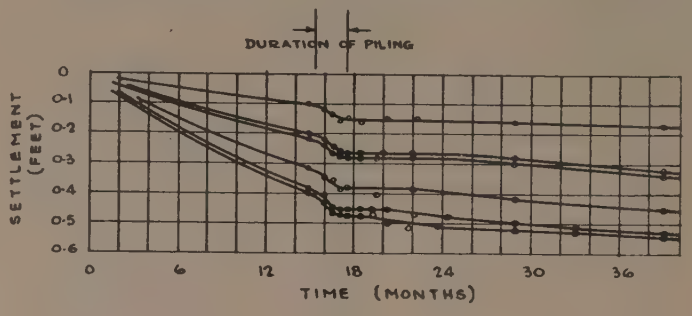


Fig. 9

A fill consolidated only according to its overburden ; near the bottom the consolidation was a maximum, and near the top, where good compaction was most required, it was a minimum. If we applied further load, we must expect additional movements.

The question of negative skin friction on the piles was also raised by Mr. French, who had asked whether the friction was negative originally or whether it had gradually become negative. Dr. Meyerhof said that the negative skin friction was induced by the movement of the fill, and immediately after the installation of the piles, the movement and thus skin friction might be relatively small ; but as the consolidation continued, so the negative friction developed more and more and started to drag the piles down, depending on the support they had at their points. On the whole, he thought that the negative friction was gradually developing as the support at the base of the piles was overcome by the drag on the sides. In that case, resistance would have to be obtained by a greater depth of piling into the

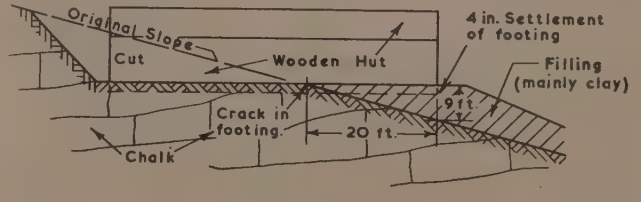


Fig. 10

underlying brown or even blue clay. It was a question of obtaining adequate resistance by skin friction and end bearing from the underlying natural ground, which did not move and could withstand the load.

With regard to the interesting information given by Mr. Maclean and his emphasis of the point concerning movements due to moisture variation and change of air content, Dr. Meyerhof said that these problems had been considered by engineers for some time. Over-compaction of cohesive material could lead to large swelling and high stresses in the underlying virgin ground. These effects had been observed for several years and were first mentioned by Knappen (1936) and later in a report on the compaction of soils by the American Society of Civil Engineers (1940).* The equilibrium moisture content was a matter on which

*Knappen, T. T. (1936). "*Calculation of the Stability of Earth Dams.*" *Second Cong. Large Dams, Washington (D.C.), Vol. 4. p. 505. American Society of Civil Engineers* (1940). "*Consolidation of Embankment and Foundation Materials.*" *Proc. Am. Soc. Civ. Engrs., Vol. 66, p. 1491.*

work was proceeding at the Road Research Laboratory, and it was of interest in showing a rational approach to the compaction of cohesive material. But Dr. Meyerhof did not think that the loss of water from a well-compacted cohesionless subgrade altered its degree of compaction in any way.

The question of repeated test loadings was one which needed consideration, especially on sites where the live loads were to be relatively large in relation to the dead loads. In structural design that situation did not often arise, for usually the dead loads represented a large proportion of the total loads, and only the first part of the load-settlement curve was of interest. Dr. Meyerhof thought, however, that the tested area was too small in relation to building foundations and extrapolation was very difficult.

In connection with road and airfield pavements, however, the dead load was negligible and the live load was very important so that the question of repeated loading then became of major importance.

On Mr. Faber's reference to the use of bored piles *versus* driven piles, Dr. Meyerhof said he had stated in the paper that bored piles might be preferred where vibration had to be avoided during installation of the piles. Of course, the driving of piles into a loose fill would cause settlement, and the subsequent movements were considerably less when the fill was compacted by pile driving than would be the case otherwise. But where there was an existing structure, especially if resting on a relatively loose granular material, pile driving in the vicinity may produce very undesirable settlement.

Dealing with Mr. Crispin's question about filling in pits and on embankments, he said that when there was a restraint on the sides of a fairly small amount of fill, the density of the material might be very much greater than if there was no restraint, as in the case of a free slope. But usually when dealing with pits and embankments the dimensions were fairly large and the effect would only be noticeable near the edges.

Site investigation, mentioned by Mr. Gell, was a matter of considerable importance, as was illustrated by the factory discussed in the paper.

Finally, he expressed gratitude to Mr. Tomlinson for his contribution relating to buildings on poor and variable fills.

At the conclusion of the discussion, the Chairman again thanked Dr. Meyerhof for his able replies to the points raised.

Written Communications

From Mr. W. T. Wilks (Associate-Member) : Dr. G. G. Meyerhof's paper " Building on Fill," is very informative for the engineer who is frequently faced with the problem of building structures on fill of one kind or another. Fig. 1 confirms the opinion, generally held, that clay and chalk, or alternatively, clay and sand, with light compaction, make a good fill with little ultimate settlement. The theory is that water forced out of the clay during its compression can be readily absorbed in the sandy layers, resulting in rapid consolidation. Regarding filling up a crater with rock or forming an embankment with rock, this is not a particularly easy task in many circumstances, and if the rock is not properly consolidated it is possible to get a large settlement which can be troublesome. In these circumstances, should the rock fill be soft, like one or two of our own natural rocks, then after a period of weathering they disintegrate and flow into the large voids, which are inevitable in such fill, to cause settlement which can be of some magnitude.

What is important in the construction of fills is to produce uniform compaction as distinct from high compaction which is generally, of course, less uniform. The latter often results in subsequent uneven settlement.

Several methods of compaction have been used which are not well known in this country. One method of compacting sandy soil by vibration has been devised by means of a process of sinking tubes into the ground to be treated, and lowering into them electrically driven vibrators. At the same time water is pumped under pressure into the tubes and forced up the outside. It is believed that the tubes at about 5 ft. centres sunk in this way produce satisfactory consolidation.

A second method of vibrating and compacting loose non-cohesive soil is by producing water hammer effects in armoured flexible hoses buried in the ground.

A third method of compacting loose granular soils is by means of driving steel tubes into the ground and filling them with sand and gravel, and while the tube is being withdrawn this fill is compacted in the hole formed by the tube.

The period of two years, given by the author as being sufficient time to lapse before commencing construction of a structure after the placing of badly compacted materials or fills consolidating under their own weight, is regarded as optimistic. Furthermore, the writer does not favour the design of rigid superstructures on any kind of fill. He calls to mind an instance where he had to provide remedial measures to a reinforced concrete framed structure of one floor carrying heavy machinery and roof, built on brick rubble. At one place, settlement of the fill had left a 14 in. reinforced concrete column suspended from the heavy beams above, resulting in serious cracking of the beams and the column. The column foundations and the ground floor slab were incorporated as one unit and were of sufficient strength to put the column in tension when the floor was loaded. Flexibility in design is suggested to be the proper solution for structures built on fill.

From Mr. F. R. Bullen (Member of Council) : This interesting paper will do much to draw attention to the necessity for skilled advice in the erection of buildings. I am particularly interested in the reference to the piled foundations. One of the lessons which I think this paper teaches is that the bearing capacity of a pile depends upon two separately ascertainable quantities.

(1) The point bearing capacity.

(2) The frictional capacity.

Whilst some piles may receive greater support from their toe capacities, others receive greater support from the friction capacity, the relative values of these two quantities being entirely dependent upon the type of piles used and the ground in which they are installed. For the factory in question it is clear that the point-bearing capacity of these piles was not very great since there were founded in London clay, and since they were only 5 to 10 feet into that clay they received relatively small frictional support. Unfortunately, however, as Dr. Meyerhof has shown, instead of receiving additional support from the friction of the fill they were dragged down thereby, or, in other words, the frictional capacity of the piles in this case was negative. Dr. Meyerhof refers to the bearing capacity of the piles ; I should like to ask Dr. Meyerhof if he could define the bearing capacity of a pile. This is a term which is used very loosely, and whilst it might be said that the intention is known, certainly the value intended is never defined. I would venture to define the ultimate bearing capacity of a pile as that load which causes the head of the pile to settle a distance equal to one-tenth of the lateral dimension of the pile. I hope to discuss this

matter on another occasion but I do think it is important in discussing these matters to define carefully the terms which are being used.

I feel sure that if the original designers of the building in question had appreciated that the ultimate bearing capacity of a pile consists of these two quantities, they would most likely have appreciated that the effect of settlement of the fill would be to impose upon the piles an additional load instead of assisting the piles to carry the superimposed loads. In these circumstances, of course, they would have installed the piles of greater length and the troubles experienced would not have arisen.

One general comment : Dr. Meyerhof suggests that buildings of the nature of the one in question might conveniently be sub-divided into small sections, and I would like to support that suggestion. It seems to me that there could be many advantages obtained by sub-dividing structurally long buildings which may be subject to different settlements.

I was a little worried by Dr. Meyerhof's suggestion that the foundations ought to be well tied in two directions. In buildings of the type under consideration, this is often very difficult, not to say costly, and I would suggest that it is quite permissible to consider the lateral support which the concrete floor of such a building offers to the columns and their foundations. On the other hand, where settlement of the floor is expected it is most important that it should not be connected structurally to the columns and wall foundations, although this does not prevent the floor slab from offering lateral support to these parts of the building.

In conclusion, therefore, I should like to thank Dr. Meyerhof for bringing forward this matter. In doing so he has done three things :

(1) He has shown the necessity for very careful and skilled consideration at the time of the initial design, to say nothing of careful investigation of the ground conditions.

(2) As a corollary to (1) he has shown the necessity for careful attention to matters of detail.

(3) He has drawn attention in no uncertain way to the fact that piles are just as susceptible to failure as any other foundation construction, and this, I think, emphasises a fact that has been glossed over in the past that the ultimate bearing capacity of a pile consists of two separate constituent factors.

Reply to Written Communications

Dr. G. G. Meyerhof would like to thank Mr. Wilks for drawing attention to some of the lesser-known methods of compacting granular materials. As regards the minimum period of natural consolidation, Dr. G. G. Meyerhof would like to refer Mr. Wilks to his reply to the oral discussion. Whether a flexible or rigid structure should be used on a fill, depended largely on the size and relative cost of the building and foundations. A low and extensive building was more economically constructed as a flexible structure which could accommodate itself to unequal settlement. A compact and high building was inherently more rigid and needed little additional stiffening to enable it to reduce differential movements to a minimum ; it should be adequately designed to resist the relatively large stresses due to load transfer and to span over any local deformation of the fill.

In reply to Mr. Bullen, the (ultimate) bearing capacity of foundations (including piles) was usually defined as the maximum load which the foundation could carry, irrespective of the settlement which necessarily occurred in reaching that load. Dr. Meyerhof could not agree to restricting the ultimate load by an arbitrary amount of settlement any more than that the ultimate (maximum) strength of building materials or structures was so restricted. Deformations had to be considered quite separately and usually on an entirely different basis, such as the tolerable deformations of the particular building element or structure concerned. The suggestion of tying foundations laterally by the reinforced concrete floor had sometimes been used in practice and frequently appeared to be adequate.

In conclusion, Dr. Meyerhof would like to thank both contributors for the interest shown in the paper.

Book Review

" Der Rammpfahl " (The Driven Pile), by W. Schenck. (Berlin : W. Ernst, 1951.) 111 pp. + viii, ill. DM.11.

The bearing capacity of driven piles is a controversial subject with an extensive literature to which the author adds his present well-illustrated book. In the first part of the book he is mainly concerned with full-scale pile loading tests and their interpretation as regards the amount of point resistance and skin friction. The method suggested consists of measuring the elastic compression of the whole pile and of a part (say one-half) of it during a loading test by means of extensometers and then evaluating from the results the point resistance and the magnitude and distribution of the skin friction.

This procedure, which is illustrated by some actual test results, is limited to steel piles because the compression characteristics of reinforced concrete and timber piles are too variable. More serious is, however, the basic assumption that the skin friction is distributed along the pile in a simple (mathematical) manner, either uniformly or steadily increasing or decreasing throughout the whole length of the pile (without intermediate changes). Nor is the method sufficiently sensitive to this distribution as shown by the loading test of a shod box pile driven into cohesionless soil where the analysis of the results gave zero skin friction at the toe and a rapid increase of friction with a maximum at the ground level !

In the second part of the book he proposes to utilise the above procedure to " calibrate " pile driving formulæ by substituting the reversible compression of the pile and soil as measured during a loading test for the corresponding dynamic value in the formula. This introduction of " static " components into " dynamic " equations would appear to be inconsistent and cannot be supported. In addition, it is now generally recognised that there is no relation whatever between dynamic and static bearing capacities except under special conditions, so that the former relations are at best of a qualitative nature.

The value of the book would have been enhanced by including, in the description of the loading tests, the results of the physical properties of the soil. This would have enabled theoretical estimates of the ultimate loads to be made by recent soil mechanics methods and for them to be checked by the results of the loading tests. Field and laboratory experience during the last few years indicates that such a procedure is likely to be the only rational approach to the problem of the bearing capacity of piles.

G. G. M.

Design and Construction of a Large Span Prestressed Concrete Shell Roof

Discussion of the Paper by Lt.-Col. G. W. Kirkland, M.B.E., and Mr. A. Goldstein*

The authors, introducing their paper, amplified the methods of construction and showed a number of slides. (Figs. 1 and 2 below illustrate the state of the building at the time of presentation of the paper when nearing completion, and were among the slides shown.)

Fig. 1

Fig. 2

Col. KIRKLAND opened his remarks by stating that they did not pretend their solution was an exact one, indeed, few structural problems were capable of exact solution. He was satisfied, however, that the solution they presented was a very close approximation, as would be shown by the comparison of the stresses obtained by strain readings with the calculated stresses. His co-author would elaborate this point. It was also gratifying to be able to report that other approximate checks applied to the problem had given comparable results.

Mr. A. GOLDSTEIN, continuing, said, when dealing with the strain readings, he was able to show the condensed mean results of strain and deflection observations. Fig. 3 showed the details of the gauge used together with the jig for placing the plugs. Fig. 4 gave the results for a typical intermediate shell. The negative deflection of $\frac{1}{8}$ in. of the right-hand beam in (a) was probably due to scaffold yield since the shuttering of the next shell had been struck between the two level

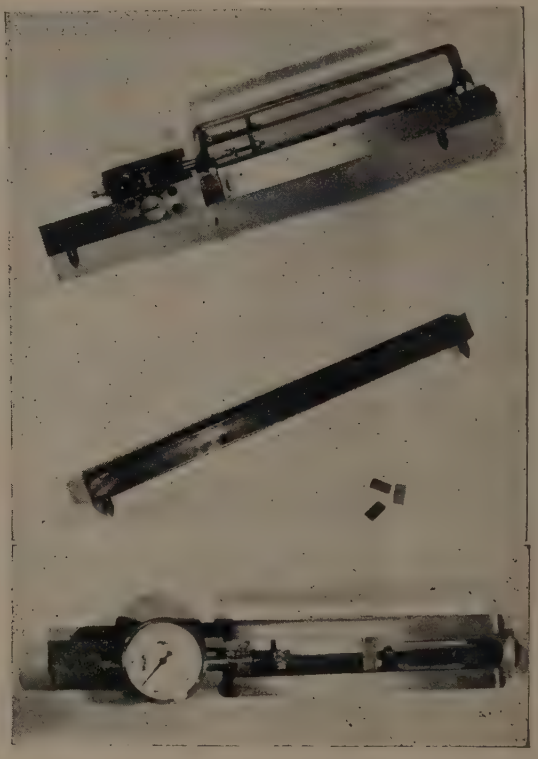


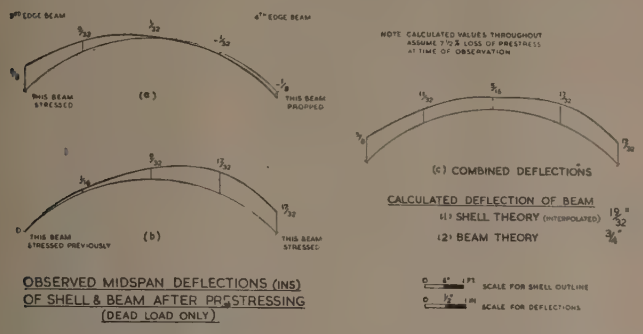
Fig. 3

observations. The comparison between the deflections due to stressing either beam was fairly consistent, and comparing (c) to the deflection curve in Fig. A5 of the paper, it was interesting to note that when a positive edge disturbance occurred, i.e., downward deflection of the springing, the movement of the crown relative to the springing is upward, whereas during prestressing, which induced a negative edge disturbance (i.e., upward) at the springing, the relative movement (observed) of crown to springing is downward, which appears consistent. No " shell " calculations as such were made for the conditions at which strain and deflection readings were taken. The calculated values were obtained by interpolation between the full prestressed case. The comparisons between the observed and calculated values are quite close, and well within the limits required to verify the reliability of the methods used.

As a point of academic interest, stresses and deflections were also calculated, using the beam theory, and

*Read before the Institution of Structural Engineers at 11, Upper Belgrave Street, London, S.W.1., on Thursday, 26th April, 1951. Mr. Walter C. Andrews, O.B.E., M.I.C.E., M.I.Struct.E. (Vice-President) in the Chair. Published in THE STRUCTURAL ENGINEER Vol. XXIX, No. 4, pp. 107–127. (April 1951).

were shown in Fig. 4. It was seen that the larger deflection thus obtained was consistent with a larger stress at the soffit and a smaller stress at the crown. The comparisons showed the limitations of the beam theory as applied to prestressed shells.*


9TH EDGE BEAM
6TH EDGE BEAM
THIS BEAM STRESSED
(a)
THIS BEAM PROPPED
NOTE CALCULATED VALUES THROUGHOUT ASSUME 7½% LOSS OF PRESTRESS AT TIME OF OBSERVATION
(c) COMBINED DEFLECTIONS
CALCULATED DEFLECTION OF BEAM
(1) SHELL THEORY (INTERPOLATED)
(2) BEAM THEORY
THIS BEAM STRESSED PREVIOUSLY
(b)
THIS BEAM STRESSED
OBSERVED MIDSPAN DEFLECTIONS (INS) OF SHELL & BEAM AFTER PRESTRESSING (DEAD LOAD ONLY)
SCALE FOR SHELL OUTLINE
SCALE FOR DEFLECTIONS

		AT CROWN	AT BEAM SOFFIT
	MEAN OF MEASURED VALUES	540	511
CALCULATED VALUES	SHELL THEORY (INTERPOLATED)	518	590
	BEAM THEORY	428	809

MIDSPAN T_1 STRESS COMPARISON AFTER PRESTRESSING (DEAD LOAD ONLY) (IN LB/SQ.IN. COMPRESSION)

Fig. 4

In the case of a long shell such as this one the beam theory might well give quite accurate results for T_1 in an intermediate shell when non-prestressed. It appeared that with prestressed shells the errors were considerably larger, and since T_1 was the force giving the closest agreement between the two theories it seemed logical to conclude that for prestressed shells the limitations of the beam theory were even more severe than for non-prestressed shells.

Errata

The following printers' errors should be noted :—

Page 107—Authors' names—" G." W. Kirkland.

,, 109—Fig. 4, title—" cores " not " corse."

,, 114—Design Loading—" g " not " go."

,, 116—Equation 1b — $\frac{L^2-x^2}{8}$ not $\frac{L^2-x^2}{\delta}$

,, 116—11th line, after equation 5—delete " Include," full stop after " shell."

,, 116—Equation 6—" $2a_s$," not " $2as$."

,, 120—Table IV
Symmetrical Loading Condition under a' and b'—" β'," not " β."

,, 120—Table IV
$Ed_s \triangle A$ for symmetrical a'.
—" 7275·7455," not " 2775·7455."

,, 120—Table IV
$Eds\eta$ for symmetrical b'.
—" 4572.7990," not " 4512.7990."

,, 124—Table XIII—bottom line, " $\triangle_H$," not "H"

,, 127—Bibliography, Reference 7—" Kalinka," not " Kalrinka."

Discussion

The CHAIRMAN, opening the meeting to discussion, said he had a very pleasant duty to perform in proposing a vote of thanks to the lecturers.

Reading through the paper, he had been struck by the general clearness and the very able way in which it had been written and illustrated, and the very nice blend of the practical and theoretical.

Dr. K. HAJNAL-KÓNYI (Member) said he had been very interested in the paper and he agreed with Mr. Goldstein's remarks that it was the first of its kind. It was certainly a useful addition to the literature on shell design after so many publications had been confined to membrane stresses and tables, giving the impression that calculations for membrane, stresses would solve the shell problem. In the paper, membrane stresses, though they might be considered primary stresses, were of secondary importance as against the stresses caused by the edge disturbances. So the paper would be appreciated by many people who wanted to go into shell calculations.

One point that struck him particularly in the general arrangement of the structure was the fact that the building was 300 feet long and there was no expansion joint. As far as the shell was concerned that was all right, but what were the consequences for the stiffening beams ?

He wondered whether alternative designs had been investigated because the relation of width to length of the barrels was rather unusual. Had the authors gone into the question of using eight, seven or even six bays, and had they made any comparisons which decided them on nine bays ?

A statement was made on page 107 that the designers decided to exploit the advantages of prestressing and thereby effected considerable saving in steel and concrete in comparison with ordinary reinforced concrete design. He agreed on the saving in steel but where was the saving in concrete ? Probably the authors thought that without prestressing it would be necessary to have a shell three inches thick as was stated on page 114 under Design Loading. With concrete strength as mentioned by Col. Kirkland, he would have had no hesitation to carry out the shell in 2½ inch thickness without prestressing.

He wondered if the effect of subsequent prestressing on the wires already anchored had been taken into account, or whether each wire had been stressed to the same extent. Once a wire was anchored the stressing of every following wire reduced its stress, so virtually the first wire should be stressed to the maximum and subsequent ones to less, so as to achieve a final balance. That was not, perhaps, of great practical importance, but it was theoretically interesting.

Then there was the effect of prestressing on the columns. By prestressing the valley beam, the distance between the columns was reduced and the columns were subjected to bending. Had that been taken into account in the design and had observations been made regarding the movement of the columns ?

He would be very much interested in the cost of the whole scheme. Just as the weight of the steel had been given in elaborate detail it would be interesting to hear about the cost of the job as carried out.

Regarding the calculations on page 117, it was stated that it was generally sufficient to calculate the forces at the springing, crown and quarter points of the shell. He wondered if that was enough to produce such curves as shown in Fig. 5. He thought it required some imagination to get those curves without having intermediate points. One knew the general shape, but he thought that three points were not sufficient.

The authors did not state what " E " had been assumed.

He had not, of course, checked the figures, but he would like to draw attention to Table VII. There was

*Discussion on "*A Method of Determining the Secondary Stresses in Cylindrical Shell Roofs*" by W. T. Marshall* — JOURNAL OF THE INSTITUTION OF CIVIL ENGINEERS — *supplement to No.* 8 1949-50. *October* 1950.

the value of $\triangle B$, the horizontal movement of the edge of the shell caused by a vertical force $V = \frac{4}{\pi}$ which must agree with $\triangle A$, the vertical movement of the edge of the shell caused by a horizontal force $H = \frac{4}{\pi}$. There was a discrepancy already in the second significant figure, which indicated a slight error. The same condition applied to the antisymmetrical case where the discrepancy started in the third significant figure. It was not serious, but if one were using a calculating machine and went to so many digits one would expect better agreement.

There was another check. There was a constant ratio between T_1, caused by $V = \frac{4}{\pi}$ and $\triangle A$ caused by $\frac{dS}{dx} = \frac{4}{\pi}$ on the one hand and between T_1 caused by $H = \frac{4}{\pi}$ and $\triangle B$ caused by $\frac{dS}{dx} = \frac{4}{\pi}$ on the other. He had worked out these figures by calculating machine. The ratio was in the first case —169,692, in the second it was —148,049. For antisymmetrical edge load the ratios were —168,970 and —166,662 respectively. All these four ratios should be the same. He would have liked the author's comment and to draw the reader's attention to the fact that there were several automatic checks that could be done during the calculations. In spite of duplicating the work errors creep in and it was always very useful to check the co-efficients before one went into the equations.

Those were minor points. He congratulated the authors on the paper and hoped that they would answer some of the questions he had put to them.

Mr. H. G. Cousins (Member) congratulated the authors on an excellent paper and thanked them for the wealth of detail, of all aspects of the construction, which they had put into it. He had enjoyed the paper because he was interested in that type of construction.

He agreed with the authors that the design should preferably be made on the elastic theory, because we had sufficient evidence from experience that shells so designed did not crack, and from many tests, which had been made, appeared to act as was to be expected from the design.

The calculations for this type of roof were lengthy and difficult when first attempted. The equations were formidable but it was essential to tackle them. He thought it a pity that the authors had used Finsterwalder's approximation, rather than what he considered the more exact equations of Jenkins ; that was purely a personal view. He thought Finsterwalder's approximation was sufficiently accurate for certain types of shell but Jenkins's equations, with their greater mathematical symmetry, were more universally applicable and quicker in use. He had worked out the characteristics for any shape of shell from these calculations and had found that four to eight hours was sufficient for a complete calculation of the first and second load terms, according to the complexity of the edge conditions.

One simplification was to deal with the disturbance at one edge without taking into account the effects of the second edge. The effects from the second edge could then be treated as a secondary disturbance at the first edge in the normal manner of relaxation techniques. Since the equations and characteristics had already been worked out this resulted in a saving in time. He would not say any more about the design because he was not accustomed to using that particular method.

He agreed with Dr. Hajnal-Kónyi that the shell could perhaps have been 2½ in. thick, but he would prefer, if he might criticise, to suggest a reduction in the size of the edge beam. A rough check showed that a reduction of the edge beam to 8 in. wide would have reduced considerably the prestressing force required, while keeping the stresses unchanged.

This point had to be watched, because shell design was so economical that it was easy to add dead weight merely in order to cover cables. He had used 8 in. wide edge beams in all cases so far and he had had no difficulty in the construction.

The external edge beam was, of course, rather large, and the diagram shown in the calculations indicated the great disturbance due to this heavy loading. He had wondered why these curves had been included because, in practice, he believed that the external columns were erected before the roof and therefore the stress disturbance was very much less.

Wherever possible he had cambered the construction longitudinally, his first idea being to use straight cables. He found in practice that the contractors prefer the curved edge beams and shells which they thought had many advantages over the straight construction.

It was interesting to note that to get the same extension on the wires, it needed about 5 per cent. less pull with a straight cable than with a parabolic cable as used in a straight edge beam.

He agreed that the construction time for a shell roof need not be great, but he thought that when prestressing was used they might have to allow perhaps another fortnight for setting up the first bay, but after that the normal rhythm could be maintained and two different contractors had been able to complete one bay per week.

He had used anchorage stresses of 140,000 lb./sq. in., which was higher than that used by the authors. He would also ask about the expansion joint, about which he was a little worried. He had wondered why they did not use portal type end frames which would allow flexibility.

He was sorry to see that they had not got roof lights in that building. In spite of the figures for light intensity, his experience was that buildings of that size, especially with exhaust fumes, presented a very gloomy appearance.

If he had been critical, it was only to develop the discussion, and in conclusion he would say that the design, erection and supervision of such a notable building was one of which anyone could well be proud.

Lt.-Col. R. F. Galbraith, M.C. (Vice-President) congratulated the authors on an excellent paper. He had found the photographs of the construction particularly interesting. One of the notable features of the building was the low weight of steel per square foot. To achieve 3.3 pounds per square foot on a 150 ft. span was remarkable. Recently he had had occasion to look at weights per square foot for various spans and the results were as follows :—

150 ft. Span	Prestressed Portal Construction	8 lb./sq.ft.
100 ,,	,, ,,	5 ,,
100 ,, Span	Welded Portal Construction	18 ,,
77 ,,	,, ,,	15½ ,,

He would like to know how the cost compared with more normal, or more traditional types of construction.

Major C. D. C. BRAINE (Member) wanted to know what had been done to deal with the bending moments at the gable-ends.

Not very long ago he had been interested in the behaviour of a very large diameter steel pipe which had spanned an opening on a water supply scheme. It had been found that at the point where the pipe emerged from the rigid abutment there would be very heavy stresses. Within the concrete wall the pipe was hardly strained at all, but on emerging from the wall, it expanded to a condition of full stress, and this abrupt change resulted in a large bending moment which was difficult to deal with. With a shell roof it seemed to him that they would have almost the same difficult conditions at the gable ends as with a pipe, but the moments would be in the reverse direction. He saw no mention of this difficulty in the paper and enquired how it had been overcome. Unless precautions had been taken on the site he would have expected to find a lot of cracking in the shell near the gable or, alternately in the gable wall itself. Schorer's paper contained a warning about the magnitude of the bending moments arising in such conditions.

Mr. A. P. MASON (Member of Council) asked was there any particular point, or any particular reason that led to the use of Gunite in the shell, as opposed to the use of concrete, which was more normal. And that decision having been taken, whether anything was done to check up whether the E value for the vibrated concrete was the same or compared reasonably with the Gunite, because they might have two E values, so that, in the calculations E would not cancel out as it should do.

Dr. J. MCNAMEE, of Building Research Station, Watford, pointed out that the labour needed to prepare an adequate tabulation of the parameters required in shell calculations might well be excessive. It might be supposed that tables for shells similar to those already existing for flat slabs could be made but the task for shells was much more difficult. The critical moments in a slab could easily be tabulated for chosen values of the length/breadth ratio and nothing more was required. In the analysis of thin shells there were four relevant variables : the length, breadth, radius, and thickness, i.e., three independent variables. To tabulate the parameters needed by designers over sufficient ranges of these variables would be a lengthy task and the tables might be very bulky. To clarify—and perhaps over-stress—this point, he suggested the following illustration : if a function of one variable required a page to tabulate, a function of three variables tabulated to the same fineness of interval would require a library. Variations in the dimensions of edge-beams would of course increase the difficulty.

He deprecated the all-too-frequent references in engineering literature to exact and inexact theories. So far as he knew, all the theories currently used in design were approximate. All that one could ask of a theory was that it should be based on plausible assumptions and that the predictions of the theory should be verified by measurements on full-scale structures.

A SPEAKER pointed out in relation to the amount of calculation required that Lundgren had published tables giving various values, and recently the National Physical Laboratory had become possessed of an electronic calculator and they would like to be given problems. He suggested the Institution should put the problem up to them and ask them to prepare the curves.

Mr. R. A. SEFTON-JENKINS said that they had heard a lot that evening about concrete shells. He would like to remind them that concrete was not the only material from which shells could be made.

Recently his firm had been very much tied up with the design and construction of a shell roof for the Thameside Restaurant at the Festival of Britain. This roof was made of a patent sandwich of two sheets of aluminium glued to 2 in. of baked cork slab.

It took the form of a wave of about 14 ft. pitch and 27 ft. span.

Using this material, they had found that a rather better kind of shell roof could be made than that used at the Festival of Britain in the form of barrel vault roofing. Here an edge beam can be made at the junction of the shells with a consequent increase in span.

The great advantage of using this material over the use of concrete for shell roofs is the extremely good insulation value (U = 0.12 B.Th.U.s/sq. ft./hr./1°F.), equivalent to five feet of concrete.

Assuming that the building about which they had just heard required heating during the winter months, the saving in fuel alone that would be made by the use of this sandwich, would be about £750 per annum.

Lt.-Colonel KIRKLAND, replying to the discussion, thanked the contributors for the interesting points they had raised.

He continued, that while the design methods they had used were capable of mathematical check, one could not apply to that type of structure the usual office checks similar to those applied to a framed structure in which one dealt with each loaded unit through to the foundations.

They who had to take the responsibility had either to be mathematicians, or to be satisfied with more approximate checks based on analagous conditions. Whilst at first sight this might appear a rough-and-ready method, it was remarkable how close those same methods could approach the actual results obtained by the more involved analysis.

Engineers were still learning the intricacies of the subject, as were the authors, and they hoped that their efforts would contribute to eventual simplification. It would probably be a long time before elementary analyses could be used to obtain the design results required, but each experience in the application of an " engineer " check brought the day nearer.

Replying to Dr. Hajnal-Kónyi on the question of expansion joints, he said this matter had been fully investigated before deciding on their omission. Calculations investigating the stresses involved had been prepared and they had been satisfied with the provisions which had been made. The building had been designed in other respects as an elastic structure, and there seemed little difference as to whether it were designed for a questionable live load condition or an even more doubtful temperature load condition. He commented that frequent references were made to " expansion joints," but of the forces involved due to temperature change in a structure of this nature, those which he considered of most importance were the forces arising from contraction. Short of calling such joints " expansion/contraction joints," would it not be preferable to refer to them as " contraction joints," when their value would at once be questionable ?

Several alternative schemes had been submitted, the selection of those responsible had certainly been influenced at the time by steel economy.

Dr. Hajnal-Kónyi had suggested the calculation of stresses at several additional points on the membrane. This could have been done, but he asked, was it necessary ? Very little had been published on the subject hitherto which could be used for guidance, and had a

previously published authority suggested the value of other additional points, consideration might have been given to this view.

Prestressed concrete was a comparatively new field and while the authors had some previous experience in this field, they found evidence that this building could profit by its adoption.

Both Dr. Hajnal-Kónyi and Col. Galbraith had made reference to cost, and in this connection he could give little information of value. Almost each week brought notification of increases in cost of materials and labour so that whatever one said about the cost of the work, it was liable to be inaccurate by the time the final account was agreed. As a general guide, based on the contract sum the cost was a little over £5 per yard super. He made no claim that this was competitive with the more traditional forms of construction but held that there was considerable difference between the quality of the cover provided as compared with that of the traditional garage. The overhead and maintenance costs should be particularly low with a building of this nature.

He thanked Mr. Cousins very sincerely for his contribution and especially for his adherence to the elastic theory. He knew Mr. Cousins went even further and was a fellow-pupil with him in the " no-crack school " of prestressed concrete.

He noted that Mr. Cousins preferred a curved concrete soffit and straight cables, and admitted the advantage so far as the elimination of some friction was concerned. On the other hand his own preference was for the apparently horizontal soffit, which he knew pleased his client. In point of fact a small camber had been provided for in both the shell and edge beams, and he had examined the contractors' costs in this connection. Had an additional camber been called for he shuddered to think what the cost might have been had it borne any relationship to the figure he had examined. The frictional resistance through curvature of the cables had not presented any difficulty on the site.

Mr. Cousins had also commented on the absence of roof lighting. The original design had provided for top lights but these were later omitted on grounds of economy. The garage was intended for housing vehicles overnight, and they would be serviced during the hours of darkness when artificial lighting would be essential. The provision of additional daylight over what was required for purely local purposes would have been superfluous, and the economy seemed justified.

He thanked Col. Galbraith for his contribution, and said that economy in steel was a prime consideration in the adoption of the scheme. Apparently steel was an even rarer commodity than money. The question of comparable cost he had already dealt with.

Replying to Mr. Mason, he said that very serious consideration had been given to the method of construction of the membrane. The human element in placing and compacting concrete was of paramount importance in regard to the quality of the finished product, and it was in consequence a decisive factor in the general success of concrete work. They had felt that the reduction in the number of human handling agencies effected by the use of a pressure sprayed concrete was a strong point in its favour, and that the impinging of the constituent particles formed a method of consolidation which could not be much improved upon for a thin membrane. Work on site proved the justice of these claims, and concrete of very high density and high compressive strength had resulted.

He thought the suggestion that the National Physical Laboratory be asked to prepare tables with the aid of their electronic calculator a sound one.

Mr. A. GOLDSTEIN said that no mention had been made of the strain reading technique and results during the discussion. He thought that an involved analysis such as this meant little unless some verification could be obtained. The value of experience in any new medium was surely enhanced by the knowledge that the stresses in the structure were of the same order as those calculated. The mere fact of stability of a structure could not, he contended, be altogether satisfactory.

He emphasised that the calculations dealt only with the non-prestressed condition. The prestressing calculations had been omitted not only for the sake of clarity but also because further experience and stress verification of prestressed shells was desirable before publication of the method of introducing the prestressing effects into the shell calculations.

He thanked Dr. Hajnal-Kónyi for his contribution. The chord width of 33 ft. was determined by the overall length of the buses. It was so arranged that a line of buses would stand longitudinally, each bus taking up one bay, and clear access space was thus available over the buses. He agreed that the length to width ratio was unusual, the shell being very long. He believed it was the largest span yet attempted for this type of shell, but one of the benefits of prestressing was that it made these ratios more practical when necessitated by conditions.

Regarding the statement on page 107, mentioned by Dr. Hajnal-Kónyi, whilst the load was taken off for a 3 in. shell it was anticipated that the shell would actually be $2\frac{1}{2}$ in. thick whether prestressed or not, as can be seen from the design on page 116, where b_s was calculated using $2\frac{1}{2}$ in.

The saving in concrete occurred in the reduction of edge beam depth. As an interesting comparison, a similar shell in Budapest in reinforced concrete had a span of 135 ft., a chord width of 39 ft., an edge beam depth of 8 ft., and an overall depth of 14 ft. A more recently constructed reinforced concrete shell in Dublin had a span of 105 ft., chord width 40 ft., edge beam depth 6 ft. 2 in., and overall depth of 13 ft.

All the wires were stressed equally, allowance being made for the stress reduction in the previously stressed wires in the manner suggested by M. Magnel.

He agreed that there was a prestressing effect in the columns. This point had been investigated at an early stage when the possibilities of forming a pin at the column head was considered. It was felt that the additional expense and trouble of constructing a pin was not warranted since haircracks at the beam-column junctions would not have had any ill-effects. No observations were made as to the movement of columns, but the junctions were examined for cracks and none had been found.

He disagreed with Dr. Hajnal-Kónyi regarding the number of points along the curve of the shell necessary to plot the stress curves. If the general shape was known the five points used were, in his opinion, adequate. Certainly the maximum values of design criteria could be obtained sufficiently accurately, and that was surely the main consideration. In the case of the end shell, where this question had greater significance due to the non-symmetry of the curves, intermediate points had in fact been evaluated after the curve had been drawn and the difference between the interpolated and calculated values was found to be negligible.

Modular ratios of 11 and 12 had been assumed for the shell and beam respectively.

He agreed that there were several automatic checks which could be used in the computation. The particular application of Clerk Maxwells's reciprocal theorem which

Dr. Hajnal-Kónyi had mentioned had been omitted in this case, which was unfortunate, as there did appear to be some slight discrepancies in Table VII. However, these were certainly not of sufficient importance to change the stresses by any but negligible amount and would not affect the design. He would, however, investigate this point, and elaborate it in the written reply.

Mr. GOLDSTEIN subsequently writes : On examination it was found that the discrepancy occurred due to the high sensitivity of the equations in Table V, particularly the 4th and 8th equation. If, for example, the solutions are applied to the 4th equation of Table V, a residual error is obtained which is very small in comparison with the coefficients (6th significant place) but not negligible in comparison with $4/\pi$. To avoid this, steps should be taken to de-sensitise the 4th and 8th equations, for example, by working in terms of $R.d_s.V$ rather than $E.R.d_s.V$. This would not give much greater "accuracy" in the final solutions but would avoid discrepancies in Table VII. It must be emphasised that the sensitivity of the simultaneous equations is a purely mathematical phenomenon and is in no way reflected by sensitivity in the actual structure as is the case in some structures such as, e.g., flat arches. Thus in a long shell changes of the edge beam sizes would produce shell stress changes of a relatively small nature, and it has been stated that quite large differences of shell stiffness factors would not affect the shell stresses in the same proportion.* It is clear, therefore, that whilst the small discrepancies in Table VII do exist, due to the above mathematical phenomena, they are quite negligible as far as the actual value of the resulting stresses are concerned.

Replying to Mr. Cousins, whilst he agreed that the calculations were at first rather lengthy as are most numerical solutions of differential equations, the actual difficulties were apt to appear greater than they are since the mathematical reasoning is involved, whereas the difficulties in any particular practical solution are computational.

The differences between the various equations were not within the scope of this paper. It was safe to say, however, that the "longer" the shell is the more simplified can the differential equation be made. Where simplifications cannot be made at all, the equations of Dischinger, Aas Jakobsen or Jenkins can be used with considerable facility. It must be remembered that the actual solution of the characteristic equation forms only one part of the total calculation.

He agreed that if the characteristics for a shell of any shape had been evaluated, the solution of any particular problem could be obtained very quickly indeed. Only the evaluation of one set of simultaneous equations would be necessary.

Dealing with disturbances from one edge only was to be recommended. In the case under discussion there would be comparatively little difference in calculation time but for problems such as north light shells, butterfly shells, etc., the above method was preferable and had in fact been used by the authors.

He would doubt whether a reduction of the width of edge beam to 8 in. would produce any great theoretical economies, and when considered in view of the difficulties encountered by the contractor in casting the 10 in. wide beam with all its reinforcement, cores, etc., he would be loth to recommend such a change.

The curves for an external edge beam without intermediate supports were shown, as had been mentioned, for the sake of academic interest and completeness. The paper was, after all, written not only for the interest of those who had experience, but also to provide information for those engineers who were relatively new to this subject.

The 5 per cent. frictional losses quoted by Mr. Cousins were, he thought, rather high. On this particular job the "clock reading" on the pressure gauge of the hydraulic pump agreed very closely with the calculated value when the predetermined extension had been obtained. This shows that the frictional losses were very small.

It was to be doubted whether any better results could have been obtained regarding the expansion-contraction problem by using portal type tie frames.

He thanked Col. Galbraith for his remarks and his most interesting comparative figures of steel weight per square foot covered. This clearly showed one of the features of this type of construction.

Referring to Mr. Braine's comments regarding the discontinuity at the tie frames, he did not think that the condition was quite as analogous as he (Mr. Braine) had mentioned. It was not clear whether the transverse moments M_2 or the longitudinal moments M_1 (considered as negligible in the analysis) were referred to. The moments M_2 would be damped to virtually zero near the tie frame and whilst small moments M_1 might exist near to the frames, it was found from experience that little trouble occurs from these moments in a shell of this shape. Certainly in this case there were no signs whatever of stresses of too great a magnitude in the tie frames ; indeed, as far as he could ascertain there was not a single crack at these locations.

Mr. Mason's query regarding the cancelling out of the "E" values could be illustrated by reference to Tables VII and VIII. It would be seen that the unit deflections for shell and beam were evaluated separately and there was no question therefore of the "E" values cancelling out. Different values might therefore be used with the same facility as equal ones.

He thought that Dr. McNamee had misunderstood his point about tabulation. It would clearly be an immense task to tabulate solutions for all values of the several variables. This, however, would not be necessary. It would be adequate to tabulate solutions on a basis of parameters, i.e., dimensionless constants combining a number of variables. In shell calculations there are only two parameters ; in this instance a_s and b_s and tabulation would be quite feasible. Edge beams need not be a feature of any such tabulation since if the required constants for the shell were available by direct reference the unit constants for the edge beam could be evaluated from formulæ.

He quite agreed with Dr. McNamee's comments regarding "exact" and "inexact" solutions.

The National Physical Laboratory, as suggested, would certainly be a most suitable place where tabulation could be undertaken. The tables given in "Lundgren" are limited in scope and practical value.

Replying to Mr. Sefton-Jenkins, he very much doubted whether the aluminium-cork sandwich board could be used for shells with spans of any magnitude. The insulation comparisons were of interest but if efficient insulation were required in the case of concrete shells quite adequate arrangements could be made by the use of "Celotex" lining, fibre spray, etc., on the soffit combined with the normal waterproofing on the extrados. He thought the figure of £750 mentioned as a saving in fuel quite unrealistic and far in excess of any such possible economy in an actual practical case.

The CHAIRMAN, closing the meeting, expressed thanks to the lecturers for the way in which they had answered the questions and handled the discussion.

Discussion on "DEVELOPMENT AND USE OF BARREL VAULT SHELL CONCRETE" by C. V. Blumfield, I.C.E. 1947–48, page 39, Structural and Building Engineering Division.

Institution Notices and Proceedings

FORTHCOMING MEETINGS

The following meetings will be held at 11, Upper Belgrave Street, London, S.W.1 :—

Thursday, November 22nd, 1951

Ordinary General Meeting for the election of members at 5.55 p.m., followed by an Ordinary Meeting at 6 p.m., when Mr. G. B. R. Pimm, M.I.C.E. (Past-President), will give a paper on "The Structural Engineer as Arbitrator, Expert Witness and Advocate."

Thursday, December 13th, 1951

Ordinary General Meeting for the election of members at 5.55 p.m., followed by a Joint Meeting with the British Section of the Société des Ingénieurs Civils de France at 6 p.m., when Monsieur L. P. Brice will give a paper entitled "Construction of the Temple Barrage on the River Lot."

Thursday, January 24th, 1952

Ordinary General Meeting for the election of members at 5.55 p.m., followed by an Ordinary Meeting at 6 p.m., when Mr. Arthur Bolton, B.Sc. (Graduate), will give a paper on "A New Approach to the Elastic Analysis of Two Dimensional Rigid Frames."

Members wishing to bring guests to the Ordinary Meetings announced above are requested to apply to the Secretary for tickets of admission.

JANUARY EXAMINATIONS

The Examinations of the Institution will next be held at centres in the United Kingdom and overseas on January 8th and 9th, 1952 (Graduateship), and on January 10th and 11th (Associate-Membership).

EXAMINATION PASS LIST—JULY, 1951

Home Centres

The Examinations were held in July, 1951, at the usual centres in Great Britain.

One hundred and ten candidates took the Graduateship Examination, and 251 took the Associate-Membership Examination, making a total of 361. Of these, 52 passed the Graduateship Examination and 98 the Associate-Membership Examination.

The names of the successful candidates are :—

Graduateship Examination

Ashford, J. A., Bailey, J., Bailey, S., Barritt, N., Beirne, T., Bennett, R. W., Bokota, J., Boston-Mammah, C. O., Boswell, P., Bramwell, H. L., Croft, G. A., Clark, T. A., Chodzko-Zajko, W. J., Emmerson, Miss M., Evans, H. S., Fasehun, E. O. O., Fox, R. T., Freiter, I. N., Hayton, J. A., Healey, W., Hutter, J. L., Jenkinson, A. R., Kiley, G., Kinder, G., Livermore, G., L'Oste-Brown, A. J., Luniewski, Z., Marriage, E. F., Morton, J., Marriott, P. S., Matthews, R., Nesbit, J. K., Norman, K. P., Newton, C. J. B., Newton, M. W., Parish, J. L. J., Puller, M. J., Rowley, N., Rogers, K. J., Steele, A. D., Skupin, H., Stocks, T., Szelepet, J., Tait, A., Teller, G., Turzynski, L., Watson, J. R., Wheeler, G. F., Williams, F. L., Wild, P., Williams, L. W., Weavers, P. J.

Associate-Membership Examination

Akroyd, T. N. W., Anchor, R. D., Armstrong, J. B., Bell, S. J., Bewick, J. R., Blackburn, K. C., Boeuf, F., Bowen, H. D., Bradford, G. E., Braithwaite, J. C., Butterfield, R., Chmielewski, S. W., Clare, F. A., Clark, J. A., Cluett, R. W., Cohen, N., Cole, E. M., Collinson, T. A., Cotton, S. A. C., Dange, S. V., Deakin, N. T., Denson, F. R., Dowe, H. W., Drew, C. J., El-Katib, M. T., Fewster, H. M., Foot, R. E., Ganguly, J. C., Garthwaite, C. H., Gillies, M. J., Goldes, A. A., Grace, J. S., Graham, L., Green, H. W., Hall, K. J., Hamp, E. J., Hancock, T. G., Hill, E., Holland, R. W., Hughes, T. A., Hunt, C. G. A., James, J. E., Jones, R., Jones, W. G., Jupp, A. H., Keeling, A. J., Lawrence, C., Lee, J. A. B., Leibowitz, B., Lester, E. G., Lever, D. L., Lewis, J. R., Lewis, M. R., Liszka, S. K., Longbottom, D. P., Lowe, J. R., Macfarlane, G. K. M., Mackrill, H. W., Mallick, S. K., Marshall, L. J., Mather, R., Molyneux, J. S., Moncrieff, M. L. A., Moss. B. J., Nightingale, R. A., O'Connell, W. N., Osborn, H. D., Owen, A. D., Pearce, D. J., Pendree, H. E., Pepper, S., Reith, I. H., Robinson, L. W., Ross, D. E. J., Rumney, S., Sage, F., Shadbolt, K. F., Sharp, W. H., Shearcroft, R., Shirlin, G. T., Spencer, D., Stokeld, P. G., Taverner, G. C., Thurlow, R. C., Tinkler, J., Topliss, D. G., Uren, J. M. L., Westall, T. R., Wheeler, E. A. B., Whitaker, A. S., Whitham, C., Whittaker, J. B., Wilby, C. B., Wilson, T. D., Woodhead, S., Woodward, C. F., Wright, J. S., Zunz, G. J.

THE MACLACHLAN LECTURE

General Conditions

Through the generosity of Mr. John MacLachlan (Retired Member), the Council was able in 1948 to institute an Annual Lecture to be competed for by Associate-Members. The conditions of the presentation are as follows :—

1. The Institution of Structural Engineers shall institute a written lecture to be known as the MacLachlan Lecture and to be held annually.

2. The subject of the Lecture may be on any aspect of Structural Engineering so long as in every second year the subject shall be confined to steel structures.

3. Entrance into the competition for the Lecture shall be confined to Associate-Members of the Institution, who are under the age of 32 years.

4. All papers entered for the competition shall be submitted to assessors to be appointed by the Council of the Institution, and all such papers (including the prize-winning lecture) shall be available for publication in the Journal of the Institution at the discretion of the Council.

5. No paper submitted shall have been published or read elsewhere.

6. The winner of the competition shall be required to present the Lecture to a meeting of the Institution at which he will be presented with the sum of £17 10s. 0d.

7. Should a competitor's paper be considered worthy of ranking second in merit, he will receive a consolation award of £5.

8. In the event of there being no winner of the competition in any one or more years, whether because no lecture is submitted or because no lecture submitted is considered to be of sufficient merit to warrant an award, or for any other reason, the Institution shall transfer these sums to the Research Fund of the Institution.

PARTICULARS OF THE COMPETITION FOR 1952

1. The MacLachlan Lecture will be given at a meeting of the Institution to be arranged towards the end of 1952.

2. The subject of the Lecture shall be on any aspect of structural engineering.

3. The work should be submitted as the script of a lecture which the author, if successful in the competition, will deliver before an audience in the course of about one hour. The development of mathematical formulæ and detailed calculations should be avoided as far as possible in the text ; if they are essential they should be embodied in appendices. Photographs, drawings, graphs, etc., which would appear as illustrations to the lecture in published form, should accompany the script. If additional illustrations would be shown as slides, a list of these should be included.

Lectures should be prepared in accordance with the requirements of the Literature Committee for publication in THE STRUCTURAL ENGINEER. Candidates may obtain a copy of these requirements on application to the Secretary.

4. Six copies of each Lecture should be submitted and should be addressed to the Secretary of the Institution.

5. The closing date for the receipt of entries by the Institution is Monday, March 31st, 1952.

LONDON GRADUATES' AND STUDENTS' SECTION

The next meeting will be held at 11, Upper Belgrave Street, London, S.W.1, on Tuesday, November 13th, at 6 o'clock, when the Section will hold an Engineers' Forum and informal discussion, presided over by Mr. Leslie Turner, B.Sc., M.I.C.E. (Past-President), Mr. S. M. Reisser, B.Sc., A.M.I.C.E. (Member), Mr. C. F. Pike (Member), and Mr. C. H. Hockley (Associate-Member).

Hon. Secretary : D. B. Rogers, 4, Portland Rise, Finsbury Park, N.4.

BRANCH NOTICES

LANCASHIRE AND CHESHIRE BRANCH

The following meetings will be held :—

Monday, November 26th, 1951

Second Ordinary Meeting, Reynolds Hall, College of Technology, Manchester, 6.30 p.m.

Tuesday, January 22nd, 1952

Annual Dinner and Dance at Longford Hall, Stretford, 6 p.m.

Wednesday, January 30th, 1952

Three short lectures by Mr. W. H. Rosier, Mr. A. S. Sinclair, and Mr. C. Thirsk (Associate-Members), followed by discussion, at the Reynolds Hall, Manchester, 6.30 p.m.

Hon. Secretary : A. S. Sinclair, A.M.I.Struct.E., 28, Kenwood Road, Stretford, Lancs.

MIDLAND COUNTIES BRANCH

The following meetings will be held :—

Monday, November 5th, 1951

Four films will be shown, namely "San Francisco Bay Bridge," "Collapse of the Remagen Bridge," "Loch Sloy," and "Plan into Action," at Derby, 7 p.m.

Friday, November 23rd, 1951

"Designing for Welding," by Mr. F. Brooksbank, B.A. (Graduate), at James Watt Memorial Institute, Birmingham, 6 p.m.

Friday, January 25th, 1952

"Notes on Soil Mechanics," by Mr. J. Kolbuszewski, Ph.D., D.I.C., F.G.S., at James Watt Memorial Institute, 6 p.m.

Hon. Secretary. E. R. Deeley, A.M.I.Struct.E., Arranmoor, Adshead Road, Dudley, Worcs.

GRADUATES' AND STUDENTS' SECTION

Thursday, November 29th, 1951

"Some Recent Midland Structures," at James Watt Memorial Institute, 7 p.m.

Saturday, December 8th, 1951

Social Function.

Monday, December 31st, 1951

The following films will be shown : "Building of the South Bank Exhibition," "Prelude to Power," "H.M.S.O., Sighthill, Edinburgh," at James Watt Memorial Institute, 7 p.m.

Hon. Secretary : M. H. Evans, B.Sc., 42, Church Hill Road, Handsworth, Birmingham, 20.

NORTHERN COUNTIES BRANCH

The following meetings will be held :—

Tuesday, November 6th, 1951

"Contract and Similarities between Ship and Land Structures," by Mr. G. M. Boyd (Associate-Member), at Cleveland Scientific and Technical Institution, Middlesbrough, 6.30 p.m.

Wednesday, November 7th, 1951

The above meeting will be repeated at the Neville Hall, Newcastle, at 6.30 p.m.

Tuesday, December 4th, 1951

"Factory Extension at Silvertown," by Mr. Frank G. Etches, B.Sc.(Eng.), A.M.I.C.E. (Member), at Middlesbrough, 6.30 p.m.

Wednesday, December 5th, 1951

The above meeting will be repeated at Newcastle, at 6.30 p.m.

Hon. Secretary : Ian MacGregor, M.I.Struct.E., 9, Ellison Place, Newcastle-upon-Tyne, 1.

NORTHERN IRELAND BRANCH

The following meetings will be held :—

Tuesday, November 6th, 1951

"The Use of Equal Strength Principles in the Design of Structures for Minimum Weight," by Dr. R. M. Kenedi, A.R.T.C.

Tuesday, December 4th, 1951

"Prestressed Concrete Beams and Slabs," by Mr. A. H. K. Roberts, B.A., M.I.C.E.I. (Member).

The above meetings will be held at the College of Technology, Belfast, at 7.30 p.m.

Friday, January 11th, 1952

Annual Dinner and Social Function.

Hon. Secretary : S. G. Duckworth, M.I.Struct.E., "Lisleen," 13, Finaghy Road North, Belfast.

SCOTTISH BRANCH

The following meetings will be held :—

Wednesday, November 7th, 1951

Chairman's Address, by Mr. R. Summers, (Member) at the Ca'doro Restaurant, Glasgow, 6 p.m.

Thursday, November 8th, 1951

Annual Dinner at Central Hotel, Glasgow, 6 p.m. for 6.30 p.m. Buffet Dance at Central Hotel, Glasgow, 8 p.m. for 8.30 p.m.

Monday, December 10th, 1951

Joint Meeting with the Institution of Civil Engineers, at Royal Technical College, Glasgow. "Some Impressions of Technical Education in the United States," by Dr. D. S. Anderson.

Hon. Secretary : D. G. Drummond, B.Sc., A.M.I.C.E., M.I.Struct.E., 11, Woodside Terrace, Glasgow, C.3.

SOUTH-WESTERN COUNTIES BRANCH

The following meetings will be held :—

Friday, November 2nd, 1951 (at Plymouth)

Chairman's Address, by Mr. L. F. Vanstone, L.R.I.B.A. (Member). The President and Secretary of the Institution will be present.

Friday, January 11th, 1952 (at Newton Abbot)

"The Reconstruction of Abertillery Bridge," by Mr. Wallace A. Evans (Member).

Hon. Secretary : E. W. Howells, M.I.Struct.E., c/o Messrs. T. L. Harding & Sons, Ltd., 10/12, Market Street, Torquay.

WALES AND MONMOUTHSHIRE BRANCH

The following meetings will be held :—

Saturday, November 10th, 1951

The Chairman's Address, which was given by Mr. G. H. Hodgson, M.I.C.E. (Member), at Cardiff, on October 30th, and at Swansea on October 31st, will be repeated at the County Buildings, Colwyn Bay, at 6 p.m.

Wednesday, November 28th, 1951

"Structural Engineering Questions Answered," at the Mackworth Hotel, Swansea, at 6.30 p.m.

Tuesday, December 4th, 1951

The above meeting will be repeated at the South Wales Institute of Engineers, Cardiff, at 6.30 p.m.

Friday, December 14th, 1951

"Piling, with particular regard to Sea Defence Works," by Mr. J. Owen Lake (Associate-Member), at the County Buildings, Colwyn Bay, 6 p.m.

Hon. Secretary : E. R. Steward, A.M.I.Struct.E., Edrom, Ashleign Road, Blackpill, Swansea.

WESTERN COUNTIES BRANCH

The following meetings will be held :—

Thursday, November 1st, 1951

"Some Interesting Bridge Reconstruction Schemes," by Mr. E. Bateson, M.I.C.E. (Member). Combined Meeting with The Institution of Civil Engineers, 5.30 p.m.

Friday, December 7th, 1951

"Some Recent Developments in the Bulk Handling of Raw Materials for Steelworks," by Lt.-Col. E. Ward, T.D., A.M.I.Mech.E. (Associate-Member), at Bristol University, 6 p.m.

Friday, January 4th, 1952

"The Fabrication of Steel Structures," by Mr. O. H. Willey, M.Inst.W., at Bristol University, 6 p.m.

Hon. Secretary : C. E. Saunders, M.I.Struct.E., Dunkery, Edward Road, Walton St. Mary, Clevedon, Somerset.

YORKSHIRE BRANCH

The following meetings will be held :—

Wednesday, November 21st, 1951

"The Baitings Reservoir," by Mr. T. E. S. White, B.Sc., M.I.C.E.

Wednesday, December 12th, 1951

"The New House of Commons—Foundations and Structure," by Mr. S. Vaughan, B.Sc., M.I.C.E. (Vice-President). Combined meeting with the Yorkshire Association of Architects.

The above meetings will be held at the Great Northern Hotel, Leeds, at 6.30 p.m.

Hon. Secretary : E. Wrigley, A.M.I.Struct.E., City Engineer's Department, Civic Hall, Leeds, 1.

UNION OF SOUTH AFRICA BRANCH

A most enjoyable meeting of the Branch was held at Barclay's Bank Buildings, Johannesburg, on Tuesday, August 14th, when Mr. Stokes (Member) showed two films and gave a talk on prestressed concrete. The films covered the construction of the bridge over the Marne and the Galleries at Rouen. The meeting was attended by 50 members and friends.

Branch Hon. Secretary : A. E. Tait, B.Sc., A.M.I.C.E., A.M.I.Struct.E., P.O. Box 3306, Johannesburg. During week-days, Mr. Tait can be contacted in the City Engineer's Department, City Hall, Johannesburg. 'Phone 34-1111, ext. 257.

Natal Section Hon. Secretary : E. G. Bennett, A.M.I.Struct.E., c/o Reinforcing Steel Co., Ltd., P.O. Box 49, Merebank, Durban.

Cape Section Hon. Secretary : R. Stubbs, M.I.Struct.E., P.O. Box 1692, Cape Town.

CORRIGENDA

The entry under the name of Mr. H. C. Husband on page 2 of the Sessional Programme 1951-52 should be amended to read as follows :—

H. C. Husband, B.Eng., M.I.C.E., M.I.Mech.E.

The entry under the name of Mr. A. P. Mason on the same page should read :—

A. P. Mason, B.Sc., M.I.C.E.

Book Review

Teach Yourself Mechanical Engineering. Volume 2 : Engineering Components and Materials, by A. E. Peatfield. London : English Universities Press, 1951. 240 pages. 7 in. by 4½ in., illustrated. 6s.

This is an excellent book for students of Mechanical Engineering ; students of other branches of engineering would be well advised to obtain a copy for reference purposes. Concise descriptions, with illustrative drawings, are given of a multitude of components which are used in the assembly of machines and engines.

To quote from the author's preface, "In this volume no attempt has been made to deal with detailed engineering design or calculations ; the sole object being to introduce to the student the various components and certain engineering practice."

The first chapter deals with bolts, nuts, washers, etc., and contains descriptions of such items as square-headed bolts, eyebolts, rag bolts, fish-tail bolts, wing nuts, thumb nuts, castle nuts, spring washers, tapered washers, set screws, grub screws, etc., etc. The other nine chapters on components are fully as comprehensive as the first chapter and a few items chosen at random are :—Cotters, rivets, clutches, bearings, gearing, springs, joints, glands, valves. Chapters XI and XII deal with engineering materials and testing of materials, and although these two subjects cover only eighteen pages, the allotted space is probably sufficient for a book of this nature. Concise descriptions are given of the ferrous and non-ferrous metals and various non-ferrous alloys ; tensile and hardness tests are described briefly.

The author is to be congratulated on the high standard of his drawings and for compressing into a small volume such a wealth of information on engineering components.

W. M.

Presidential Address*

By Walter C. Andrews, O.B.E., M.I.C.E., M.I.Struct.E.

Gentlemen : My first, most pleasant, and probably simplest duty this evening is to thank you and all other members of the Institution for having elected me President for the coming year.

My pride in the honour you have conferred on me, is tempered by a consciousness of the high standards set by my predecessors. It will be my earnest endeavour to live up to these standards and, God willing, to set them still higher and still further enhance the prestige of this Institution. I am fortified in the belief that these hopes may be realised, from a long acquaintance of the help so readily given by Members of the Council and the Institution's officers.

My duties as President officially begin this evening, but they really commenced soon after I attained the honour of being your President-Elect, because then the probability of having to prepare a Presidential Address became an obligation. In a belief that a study of the past is an aid to the future, and is a help in getting things into proper focus, I offer you a brief historical review, and trust that it will be as interesting for you to hear as it has been for me to prepare.

The subject of my address then, is the development of Structural Engineering as applied to buildings during the past hundred years.

My original thought was to take a much shorter period, say my own life, or that of this Institution : but this being Festival Year, with its reminder of the Exhibition of 1851, I was led to study the design of the Crystal Palace, so-called. I found this so interesting that I felt impelled to take that date as a starting-point. Of course the objective design of structures did not commence then, but in the time at my disposal I had to set some limit, however arbitrary, and the one I have chosen has at least the merit of having some connection with this current year. I hope to show, further, that there are several other connections, some of them, to me at least, surprising. I should also say that I have limited my survey to developments in this country, not with any idea of suggesting that development and invention in Structural Engineering are confined to the British Isles, for that is far from being the case, but again to set a limit to the range of my survey.

Sequence of Principal Materials

In 1851 the principal materials in use for building were stone, brick and timber, and construction was relatively heavy in form. Cast iron and wrought iron were mainly used for purely engineering structures, although C.I. and W.I. beams were being introduced into mill buildings. Probably the greatest fillip given to the application of iron to building was the enormous expansion of the railways, then taking place. Steel was available but only in small quantities at high prices, so that its use was then restricted to the making of tools. Experiments were being made with lime and clay to produce stronger cements, but many years were to elapse before the mixture now known as Portland cement was discovered. Reinforced concrete and light alloys had not been thought of. The sequence, therefore, of materials used in construction, and, to some extent, the sequence of decline also, has been :—

Brick and stone
Timber
Cast iron and wrought iron
Steel
Reinforced concrete
Light alloys.

Brick and Stone

For the greater part of the period under review the methods of brickmaking and of hewing stone remained static and their use followed traditional forms. Stone has now almost ceased to be used as a structural, as distinct from an ornamental, material. The low outputs of clay bricks and rates of laying them of recent years have led to a more scientific use of brickwork and the mechanisation of the methods of winning clay, of its preparation, drying and kilning. This mechanisation has all taken place since 1920.

Bricks are still laid by methods and with tools which have hardly changed since their first application in this country. I was recently shown some old prints of work in progress in the early seventeen hundreds, and as far as I could see the only difference from what one sees to-day was the fashion in the workmen's wearing apparel. Although some investigations with a view to improving and speeding up methods of construction have been made, much more could be done in this direction. Brickwork, like stone, however, except for the smaller domestic dwellings, seems to have had its day as a structural element.

Timber

Timber as a building material is of such antiquity and, up to the recent past, supplies were so abundant, that the empirical technique of construction that was developed in this material over long ages before the basis of scientific and mathematical design was laid down, has persisted up to very recently ; timber construction has thus tended to remain a craft. Even in this country, where for years timber of building quality has been lacking in any reasonable quantities, the need for economy in its use was marked by first, the ease with which we could exchange our manufactured goods for abundant supplies from abroad, and later by the economical application of a number of alternative materials.

It is only in the last decade, when we as a nation are no longer rich and when these alternative materials are themselves in short supply, that the need for real economy by the application to timber of design methods developed in other directions has been driven home. We thus find that a material which once acted as a model for the application of new materials, is now being remodelled on the lines of its original copiers.

This more scientific approach in the use of timber has not been easy since many of the basic assumptions used in design are even less valid for timber than they are for other materials of more uniform quality. This difficulty has been reduced in importance by improvements in methods of jointing and improvements in the grading of structural timber.

Perhaps the most striking modern development in timber has been its transformation into plywood, and in laminated construction. Originating from decorative

*Given at a General Meeting of the Institution of Structural Engineers, held at 11, Upper Belgrave Street, London, S.W.1, on Thursday, October 11th, 1951.

veneers, the term plywood has been in use for slightly over thirty years. Resin bonding, which permits plywood and laminated construction to be used for external purposes, is of very recent application.

It is probable that plywood, laminated boards and laminated frames offer the greatest opportunities for the scientific use of timber.

While many interesting developments in these directions are taking place abroad, the necessity of importing the raw material, in our present economic condition, must restrict opportunities in this country of designing and constructing complete structures in this most pleasant of materials. The use of plywood as formwork for reinforced concrete seems to me to be worthy of much more consideration than has so far been given to it.

Iron and Steel

I need hardly dwell on the phenomenal development in iron and steel. It is too well known to you all. It may, however, interest you to be reminded of a few salient facts. The manufacture of pig iron is, basically, carried out as it was in 1851. Plant has, of course, increased vastly in size and output increased. It is the use to which it is put that is different.

As I have already mentioned, although steel was not unknown in 1851, its manufacture was arduous and expensive. Large-scale production by new processes was started by Bessemer in 1856, and developed by Thomas in 1878. In quite a short time steel, originally costing £60 a ton, was ultimately produced by the new methods for £12 a ton or less.

The changes that have taken place in production and its relationship to growth of population, Great Britain figures only, of course, are shown in the following graphs.

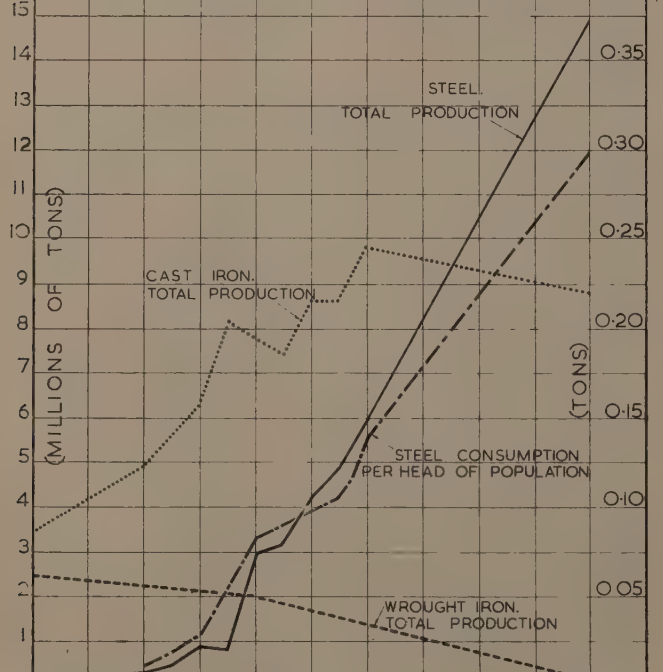


Fig. 1

The steel output figures are totals for all purposes, only about 12 per cent. being used structurally, but the trend in the structural application of steel is the same, if not greater.

In the early years of the nineteenth century much ingenuity was displayed in the design and erection of structures combining cast and wrought iron with and without timber. These combinations were then almost entirely restricted to purely engineering structures, such as railway bridges and terminal station roofs, either as arches or triangulated frames. The application of theories of flexure to beams brought about the development of new types of beam sections suitable first to cast, and then to wrought iron. This permitted the first step in a steady increase in possible floor spans. For example, in the case of mill buildings column spacing could be increased from bays of 9 ft. × 15 ft. to 30 ft. spans in two-bay buildings, that is one line of columns instead of two. Gradually W.I. entirely superseded C.I. except for columns, and subsequently steel superseded W.I.

This latter substitution has not been so obvious because of the great similarity in the sections used for the two materials at the period of the changeover. Except for roofs, however, iron and steel did not play a very prominent part in the construction of buildings. It is less than fifty years ago that the first fully-framed multi-storey building was erected in this country. The next stage in development was the application of welding for the formation of joints. This expedited the acceptance of the rigid frame, although its use is as yet severely limited. In spite of the great strides which have been made in methods of design of welded structures and in the technique of welding, riveted and bolted construction is still the most popular form.

Cement and Concrete

While the original patent for Portland cement, more or less in the form with which we are now familiar, was taken out before the beginning of the period under review, its production in really reliable quality is much more recent ; I think it is fair to say that the uniformity of quality has been brought about by the necessity for reliability in the making of reinforced concrete.

The development of concrete reinforced with steel has followed a steady growth since 1870, but it was much later that designs were anything but extremely empirical. The most rapid development has taken place since the turn of the century. It is interesting to note how close in point of time has been the birth of modern steel, modern cement and modern reinforced concrete production. This parallel development has not taken place in any one part of the world. Until fairly recent times steel has been the material to which most attention has been paid in this country. Reinforced concrete has been largely a Continental affair, and its application in the British Isles lagged far behind that of plain steelwork. Figures of cement output, while phenomenal, since as compared with steel it is almost entirely consumed in building, do not give a true picture of the growth of reinforced concrete : much cement is used in plain concrete, in mortars instead of lime and so on. The general trend, however, is shown in the following curves.

The short history of reinforced concrete has been one of steady progress both in knowledge of the properties of the constituent materials and in the form and design of structures. It is true that we are not now so confident as were some of the pioneers of this material at the turn of the century regarding the waterproofing and fire resisting qualities of concrete. Experience has shown that much more care and skill is required in making good concrete and producing sound structure than was then thought necessary. On the other hand, our ideas of continuous and rigid or portal frames, applicable in principle to any material, are due to developments in reinforced concrete structures and observations on their behaviour. These ideas and observations have in turn encouraged a number of new methods of calculation

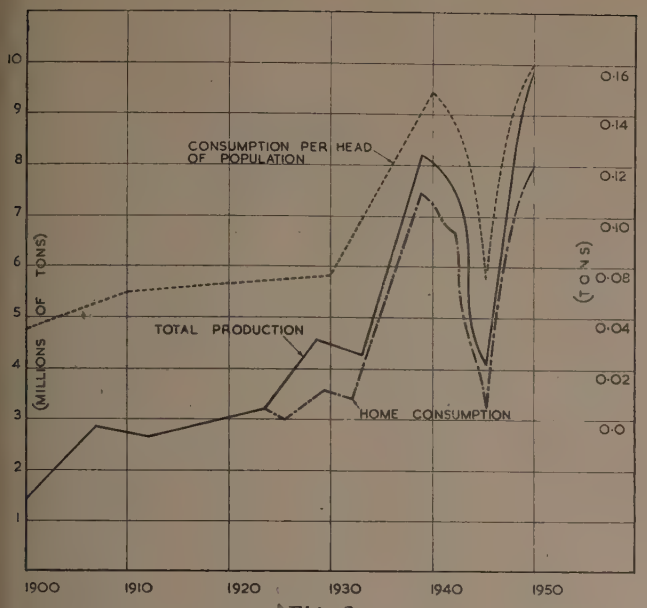


Fig. 2

There has also been a greater realisation of the three-dimensional nature of many structures with consequent economies in design. More recently there has emerged "shell" construction and prestressed concrete, both of which are so constantly kept before us that I need hardly comment further on them here.

Light Alloys

The basis of most commercial light alloy is aluminium. It was virtually a precious metal until 1886, when an electrolytic process of separation was discovered. Even then cost of production restricted its use to specialised work outside the structural field. Really serious consideration for building work did not begin until approximately 20 years ago. Although the last ten years have seen a threefold growth in its production, and several interesting and important structures have been built with frames of aluminium alloy, its use is still very small compared with other materials to which I have referred.

Bases of Design

Until well into the nineteenth century, design of structures was based on "rule of thumb" or tradition, thicknesses of walls, depth and curvature of arches, dimensions of piers and beams (generally of timber) were tabulated in building regulations or text-books such as Tredgold's. Practice rather than theory held sway. As frames, both in timber and iron, were developed, necessitating the evaluation of direct forces as distinct from moments, graphic statics was the means employed. I am here concerned, of course, with knowledge as it was applied, not, necessarily, as it existed in the realms of pure mathematics. The elastic theories of beam sections developed at the end of the eighteenth century were not adopted for practical application until about 1860.

As structures became more and more complicated, much ingenuity was used in applying graphical methods of analysis. So much so, that I think it fair to say, by the beginning of this century, it had become a fine art, even if much of it was involved in method and tedious in application.

Overlapping this period was a gradually increasing application of mathematics to the solving of structural problems. Engineers began to study the derivation of formulæ as distinct from merely using them largely without question.

The development of mathematical analysis was for many years limited to the design of individual parts of a structure. That is, beams were considered separately from columns and from each other; slabs and roof coverings, as distinct from the beams or purlins supporting them. It is true that connections between beam and beam, and beam and column had to be designed, but they were considered as connections or brackets rather than as joints. This outlook continued into the early days of reinforced concrete, but rather more obvious errors in this individual element method of approach eventually brought about the idea of the rigid frame. It was but a small step further to the consideration of frames in three dimensions. It is this latter aspect which is most in our minds to-day.

The practical application of elastic theories and other mathematical concepts were evolved from one or more assumptions. These assumptions have been the subject of continuous query and investigation, and, while many of them are suspect or known to be untrue, they still serve their purpose in everyday work. Most of them erred on the side of safety. While one would wish to use methods of design which have been proved more accurate and economical in the use of materials, their application is as yet lengthy and cumbersome, and therefore costly in the design stage. Much thought is at present being given to this problem, and I venture to prophesy that it will not be long before a revolution in design methods takes place.

Practitioners

At the beginning of the period under review, building was an art, and as such was largely in the hands of architects. As the Industrial Era developed mechanics, such as wheelwrights, applied their knowledge to the construction of mill and similar industrial structures, as well as railway buildings and bridges. This led to the emergence of the civil engineer, who concerned himself, as one branch of his work, with industrial structures, leaving the design of public and domestic buildings still in the hands of the architect. As, in turn, the work of the civil engineer grew in magnitude and complexity, individual engineers found it necessary to limit the kinds of work they would undertake, and thus the present practice of specialisation began to take shape. Parallel with this development among civil engineers, the application of science to and the use of new materials in buildings, made it more and more difficult for the architect to design a structure without employing an engineer in some capacity, be it professionally or commercially. Thus we get the Structural Engineer as such.

The Institution

For the benefit of the considerable number of new members who join year by year, I make no apology for reviewing briefly the history and growth of the Institution. The increasing application of reinforced concrete in the early years of this century led a number of interested people to form in 1908 the Concrete Institute, with the object of advancing the knowledge of concrete and reinforced concrete and direct attention to its best application; affording means of communication between persons engaged in design and execution of reinforced concrete work, and arranging periodical meetings to discuss practical and scientific subjects bearing upon reinforced concrete. It is interesting to note that while the scope has widened, the objects are the same to-day as then. The membership in those early days consisted of persons drawn from a number of different professions. They comprised engineers, archi-

Fig. 3

tects, surveyors, builders and contractors, the only qualification for membership being an interest in the subject of reinforced concrete, however casual.

It was soon found that the original scope was too narrow and that there was a need for a common meeting-ground for persons interested in all types of structures, whatever their material of construction. This widening of interest was paralleled by a narrowing of qualification for membership, until eventually the main body consisted entirely of engineers. These developments were noted in 1922 by a change in name, the present one of the Institution of Structural Engineers, which in turn led to a considerable spread of influence and the formation of a number of branches. This spread of influence was recognised in 1934 by the grant of a Royal Charter. From the commencement the Institution was recognised by local and other regulation-making authorities, and the opinions of its members sought when regulations were being drafted. The first Science Committee was formed in 1908. Notwithstanding the continuing up-grading of conditions of entry, membership has grown steadily in numbers over the years, and, in fact, the rate of growth increases year by year. Our membership now stands at over 5,900, a testimony to the value placed upon our certificate of membership.

For many years the Institution's publications have been accepted as standards for design and its Science Committee has been in the forefront in rationalising the application of new forms of construction and the use of new materials. Since the war, and in the national interest, it has sunk its individuality to some extent by joining with other Institutions and Societies in producing Construction Codes which may be accepted on a national basis.

Types of Structures

Having briefly considered the development of materials and the types and progress in organisation of the persons using them, I now turn to the application of these materials.

It is somewhat difficult to establish any well-defined changes. Broadly, prior to the application of steel to structures, building structural elements consisted of the post, the beam and the arch. The advent of iron permitted much greater beam spans, but the most noticeable development was the frame, first in W.I. in conjunction with cast iron, then steel and so on. In each material the form of any particular frame followed that in use for the immediate predecessor, until a type more suitable to the properties of the new materials was discovered. Thus, early steel frames have the heavy look and detailed workmanship previously put into W.I. frames. At the present time aluminium alloy frames follow the lines of their steel counterparts, and there has yet to be developed a strictly light alloy type of construction.

The first completely steel-framed building in this country was erected as recently as 1904, but even for this, the walls had to be of regulation thickness as if the steel frame was not there, and in spite of the steel being designed to carry the wall at each floor level. In London, frames in the form we now know them did not become legal until 1909.

Apart from the development of the frame it cannot be said that there have been any great changes in, or additions to, the various forms of structure since 1851. What has happened from time to time has been the application of new materials or new techniques to old forms.

The following illustrations may amplify this point. My first, Fig. 3, is that of the Crystal Palace, opened to the public in 1851. It was appropriately 1,851 ft. long and 408 ft. wide, with a total floor area of 755,208 sq. ft. The building was erected in 26 weeks from date of possession of the site, at one period 2,000 men being employed. The ridge and furrow arrangement of the glazing seems to have given a rigidity to the whole structure which the form of construction otherwise lacked.

Fig. 4

Fig. 4 shows a view during construction. A travelling frame was used to lift the main arch members in one piece. The whole building was a model of prefabricated construction.

Fig. 5 is that of a timber arched roof of 130 ft. span, being of laminated construction.

Fig. 6 illustrates a lattice type of iron station roof.

Fig. 7 you will probably recognise as York Station, with arch ribs forming a vault curved in plan.

Fig. 8 shows the arched roof over St. Pancras Station.

Fig. 9 is another arched roof, that of Olympia, called originally the National Agricultural Hall.

And, finally, in my first group, Fig. 10 shows another lattice type truss construction. Most of these were built round about 1851 ; all were in existence before 1880.

Fig. 11 shows a fairly recent example of traditional steel truss construction, its spider web appearance being in marked contrast to an equally recent example of a steel portal frame illustrated in Fig. 12.

Turning now to reinforced concrete, Fig. 13 is an example of a truss hardly distinguishable from one in timber, while Fig. 14 shows the Belfast type of truss used for Kelvin Hall.

Fig. 15 is an example of precast reinforced concrete portal frames of varying spans, and finally in this section, Fig. 16 illustrates the spaciousness associated with a " shell " roof.

Fig. 17 is a very recent application of modern design in timber, whilst Fig. 18 shows the possibilities of laminated board construction as used in the ceiling of the Royal Festival Hall.

I have endeavoured to illustrate different approaches in different materials to one particular problem—that of roofs, these being the easiest examples to select in their naked form.

Research

The idea of research in building is neither new non modern. It is probably as old as building itself, ever if sometimes the research was being unconsciously pursued. Objective research with the intention of applying the results can reasonably be dated some 200 years ago. What is to be particularly noted with respect to research in the period under review is the development of the idea of co-operative research on an ever-increasing scale, involving tests of structures as distinct from individual members, and the relation of materials to influencing factors, in addition to assessing merely the properties of those materials.

The building industry is made up of a number of elements, many of them having few points of contact with the others. It is basically an assembling industry. The steel producer has little in common with the concrete maker. The timber merchant nothing in common with the producer of aggregates. The engineer is faced with many craft techniques both individual and geographical. It is only by having a knowledge of all these differences in outlook and interest that he can so fuse the conglomeration as to achieve the result he requires. Co-operative research gives much help to this end.

There appears to have been very little systematic research before 1914, although many people and organisations had begun to realise the need. The Department of Industrial and Scientific Research was formed in 1915, the section which mostly concerns us, the Building Research Station, in 1917, and the Forest Products Laboratory in 1921.

From small beginnings and working on day-to-day problems, the B.R.S. has evolved into a very important

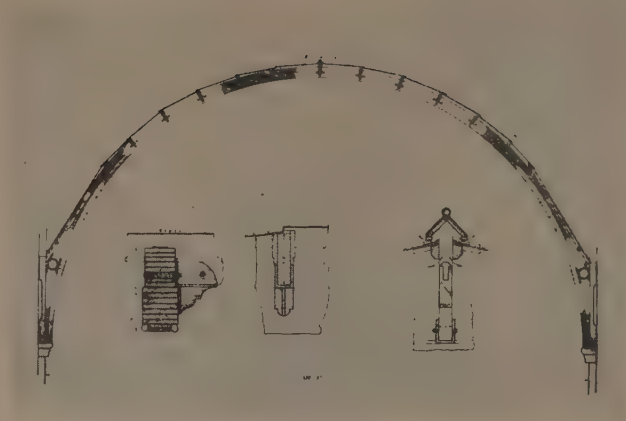


Fig 5

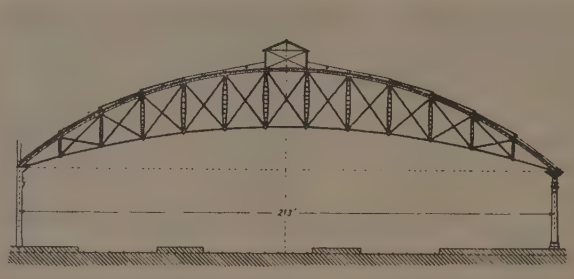


Fig. 6

Fig. 7

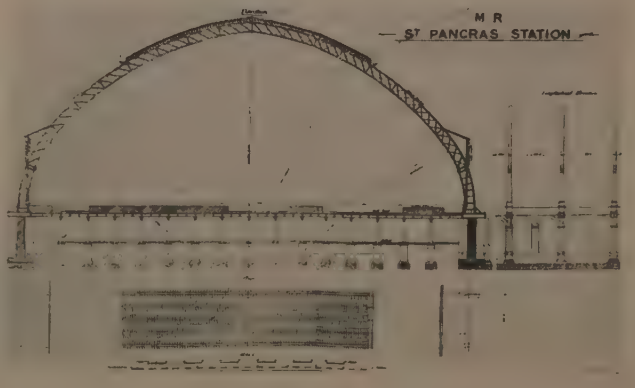


Fig. 8

organisation, with programmes both of immediate and fundamental research. Colleges, Technical Institutions and Industrial Associations also have important programmes of research.

This Institution has played a very useful part in indicating lines of research, assisting in co-ordinating the work of the various research bodies and, in short,

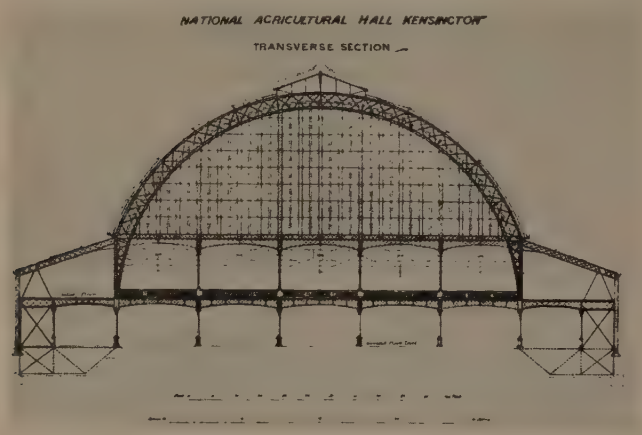


Fig. 9

in keeping the building research worker in touch with engineers. A committee was formed for this purpose in 1946. After having done much useful work it has this year been amalgamated with the Science Committee under the new title of Science and Research Committee.

The recent Building Research Congress has been a further step in the encouragement of the application of Science to Building. I am happy to have been the representative of this Institution assisting to bring about this first congress of its kind, and I am hopeful that much will result from it.

Regulations

There has been a kind of ebb and flow in the effects of Building Regulations on construction. Prior to, and for some years after, 1851, the detailed form of Building Acts suited the empirical nature of design of those times. Such Acts were not of national application and were limited to requirements for masonry and timber and to domestic and public buildings. The advent of iron and the enormous call for industrial structures attracted little attention from Public Authorities, and it was free to develop without control.

Since about 1880, however, and until recent years, where construction has been subject to control, its effects have been restrictive and have been a brake on progress. As an example, the London Building Act, 1879, required that walls of Portland Cement concrete should be one-third thicker than corresponding walls of brick. That may have been correct for 1879, but I found that to be a requirement in one urban district for 1.2.4 concrete as recently as 1928.

The detailed requirements of the L.C.C. 1909 Act, for Steelwork and 1915 for Reinforced Concrete, and similar bye-laws in other large towns followed the tradition of earlier Acts in respect of masonry and timber in laying down most detailed requirements. They naturally could not be in advance of the knowledge existing at the time of drafting, so that they were out of date almost as soon as they were published. The illogical nature of regulations in such a form happily now seems to be realised in the more general character of the requirements of

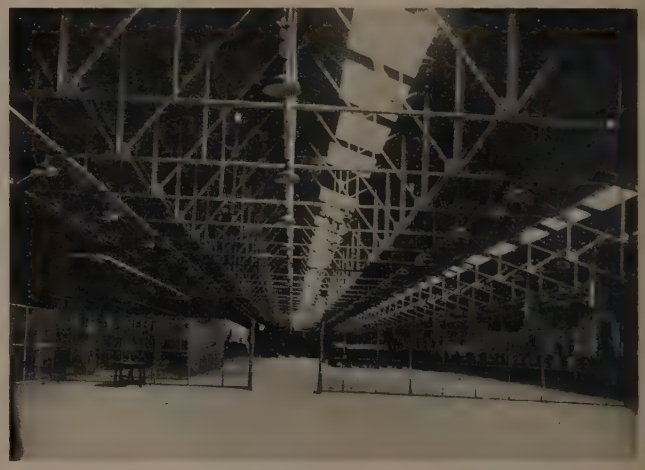


Fig. 11

recent building legislation. Members of this Institution, past and present, acting both corporately and individually, have played a large part in bringing about this change of outlook. There is, however, a danger that the recently formulated Codes of Practice may lead to a stifling of progress, and we should be on our guard against this, always remembering that the Codes are recommendations for the guidance of engineers, and are most definitely not intended as Specifications to be followed rigidly and blindly.

Standards

In any appraisal of our branch of engineering the value of standards must not be overlooked. When buildings

Fig. 10

Fig. 12

were built of masonry and timber, the materials were generally local in origin. The enormous expansion in trade, improvements in transport and changes in forms of construction which took place from 1851 onwards, made evident the great diversity in size and quality of the various materials and components which go to make up a structure. Gradually it was realised that in the interests of economical production some relaxation of the rugged individualism of the period was necessary. As with other aspects of my subject, the use of standards is very ancient, but there is a wide gap between the idea of standards of length and weight and those standards of function, quality and dimension with which we are now so familiar.

Without a reasonable certainty of size and quality, it would be an almost impossible task for the engineer of to-day to make the right choice from the great diversity of materials and range of forms which are at his disposal.

Fig. 13

Towards the end of the nineteenth century, W.I. and steel sections could be obtained in practically any size desired, and there were a number of screw threads in use. This apparent freedom greatly retarded full mechanical development of production and kept up costs.

The British Standards Institution is just 50 years old. One of the first results of its formation was the standardisation of structural steel sections, which brought about a reduction by one-third of the number in use at that time, including a revision in the shapes of the sections generally to the form we now know.

I need not enlarge on the growth of British Standards, their widespread application or the important part played by members of this Institution in their production.

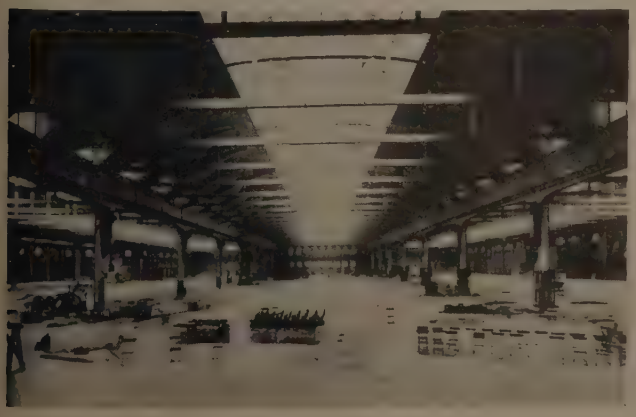


Fig. 14

Working Conditions

A review such as this would not be complete without some reference, however brief, to the great changes for the better that have come about in working conditions.

Fig. 15

The production of wrought iron in 1851 was heavy work. The hours were long and pay poor. Puddlers usually died before reaching 50 years of age, frequently of pneumonia induced by extremes of heat and cold. Eye trouble and cataract were common. A chief puddler was paid 6½d. per hour and would work 60 hours per week. In brick and cement works the atmosphere was foul and, in the latter, dust laden, causing various lung diseases. On sites work was arduous, dangerous and uncertain, and wages very much at the mercy of our variable climate. Tests on structures were often made with human beings. The great advances in the use of mechanical aids have much lightened most operations, particularly in factories, and an awakened social conscience has brought about a great reduction in the incidence of industrial diseases. Rules for scaffolding mitigate the hazards on site. Against these improvements productivity on site has steadily declined. This is a social phenomenon which is outside the scope of an address such as this. Much could be done, however, to balance this state of affairs by the greater use of mechanical power. While the direct application of machines to site work is the responsibility of few of the members

Fig. 16

of this Institution, more attention could be given by the designer of structures to methods of construction or erection on site than sometimes appears to be the case.

Prophecy

It is natural to be cautious in adopting new ideas or materials, but such caution should be based on reason and not simply bias. Prophets are so often confounded and objectors to change merely nuisances.

Fig. 17

I am led to make this somewhat sententious comment by a number of statements by such objectors which I have come across in my search for material for this address. The following are examples:

Just prior to the opening of the Exhibition of 1851, it was said of the Crystal Palace "A gigantic building of glass and iron could not be made stable. There would be a stupendous disaster. The first gale that blew would reduce it to a shapeless wreck." Or again, "Even if the glass case managed to resist the gales, the heat engendered by the sun when it poured its rays upon the glass would be so terrific that no human being could hope to withstand it, consequently if they escaped the avalanche of glass they would be roasted to death inside it."

Even a group of eminent engineers proved to their own satisfaction that the structure was unsound, yet it stood for nearly 90 years.

In an introduction to a text-book on Iron published in 1872, the author stated that "It is questionable whether the substitution of iron for wood, bricks and stone is not being carried too far, and whether the new perishable substance is not frequently adopted for the sake of cheapness or facility of construction in cases where the more durable but less tractable material, stone, would be more appropriate, more noble and more worthy of the profession."

Such attitudes of mind are still with us. I need only remind you as an example applicable to the present Exhibition, of the fear of standing immediately under the Skylon!

I have now completed my brief review. Much of what I have said will be well-known to some of you, but not I hope to all, particularly the younger members. If this historical study brings about some spirit of humility and acts as an inspiration to even a few, then I shall feel well repaid. May I suggest that the conclusions to be drawn are:

1. There has not been a great change in the types or form of construction, but rather an increase in the number of materials at our disposal and improvements in techniques.

2. That engineers should beware of thinking they have the final answer to any problem, but rather to keep an open mind.

3. That each individual member should avoid, as far as lies in his power, specialising in one branch of structures, or one material, at too early an age—in short, to endeavour to become an engineer rather than a mere designer of structural elements.

4. To realise that structural engineering is a form of applied economics, and that there is an urgent need for the conservation of all materials and a saving of labour.

5. That every job we do has a social significance.

6. Appreciate that we are not more ingenious than our predecessors of 1851. Rather we have greater opportunities and better tools. They appear to have been light of heart and sure of themselves—we not so. What we most require is the opportunity to apply new knowledge, use new materials, and develop new techniques, without restraint or restrictions—but they are conditions somewhat outside our ability to bring about.

May I conclude by quoting the principal objects of our Charter:

"To promote the general advancement of the science and art of structural engineering in any or all of its branches, and to facilitate the exchange of information and ideas relating to structural engineering amongst the members of the Institution and otherwise."

Fig. 18

By following these objects in their spirit as well as their letter, our members will do work of service to the community at large, the Institution in particular, and of satisfaction to themselves.

Acknowledgements

Figs. 5 and 6. "Practical Railway Engineering," by Dempsey (1855).

Figs. 7-10 incl. "Iron Roofs," by A. T. Walmisley (1888).

Fig. 12. Messrs. Dorman Long & Co., Ltd., and Messrs. C. A. Parsons & Co., Ltd.

Fig. 14. Cement and Concrete Association.

Figs. 17 and 18. The Timber Development Association.

Bond Stresses in Reinforced Concrete Columns and Beams

By J. M. Hawkes, Ph.D., B.Sc., A.M.I.Struct.E. and Professor R. H. Evans, D.Sc., M.I.C.E., M.I.Mech.E., M.I.Struct.E.

Summary

Pull out tests have shown that the distribution of bond stress along the column is an exponential function, and that the maximum value of bond stress occurs at the loaded end of the steel before any slip takes place, the order of the maximum value being 700 lb./in.². After initial slip, the position of maximum bond stress moves along the steel, its value remaining sensibly constant.

If 300 lb./in.² is taken as the allowable bond stress, an embedded length of 20 diameters is theoretically sufficient and this length is not proportional to the tensile stress in the reinforcement.

The production of a crack in a reinforced concrete beam causes the bond stress over the crack to rise rapidly, and by comparison the bond stress over the uncracked concrete can be ignored. The maximum value of the bond stress in beams, namely, about 600 lb./in.² for BRC and MS reinforcements and 1,300 lb./in.² for twist steel, is always reached at approximately half the failing load.

Only mild steel showed complete load failure, while twist steel exhibited exceptionally good bond qualities, accompanied unfortunately by comparatively larger deflections and crack widths. The bond failure with mild steel only occurred when the yield point of steel had been reached, hence, if steel stresses are maintained well below the yield point, the question of bond failure is never serious.

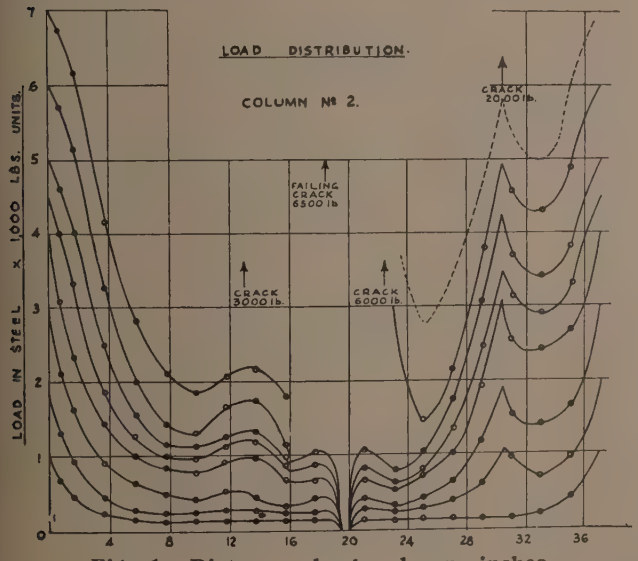


Fig. 1.—Distance along column, inches

Introduction

The phenomenon of bond between concrete and steel is believed to be the result of two physical actions, firstly, the true adhesion between the concrete and steel surfaces, and secondly, the frictional resistance due to the contraction of the concrete during setting.

To find the nature of the adhesion, experiments have been carried out in the past, in which the shrinkage has been eliminated by curing under water. But the inconsistency of such results has never been explained, and in the present investigations it has been assumed that both adhesion and friction are present and that the combined effect produces "bond."

In order to obtain bond distribution and critical values of bond stress which can be applied to normal reinforced concrete beams, pull-out tests were carried out during the years 1942-44 on reinforced concrete columns in which both the steel and concrete have been in tension, and artificial cracks introduced in beams reinforced with various types of mild steel reinforcement have made it possible to study the bond stress distribution in beams up to failure.

Pull-out Tests

Two lengths of mild steel bar were embedded in concrete columns of square section, and separated and aligned by brass bushes. In order to obtain an accurate and continuous set of strain measurements in the steel, 5/16 in. diameter legs were silver soldered to the main bar; these legs projected outside the column, always clear of the concrete, and were placed in pairs diametrically opposite. The elongation between the legs was measured by dial gauge extensometers.

The details of the three columns are shown in Table I.

TABLE I

Column No.	1	2	3
Embedded length of $\frac{5}{8}$" diameter bar	Two 18.5"	Two 18.5"	One 10" One 8"
W/C ratio	·375	·375	·375
Av. tensile strength of concrete ...	670 lb./in.²	660 lb./in.²	680 lb./in.²
Av. Comp. str. of concrete	12,300 lb./in.²	11,800 lb./in.²	12,900 lb./in.²

After curing under damp sacks for 14 days the columns were tested in a Universal Testing Machine, the mild steel (M.S.) bars being gripped in the jaws of the machine, when each column failed in tension at about 6,500 lb.

Results and Discussions

By averaging the strain readings on two diametrically opposite faces, thus eliminating the effect of eccentricity, the average bond in the steel was calculated to each gauge length, and assuming that the distribution of load is roughly parabolic, the calculated values are plotted at 1/3 pts. of the gauge length. Fig. 1 shows the load

distribution for column 2, and it is seen that the load in the steel drops rapidly over the first 6 in. of embedded length, and afterwards, except for an increase where cracks occur, the load remains constant over the remain-

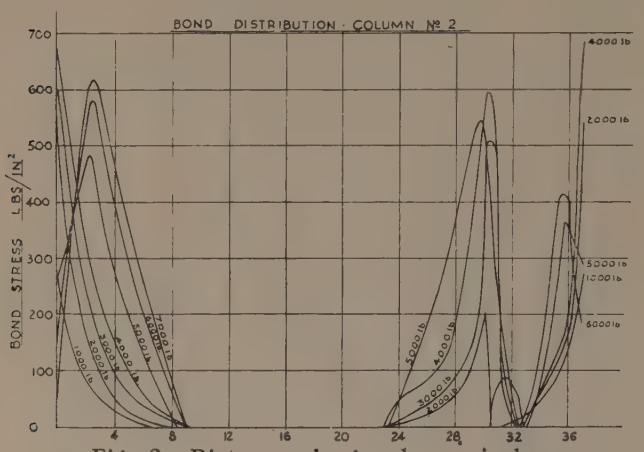


Fig. 2.—Distance along column, inches

ing length. This "residual" load in the steel conforms to theory where cracking has not taken place, e.g., for applied load of 1,000 lb., m (modular ratio) = 7.

average area of concrete in tension = 13 in.²

"equivalent" area of steel =

$7 \times .305 = 2.135$ in.²

total area resisting tension = $13+2.135 = 15.135$ in.²

$$\text{load carried by steel} = \frac{1{,}000 \times 2.135}{15.135} = 140 \text{ lb.}$$

whereas the actual load carried by the central steel = 130 lb. The bond stresses have been calculated by dividing the reduction in load over an increment of length by the effective steel area over the same increment, and are shown plotted on Fig. 2, except over the

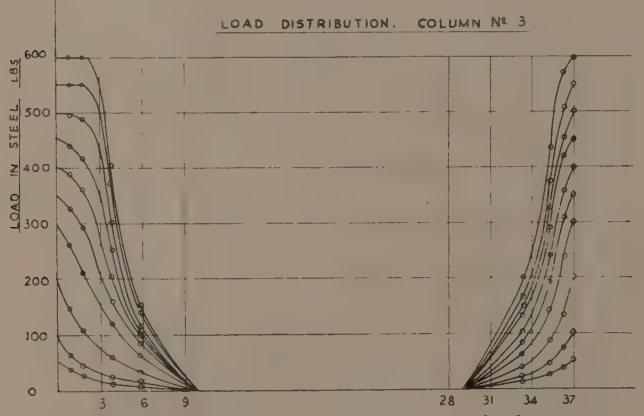


Fig. 3.—Distance along columns, inches

central portion, which has been omitted for clarity. From this graph it can be seen that for loads up to 4,000 lb., the bond distribution closely follows the load distribution, the maximum bond stress being at the "free" end. This maximum approaches a value of 700 lb./in.² with a load of 4,000 lb. Above this load, slip occurred, but not so as completely to destroy the bond, some sort of adhesion or drag between the concrete and the steel remaining. As the slip takes place, the position of the maximum bond stress moves along the column and its value is slightly less than the figure of 700 lb./in.².

Fig. 2 shows that practically all the load has been transferred to the concrete in the first 10 in. of embedded length. As the load is increased the amount of slip increases, and the point of maximum bond stress moves down the column, and thus the maximum load which could be carried by the column would be equal to the load carried by the last 10 in. of steel where the true

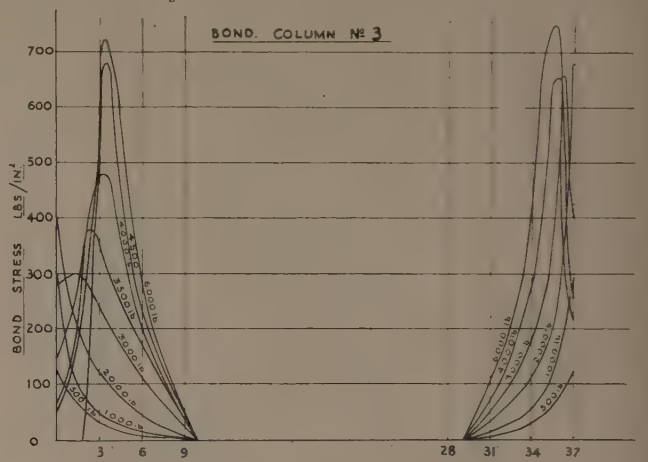


Fig. 4.—Distance along column, inches

bond would remain intact plus the "drag" over the length where the slip has taken place. This is shown by Gilbey, Chamberlain and Beal, who found that the load still increased after initial slip. Figs. 3 and 4 show similar load and bond distribution for column 3, and it can be seen that the maximum bond stress for the 10 in. length is 390 lb./in.² at 2,000 lb. load, while it is 670 lb./in.² for the 8 in. length. It is, therefore, clear that the 8 in. length was just on the point of slipping at 2,000 lb. load, whereas the 10 in. length would not slip until the load is nearer 3,000 lb.

In both cases the position of maximum bond stress moved along the column with increased load, and the value at the free end diminished.

Gilbey, Chamberlain and Beal have stated that the total applied load at "first slip" was approximately

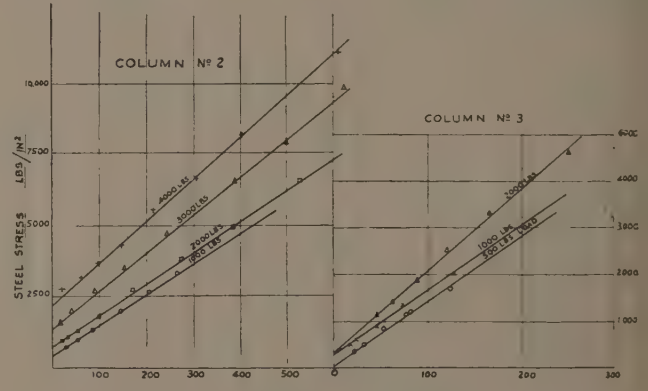


Fig. 5.—Bond stress, lb./in.²

proportional to the length up to an l/d ratio of 30. The maximum loads of 4,000—5,000, 2,000—3,000 and 2,000 lb. for the 18½ in., 15 in. and 8 in. embedded lengths respectively appear to agree with their statement.

Readings of bond stress and the corresponding steel stress before slipping took place are shown plotted in Fig. 5. From this it can be seen that for any particular applied load, the bond stress bears a linear relation to the steel stress.

Using the following notation :

P = applied load at the "free" end of the bars—(lb.)

f = bond stress at the "free" end of the bars—(lb./in.²)

x = distance in inches of any point, measured from the "free" end

δx = small increment of distance

Table II

Beam No.	1	2	3	4	5	6	7	8	9	10	11
% steel	.59	.135	.41	.102	.205	.102	.1725	.158	.204	.408	.258
Type of steel	BRC	BRC	MS	BRC	MS	MS	MS	threaded MS	TS	TS	TS
Allowable steel stress (lb./in.²)	27000	27000	18000	27000	18000	18000	18000	18000	27000	27000	27000
Calc. allowable load (tons)	.875	.21	.43	.17	.21	.44	.19	.18	.39	.65	.43
Failing Load (tons) ...	3.7	.49	1.1	.3	.65	1.04	.32	.622	1.0	2.10	1.0
Act. steel stress at allowable load (lb./in.²)	39600	24900	16000	23500	9550	13700	14600	13600	36500	27400	43000
Max. bond stress at allowable load (lb./in.²)	550	580	260	380	310	320	450	220	1070	1330	1120
Max. bond stress (lb./in.²)	650	650	650	700	590	490	550	700	1200	1400	1350
Type of failure	Comp.	Steel	Bond	Steel	Bond	Bond	Bond	Comp.	Comp.	Comp.	Steel

BRC = British reinforced concrete fabric.
MS = Mild steel.
TS = Twist steel.

P_a = load in steel at distance x—(lb.)
f_a = bond stress at distance x—(lb./in.²)
δP_a = small decrease in load over distance δx
δf_a = corresponding small decrease in bond stress over distance δx
f_s = steel stress corresponding to steel load P_a lb./in.²
δf_s = small increment of steel stress
d = diameter of bar in inches
A = a constant

from Fig. 5 it will be seen that for a particular load the slope of the bond stress to steel stress curve is constant,

$$\text{or } \frac{\delta f_s}{\delta f_a} = \text{a constant} = A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{or } \delta f_a = A\,\delta f_s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\text{Now } f_s = \frac{4P_a}{\pi d^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\text{and } \delta f_s = 4\frac{\delta P_a}{\pi d^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Combining (2) and (4)

$$\delta P_a = \frac{\pi d^2 \delta f_a}{4A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Now since an increment of load along the bar must be balanced by the bond stress multiplied by the effective surface area of the steel, we have for length δx :—

$$\delta P_a = -f_a \pi d\ \delta x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$\text{from (5) and (6)} \frac{\pi d^2 \delta f_a}{4A} = -f_a \pi d\,\delta x \quad . \quad . \quad . \quad (7)$$

$$\text{or } \frac{\delta f_a}{f_a} = -\frac{4A\,\delta x}{d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$\text{Integrating } \log_e (f_a) = -\frac{4Ax}{d} + B \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$\text{or } f_a = Ce^{\frac{-4Ax}{d}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where B and C are constants of integration when $x = 0$; $f_a = f$ and equation (10) becomes

$$f_a = fe^{\frac{-4Ax}{d}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Also, P, the applied load, must be equal to the sum of the total bond stress multiplied by the surface area of steel

$$\text{or } P = \int_0^l \pi d\, f_a\, dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$\text{Hence } P = \frac{\pi d^2 f}{4A}\left(1 - e^{\frac{-4Al}{d}}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where l = distance at which the bond stress becomes zero.

From equation (11) it will be seen that the bond stress distribution before slip occurs follows an exponential

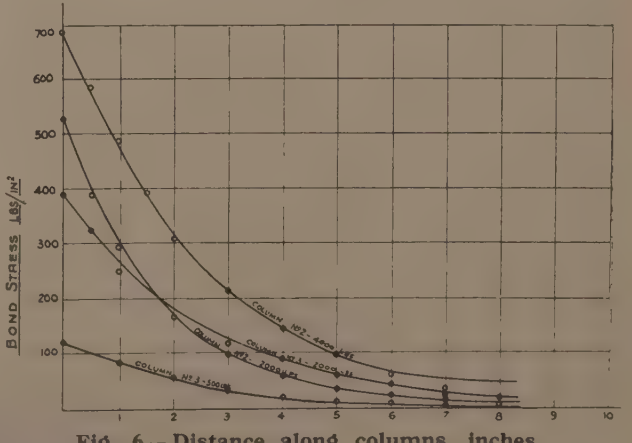


Fig. 6.—Distance along columns, inches

law, and its value at any point will depend on the stress at the free end, the diameter of the bar and a constant.

Using values of A found from Fig. 5, the bond stress has been evaluated by means of equation (11) for various

loads at various points along the column. These are shown plotted in Fig. 6 as theoretical curves, the circles representing the actual stresses. It will be seen that equation (11) gives exceptionally good values for the bond stress distribution. The reason for the discrepancy between the calculated and actual stresses over the range 6 in.—8 in. for column 2 (4,000 lb.) is the crack in the concrete which has affected the bond stress distribution.

Much work has been done on the variation of bond stress with (a) the quality of the concrete, and (b) the W/C ratio, and while it is accepted that bond is little affected by the normal variations in quality of concrete, Abrams found a decided decrease in average bond strength with increase in W/C ratio.

Assuming a maximum bond stress of 600 lb./in.², equation (11) gives $A = .07$ for the load producing such a bond stress at the free end. Taking a factor of safety of 2, the transmission length (length required to transfer

Steel Load Distribution
Beam reinforced with MS (No. 3)

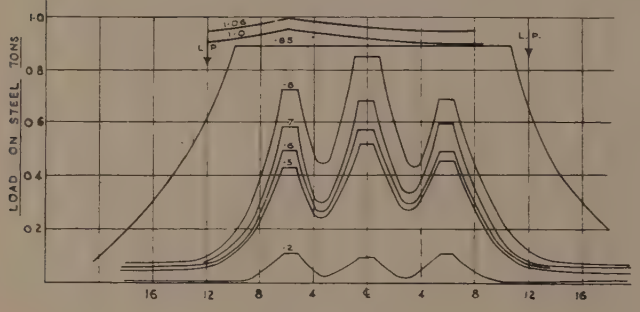


Fig. 7.—Inches

the steel load to the concrete without slip), is given by x where

$$1 = 300\, e^{\frac{-.28x}{d}}$$

from which $x = 20.3d$, f_s being taken as 1 lb./in.². This value will often in practice include a safety factor of 6, as for example in column 3, where a 10 in. embedded length safely withstood a 6,000 lb. load, which is six times the load required to produce a bond stress of 300 lb./in.² and "x" as given by above formula = 12.7 in., thus the factor of safety is greater than 6, when a maximum bond stress of 300 lb./in.² is given. Such a value as $20.3d$ compares favourably with $45d$ given by the old B.S.S. and $22.5d$ given by B.S.S. No. CP114 (1948), which uses 18,000 lb./in.² as the maximum allowable stress in steel.

Tests on Beams

Details of the beams cast are shown in Table II. All the beams, except No. 6, were of 4 in. × 12 in. rectangular section, 72 in. span, and in order that readings of strains in the concrete surrounding the cracks could be obtained, two or three artificial cracks were introduced by two thin vertical plates set in the lower part of each beam, and the reinforcement threaded through.

While each beam was loaded cyclically up to failure, measurements of strains in the concrete were taken over an 8 in. gauge length in addition to those taken over a 1 in. gauge length over the induced cracks.

Results and Discussion

Assuming a linear stress/strain relationship for the concrete up to a compressive stress of 2,000 lb./in.² and

Bond Stress
Beam reinforced with BRC (No. 2)

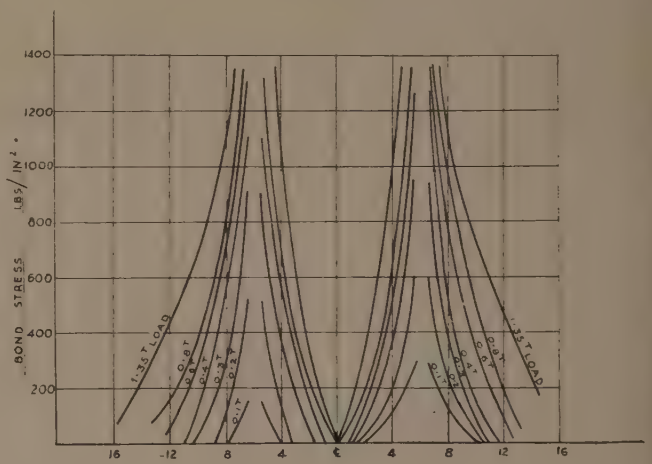


Fig. 8.—Inches

that the concrete in tensions assists the steel, the amount of slip can be calculated as follows.

Let the bond be eliminated over a distance $2x$,
then if (a) = extension of steel over this distance = $2x$ times steel strain
(b) = contraction of concrete over this distance due to failure of the bond = $2x$ times concrete strain
(c) = allowance for shrinkage = .0005 in./in.,
width of crack = (a+b+c).

Bond Stress Distribution
Beam reinforced with Twist steel (No. 11)

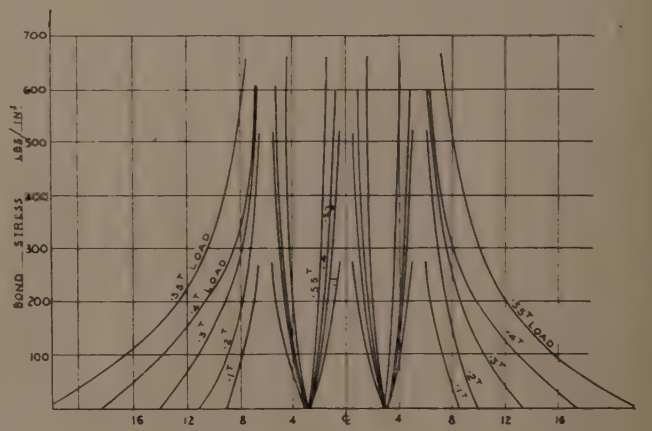


Fig. 9.—Inches

Having measured the width of the crack by means of a travelling microscope, the above linear equation in x can be solved.

The steel extension over this distance $2x$ has been subtracted from the extension recorded over the 8 in. gauge length and an average steel load over the remaining

gauge length calculated and plotted against 1/3 points as before. By joining such points exponentially to the theoretical values where no cracking had taken place, the graphs of load distribution were completed and an example of such a graph is shown in Fig. 7 : the curves were only drawn for the central portion of the beam, and the applied load is shown in tons units by each curve. The load distribution for BRC fabric, twist steel, and threaded steel are similar to the distribution for mild steel at low loads, but complete load failure never occurred with any reinforcement except mild steel.

Bond Stress at Crack Plotted against Steel Stress

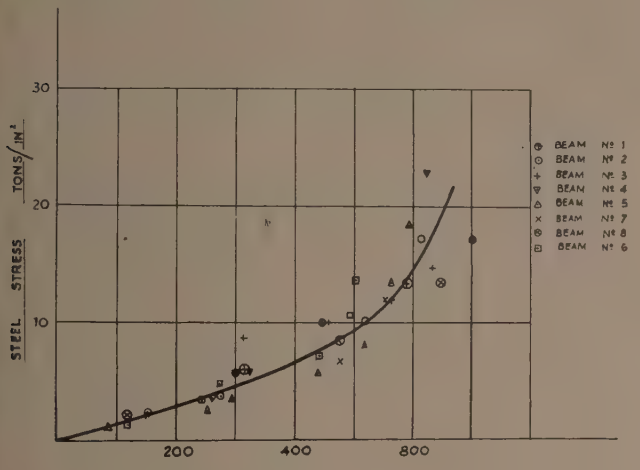


Fig. 10.—Bond stress. lb./in.²

From such load distribution curves the corresponding bond stress distribution was calculated and curves as in Figs. 8 and 9 obtained. It should be noted that the extremely small values of bond stress produced by the small changes in steel load outside the loading points have been ignored and the bond stress distribution curves produced to zero.

In the case of beams reinforced with BRC fabric, the bond stress increased with increase of load to the maximum load with no sign of bond failure. The maximum bond stress was 600-650 lb./in.².

Mild steel reinforcement gives consistent steel load distribution up to the yield point of the steel, and here yielding produced complete bond failure over the central portion, thereby making the beam almost a tied arch. The maximum bond stress was reached at approximately half the yield load, and its value was about 500 lb./in.², while further loading caused slip and the outward movement of the point of maximum stress.

The threaded steel, in spite of considerable slip at higher loads, did prevent complete bond failure. The concrete filling the threads probably cracks under the high tensile stress and produces a condition similar to that of a rough bar surrounded by concrete. The maximum value of under 700 lb./in.² bears out this suggestion. Threaded steel therefore prevents sudden failure due to elimination of bond.

Twist steel showed very little slip and had exceptionally high values of bond stress (1,200—1,700 lb./in.²).

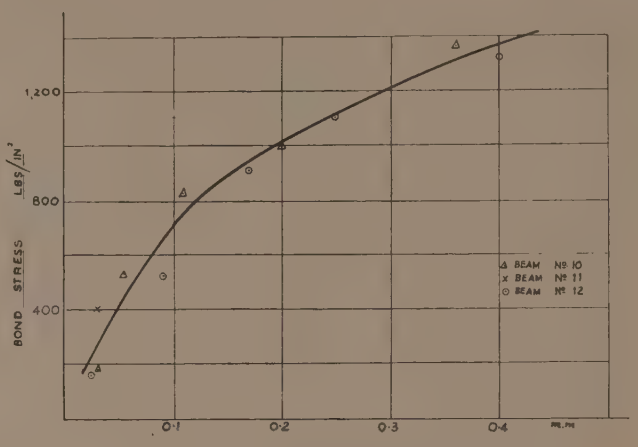


Fig. 11.—Width of crack

this maximum value again being reached at approximately half the load producing yielding of the tension reinforcement.

Table III has been drawn up to show that the advantages of twist steel are offset by the larger deflections and crack width which the allowable load produces with twist steel.

Fig. 10 shows that within the working stress for steel, the bond stress bears an approximately linear relationship to the steel stress. A similar curve for twist steel could be drawn, and for a steel stress of 10 tons in.² the corresponding bond stresses would be 500 lb./in.² for BRC, MS, and 1,000 lb./in.² for twist steel.

While Fig. 11 refers only to the rectangular slab type of beam, it indicates the relationship of bond stress and crack width, and such a set of curves should be very useful in the design of structures subjected to water pressure.

TABLE III

Beam No.	1	2	3	4	5	7	8	9	10	11
Central deflection at allowable load	BRC 1″	BRC .05″	MS .05″	BRC .025″	MS .02″	MS .025″	threaded MS .025″	TS .11″	TS .10″	TS .1″
No. of artificial crack	3	3	3	2	2	3	3	2	2	2
Width of crack at allowable load	.14 mm.	.11 mm.	.05 mm.	.05 mm.	.05 mm.	.05 mm.	.08 mm.	.25 mm.	.10 mm.	.25 mm.
Width of crack at Max. bond stress ...	.145 mm.	.15 mm.	.14 mm.	.12 mm.	.13 mm.	.10 mm.	.2 mm.	.43 mm.	.20 mm.	.40 mm.

Why Not Use Three Moments?

By C. Marsh, B.A. (Graduate)

Introduction

The three-moment equation is, undoubtedly, the best-known and most popular among designers of all the expressions relating to rigid frames, but its full power seldom appears to be utilised, or even realised, and it is often discarded in favour of rather more exotic methods for complicated frameworks.

This paper sets out to show that it supplies perhaps the simplest and yet the most powerful of all the methods of analysis as applied to rigid frames composed of members of prismatic sections.

Standard Formulae

The derivation of the three-moments equation is given in most engineering text-books and need not be re-stated. The final expression relating to the bending moments in two bays of a continuous beam loaded thus :

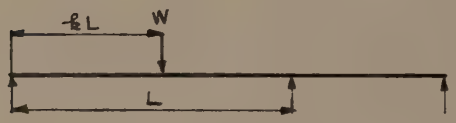
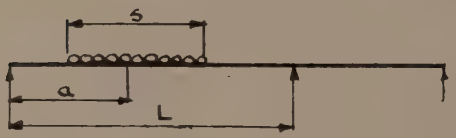
Fig. 1

is :—

$$M_A\frac{L_1}{I_1}+2M_B\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right)+M_C\frac{L_2}{I_2}=\frac{6A_1\bar{x}_1}{L_1I_1}+\frac{6A_2\bar{x}_2}{L_2I_2}+\frac{6E\,\delta_A}{L_1}+\frac{6E\,\delta_C}{L_2}$$

where M_A, M_B and M_C are the hogging moments at the supports.

Table 1

	$\frac{WL^2}{4}$
	$WL^2(k-k^3)$
	$WLa\left(1-\frac{4a^2+S^2}{4L^2}\right)$
	$\frac{4}{15}WL^2$
	$\frac{7}{30}WL^2$
	$\frac{5}{16}WL^2$
	$\frac{3}{16}WL^2$
	$\frac{W}{SL}\left(\frac{L^2(2b^2-a(a+b))}{3}-\frac{(4b^4-a(b^2+a^2)(b+a))}{10}\right)$
	$\frac{W}{SL}\left(\frac{L^2(b(b+a)-2a^2)}{3}-\frac{(b(b^2+a^2)(b+a)-4a^4)}{10}\right)$

L_1 and L_2 are lengths AB and BC respectively, I_1 and I_2 are the moments of inertia of the sections AB and BC respectively, $A_1\bar{x}_1$ and $A_2\bar{x}_2$ are the moments of the areas of the bending moment diagram for the loading on AB and BC about A and C respectively, and δ_A and δ_C are the downward deflections of the supports A and C respectively relative to B.

This expression, for a beam of uniform moment of inertia with no deflection of the supports, subjected to uniform loading, reduces to its most familiar form :—

$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = \frac{wL_1^3}{4} + \frac{wL_2^3}{4}$$

$\frac{w_1 L^3}{4}$ is the evaluated term $6A\bar{x}/L$. The evaluation of this term for less simple cases of loading, sometimes occasions difficulty, and to simplify the analysis a list is given in Table I of this term $\frac{6A\bar{x}}{L}$ for various conditions of loading.

These expressions can be superimposed directly for any type of loading that can be obtained by combining them.

In more unusual cases not accounted for in Table I, it is possible to calculate the required expression by direct integration using the expression for a single point load.

FIGURE 1

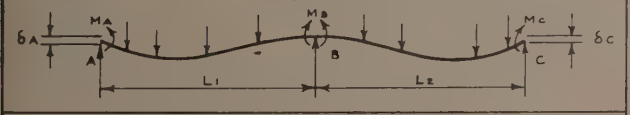


FIGURE 2

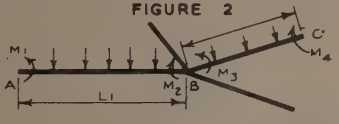


FIGURE 3

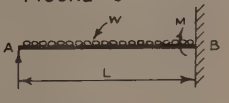


FIGURE 4

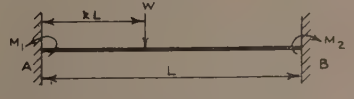


FIGURE 5

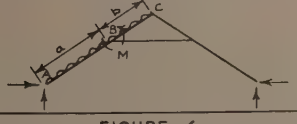


FIGURE 6

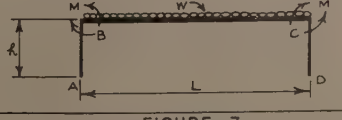


FIGURE 7

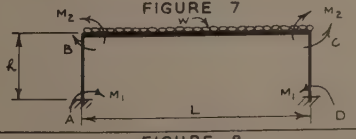
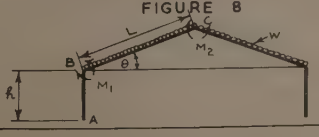


FIGURE 8



If w_x is the loading along an element δx of the beam distance x from the end, then the term for this element of loading is :—

$$w_x\ \delta x\ L^2 \left(\frac{x}{L} - \left(\frac{x}{L}\right)^3 \right)$$

and the expression for the total load is :—

$$\int_0^L w_x\ L^2 \left(\frac{x}{L} - \left(\frac{x}{L}\right)^3 \right) dx$$

For a loading following the sine law for example :—

$$w_x = w \sin \frac{\pi x}{L}$$

thus :—

$$\frac{6A\bar{x}}{L} = \int_0^L w L^2 \sin \frac{\pi x}{L} \left(\frac{x}{L} - \left(\frac{x}{L}\right)^3 \right) dx$$

$$= \frac{6wL^3}{\pi^3}$$

$$= \frac{3WL^2}{\pi^2}$$

where W is the total load.

Where more than two members meet at one point the three-moments equation is extended to become the four-moments equation.

Fig. 2

Any two of the members meeting at the point are selected, such as AB and BC in the figures above, and an equation formed thus :—

$$M_1 \frac{L_1}{I_1} + 2M_2 \frac{L_1}{I_1} + 2M_3 \frac{L_2}{I_2} + M_4 \frac{L_2}{I_2} = \frac{6A_1\bar{x}_1}{I_1 L_1} + \frac{6A_2\bar{x}_2}{I_2 L_2} + \frac{6E\ \delta_A}{L_1} + \frac{6E\ \delta_C}{L_2}$$

This equation can be seen to reduce to the standard three-moments equation if $M_2 = M_3$, i.e., if AB and BC are the only members.

Using such expressions $(n - 1)$ independent equations are obtainable where there are n members meeting at the point.

With these weapons in hand the greater majority of the varieties of rigid frames can be attacked and dealt with without any appreciable difficulty.

Applications—continuous Beams

The use of the three-moments equation for continuous beams requires no further treatment here, except where encastré ends occur. In these cases one can either "reflect" the beam about the encastré point, or, preferably, treat the beam beyond this point as being of infinite rigidity, i.e., $\frac{L}{I} = 0$, the resulting equation is then exactly half that obtained by reflecting.

Thus in the case of a propped cantilever subjected to uniform loading :—

Fig. 3

$$AB— \; 0 + 2M(L+0) + 0 = \frac{wL^3}{4} \therefore M = \frac{wL^2}{8}$$

(*AB* indicates the spans for which the equation is formed.)

For a beam encastré at each end with a single point load :—

Fig. 4

$$—AB \quad 2M_1L + M_2L = WL^2((1—k)—(1—k)^3)$$

$$AB— \quad M_1L+2M_2L = WL^2(k—k^3)$$

from which :—

$$M_1 = WLk\,(1—k)^2$$
$$M_2 = WLk^2\,(1—k)$$

If a simple collar roof behaves as an arch the rafter can be treated as a simple continuous beam, thus :—

Fig. 5

$$2M(a+b) = \frac{w}{4}(a^3 + b^3)$$

$$\therefore M = \frac{w}{8}\frac{(a^3+b^3)}{a+b}$$

From the knowledge of this moment the redundant force can be calculated.

These are simple and standard results but the ease of analysis and the simplicity of the method of attack becomes most evident in plane frames in which no displacement of the nodes occurs.

Single Span Portal Frames

The solution of a simple portal frame by three moments is well known :—

Fig. 6

$$ABC \quad 2M(h + L) + ML = \frac{wL^3}{4}$$

$$\therefore M = \frac{wL^3}{4(2h+3L)}$$

If the frame has encastré bases it is usual to assume points of inflexion at one-third the way up the legs and treat it as for a simple portal.

This does not yield an exact result owing to the displacement of this point of inflexion, but the exact solution is not difficult to obtain.

Fig. 7

$$—AB \quad —2M_1h + M_2h = 0$$
$$\therefore M_1 = \frac{M_2}{2}$$

$$ABC \quad —M_1h + 2M_2(h+L)+M_2L = \frac{wL^3}{4}$$

$$\therefore M_2 = \frac{wL^3}{6(h+2L)}$$

A haunch tied portal frame which contains two redundances, succumbs quickly to this method of attack.

Fig. 8

$$ABC \quad 2M_1(h+L) + M_2L = \frac{w\cos^2\theta \; L^3}{4}$$

$$BC— \quad M_1L + 2M_2L = \frac{w\cos^2\theta \; L^3}{4}$$

$$\therefore M_1 = \frac{w\cos^2\theta\, L^3}{4(4h+3L)} \text{ and } M_2 = \frac{\cos^2\theta\, L^2}{4}\,\frac{(2h+L)}{(4h+3L)}$$

Knowing these moments the redundant forces can be evaluated at once.

Where the frame is unsymmetrical, or is subjected to unsymmetrical or side loading, sway occurs and must be taken into account in the analysis. The general three moments equation contains terms relating to deflections. In order to establish the necessary number of equations, however, equilibrium conditions have usually to be included.

In the frame shown below both the frame and the load are unsymmetrical :

Fig. 9

Let the deck, *BC*, sway to the right a distance δ. Whether this is the true direction or not is of little importance, as if it is wrong, it will merely be negative when evaluated.

To form the equation for the bays *ABC* and *BCD*, it is necessary to know the relative signs of δ. This can best be done by opening out the frame to see in which direction *A* and *D* move relative to *B* and *C*, thus :

Fig. 10

A moves up relative to *B*, whilst *D* moves down relative to *C*, thus δ is negative and positive respectively.

$$ABC \quad 2M_1(a+L)+M_2 \; L = WL^2((1—k)—(1—k)^3) \; —6EI \; \frac{\delta}{a}$$

$$BCD \quad M_1L+2M_2(L+b) = WL^2(k—k^3)+6EI \; \frac{\delta}{b}$$

There is also the equilibrium condition :—

$$\frac{M_1}{a} = \frac{M_2}{b}$$

from which :—

$$M_1 = \frac{WL^2ak(1—k)\left\{(2—k)a+(1+k)b\right\}}{2(a^3+a^2L+aLb+b^2L+b^3)}$$

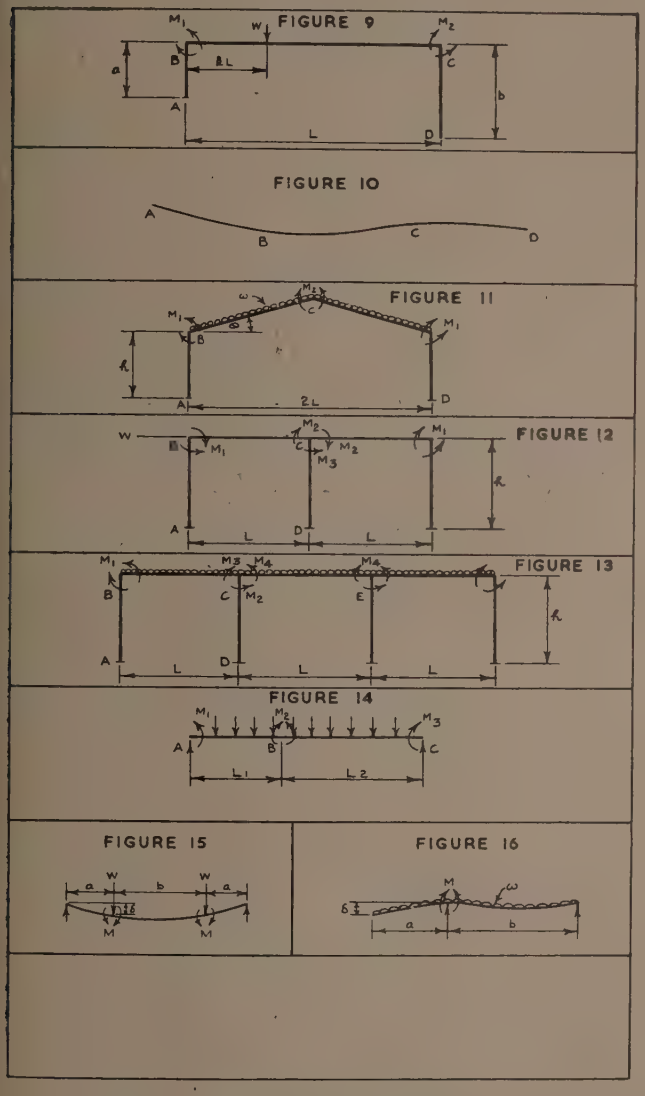
This problem instances a case that would probably have been more readily solved arithmetically in actual practice, but the extra labour required for the algebraic analysis is well repaid when, in the future, similar cases arise.

The power of the three moments equation is most forcibly shown in the analysis of a pitched roof portal frame.

Fig. 11

Both C and B deflect relative both to A and to each other. Let B move outwards δ, C will then drop

$$\frac{\delta}{\tan\theta}$$



The necessary three-moments equation can now be formed :—

$$ABC\ 2M_1\left(h+\frac{L}{\cos\theta}\right)+M_2\frac{L}{\cos\theta}=\frac{wL^3}{4\cos\theta}+6EI\left(\frac{\delta}{h}+\frac{\delta}{L}\tan\theta\right)$$

$$BCD\ \ M_1\frac{L}{\cos\theta}+2M_2\left(\frac{L}{\cos\theta}+\frac{L}{\cos\theta}\right)+\frac{M_1L}{\cos\theta}=\frac{WL^3}{4\cos\theta}+\frac{WL^3}{4\cos\theta}-6EI\left(\frac{\delta}{L\tan\theta}+\frac{\delta}{L\tan\theta}\right)$$

There is also an equilibrium condition relating M_2 to M_1 :—

$$M_2=\frac{M_1}{h}(h+L\tan\theta)-\frac{wL^2}{2}$$

From these equations we get :—

$$M_1=-\frac{wL^3h(h+\frac{5}{8}L\tan\theta)}{h^3\cos\theta+3Lh^2+3L^2h\tan\theta+L^3\tan^2\theta}$$

a very useful standard solution.

Multi-span Frames

The solution of frames in which more than two members meet at one node requires the use of the four-moments equation. This is most simply illustrated by a twin-span portal subjected to side load at deck level.

Fig. 12

Let the deck sway through a distance δ, then :—

$$ABC\ \ -2M_1(h+L)+M_2L=-6EI\frac{\delta}{h}$$

$$BCD\ \ -M_1L+2M_2L+2M_3h=6EI\frac{\delta}{h}$$

which, together with the equilibrium conditions :—

$$2M_1+M_3=Wh$$
$$M_3=2M_2$$

give :

$$M_1=\frac{Wh}{12}\cdot\frac{(3L+4h)}{(L+h)}$$

$$M_2=\frac{Wh}{12}\cdot\frac{(3L+2h)}{(L+h)}$$

Similar expressions are readily obtained for multi-span portals under down load.

For example, a three-span portal :—

Fig. 13

$$ABC\ \ 2M_1(h+L)+M_3L=\frac{wL^3}{4}$$

$$BCE\ \ M_1L+2M_3L+2M_4L+M_4L=\frac{2wL^3}{4}$$

$$BCD\ \ M_1L+2M_3L+2M_2h=\frac{wL^3}{4}$$

The only equilibrium condition is :—$M_2=M_3=M_4$.

These equations give :

$$M_1=\tfrac{3}{4}\cdot\frac{wL^3(2h+L)}{(20h^2+30hL+9L^2)},$$

$$M_2=\frac{wL^3h}{2(20h^2+30hL+9L^2)}\qquad M_3=\frac{wL^3(8h^2+12hL+3L^2)}{4(20h^2+30hL+9L^2)}$$

The use of the three-moments equation provides a rapid means of supplying an approximate answer for the redundant forces in a latticed frame where a preliminary design is required to give some idea of the size of the members before a full strain energy analysis is conducted. This "preliminary" design will often be found to be sufficiently accurate to be treated as a final one.

Where a large number of equations have to be solved the fundamental expressions can be inserted into a matrix to facilitate their solution, but usually the simple straightforward successive elimination of unknowns is as quick and more readily understood.

Deflections

Although the deflections for most of the cases of loading are tabulated and easily looked up, occasionally it is necessary to work them out, and here again the three moments equation supplies the complete method of calculation.

In the beam in Fig. 14, the deflection at the point B is given immediately by the equation :

$$M_1L_1 + 2M_2(L_1 + L_2) + M_3L_2 = \frac{6A_1\bar{x}_1}{L_1} + \frac{6A_2\,\bar{x}_2}{L_2} - 6EI\,\delta\left(\frac{1}{L_1} + \frac{1}{L_2}\right)$$

Fig. 14

Two striking examples of the facility by which deflection can be determined are given below.

In Fig. 15 two symmetrically placed loads are applied to a simple beam. It is required to find the deflection at the load.

Fig. 15

$$-2M(a+b) - Mb = -6EI\,\frac{\delta}{a},\quad M = Wa$$

$$\therefore\ \delta = Wa^2(2a + 3b)/6EI$$

The continuous beam in Fig. 16 is cantilevered from one support. The deflection at the end of the cantilever is given by :

$$2M(a+b) = \frac{w(a^3+b^3)}{4} + \frac{6EI\,\delta}{a}$$

$$M = \frac{wa^2}{2}$$

$$\delta = \frac{wa}{24EI}(3a^3 + 4a^2b - b^3)$$

Conclusion

From the foregoing range of analyses it is evident that once one has a grasp of the fundamental three and four moment equations, as most structural designers have, the solution of any rigid frame can be obtained, and the need for the more exotic methods can be restricted to very complex frames where exact analysis is, by any means, impracticable.

A Comparison of the Bearing Power of Footings on Dry and Inundated Sand*

Discussion on Dr. W. Eastwood's Paper

Introduction

In introducing his paper, Dr. Eastwood said that there were two principal conclusions to be drawn from the tests described in the paper. The first was that the bearing power of foundations on dense inundated sand was higher than would be expected from the usually accepted theories, and the second was that the mechanism of failure was very different from that assumed in any of these theories. Because of this second conclusion it would appear that any agreement between these theories and the results of experiment was quite fortuitous, and was unlikely to hold except over a narrow range of conditions.

It was perhaps a little unsatisfactory to leave the matter rather in the air by not suggesting an alternative general theory, but he believed that from the tests so far carried out he would not be justified in doing so. It was only possible to say, as had been said in the paper, that the inferred mechanism of failure was confirmed by simple statical analysis, but it was quite possible that for larger footings, or for non-surface footings, or for lighter and less frictional bearing materials, the mechanism would not be the same.

It may be thought that the use of the stress distribution according to the elastic theory was an over-simplification, but to use any so-called plastic theory would be no more correct since the inherent faults were just as great.

Further, it was made clear in the paper that the calculations were intended to serve more as an indication of feasibility than as positive proof.

Referring to the ⌀ value, it was stated in the paper that the value of ⌀ was possibly smaller for the inundated sand than for the dry sand. Since the paper was written, some further tests had been made which showed that for shear tests on saturated sand carried out in about the same time as one of the footing tests, ⌀ is about 1 deg. lower at small pressures but only slightly lower at high pressures than for dry sand. For a quick shear test the ⌀ values were hardly altered by inundation, probably because the inundated ⌀ value was raised slightly by the greater pore water stresses.

Dr. Eastwood then went on to describe a more recent investigation into the effect of vertical vibration on the failing load of footings. This investigation has provided further confirmation of the mechanism of failure described in the paper.

Finally, he apologised for not having carried out tests on footings below ground level. Such tests were obviously necessary, but there was a limit to the amount of work which a solitary worker could get through without the aid of skilled soil technicians.

Discussion

The PRESIDENT, having proposed a vote of thanks from the Chair to Dr. Eastwood, called on Mr. F. R. Bullen (Member of Council) to open the discussion.

*Paper read before the Institution of Structural Engineers at 11, Upper Belgrave Street, London, S.W.1, on Thursday, January 11th, 1951. Mr. J. E. Swindlehurst (President) in the Chair. published in THE STRUCTURAL ENGINEER, Vol. XXIX, No. 1, Pp. 1-11. (Jan., 1951.)

Mr. BULLEN said that inundation of foundations seemed to be particularly appropriate at the present moment in view of the abnormal rainfall, whether they were on sand or anything else. A short time ago he had had to go to a site to see how the work was proceeding, and was confronted with a perfectly appalling mess ; the contractor assured him everything was perfectly all right, but he himself had grave misgivings and, in fact, had to have a considerable amount of extra reinforcement put in the work.

Without discussing it he would like to refer to a paper which was presented to the Institution of Civil Engineers some months ago by Mr. Guthlac Wilson. That dealt with foundations in sand and clay and seemed to him to constitute a powerful contribution to soil mechanics. What he would like to have seen in Dr. Eastwood's paper was a diagram relating the pressure or load on the footing to the settlement actually measured. He would like to see such a graph of settlement under load to see the shape it took. He believed that such a graph would be found to be in two parts. The first part would be approximately linear and then at some point fairly clearly marked, he thought they might find that it departed from a straight line and the second part became something of the order of a parabola. Had Dr. Eastwood any information in that regard ? He would also like to ask Dr. Eastwood if he would define the term " ultimate bearing capacity of a foundation." This term was often used but he had never seen it carefully defined.

Dr. G. G. MEYERHOF (Associate-Member) said that in the design of foundations for large structures an estimate of the allowable bearing pressure was of great importance and that depended on the ultimate bearing capacity of the soil and the permissible deformation of the structure by unequal settlement. The author's work on the bearing capacity of surface footings on dense sand was, therefore, of considerable interest, particularly as little published information existed on the main problems of his investigation, namely, first the effect of the shape of footings of equal width and second, the effect of inundation or submergence of the material on the bearing capacity of sand.

The influence of the shape of surface footings of equal width on the bearing capacity of sand had been controversial for a long time, and the author suggested that this was partly due to inadequate attention in the past to the size of the model footings in relation to that of the box containing the soil. Extensive research at the Building Research Station[1,2] had led to the same conclusion and indicated that for surface footings on dense sand a minimum clear distance of 10 times the footing width was required at the side of rectangles, and five times the footing width at the end of rectangles, the side of squares or circles and the base of footings of any shape. These distances represented two to three times the dimension of the actual failure surface in a very large box so that on the author's own admission only his 3 in. footing tests were reliable.

The B.R.S. investigations also showed that for surface footings on dense sand the bearing capacity increased slightly from circular to square areas and very rapidly from square to short rectangular footings, and that for rectangles of a length/width ratio exceeding 6, the increase was small. For compact sand the variation with shape of footing was similar but less pronounced, while for loose sand no difference whatever was observed between circular, square and rectangular areas up to a length/width ratio of 36. From this investigation it might be concluded that the shape effect was a function of the relative density of the sand or its angle of internal friction, which would explain why some investigators found less difference between square and rectangular areas than others. With the limitations given above in regard to the relative size of footings and box, previous investigations appeared to be quite consistent with these findings. They were also supported by the author's results when restricted to 3 in. wide footings for which unfortunately no squares had been tested ; the bearing capacity of the latter was expected to be about three-quarters of that of long rectangles[1,2].

The influence of inundation of sand on the bearing capacity of surface footings had previously been investigated in some detail[3] and for a uniform fine and medium sand (like the author's) it was found that a rise of the water table to the surface reduced the bearing capacity of dry sand by about 40 per cent., which was in accordance with the effective unit weight of the material. Extensive research at the Building Research Station[2] led to the same conclusion as shown in Figs. 1 and 2, where the " wet " bearing capacity was compared with 0.6 times the " dry " bearing capacity for surface and deep foundations, respectively, and the results showed reasonable agreement. Additional tests of footings on sand subjected to various rates of upward ground water seepage, which further reduced the effective weight up to piping of the soil, were also in fair agreement with the theory. These results were further confirmed by a series of loading tests, which they had carried out in the field, using plates of different sizes up to 2 ft. square on sand in a compressed air caisson in London.

In view of all this laboratory and field evidence it was somewhat surprising that the author found that the dry bearing capacity was reduced by not more than 20 per cent. or one-half of the theoretical amount when the sand was inundated. The difference might be due to an excessive rate of loading or an insufficient cover of water at ground level at the ultimate load. Dense sand dilated when loaded and this expansion set up a tension in the pore water leading to an increase in bearing capacity, unless the tests were carried out very slowly so that no tensions were induced. Further, if at the ultimate load the water table was only very slightly below the ground level, the bearing capacity was increased by the same phenomenon as above and in addition by a greater effective weight and an apparent cohesion.

Finally, in connection with the author's analysis of his test results Dr. Meyerhof suggested that the ultimate bearing capacity of soils could not even approximately be estimated by the theory of elasticity. At the plastic state of stress existing in the failure zone the elastic theory no longer applied so that the magnitude and direction of the actual stresses were completely changed and must be estimated by the theory of plasticity. This theory[4] showed that the bearing capacity was unaffected by ground failure to one or both sides of strip foundations, and it gave results in fair agreement

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[1] *G. G. Meyerhof : " An Investigation of the Bearing Capacity of Shallow Footings on Dry Sand." Proc. Sec. Int. Conf. Soil Mech., Rotterdam, Vol. 1, p. 237, 1948.*

[2] *G. G. Meyerhof : " The Bearing Capacity of Sand." Building Res. Sta. Note, No. C 159, 1950 (to be published as Nat. Building Study Research Paper).*

[3] *J. P. Luby and A. Woolf : " An Experimental Investigation of the Relation between the Bearing Capacity of Dry and Wet Sands." B.Sc. Thesis, M.I.T., Cambridge (Mass.), 1928. (Trans. Am. Soc. C.E., Vol. 109, p. 452, 1944.)*

[4] *G. G. Meyerhof : " A General Theory of Bearing Capacity." Building Res. Sta. Note No. C. 143, 1949. (To be published in* GEOTECHNIQUE.)

not only with the magnitude of the ultimate load, but also with the mechanism and extent of failure observed in the author's and previously mentioned tests.

Mr. R. H. Squire (Member) hoped Dr. Eastwood would be able to carry out further experiments on the same lines on a larger scale. He thought, without in any way decrying the work, that the scale was far too small, on account of the limited range. Perhaps the author might elaborate on that a little.

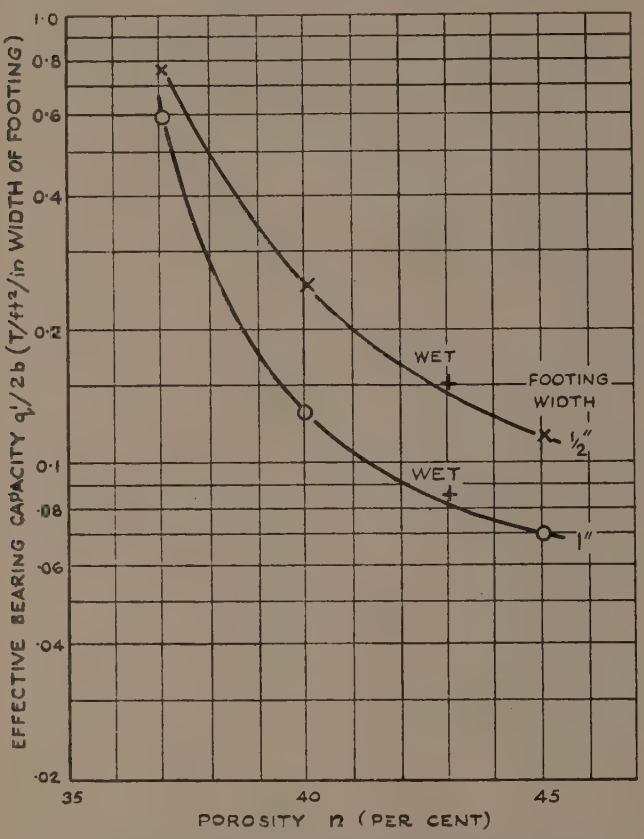

Fig. 1.—Relation between bearing capacity and porosity for surface footings on submerged sand

In Fig. 11 they had a graph showing the ultimate pressure per square foot plotted against the equivalent width or diameter in inches. Extrapolating from that graph they would have, say, twelve tons per square foot at three feet wide, and 24 tons at six feet wide, but quite obviously it seemed to him there was a limit. That was one of the things to which he referred when he said that that work was on too small a scale. He hoped Dr. Eastwood would comment on that in his reply.

At the top of page 7 the author remarked that the footing did not tilt more than one or two degrees, despite the fact that the loading arrangements did not impose any restraint on rotation. That was a point he would like to query. He estimated that, judging from Figs. 8, 14, 15 and 17 that there was definitely a restraint—a certain amount of restraint—against rotation. Suppose one side tilted a little bit the edge of the proving ring against the top support, or the top of the jack, or the base of the jack against the baseplate would all tend to throw an eccentric load which would immediately produce a rotating effect. That was what he meant by restraint. A little explanation on that point would perhaps clear the air.

Some of the tests did tilt a little. Which way did they tilt? Did the side nearest the upheave in a single failure go down or did the other side go down? If the side removed from the heave went down it would support the theory of a circular movement, as indicated in Fig. 4. If the side near the upheave sank, one might say that the wedge portion CDE had been pushed up, but the sand pushing it up had simply spread away from the side B (Fig. 4), and that side would drop. That, in his limited experience, was what happened in foundations which had failed. All the settlement had occurred on the same side as the upheave. There was a large and very well-known example of that in the silos at Winnipeg which, he thought, must be known to most of the members. The silos canted over considerably, and the side to which they canted was that on which the upheave occurred.

From Fig. 16, the distance from the edge of the three-inch footing to the point where the rupture broke the surface was 15½ inches and the depth 5.6 inches; with a four-inch footing it was 20¾ in. and depth of 6½ in. The wider the footing the greater that dimension, so that for a six-inch wide footing they could expect something considerably in excess of 20 inches—say perhaps 24 in. But where the 6-inch footing failed on both sides the distance was given as 16 and 21 inches to where the surface was broken. Why that interested him was that with a double upheave he would expect a shorter distance from the edge of the footing than one did in the case of a single upheave on one side only. It would rather indicate that the method of failure was as shown in Fig. 4 for a single heave and as Fig. 6 for a double failure, which would give less depth to the lowest point and less width from the edge of the footing to the outcrop.

Dr. H. Q. Golder said that he had himself done some work on bearing tests on sand to which Dr. Eastwood had referred. With some of the results Dr. Eastwood was not in agreement and on this count Dr. Golder would defend himself.

To illustrate that the problem was by no means simple or obvious, Dr. Golder read two extracts from papers which he had delivered to the Institution of Civil Engineers. In the first he had said (referring to the bearing capacity of inundated sand), "it would mean that the strength of a sand foundation would be approximately halved if flooded, but that was not the case." From another paper written several years later he quoted: "However, the tests demonstrated in a striking manner that the ultimate bearing capacity of a frictional material is approximately halved when the material is inundated." This latter statement was based on the results of some careful laboratory tests.

Dr. Eastwood had claimed that his (Dr. Golder's) tests were carried out in a box which was too small and were therefore unreliable. Dr. Golder agreed that the box was possibly on the small side, but did not consider that this invalidated the results since the surface of failure observed with a footing 3 in. wide gave a value of 3 to 1 for the ratio of the distance from the edge of the footing to the surface of failure, and the width of the footing. The same ratio was observed in some smaller tests done recently. The variations obtained in tests of this type were such that in his opinion it was not possible to say the angle of friction was so much and we could therefore calculate the bearing capacity. It was only possible to work back from a measured value of the bearing capacity to an angle of friction which gave agreement with a theory and to say that this angle was within the possible range. Using this technique on tests 1 ft. square and 2 ft. square on natural sand (therefore no questions of box restraint came in) he had obtained results in agreement with the Prandtl and Terzaghi Theories. He did not think that this agreement was purely fortuitous.

As to the position of the surfaces of failure, in his original tests he did not expect such a high angle of internal friction for the sand as was in fact measured. It was only this large value of angle of internal friction which forced the theoretical surface of failure so far away from the footing. In practice the surface of failure was often much closer to the edge of the footing. This was substantiated by Dr. Meyerhof's results, and it was noticeable that his theory gave a surface of failure well inside that given by Prandtl for the same angle of friction. Dr. Golder suggested, and pointed out that it was only a suggestion, that it might be possible that the resistance depended on the angle of friction in the dense state, but that the position of the final failure surface depended on the angle of friction in a loose state, because failure could not occur until the sand in the zone of failure was in a loose state.

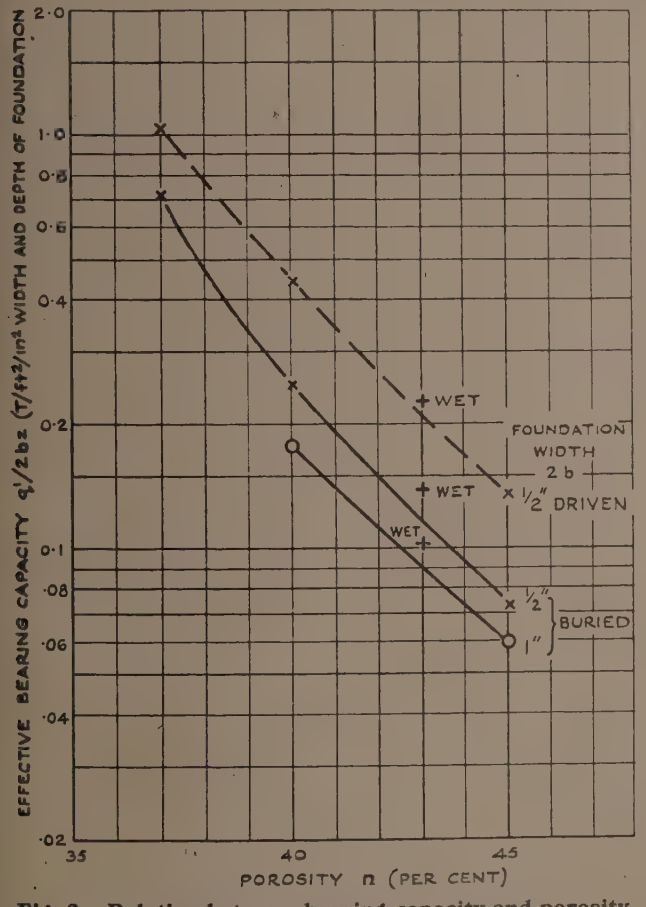

Fig. 2.—Relation between bearing capacity and porosity for buried and driven foundations in submerged sand

Dr. Eastwood had stated that there was no experimental confirmation of Terzaghi's statement that the bearing capacity was approximately halved when the sand was inundated. Dr. Meyerhof had already produced his own test results and those obtained by Woolf in 1928. Although not published until 1944, they show that the ratio of the bearing capacity inundated to that dry varied from 0.36 to 0.66 depending on the grading of the sand. Since Dr. Eastwood's paper had appeared, Dr. Golder had repeated his tests on dry and saturated sands and had obtained a ratio varying from 0.46 to 0.67 giving a mean of 0.55 on a circular footing 2 in. dia. He then did a series on a rectangle measuring 4½ in. by 1½ in. and the ratio was 0.53. The mean for all the tests carried out was 0.57, the theoretical answer for the sand in question was 0.61. Thus four sets of tests were in agreement with the theory, but Dr. Eastwood did not agree. It was therefore necessary to try and discover where Dr. Eastwood had gone wrong. Dr. Eastwood's own tests showed no scale effect with increasing width of footing so that the smaller tests could not be ruled out on this basis. One further point had already been mentioned by Dr. Meyerhof, namely, the rate of loading. Dr. Eastwood did not state the rate of loading in his test, but stated that all the tests had been carried out at the same rate of loading. He had used a sand which was finer than that employed by Dr. Golder in his own tests and Dr. Golder thought that possibly he (Dr. Eastwood) had obtained an increased bearing capacity by using too high a rate of loading. This was just a suggestion. There was a photograph in Dr. Eastwood's paper which showed a motor-car type jack which had been used for the loading. In Dr. Golder's opinion this was too crude an apparatus for this type of work.

In looking through Dr. Eastwood's results it would be found that a variation of 30 per cent. in the mean value was all that was required to bring them into agreement with the theory. This variation was equivalent to a variation of 1 deg. to 1½ deg. in the angle of friction, which was 1 to 2 lb./cu. ft. in the weight of the soil. The densities quoted by Dr. Eastwood varied from 106 to 110 lb./cu. ft., but he did not state the density in his saturated box-shear tests, but just stated that his aim was to get the same density. Although the average density in the box may have varied very little, the density from point to point may have varied considerably, as was suggested by the densities measured in the buried tins. It had been shown by Mr. Yassim, of Imperial College, that it was necessary to control the density to within 1/10th of a lb. per cu. ft. in work of this sort. It was exceedingly difficult to make sure that the density was uniformly controlled within these limits. That was why Dr. Golder suggested that it was really only possible to work back from a bearing capacity to a calculated angle of internal friction. There were two ways to succeed in solving this problem. One was to work at different densities measuring the densities every time and plotting resistance against density, and the other was to work with a frictional material, not sand, in which the angle of friction did not vary greatly with the density.

There were one or two other points on which one could differ from Dr. Eastwood. The first was that he did not state correctly the mechanism suggested by Krey, who had not assumed a rotation, and who showed a vertical junction between the two sections, and carried out a complete statical analysis for the circular section. Further, Dr. Golder was of the opinion that the placing of the paper strips would cause some disturbance of the sand. What the effect on the final answer would be he did not know. Again, on page 8, a stress distribution was shown calculated on elastic theory. At the point where the curve for V crossed that for H it was implied that the angle of friction was 45 deg. and below this point it was greater, so that the pressure distribution suggested was an impossible one which indicated that elastic theory could not be used in this case.

Dr. Eastwood suggested that the only way to find the bearing capacity on the sand was to do a loading test. Although Dr. Golder agreed that loading tests were of great value, when carried out on large footings they were exceedingly expensive, and most engineers would not face up to this expense. Soil engineers had not yet been able to provide the best solution to this problem.

Dr. Golder suggested that to use a factor of 0.75 for inundation instead of the theoretical 0.5 was an extremely risky thing to do if based simply on the test results

given in this paper. There was always the chance that some engineer reading the paper might carry out this suggestion, design a building based on it, and that an accidental inundation of this foundation might cause the collapse of the building.

Dr. B. H. KNIGHT (Member) said that he felt that after Dr. Golder there was not very much for him to say, but at the Air Ministry they were interested, when they read on the first page of the paper that the results would be useful in designing road and runway pavements on frictional soils by the Golder method. He thought at first that he was perhaps in some agreement with the author, but somehow he could not see it as he went along. Fig. 12 rather shook him. He felt that the "scatter" on that figure was such that he would hate to make any deductions at all from it; he looked at it, and decided it was not right, and he would ask Dr. Eastwood if he really thought he could draw any conclusions from such a scatter?

He had listened very carefully to Dr. Meyerhof's remarks and that was his own line of thought, and he thought others might have a similar point of view, because they read on page 6, "The ultimate load was not nearly so distinct for the square and circular footings as in the tests on rectangles." When he had finished reading the paper he was still in a fog. There must surely be some connecting link between those three shapes of footing. That was very important, so that he asked for a further explanation of page 6.

He had read the paper several times, but the fact was that he was still quite unable to accept the conclusions. His physics at school had prepared him for the effects of inundations on bearing capacities of soils, but he felt it was physically impossible to get a reduction in bearing value of the small magnitude quoted by the author. It was extremely important from a design point of view if it was right. When he reached that point he felt he had missed something in the reasoning, which he could not see. Perhaps the author could give them some explanation, which he had not been able to find in the paper, for that great divergence from the accepted reduction in bearing capacity caused by flooding of a soil.

There was one final point, and he thought it applied to a great deal of other research work in that field, done with a rigid type of bearing plate. He would like to know if the author thought he would get results anything like that if his stresses were applied hydraulically to a river dam. It was stated in the first paragraph that those loads applied to any loaded surface. Aeroplanes did not have steel plates underneath them when they came down. He would like to know how much those results applied in the case of a flexibly applied load, such as an aeroplane tyre. He hated to have to draw attention to it, but runways were mentioned, and at the Air Ministry they would like more information about that.

He thought the clarity of the paper was the only thing he had got out of it. It was extremely clearly written, and even if the ideas were wrong, which he thought they were, he would recommend it for study purposes.

Mr. P. L. CAPPER (Associate-Member of Council) thought the previous speakers had dealt fairly exhaustively with some of the main controversial points in the paper, but there were one or two things he would like to bring up. There was the question of applying these results to a runway, and he thought even Dr. Golder would agree that the conditions there were quite different from those in the tests. In these there was a definite loading on a precise area, and it was allowed to penetrate into the surface of the sand, and the sand slipped up on one or both sides, which it could not do in the case of a runway. The results of the paper, however good they might be, could not be applied very accurately to those conditions.

Some of the speakers had mentioned the question of the relatively small size of the footing used in those tests. Although a lot of valuable information could be obtained from small scale tests, factors like the variation in density and other conditions probably had a relatively much greater effect on the results than they would have on bearing tests with a larger area.

Turning to Fig. 11 which showed some rather curious curves he could not help thinking that if he had been confronted with a series of points as shown there, they would have indicated a straight line to him—one for the

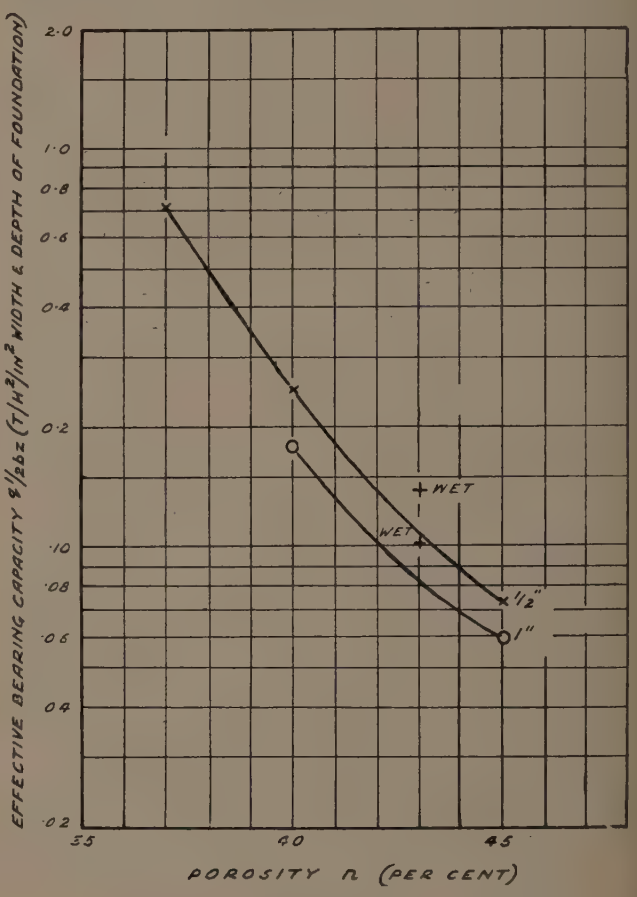

Fig. 3

squares and rectangles and the other for the circles, in this form—(and Mr. Capper drew on the blackboard the two lines in question).

The author had mentioned the question of the sand heaving up on one side or both sides and suggested that the Terzaghi and Pradtl theories were not applicable because they assumed heaving up on both sides. He suggested it was a question of symmetry of loading. If they could have ideal conditions, loading absolutely axial and the material perfectly homogeneous one would expect heaving up on both sides. In those tests and in practice there was always some slight variation from those conditions and therefore one-sided failure was much more likely, as had been shown by the tests.

Towards the end of the paper the author introduced some of the elastic theory, which he thought had been commented on by Dr. Golder. He suggested that for sand on the point of failure the assumption of elastic conditions was not accurate, and also, he would say that the theory of plasticity was also inaccurate in that case.

It seemed to him that with that sand two things might happen. They might have bodily displacement of a volume of sand which actually heaved up. That seemed to indicate that a chunk of sand might be pushed up, but in some part of the area of failure there must be movement of the grains over one another—granular flow—of which he thought no one had ever tried to apply a theory of that type. As he saw it the theories which had been put forward by Prandtl and others assuming some sort of slip failure and trying to account for it, were probably the best they could do and it remained for the experimenters to discover whether those assumptions were near the mark or not. The author had made a very useful attempt at solving that problem and he did not wish to discuss the question of whether he had got the right answer or not. That had already been fairly discussed by other speakers.

There was one other point—the direction in which the footing turned when the failure took place.

(Turning to the blackboard, Mr. Capper illustrated his point with a diagram.) If one had a building which failed owing to the material heaving up it would, he thought, only fail in one direction. If, however, it was

Fig. 4a.—Rectangular footing

restrained at the top, as would be the case in that experiment, then it would tend to swing round and the opposite corner would go down.

Mr. B. L. Clark (Associate-Member) thought all these experiments were based on the assumption that the sand would heave up on either side of the foundation ; surely there were few buildings where such things could happen, the sand being restrained ; even on runways, the slab on either side restrained the soil from heaving up. A different action must therefore be the case with sand under compression only and would therefore alter the bearing capacity of the sand considerably, the paper did not seem to refer to these conditions. The only time conditions such as that indicated in the paper could apply was on a flat surface such as a desert or similar case where there was no earth or restraining soil on either side of the footing. Perhaps Dr. Eastwood could explain and indicate the actual bearing value of sand when it was restrained on either side.

Mr. John Faber (Associate-Member) said it was mentioned in the conclusions that a loading test was a safe basis for designing footings, but Dr. Golder appeared not to agree with this. He had said that the soil mechanicians should be able to do something better than rely on tests and should be able to do something by way of calculation. That, Mr. Faber thought, was the meaning of Dr. Golder's remarks . . .

Dr. Golder interposed to say, "Not quite." He did not imply that they had got the answer yet.

Mr. John Faber said that that confirmed what he had already believed, and that was why he could not understand Dr. Golder's objection to the use of the loading test, the cost of which was normally a very small proportion of the total cost of a project and indeed in many cases gave information which enabled there to be a saving in cost.

In Mr. Gower Pimm's excellent paper, "The Limitations of Soil Mechanics as Applied to Foundations,"* Engineers had been warned of the difficulty of getting proper samples and the unlikelihood of these small samples being representative of the large mass from which they had been taken. No doubt this was what Dr. Eastwood had in his mind when he said that the practical Engineer's way of dealing with this problem

Fig. 4b.—Square Footing

was to make a loading test wherever possible and see just what happened, and he had not pretended to rely too much on those long formulæ, so popular in some fields, which many people admitted were not yet fully understood.

Dr. Eastwood's Reply

In replying to the discussion, Dr. Eastwood thanked Mr. Bullen for his kind vote of thanks. Mr. Bullen was not alone in trying to understand what was meant by ultimate load since it was difficult to realise without seeing an actual test that a peak load could be reached as the foundation was pushed further into the soil. That was the case, however, and for rectangular footings the load which the footing could withstand would suddenly drop at the instant when definite planes of sliding were first formed.

In the discussion there had been no disagreement with one of the two main conclusions, the one concerned with the mechanism of failure. It appeared from Dr. Meyerhof's report that failure had occurred in exactly the same way in his tests. In view of this it seemed unfortunate that Dr. Meyerhof was trying to

*THE STRUCTURAL ENGINEER. *Vol.* XXVI, *No.* 10, *p.* 589.

propagate his plastic theory in which the assumed mechanism of failure was very different from the observed phenomenon. Dr. Meyerhof was to be congratulated on the elegance of his method, but it had defects which were just as serious as any in the so-called elastic theory. Even if the soil behaved in the same way as mild steel, exhibiting perfect elastic properties up to a definite yield point and then becoming almost perfectly plastic, it would not be possible to forecast the stress distribution and the boundary of failure zones under the foundation. When the problem was complicated by the soil having a non-linear stress-strain relationship up to the "yield point," a sudden reduction in the shear strength of an unknown value after the "yield point," and the unknown effect of dilatance on the stresses in the surrounding areas of soil, it was quite futile to suggest that any plastic theory was correct, even if the boundaries of failure zones fitted those in the experiments. In Dr. Meyerhof's case his arbitrarily chosen boundaries did not fit those observed in the experiments.

Dr. Golder had been quite right to point out that locally a value of $\phi > 45°$ would have to exist for the author's stress distribution on the face of the wedge AB of Fig. 20 to be correct, and that the stress distribution must be modified, therefore, by plastic flow. It was, however, generally considered quite legitimate in many soil mechanics problems to ignore local overstressing in shear and to assume that neighbouring areas which were understressed took part of the load. The ϕ circle method of analysing the stability of slopes was a good example of this. It must be borne in mind too that Dr. Eastwood did not claim that the statical analysis was exact. In introducing the paper he had said "the calculations were intended to serve more as an indication of feasibility than as positive proof."

Turning to the question of the reduction in failing load caused by inundation, Dr. Meyerhof had said that his results had agreed with the generally accepted theories. It should be pointed out, however, that there were many serious defects in the design of Dr. Meyerhof's experiments. First, Dr. Meyerhof had not carried out his inundated tests at the same density as his dry sand tests, and had to interpolate between two bearing pressures obtained for dry sands, one of which was more than four times the other. Secondly, he had not measured ϕ in the completely inundated state, and if in the case of his sand ϕ had been reduced by only 1° by inundation, his measured bearing pressures would be affected by as much as 20 per cent. Finally, his experiments had been carried out on a very small scale, the footings being only 0.5 inches and 1 inch wide. Dr. Meyerhof had reported elsewhere that oil on the base of these footings had reduced the bearing power considerably, whereas this was not the case with the author's larger footings. Might it not be that the water had acted as a lubricant also and reduced Dr. Meyerhof's values?

Even if Dr. Meyerhof's experimental results could be taken to be without defect his analysis of them was a little difficult to understand. Referring to Fig. 2 shown by Dr. Meyerhof, it would be seen that for the two buried footings (the driven one should not be considered because it was not known what the relative effects of driving the footings in dry and wet sands would be) the bearing power on the "wet" sand was quite considerably above the curve obtained by multiplying the bearing values for dry sand by 0.6 Even as Dr. Meyerhof had drawn these curves, the wet results for the 0.5 and 1 inch footings were about 18 per cent. and 11 per cent. high respectively, although at first sight this fact is rather masked by the logarithmic plotting. Further, not everyone would draw the two curves as Dr. Meyerhof had drawn them. The author's version would be as in Fig. 3, from which it will be seen that his "wet" points are 25 per cent. and 20 per cent. too high respectively. Thus the results are nearer to agreeing with the author's than they are to the theory which Dr. Meyerhof says they confirm.

Further doubt is cast upon the interpretation of Fig. 2 by the fact that Dr. Meyerhof has rounded off the porosity values to the nearest 1 per cent. before plotting. It can be seen that if the porosity of any of the points on the "dry" curve is altered by only 0.5 per cent. the interpolated bearing value corresponding to the "wet" tests is affected by as much as 10 per cent.

Dr. Meyerhof had also stated that further reducing the effective weight of the soil by subjecting it to upward seepage gave results in fair agreement with the theory. Dr. Meyerhof's results were not published here, but the author had seen them and all the results at low rates of seepage were too high, in some cases the average for a particular rate of seepage being 15 per cent. out.

To sum up, the author thought that not only were Dr. Meyerhof's experiments badly designed as far as the inundated tests were concerned, but his results were capable of a very different interpretation from the one he had put on them.

Dr. Golder had also carried out a few small-scale tests on a single 2 in. diameter footing and a rectangle $4\frac{1}{2}$ in. by $1\frac{1}{2}$ in. These tests were a repeat of some tests carried out some time ago, and since no further details were given of the method of test it must be assumed that the testing technique was the same. In the earlier tests it had not been possible to compact the sand in the wet state to the same density as in the dry state, and as Dr. Golder had just stated himself, this would reduce the bearing power in the wet state enormously.

Further, it would appear from his description of the tests that the wet sand was compacted first and then inundated. It was virtually impossible using such a procedure completely to saturate the soil and it was possible that sufficient air was trapped in the sand seriously to alter the effective density.

There was also the point that Dr. Golder's sand was not very compact, whereas the author's sand was very dense. As the author had said in his introduction, it was not possible to generalise on the behaviour of sand at different densities from one set of tests on dense sand and there was always the possibility that despite the shortcomings of Dr. Golder's tests, his results were just as correct for the conditions of test as the author's.

Regarding the size of the box in which the tests were carried out the author had been assailed on two sides, first by Dr. Meyerhof, who said that the author's box was too small for the 4 in. footings, and then by Dr. Golder, who insisted that an even smaller box had been big enough for tests on 6 in. footings. The truth, of course, lay between the two. Dr. Meyerhof's rules regarding the size of box had been obtained using very small footings, and as was generally accepted the disturbed area of sand decreased relative to the size of the footing as its size increased. The author had no doubt whatever, and Dr. Meyerhof would agree if he saw one of the tests carried out, that the 4 in. footing failure surfaces were not interfered with in any way by the boundaries of the box in the present tests.

It was also equally obvious, as stated in the paper, that the box was too small for 6 in. footings, and he thought that Dr. Golder had probably been misled by the mechanism of failure being completely changed in his even smaller box.

Dr. Golder had made some very important remarks regarding the necessity for maintaining the density of the sand at a constant value in tests of this sort. This had been realised from the beginning and in the present tests the density had not been allowed to vary by more than 1/20 lb. per cubic ft. from the first test to the last. Dr. Golder had misunderstood the purpose of the density tin tests, which was merely to show whether extra compaction was needed in the corners and near the edges of the box to get the same density as in the centre. The density tin had given the same average density wherever it was located. The variations in the individual measurements did not reflect an actual change of density which was known to be constant, but were merely the product of experimental error.

Dr. Golders' fear that placing the paper strips might have upset the results was not justified. The disturbance of the sand was very local, and the average test result when using the paper strips had been almost exactly the same as when no strips were used.

Referring to Dr. Knight's contribution, the author said that he could not understand Dr. Knight's difficulties about the reference to pavement design. All that was said in the paper was that the test results would be useful in connection with any method of pavement design, such as for example the Golder method, in which the runway thickness was related to the ultimate bearing capacity of the soil. This statement was so simple that it was almost platitudinous. The merits of different methods of pavement design were not discussed and Dr. Eastwood had not taken sides in the paper. The question of the possible effect on the bearing capacity of applying a load through a plane tyre instead of a steel plate should really be directed at Dr. Golder. But although the author agreed that this was one of the inherent weaknesses in the Golder method of pavement design, that method must have some semi-empirical value at least, and it was up to people like Dr. Knight who were engaged in runway design and construction to find out what that value was by experiment.

With regard to the scatter of the points in Figs. 10 and 12, Dr. Eastwood thought that there was nothing unusual. If Dr. Knight had carried out tests of this kind he might think that the scatter was far from abnormal as the published results of Dr. Meyerhof and Dr. Golder would bear witness. Regarding the interpretation, all that had been done was to take the mean value. It was as simple as that.

The statement on page 6 regarding the interpretation of the ultimate load could best be explained by reference to Fig. 4. Part (a) showed the relation between load and settlement for a rectangular footing, and part (b) showed the same relation for a circle or square.

It was often quite difficult to determine the ultimate load for these latter, but there was never any difficulty with the rectangles.

Dr. Eastwood thought everyone would agree with Dr. Knight that it was a pity that school physics did not solve all our engineering problems. Dr. Knight could surely see, however, that any factor which was not affected by buoyancy could prevent the ultimate load of the foundation from being reduced proportionally to the loss of effective density. A simple example was cohesion, although that was not playing an important part in this case. In this instance what upset the ordinary physical reasoning was the fact that the pressures on the sand were lower, and hence the plastic deformations in the sand would not be geometrically similar in the two types of test. This had resulted in dissimilar stress distributions and hence slightly dissimilar failure mechanisms.

Mr. Capper and Mr. Squire had asked about the direction of tilt of the footing. As Mr. Squire had pointed out, the loading arrangement as shown in the photographs might impose some restraint on rotation. A number of the tests had been carried out, however, with the proving ring turned through 90° so that restraint was not possible and the amount of tilt was not affected. Some recent tests to determine the amount and direction of tilt had shown that at the instant of failure, the footing might be tilted either towards or away from the direction of sliding, and did not in general exceed $\frac{1}{2}°$. For all practical purposes there was no rotation either way. If the footing were driven further into the sand after the ultimate load had been passed, it began to tilt away from the surface of failure even if at the instant of failure it had been tilted the other way. Again the amount of tilt was very small, amounting to 2° or less when the footing had penetrated a depth equal to half its width (i.e., considerably deeper than at the ultimate load). After failure, therefore, the direction of tilt was always opposite to that of the Winnipeg grain silo mentioned by Mr. Squire, but it should be pointed out that the silo was founded on clay, not sand, and that its mode of failure is generally regarded as unusual since the normal rotational failure of a foundation on clay was prevented by an increase in the strength of the strata with depth.

Both Mr. Clark and Mr. Capper had asked about the effect of surcharge, particularly in the form of a pavement. This question was rather beyond the scope of the present paper, but just as the Terzaghi and other theories could be modified to allow for surcharge, it should be possible to allow for it in considering the equilibrium of wedge ABC of Fig. 20. First, however, it would be necessary to carry out further tests to determine whether the shape of the wedge was altered when surcharge was applied. No attempt had been made by anyone to allow for the effects of tensile and shear strength when the surcharge was in the form of a pavement, and it seems that the solution will only be obtained by experiment under field conditions.

Mr. Squire had commented that he thought the tests should be repeated to a larger scale. The author agreed heartily with this view. Mr. Squire's comment on the seemingly high bearing powers given by extrapolation of the small scale test results was also justified, but the author pointed out that, as stated in the paper, bearing pressures for large foundations on sand were determined by the permissible settlement rather than ultimate failure.

Dr. Golder's statement that it should be possible to do something better than rely on bearing tests appeared to have puzzled Mr. Faber as much as it had puzzled the author. Surely there would never be a *better* solution than bearing tests, and whatever the expense, they *must* be carried out in any instance where accepted theories showed that there was the slightest danger of overloading.

A Fundamental Approach to Prestressed Concrete Design

Discussion on Mr. J. W. H. King's Paper*

Introduction

The AUTHOR, introducing the paper, apologised for inflicting on a long-suffering profession yet another method of dealing with prestressed concrete, but explained that he had been horrified at the tendency to overwhelm the subject with formulæ and notations, to say nothing of fundamentally unsound assumptions, such as that of using the standard moment of inertia for a beam for calculations outside the non-linear stress strain range. As his job was to teach students, with whom he had already taken some pains to train to think for themselves, he just dared not try to teach them the current unsound theories, and evolved the present technique as a theoretically sounder method.

He then ran rapidly through one or two examples from the paper, and went on to give in addition a full calculation for a non-rectangular beam built up of rectangles, and an indication of the methods of allowing for self weight moment, and the cavity for steel, and of estimating rapidly and conservatively the cracking moment for a designed beam.

He explained that the stresses used in the paper gave a compromise between heavy concrete and heavy steel, and that they could be modified by the designer, as could the stress/strain curves, if more acceptable values were available. He also showed that a more deformable concrete, if proved, would lead to economy of both steel and concrete, for the normal range of designs, whilst the more rigid it was assumed the safer the design. It was thus of importance to use the curve with the biggest strains that would be certain to occur under the most rapidly loaded conditions with concrete of the quality proposed for the design, if maximum economy commensurate with safety was desired.

Discussion

Dr. P. W. ABELES (Member) felt that Mr. King's method represented an interesting approach which should be of interest for laboratory investigations when exact stress/strain curves were available. It should be useful, if applicable, for determining the ultimate load for over-reinforced beams and it would be interesting to investigate whether it would be successfully employed for non-bonded steel for which there was no simple design method available. The speaker agreed that non-bonded steel should be avoided and that the modular ratio should not be re-introduced, since it caused only unnecessary complications. However, as far as under-reinforced sections were concerned, Mr. King's method was rather lengthy, even for simple rectangles, and finally led to approximately the same results, as the simple approximation suggested by the speaker by which a rectangular concrete compressive stress distribution of the prism strength was balanced with the ultimate steel resistance. This had been ascertained, when Mr. King calculated by his method the failure loads for the various beams tested at the Brixton School of Building in 1949/50[1], the results of which agreed with Dr. Abeles' approximation and the actual test results.

When considering the application of Mr. King's method to the design of sections, objection must be made to his approach. He assumed as basis for his computations the position of the neutral axis at failure and positioned the steel rather far away from the tensile fibre, which was not economical. Professor R. H. Evans pointed out as early as 1942[2] that it was advisable to place the tensioned steel members as near as possible to the outer tensile fibre, at the same time providing tensioned wires near the compressive fibre if necessary. When considering example No. 2 of Mr. King's paper and assuming a rectangular section of a depth of D = 18 in. (instead of 18.15 in.), one would place the steel near to the bottom fibre, say at a distance of 3 in., resulting in a depth of the steel from the top fibre d = 15 in. Provided that 14 post-tensioned wires 0.2 in. diameter of a strength of 210,000 lb. per sq. in., as in example 2, were provided, and that the reduction due to inefficiency of the grout amounted to 0.95, the ultimate tensile resistance of the steel amounted to T_{ult} = 0.44 × 210,000 × 0.95 = 88,000 lb. When balancing the forces the depth of the concrete stress distribution n_p could be computed from $C_{ult} = n_p.b.c_p = T_{ult}$ as $n_p = \frac{88{,}000}{6 \times 6000} = 2.53$ in. (see Fig. 11), where c_p = 6,000 lb./in.^{2} was the prism strength, resulting in a_p = 15 — 1.26 = 13.74 in. and M_{ult} = 13.74 × 88,000 = 1,210,000 lb.,in. ensuring a factor of safety $FS = \frac{1{,}210{,}000}{537{,}600} = 2.25$. This should suffice, but if required, 16 wires, instead of 14, of a steel area of 0.566 sq. in. could be provided, ensuring a factor of safety of 2.55. When comparing these steel areas of 0.44 and even 0.566 sq. in. with 0.768 sq. in. according to Mr. King's example 2, one could see the possibility of saving steel if the required steel area was placed at its suitable position, instead of assuming a position of the neutral axis at failure as basis for the design according to Mr. King's approach.

It was also important in Dr. Abeles' view to investigate the state of stresses under working load conditions. Mr. King had stated that he did not like to assume an elastic behaviour when considering the stress/strain curve for concrete. However, from the diagram plotted by Mr. King it was seen that under the stresses corresponding to the working load the curve was still straight and it could therefore be well assumed that a straight line stress distribution of a homogeneous material was appropriate. This was also proved by many test results. Now, when investigating example 2 for working load conditions, the following equations could be set up, using the symbols and formulæ recommended by the speaker[3].

Read before the Institution of Structural Engineers at 11, Upper Belgrave Street, London, S.W.1, on Thursday, January 25th, 1951, at 6 p.m., Mr. Walter C. Andrews, O.B.E., M.I.C.E. (Vice-President), in the Chair. Published in THE STRUCTURAL ENGINEER, *Vol.* XXIX, *No.* 1, *p.* 12. (*January*, 1951.)

[1] *"Small scale tests on prestressed concrete beams" by P. W. Abeles and C. H. Hockley,* THE STRUCTURAL ENGINEER *June* 1950;

[2] *"Relative merits of wire and bar reinforcement in prestressed concrete beams", by Prof. R. H. Evans,* JOURNAL INST. OF CIVIL ENGINEERS, *No.* 4,1941-2, *February.* 1942;

[3] *"Principles and practice of prestressed concrete" by P. W. Abeles, Crosby Lockwood and Son Ltd., London* 1949 & *E. F. Unger Publ. Ltd., New York* 1949.

According to Fig. 1, des section modulus Z could be approximated to $\frac{6\times18^2}{6} = 324$ in.^{3}, when ignoring the hole of the cable, resulting in a stress $\frac{M_w}{Z} = \frac{537{,}600}{324} = 1{,}659$ lb./in.^{2}. If this section was to be fully prestressed and the reduction of the prestress due to shrinkage and creep was to be assumed as 10 per cent., (reduction factor $R_o = 0.90$), at the bottom fibre of the rectangle at transfer stress $f_{1t} = \frac{1659}{0.9} = 1{,}843$ lb./in.^{2} was required, while with partial prestressing permitting a tensile stress under working load $f_{tw} = 600$ lb./in.^{2} the required prestress at transfer was reduced to $f_{1t} = \frac{1659 - 600}{0.9} = 1{,}176$ lb./in.^{2}. To compute the prestressing force, special factors k_1 and k_2 were advantageously introduced, viz., for the rectangle in question, when ignoring the hole of the cable : $k_1 = 1 + 3\frac{e_s}{\frac{D}{2}} = 1 + 3\frac{6}{9} = 3$ and $k_2 = 1 - 3\frac{6}{9} = -1$.

It was seen that the tensile stress at the top fibre amounted to a third of the compressive stress at the bottom fibre, as indicated in Fig. 1 for full and partial prestressing. A tensile stress of 614 lb./in.^{2} might be considered as too high, in which case a small area of tensioned steel at the top wire would suffice to reduce this stress. It was possible to calculate the prestressing force required at transfer P_t from the following relation $f_{1t} = \frac{k_1.P_t}{A}$, where A was the concrete area $6 \times 18 = 108$ in.^{2}.

For full prestressing, requiring $f_{1t} = 1{,}843$ lb./in.^{2}, a prestressing force of $P_t = 66{,}350$ was required, corresponding to a sectional area of tensioned steel of $A_{ts} = 0.44$ in.^{2} for a tensioning stress at transfer $p_t = 150{,}000$ lb./in.^{2}. In this case the same area was necessary to obtain the required prestressing force and to ensure a factor of safety against failure of 2.25. In the case of partial prestressing, requiring $f_{1t} = 1{,}176$ lb./in.^{2}, a prestressing force $P_t = 42{,}350$ lb. was required, corresponding to an area $A_{ts} = 0.284$ in.^{2} (9 wires 0.2 in. dia.). In this case nine tensioned and five untensioned wires were required. It might be pointed out that a bending tensile stress of $f_{tw} = 600$ lb./in.^{2}, was very moderate and ensured freedom from cracks even under repeated loading. A test had proved that freedom from cracks was ensured even under sustained loading during 30 days of a load, corresponding to a bending tensile stress of 775 lb./in.^{2}. Mr. King's statement that the effect of allowing tensile stresses was small was not correct. In fact, when considering the economy, both the shape and the size of the section were of great influence, the rectangle not being a very suitable section for prestressed concrete.

Mr. KING commented that Dr. Abeles, being a good friend, had warned him of his intentions ; so that he had been able to prepare some answers in advance.

Dr. Abeles had worked to a factor of safety of 2.25, for his design example, and needed only 0.44 sq. inches of steel, which he had put three inches up from the bottom of an 18-inch beam. Mr. King considered what

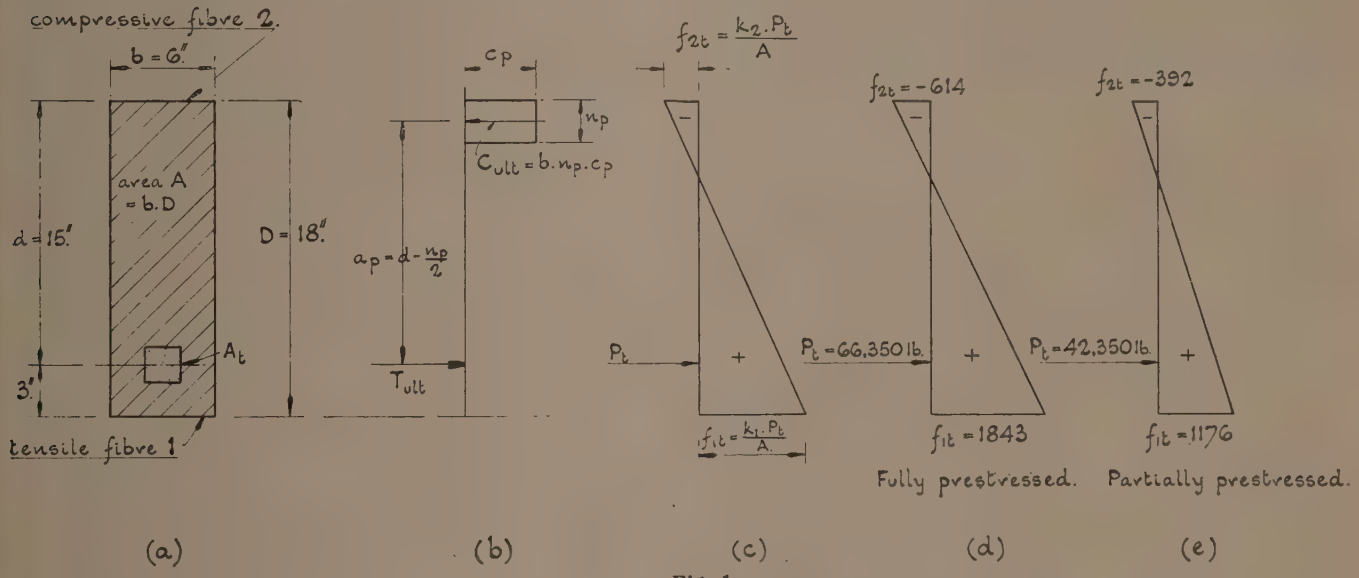

Fig. 1

would happen if he re-calculated some of his examples, using different stress/strain curves for the concrete. In the paper he had commented on the impossibility of using high steel stresses when working with a low neutral axis on the short term curve, but in his introduction he had indicated higher ultimate steel stresses could be used with a more plastic concrete. He went into the case of a neutral axis only 0.2 d. down, using first a concrete curve with 0.002 maximum strain and, secondly, one with 0.004 maximum strain. In either case the total compression worked out at 4800 d., and the lever arm worked out at 0.925 d. So that if one equated to 2.25 times the working load moment, instead of $2\frac{1}{2}$ times, as he had done, i.e., to 121,000 lb. in., one would get $d = 16.5$ in. That did not depend on the particular stress/strain curve used.

Coming to the correlation, one found that, using the same concrete stresses as in example 2, i.e., permitting 2,200 lb. per sq. in. as maximum compression in concrete at transfer, but using first of all 0.95 of the ultimate stress for steel, i.e., 200,000 lb. per sq. in., the following figures resulted. The strain in steel at failure was 0.019, which was quite a large figure.

The steel area required was 4800 × 16.5/200,000 = 0.4 sq. in. Using 0.002 in the concrete as a maximum strain, the strain change would be 0.00028 from transfer stresses to zero and 0.002 × 4 for subsequent loading to failure, giving a total strain change of 0.00828. The difference was 0.01072, which led to a transfer steel stress of 184,000 lb. per sq. in. and a total tension of 73,600 lb. The transfer compression zone depth worked out to 11.0 in. with 3.71 in. below the steel, so that he arrived at a total depth of 20.21 in.

In the slowly stressed laboratory case, using 0.004 strain in concrete at 6,000 lb. per sq. in., the strain

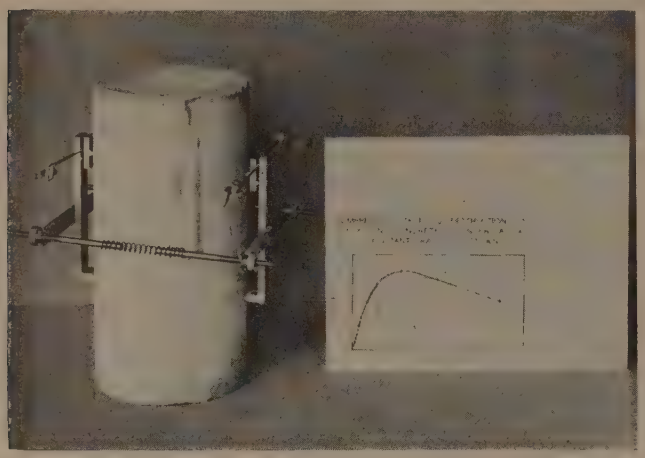

Fig. 2

change would be increased to 0.0166, which, taken from the final steel strain of 0.019 resulted in an initial steel strain of 0.0024 corresponding to 68,000 lb. per sq. in., and a total tension of 27,000 lb., "h" was then equal to 4.05 in. and the total beam depth 16.5 + 1.37 = 17.87 in. Steel area was still 0.4 sq. in., so that the design was lighter than that of Dr. Abeles.

It was easy to see that by using concrete so slowly stressed under load that the strain at steel level amounted to 0.019 (i.e., a maximum strain at 6,000 lb. per sq. in. of 0.00475 in the top fibre, which is quite a possible figure) the necessity for prestress did not exist. A slightly higher neutral axis would have the same effect.

Again the figures emphasised the importance of taking in the proper value of the stress/strain relationship for concrete, and that was the main point on which he disagreed with Dr. Abeles' method of design ; Dr. Abeles had ignored the stress/strain relationship, assuming that the concrete would behave as he wanted it to behave, whatever the stress/strain curve. So far he had got away with it ; none of his structures had fallen down !

Provided he used his present limits to the application of his method, they were not likely to do so. Danger would only arise from unrestricted application to heavily reinforced beams, which Dr. Abeles did not allow for that very reason.

Mr. King still maintained that to use *I* as Dr. Abeles had done was unsound, and that the effect of allowing tension in concrete was dangerous. Analysis in that way might lead to apparent tensions of, say, 700 lb. per sq. in., in two instances, which analysed on a cracked section basis indicated sound design in one case, and unsound in the other. It would thus be dangerous to say that because the former did not fail that concrete could always be allowed to take 700 lb. per sq. in. in tension. He much preferred safety based on sound principles, and a recognition that concrete was not a good tensile material.

It was agreed rectangular sections were far from ideal for prestressed concrete work. They were used for a first approach because they provided easier examples of the design method. An example of more orthodox design had been given during the introduction of the paper, and had also been published elsewhere.*

Mr. H. J. COWAN (Associate-Member) referred to Fig. 3 of the paper, where the curves showed several features which did not accord with his own experience. In the first place a concrete block loaded at a fast rate would give a much higher maximum stress than a block loaded at a slow rate[1], [2]. Mr. King's maximum stress did not appear to be affected by the rate of loading, and that had a bearing on the solution of the numerical examples. In the second place, Mr. King's curves showed a steady rise to a maximum stress, where the curves stopped. This was what one would expect to obtain from a concrete block tested at a *constant rate of stress*, but it did not represent the behaviour of the concrete in a beam. One assumed that plane sections remained plane, and therefore the strain varied from a maximum strain at the top of the beam to a zero strain at the neutral axis. To obtain the stress distribution corresponding to this constant variation of strain one must test the concrete block under identical conditions, i.e., at a constant rate of strain, and not at a constant rate of stress as was customary for standard testing.

If one tested at a constant rate of strain one obtained results differing very considerably from Mr. King's curves. Mr. Cowan then showed Fig. 2, pointing out the long tail of the descending portion of the curve after the maximum stress was reached. The curve was not a parabola, as Mr. King's curves might well be, but on the other hand it could be replaced by two straight lines without serious loss of accuracy.

Referring to Mr. King's use of Simpson's rule, Mr. Cowan said that the rule assumed that the curve might be replaced by the arc of a second degree parabola, and in using it Mr. King was therefore putting forward another parabolic theory which differed only in detail

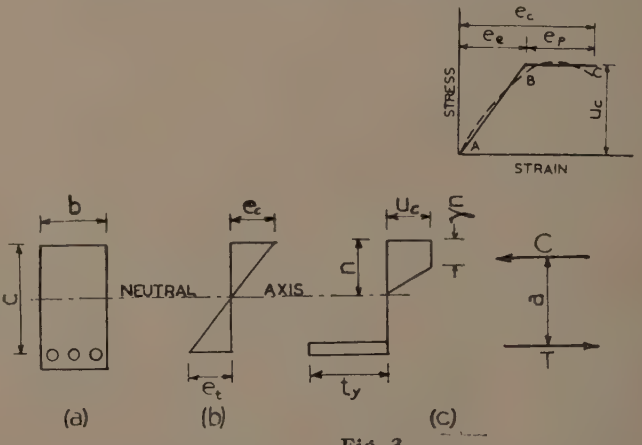

Fig. 3

(b) Strain distribution ; (c) equivalent stress distribution.

from Prof. Talbot's theory of 1906[3]. We had, however, travelled a long way in our approach to ultimate strength design since Prof. Talbot proposed his theory. Mensch[4] suggested a cubic parabola in 1914, Kempton-Dyson[5] proposed a quarter-ellipse in 1922, and Prof. Lyse[6] a fifth degree parabola in 1937.

Mr. Cowan did not think that any of these methods represented a solution to the problem of ultimate strength. Even if we succeeded in finding a single mathematical equation which adequately represented the stress/strain diagram of concrete, it was likely to

*"*The Design of Prestressed Concrete Beams from Fundamental Principles."* CONCRETE & CONSTR. ENGINEERING, *September*, 1950.

lead to results which were excessively complicated, as did Mr. King's method, as soon as he progressed from the simple rectangular section to the more complicated *I*-section commonly used for prestressed concrete beams.

A solution was more likely to be obtained by concentrating attention on one or two simple properties of the stress/strain curve rather than on the whole curve. One could, for instance, measure the ratio of the total elastic strain to the total inelastic strain, and then replace the dotted curve by two straight lines (Fig. 3 i), and the curved stress block by a trapezium (Fig. 3 ii)[7] and [8]. Or one could replace the curved stress block by a rectangle

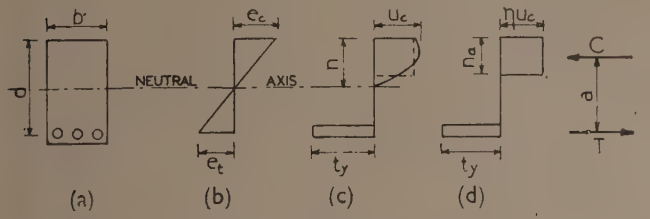

Fig. 4

(b) Strain distribution ; (c) stress distribution ;
(d) Equivalent rectangular stress block.

of the same area as the curved stress block and with its centre of gravity at the same depth.[9], [10], and [11]. The rectangle was then fully equivalent to the curved stress block (Fig. 4).

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[10]K. Hajnal-Konyi, Concr. and Constr. Eng., Vol. 32, pp. 11-26, 129-32, and 189-208 (1937).
[11]C. S. Whitney, Trans. Am. Soc. Civ. Eng., Vol. 107, pp. 251-326 (1942).

Mr. KING, replying to Mr. Cowan, reiterated that he had obtained his curves from his own work in the laboratory, and they had fitted experimental beams ; they had been idealised a little, and the obvious idealised curve to fit the experimental data was the parabola ; they just happened to fit that way.

As stated in the paper, he was not at all concerned whether one used those curves or used something else which one had proved to be better. The method of approach was not dependent on the shape of the curve, and he had never said at any time that the curve was a parabola ; Mr. Cowan had inferred that, and had inferred it rightly.

About the straight line idealising of the curve, he said one had to be somewhat careful in using a straight line elastic portion and a horizontal straight line plastic portion, because one could not then work with one curve, but must have a series, since at very low loading plastic flow was still present. There was a marked difference with ultrasonic testing between the dynamic modulus and the initial tangent modulus one obtained by standard methods which indicated plastic flow at very low loads. Simpson's rule actually fitted cubic curves for the area, but the formula given was then not quite accurate for the centre of gravity. Both were accurate enough for design purposes using any smooth curve.

With regard to the non-rectangular section, he had stated in the paper that with a rather irregular section one could not calculate the total compression and bending moment quite so easily. For sections not easily split into rectangles one should plot a breadth stress curve, and it was then perfectly easy to use it in the same way as the stress curve had been used in the examples given. He had also stated in the paper that, if one did not consider the approximate methods given were satisfactory for the centre of gravity and area, one should use more rigorous methods. There was not sufficient space available to enable him to put everything in the paper ; but he emphasised again that he had not tried to pin his method down to any particular curve, and that one should use whatever curves one considered to be applicable to the case.

The point about which he was most concerned was that people should not go in for fixed values of the modular ratio, but should get an idea of what was happening to the structure and should obtain some correlation between ultimate and transfer conditions as a basis of design.

Incidentally, the concrete in the compression zone in a beam was not strained at a uniform rate, as stated by Mr. Cowan. The rate of strain was proportional to distance from the neutral axis, so that Mr. Cowan's contant rate of strain curve was no more correct than one with a constant rate of stress. Mr. King could only affirm that his curves had given the right answer in a limited number of beam tests, and were therefore considered to be of about the right shape for practical purposes. A maximum strain of 0.004 at 6,000 lb. per sq. in. was a typical figure for laboratory tests on beams and for cubes loaded at about the same average rate of about 50-60 lb. per sq. in. per minute.

Mr. O. J. MASTERMAN (Associate-Member) said that the approach to the subject by Mr. King was very refreshing ; one had enjoyed listening to him ; and it was important that engineers should examine very thoroughly such suggestions as he had put forward whereby we might overcome some of the difficulties experienced with reinforced concrete design and prestressed design in the past. Mr. Masterman considered that prestressing did provide considerable simplification of reinforced concrete design, and that perhaps it was not so very necessary, once we had washed out elaborate formulæ and notation, to look for much more simplification. But the fundamental approach was surely the right approach, provided it was simple and that it did not involve the making of assumptions which were less accurate than those already made.

Enlarging on that point, he asked, " Is this approach simple, and are the assumptions more accurate than those to which we have been accustomed ? " He liked the idea of using the stress/strain diagrams, of using the diagram obtained when one stressed the concrete. But the trouble arose that one was not quite sure which of the diagrams to use ; it made quite a considerable difference to the result. It seemed to him that, especially in connection with failure, one ought to use the long-term figures. He could not conceive of a failure happening in normal circumstances due to a sudden load ; failures were more likely to occur after perhaps a long period of overloading. That was the first difficulty.

There was also the point that Mr. King had imagined the steel and the concrete moving together, being constrained together, along the steel lines ; in other words, he assumed good bond. One supposed that in the majority of actual jobs one could not be sure that that was true. All the good designers who used post-

tensioning certainly grouted in, and they claimed that that was a very valuable " something up their sleeves," as indeed it was. But. Mr. Masterman did not think any of them claimed to know just what was the value of that bond. Thus there was perhaps another uncertainty.

On the question of simplicity, Mr. King had dealt with his calculations in an impressive way. But he had shown calculations for rectangular beams. Mr. Masterman could not remember having come across in practice any rectangular prestressed beam, and he pointed out that the calculations were not nearly so simple for other kinds of beams which were perhaps of " T " shape or some of the other odd shapes which were much more common.

There seemed to be some valuable advantages in Mr. King's proposed method, and perhaps the most important was that it meant that one was dealing with strains, and taking notice of strains, which in the past had been rather overlooked. Thereby one obtained a much better picture of what was happening to the concrete section or concrete unit ; and strains could be measured, which was very helpful for a check.

It was interesting that, throughout this paper, on prestressed concrete design, there was hardly a mention of creep. One supposed that the creep was really lost in the selection of the one stress/strain diagram out of the three which were put forward. Perhaps Mr. King would say something more about it.

It was of great benefit to be able to convert the strains quickly into deflections and stiffness values, and it was excellent, of course, if one could deal readily with composite designs by Mr. King's method.

From the examples given, Mr. Masterman agreed that the tensioning errors that one normally made in doing prestressing were not very important. That was found to be so at the Thatched Barn. They had started with a steel modulus of 30 millions, and afterwards they had made it 26 millions and had changed the extensions accordingly, but it did not make any significant difference to the strength of the components. Mr. Masterman felt, however—and his remarks applied to example 6—that the tensile stress in concrete was valuable and ought to be used in prestressing, although not in reinforced concrete.

After calling attention again to his point that the use of the long-term curve seemed more appropriate, although, as Mr. King had pointed out, it was not nearly so safe, Mr. Masterman said he hoped his remarks did not appear to be too critical, for he had very much appreciated the paper.

Mr. KING, dealing with the reference to the long-term curve, said Mr. Masterman's original statement and his last one did not quite tally. The point about using the short-term curve was that one ensured safe design. More economy and less safety resulted from use of the other, longer term, curves. The creep was the change from one curve to the other under no increase of stress, and the effect of that under heavy loading, approaching the ultimate, was to give safer design by relieving and redistributing the stress. He had given a rough example on the blackboard, when introducing the paper, where he had taken a beam as designed and had shown what would happen if he prestressed it quietly and allowed it to be tested over two hours, thus justifying the use of a longer term curve. The moment went up from 1,344,000 to 1,400,000. The reply to Dr. Abeles considered the matter from the design aspect, and showed the economy resulting from the use of long-term curves.

He was unable to answer the question concerning good bond ; one just had to assume something. If one assumed there was no bond, then one was back again to the non-bonded beam, and the problem would not be easy. He did not agree with Professor Baker's easy approach of that problem, for he felt it was a little wrong ; but he had not found a good substitute. Both he and Professor Baker were in agreement that it was a design to be avoided if possible.

It was more difficult to deal with non-rectangular than with rectangular beams ; but he had given an example during the introduction of the paper and he did not think the added difficulty was great. At any rate, it was less difficult to deal with such beams in the manner suggested than to try to work with a lot of formulæ.

Dr. K. HAJNAL-KONYI expressed agreement with Mr. King's rejection of the modular ratio in ordinary reinforced concrete design, and also in prestressed design, so far as the ultimate load was concerned. But he disagreed with Mr. King's fundamental approach, and his disagreement was fundamental. Mr. King had emphasised the necessity for investigating what actually happened in a beam, and that was the reason why Dr. Hajnal-Konyi objected to his method.

Discussing fully bonded beams, he said he had no personal experience of post-tensioning and did not know how far injection could secure bond, but he did know that in a pre-tensioned beam, with wires up to .2 in. diameter, one could fully rely on the bond. Thus, his remarks were based on the assumption that the bond was fully efficient, and Mr. King agreed with that.

In a well-designed prestressed beam, cracks occurred before failure, he continued, and he illustrated the appearance of a beam at that stage. When using bonded wires, the bond was not destroyed between cracks and it would be maintained fully until failure. Even in non-prestressed beams one could break the wires, which was evidence of full bond. Consequently, the stress distribution in the steel was by no means uniform ; between the cracks there were peaks of stress in the steel. That could be verified quite easily by checking the results of tests. The straight line strain relationship, which was the basis of both of Mr. King's paper and of Professor Baker's method has been proved by many experiments. That was perfectly correct if applied to a certain length of the beam, as would be found from strain readings. But he emphasised that it was an *average* strain, and one could not work out the maximum stress in the steel from the average strain. The theories of Professor Baker and Mr. King led to a very gross underestimation of the steel stress at failure. They created the impression that the ultimate load depended on the amount of prestress, although in fact it did not, except in the case of over-reinforced beams.

Dr. Hajnal-Konyi then showed the stress/strain diagram for high tensile wire. Within a crack the strain could be as much as 3 per cent., whereas the average elongation was sometimes not more than a small fraction of that amount.

Thus he considered that the whole approach based on the straight line relationship and on stress-strain diagrams, attractive though it might appear, did not agree with the facts obtained by experiment, and that was why he did not think it was acceptable as a basis of design.

The basis of design should be the ultimate strength in the steel, so long as it could be balanced with a rectangular distribution of concrete stresses of about 60 per cent. of the cube strength, not going beyond a certain depth. One could predict the failing load very accurately if one knew the ultimate strength of the steel and the prism strength of the concrete without

going into details of "short term" or "long term" stress-strain diagrams, which in most practical cases were not available anyway. With pre-tensioned bonded wires one was perfectly safe in using that assumption.

There was a point which Mr. King appeared to have missed in the paper. In his example of pre-tensioned work he had not taken account of the loss due to the elastic shortening of the concrete at transfer. It was of no significance in respect of ultimate load, but it had significance if one wanted to assess the cracking load.

Coming to composite sections, he believed that in every practical application of the composite construction the precast beam must be over-reinforced, whilst the composite beam was under-reinforced. He could hardly conceive of a case where the top of the original beam failed before the top of the *in situ* portion failed. Such a beam should always be so constructed as to use the full strength of the steel.

Mr. KING said he agreed with quite a lot that Dr. Hajnal-Konyi had said. Naturally the tendency was to get higher stresses at the cracks than anywhere else, but the increase in steel stress locally due to that was very small, due to failure of load on either side of the crack. He did not think one could always design on a basis of stressing steel up to its ultimate strength. One could manage it with very much under-reinforced beams, and he had shown the trend in his examples. One could quite often use nearly the ultimate stress. He had used 200,000 p.s.i. during the discussion because Dr. Abeles had suggested it in a problem he had put forward ; but one could equally go to 212,000 p.s.i., if there were a high-enough neutral axis at failure and a good deal of deformation before failure. The method would take into account anything of that sort and would stress steel to the ultimate, so long as one allowed ample deformation by slow stressing or the use of very deformable concrete, or a high enough neutral axis. He agreed that pretension in such a case, where a beam was under-reinforced, would not affect the ultimate load. It would crack at a different moment ; but that was a somewhat different viewpoint.

With heavily reinforced beams, however, concrete failure would normally occur before the steel stress increased very much above the transfer stress value, i.e., the beam would fail at a steel stress well below the ultimate. In such a case the amount of prestress would affect the ultimate moment of the beam.

As regards strain distribution across a section having cracks, one would often find on plotting strain against location in beam, that two straight lines resulted, there being a bend at the neutral axis. The strains below the neutral axis were lower than normal between cracks, and above normal at cracks, averaging out over a long enough length to the normal value given above the neutral axis, which was assumed for design.

The failure of a beam before a slab was possible. He had mentioned a case in the paper where that stage was almost, but not quite reached.

Elastic shortening at transfer *was* taken into account in example 5, in para. 3, a change in "h" resulting since concrete stress at steel level was arranged to be the same as example 2, after transfer.

The CHAIRMAN thanked Mr. King for the very able manner in which he had dealt with the discussion.

Written Communications

From Mr. J. BAK (Member) : I feel that Mr. King's approach to the subject is substantially correct and is in my opinion a step in the right direction.

I would, however, like to know to what extent have his assumptions been corroborated by actual tests carried out by him on prestressed concrete *beams*.

The AUTHOR replies : Mr. King thanks Mr. Bak for his complimentary remarks. His own tests have been confined to over-reinforced non-bonded beams, whilst he has also checked up on a number of Dr. Abeles' pre-tensioned and partly pre-tensioned beams both before and after test. These latter were mainly under-reinforced. In all cases, the agreement was very good, although full information on concrete was not obtainable in the case of Dr. Abeles' beams.

In the cases of his own beams, stress/strain curves obtained for cubes and for beams were found to agree very well, if the stressing rates were similar.

From Mr. S. JAMPEL : Mr. King states in his paper that the use of the $M = fZ$ formula in prestressed concrete calculations was leading designers into considerable difficulties, particularly when trying to assess the value of the modular ratio. Within the intended range of application of the $M = fZ$ formula, that is above the cracking load, the concrete stresses rarely exceed 3,000 p.s.i., and the stress-strain relationship as shown by Mr. King in Fig. 3 is practically linear up to this value. If we add this assumption to those stated by Mr. King (and Mr. King uses it himself in some of his examples) all fundamental objections to the $M = fZ$ formula disappear.

As far as the value of the modular ratio is concerned when calculating the equivalent Z (or I) of a section, it should be remembered that the majority of the steel is always located in the precompressed concrete zone. On loading, this zone is subject to relief of stress for which the stress/strain relationship yields an appreciably higher concrete modulus than for increasing stress, which in turn results in a very low modular ratio (4 to 6). As in addition in prestressed design the percentage of steel is very small, the total effect of the steel on the equivalent Z is only of the order of 10 per cent. or less. Thus an error of even 30 per cent. in the value of the modular ratio will only produce an error of about 3 per cent. in the equivalent Z, which is of the same order as the error liable to occur during the stressing operation and hence of little consequence.

I feel that one of the most serious drawbacks of the $M = fZ$ formula, which it has in common with most "elastic" design methods, is that it does not ensure a definite factor of safety against failure. At the same time I would like to add that I have never yet come across a practical case where it has yielded too low a factor of safety when using the usual design stresses. In this connection I was very much interested in Mr. King's method for calculating the failure bending moment, which is rather different from the usual rectangular block method. I note that Mr. King uses the 28 day cube strength instead of the prism strength and the failure stress in the steel dependent on the original prestress and increase of average strain between transfer and failure rather than the usual 90 to 100 per cent. of the ultimate. I should be interested to hear of any loading test which would confirm these assumptions.

The AUTHOR replies : Mr. Jampel's last paragraph, I think, answers his first. With prestressed concrete design, one must not work on "factored" stresses, but on a "factored" failing load. Hence concrete stresses must be well above 3,000 lb./sq. in. during the design, and as Mr. Jampel admits, $M = fZ$ does not apply. If it is therefore not used, one does not need to worry about I or Z and their relative accuracy. Admittedly the effect of steel percentage or modular ratio would be small in either case, but the important point, which has been avoided by Mr. Jampel's apparently considering the working load state of the beam, is that the extent

of cracking would have a very great effect. I have in all cases taken this into account, and I maintain that it is necessary to do so. Normal $M = fZ$ calculations are then useless.

I commented, during presentation of the paper, on my choice of steel and concrete stresses. Using a very rigid concrete, so as to give designs erring on the side of safety made it necessary with the examples given to choose between high ultimate steel stress, achieved either by a high neutral axis or a high concrete stress at transfer, or else a lower one, necessitating rather more steel, but considerably less concrete. It is purely a matter for the designer. Calculations given briefly towards the end of examples 1 and 2, examples 3 and 4, and the reply to Dr. Abeles above, should indicate the variations that can be achieved by permitting more concrete deformation. The more deformable a concrete, the greater economy in design for all normal cases provided the strength is still present.

From Mr. F. W. GIFFORD (Associate-Member) : Mr. King's paper is particularly interesting in that he considers the ultimate strength of prestressed beams, and in doing so correlates strains.

The natural tendency when considering an ultimate load theory for prestressed beams is to consider the simplest, viz., a rectangular concrete compressive stress block and steel at its ultimate strength, as propounded by Drs. Abeles, Hajnal-Konyi and others. The fundamental error made is to assume the steel reaches its ultimate strength, which involves ignoring strains. With normal reinforced concrete, the use of mild steel having a very well defined yield point enabling considerable straining to occur without change of stress, permits this assumption, but there is no parallel with the steels used in prestressing which have no definite yield, and for which the range of strain will be smaller. In consequence, circumstances arise where the straining of the concrete produces failure before the steel reaches its ultimate strength, and in consequence any factor of safety based on the ultimate strength of the steel, is inaccurate, and on the side of danger. This is particularly the case with non-bonded end anchored beams.

The fundamental difference between bonded and non-bonded beams was first indicated by Professor A. L. L. Baker, who pointed out that change of strain in steel = F x change of strain in adjacent concrete at critical section, where F is unity for bonded beams (ignoring local slip, etc.), and less than unity for unbonded beams, since it is necessary to equate change of length between anchorages, of both steel and concrete, because steel strain is constant over its length whereas concrete strain has a peak value at the critical section. The values of 2/3 and ½, first suggested by Professor Baker, where shown by the writer to be high, a theoretical investigation indicating values of 0.3 to 0.1 to be more reasonable. Subsequent tests have confirmed this, the lowest value for any beam tested at Imperial College being 0.1. Tests indicate that the bonding of steel by grouting appreciably increases the factor of safety. From theoretical considerations it would appear that the higher the position of the neutral axis, the lower the F factor.

Further arguments against the rectangular stress block coupled with steel at its ultimate stress, are the fact that no differentiation can be made for variation of initial steel stress, and that it immediately follows from the assumption that adding further steel to a beam continually increases strength.

There is little doubt that experienced engineers who have carried out extensive tests on beams may satisfactorily calculate by the above method the ultimate strengths of *similar beams within the ranges of the tests*, but even they may make serious errors should they stray far from the range of their tests, unless they correlate strains. On the other hand, an ultimate load theory which ignores strains may be a snare and a delusion for engineers who have not the time to spend studying test data and carrying out tests themselves.

Realising the inadequacy of ignoring strains, the writer extended the theory of Professor Baker, to cover rectangular and tee beams, and has converted it into a form for direct use in design, working on the stress/strain curve for steel and using various factors obtained by test—viz., F γ $\propto$ and concrete crushing strain. The latter three could be obtained from a stress/strain curve in the absence of more suitable test data.

With regard to Mr. King's method of design, I feel that he is starting with the wrong assumptions ; to assume the depth of the neutral axis at ultimate load is not necessary—surely assume a factor of safety, and let "n" value sort itself out. Also to assume top fibre strain zero as of pre-stress is not necessarily best. For beams with high live/dead load ratio this is satisfactory, but it will not lead to an economical design for a long span beam having low live/dead load ratio. Also, the method ignores stresses under full working load, and the design has to be checked for these conditions, which are, in the writer's opinion, essential.

A point which very few engineers seem to realise is that if limits are specified for concrete stresses under working loads, an "economic" concrete section can be completely designed and, in addition, the *steel force* will be known. To obtain the steel area it is merely necessary to specify a factor of safety and thus get the required steel stresses and hence area.

The method of design adopted by the writer is as follows :—

C_{t1} C_{b1} t_L are top and bottom fibre concrete stresses and steel stress under full working load M_L, assuming full creep and shrinkage have occurred.

$'C_{ts}$ $'C_{bs}$ $'_{ts}$ are top and bottom fibre concrete stresses and steel stress under M_s, the sustained load acting at time of pre-stress of concrete. Conditions of creep and shrinkage as at time of pre-stress ($M_s = O$ if self weight is negligible, and no superstructure carried at time of pre-stress.)

Generally the above are the critical concrete stresses under working load conditions.

Assuming $C_{tL} = C\mathbf{a}$ = working stress in concrete at 28 days.

$'C_{bs} = \propto Ca$ = working stress in concrete at time of pre-stress.

$C_{b1} = O$ (possibly an allowable tension).

$'ts = (1+\gamma)t_L$ γ = a factor circa 0–0.15.

$'C_{ts} > O$ (or small tension).

A section may be readily designed which gives these stresses. (Note : $'C_{ts}$ cannot be assumed at an exact value but it can be ensured that it is not tension. In certain circumstances it may be possible to reduce C_{tL} and C_{ts}, thus saving in steel force.)

From the above calculation for working load stress, the steel force is known—not steel area or stress.

Finally, using the factor of safety, "n" may be *calculated* and hence the steel stresses and area. (The time required for the latter calculation is less than a minute !).

In connection with the point raised by Dr. Hajnal-Konyi, with regard to his believed large increase of steel stress at cracks in bonded beams, enabling steel stress always to reach very near to ultimate stress. To

the writer this appears to be a false conception—there *must* be slip between wires and concrete, adjacent to a crack—otherwise an infinite stress would develop in the steel at the crack, since the crack opening is a deformation over a length of steel of zero! Probably slip is appreciable to the extent of more or less equalising the stress in the steel. If the stability of a section adjacent to the section of failure, but between it the position of next crack, is considered, it is reasonable to consider an uncracked section with perfect bond—otherwise appreciable slip will occur and an F factor will come in to consideration. If the section is assumed to carry the ultimate moment approximately, and forces and strains are balanced, it will be found that an appreciable increase of steel stress is *not* possible.

The AUTHOR replies: The correlation of strains is, of course, a fundamental point of the method suggested in the paper, and, as indicated in the reply to Dr. Abeles, it is the lack of this correlation which makes the author reluctant to give full approval to Dr. Abeles' calculations for ultimate strength of prestressed beams. He is even more reluctant to approve of the "F" factor of Professor A. L. L. Baker and Mr. Gifford, since it is very dangerous indeed to assume that because experimentally a figure as low as 0.1 has been found, that it cannot be even smaller. The author indicated during the presentation of the paper that F can easily have very large negative values, as a slight study of the implications of example 12 and the curve of Fig. 8 will make clear.

One must normally start somewhere in a design, and the author chooses to elect a value for "n." This is preferable to assuming F, γ and α as does Mr. Gifford, since he cannot be sure that his final design will, in fact, agree with all those factors. F can vary widely as indicated already, and γ and α depend on the precise part of the concrete and steel stress/strain curves which are finally used—a fact not known precisely in advance. Incidentally, the author did not assume top fibre strain zero at prestress in any examples given, and never makes a point of so doing.

Turning to Mr. Gifford's indication of his design method, any given set of constant conditions can be compacted into formulæ which will give size of beam, steel area, and prestress in a few seconds. The author's method of design is just as amenable to this as any other. However, the object of this paper was not to establish formulæ based on assumptions of arbitrary constants, but to present a logical approach from a study of fundamental behaviour, and to get away from formulæ altogether until a proper understanding of the functioning of prestressed beams has been grasped.

The author is pleased to see Mr. Gifford supports his point in reply to Dr. Hajnal-Konyi, that steel stress is not materially increased at cracks above the value between cracks in the tension side of a concrete beam.

Institution Notices and Proceedings

GENERAL MEETING

A General Meeting of the Institution of Structural Engineers was held at 11, Upper Belgrave Street, London, S.W.1, on Thursday, October 11th, 1951, at 6 p.m., when the Presidential Address for the Session 1951-52 was given by Mr. Walter C. Andrews, O.B.E., M.I.C.E., M.I.Struct.E.

The Chair was taken by the retiring President, Mr. J. E. Swindlehurst, O.B.E., M.A., M.I.C.E., M.I.Struct.E., who welcomed the guests. He then invested Mr. Andrews with the Presidential Badge and called on him to address the meeting.

Mr. Andrews took the Chair and gave the Presidential Address for the Session, which is printed in this issue.

At the conclusion of the Address, Mr. S. Vaughan, B.Sc., M.I.C.E., M.I.Struct.E. (Vice-President), proposed a vote of thanks to the President, and this was seconded by Mr. J. Stitson, President of the Institution of Heating and Ventilating Engineers.

The President, in his response, expressed thanks for the appreciation accorded to his address.

ORDINARY GENERAL MEETING

An Ordinary General Meeting of the Institution was held at 11, Upper Belgrave Street, London, S.W.1, on Wednesday, October 24th, 1951, at 5.55 p.m., Mr. Walter C. Andrews, O.B.E., M.I.C.E., M.I.Struct.E. (President), in the Chair.

The Minutes of the Ordinary General Meetings held on May 24th and June 28th, 1951, as published in the Journal, were taken as read, were confirmed and signed.

The following members were elected in accordance with the Bye-Laws. Will members kindly note that the elections, as tabulated below, should be referred to when consulting the Year Book for evidence of membership?

GRADUATES

ASHFORD, John Albert, of Nazeing, Essex.
BAILEY, Jack, of Batley, Yorks.
BAILEY, Stanley, of London.
BENNETT, Robert Walter, of London.
BOSTON-MAMMAH, Collins Olubi, of Huddersfield, Yorks.
BRAMWELL, Henry Leslie, of Birmingham.
CROFT, Geoffrey Arthur, of London.
EVANS, Hugh Stanford, of London.
HAYTON, James Albert, of Pudsey, Yorks.
HEALEY, Wilfred, of Bristol.
HUTTER, James Louis, of Oxhey, Herts.
KILEY, Gerard, of Swansea.
LUNIEWSKI, Zdzislaw, of London.
MATTHEWS, Roy, of Doncaster.
PARISH, Jack Leonard James, of London.
SZELEPET, Jan, of London.
TELLER, George, of London.
TURZYNSKI, Leon, of London.

ASSOCIATE-MEMBERS

BRADFORD, Geoffrey Ernest, of Maidenhead, Berks.
BRAITHWAITE, John Capill, of London.
CHMIELEWSKI, Stanislaw Witold, of London.
CLARE, Frank Austin, of Stanmore, Middlesex.
DANGE, Sudhakar Vishwanath, of Bedlington, Northumberland.
GILLIES, Murdo John, of Winnington, Northwich.
GOLDES, Anton Abe, of London.
HUNT, Cyril George Alexander, of Worthing, Sussex.
JAMES, John Elwyn, of Swansea.
JONES, Royston, of Leeds.
JONES, William George, of Swansea.
KEELING, Albert Joseph, of Newcastle-upon-Tyne.
LEWIS, John Rees, of Edinburgh.
LISZKA, Stanislaw Kazimierz, of London.
MOSS, Brian James, of Crewe, Cheshire.
NIGHTINGALE, Raymond Alfred, of London.
PENDREE, Howard Edmund, of Newport.
TAVERNER, Geoffrey Clifford, of Ilford, Essex.
WHITAKER, Alan Simpson, of Blackburn, Lancs.
WILSON, Thomas Dunlop, of Glasgow.

Transfers

Students to Graduates

Barritt, Norman, of Newcastle-upon-Tyne.
Beirne, Terence, of London.
Bokota, Jerzy, of London.
Boswell, Peter, of Dewsbury, Yorks.
Chodzko-Zajko, Witold Jozef, of London.
Emmerson, Miss Maureen, of Monkseaton, Northumberland.
Fasehun, Ebenezer Oladipo Olawanle, of East Croydon, Surrey.
Fox, Ronald Theodore, of Birmingham.
Freiter, Ian Neville Ferguson, of London.
Jenkinson, Anthony Richard, of Egham, Surrey.
L'Oste-Brown, Anthony Joseph, of Burton-on-Trent.
Marriott, Philip Stanley, of London.
Morton, James, of Glasgow.
Newton, Christopher John Broughton, of Thornton Heath. Surrey.
Newton, Michael William, of Bromley, Kent.
Norman, Kenneth Peter, of London.
Rogers, Kenneth Jordan, of Findern, Nr. Derby.
Rowley, Noel, of Derby.
Skupin, Henry, of London.
Steele, Alan David, of London.
Watson, John Rex, of Bristol.
Weavers, Peter John, of Kidderminster, Worcs.
Wheeler, Gordon Francis, of London.
Wild, Peter, of Scunthorpe.
Williams, Lionel Woolf, of Pinner, Middlesex.

Student to Associate-Member

Cotton, Stanley Arthur Charles, of Goodmayes, Essex.

Graduates to Associate-Members

Ackroyd, Thomas Norman Westhead, of Wallasey, Cheshire.
Anchor, Robert Dudley, of Birmingham.
Bell, Stanley John, of Birmingham.
Bewick, John Raymond, of Manchester.
Boeuf, Frank, of Waltham Cross, Herts.
Cluett, Reginald Walter, of Andover, Hants.
Cohen, Nathan, of London.
Deakin, Neville Teare, of Altrincham, Cheshire.
Dowe, Horace William, of London.
Fewster, Howard Morgan, of Llanelly, Carm.
Ganguly, Jagadish Chandra, of London.
Garthwaite, Charles Henry, of London.
Graham, Leslie, of Bury, Lancs.
Hill, Edward, of Aughton, Lancs.
Hughes, Thomas Alun, of Flixton, Lancs.
Jupp, Arthur Harry, of Horsham, Sussex.
Lawrence, Cyril, of Swansea.
Lee, John Anthony Brian, of London.
Leibowitz, Bernard, of London.
Lowe, John Robert, of Neath, Glam.
Mackrill, Hubert Wray, of North Shields.
Marshall, Leon John, of Ilford, Essex.
Molyneux, John Stanley, of Liverpool.
Osborn, Harry Denis, of Old Kilpatrick, Glasgow.
Owen, Alun Deiniol, of Ossett, Yorks.
Pepper, Sidney, of Mansfield, Notts.
Robinson, Leslie William, of Birmingham.
Rumney, Stanley, of Manchester.
Sage, Frederick, of Warrington, Lancs.
Sharp, William Henry, of Preston, Lancs.
Shearcroft, Richard, of Kensworth, Beds.
Shirlin, Gilbert Thomas, of Stafford.
Spencer, Derek, of Todmorden, Yorks.
Stokeld, Philip Gawthorpe, of London.
Thurlow, Raymond Charles, of London.
Tinkler, John, of Billingham, Co. Durham.
Uren, John Michael Leal, of Pinner, Middlesex.
Westall, Thomas Rimmer, of Warrington, Lancs.
Whitham, Cecil, of Eccles, Lancs.
Whittaker, John Basil, of Ilkley, Yorks.
Wheeler, Edwin Alan Barton, of Bridgwater, Somerset.
Woodhead, Stewart, of Coventry.
Zunz, Gerd Jack, of Crews Hill, Middlesex.

FORTHCOMING MEETINGS

The following meetings will be held at 11, Upper Belgrave Street, London, S.W.1 :—

Thursday, December 13th, 1951

Ordinary General Meeting for the election of members at 5.55 p.m., followed by a Joint Meeting with the British Section of the Societe des Ingenieurs Civils de France, at 6 p.m., when Monsieur L. P. Brice will give a paper entitled, " Construction of the Temple Barrage on the River Lot."

Thursday, January 24th, 1952

Ordinary General Meeting for the election of members at 5.55 p.m., followed by an Ordinary Meeting at 6 p.m., when Mr. Arthur Bolton, B.Sc. (Graduate), will give a paper on " A New Approach to the Elastic Analysis of Two Dimensional Rigid Frames."

Thursday, February 14th, 1952

Ordinary Meeting at 6 p.m., when Mr. D. I. Lawson, M.Sc., M.I.E.E., Mr. C. T. Webster, F.R.I.C., and Mr. L. A. Ashton, B.Sc., will give a paper on " The Fire Endurance of Timber Beams and Floors."

Thursday, February 28th, 1952

Ordinary General Meeting for the election of members at 5.55 p.m., followed by an Ordinary Meeting at 6 p.m., when Dr. W. B. Dobie, M.Sc., F.R.S.A., A.M.I.C.E., A.M.I.Mech.E., will give a paper on " The Torsional Strength of Structural Members."

Members wishing to bring guests to the Ordinary Meetings announced above are requested to apply to the Secretary for tickets of admission.

JANUARY EXAMINATIONS

The Examinations of the Institution will next be held at centres in the United Kingdom and overseas on January 8th and 9th, 1952 (Graduateship), and on January 10th and 11th (Associate-Membership).

HONOURS AND AWARDS

In offering their sincere congratulations to the following member on the distinction recently conferred upon him, the Council feel that they are also expressing the good wishes of the Institution.

Order of the British Empire—C.B.E.
Dr. H. Schofield (Member)

PERSONAL

Mr. A. L. Knighton, L.R.I.B.A. (Associate), has been appointed Surveyor of the Fabric at the University of Leeds.

Mr. M. Cawley (Associate-Member) has been appointed Borough Surveyor, Water Engineer and Planning Officer to the Doncaster County Borough Council.

THE MACLACHLAN LECTURE

General Conditions

Through the generosity of Mr. John MacLachlan (Retired Member), the Council was able in 1948 to institute an Annual Lecture to be competed for by Associate-Members. The conditions of the presentation are as follows :—

1. The Institution of Structural Engineers shall institute a written lecture to be known as the MacLachlan Lecture and to be held annually.

2. The subject of the Lecture may be on any aspect of Structural Engineering so long as in every second year the subject shall be confined to steel structures.

3. Entrance into the competition for the Lecture shall be confined to Associate-Members of the Institution, who are under the age of 32 years.

4. All papers entered for the competition shall be submitted to assessors to be appointed by the Council of the Institution, and all such papers (including the prize-winning lecture) shall be available for publication in the Journal of the Institution at the discretion of the Council.

5. No paper submitted shall have been published or read elsewhere.

6. The winner of the competition shall be required to present the Lecture to a meeting of the Institution at which he will be presented with the sum of £17 10s. 0d.

7. Should a competitor's paper be considered worthy of ranking second in merit, he will receive a consolation award of £5.

8. In the event of there being no winner of the competition in any one or more years, whether because no lecture is submitted or because no lecture submitted is considered to be of sufficient merit to warrant an award, or for any other reason, the Institution shall transfer these sums to the Research Fund of the Institution.

PARTICULARS OF THE COMPETITION FOR 1952

1. The MacLachlan Lecture will be given at a meeting of the Institution to be arranged towards the end of 1952.

2. The subject of the Lecture shall be on any aspect of structural engineering.

3. The work should be submitted as the script of a lecture which the author, if successful in the competition, will deliver before an audience in the course of about one hour. The development of mathematical formulæ and detailed calculations should be avoided as far as possible in the text ; if they are essential they should be embodied in appendices. Photographs, drawings, graphs, etc., which would appear as illustrations to the lecture in published form, should accompany the script. If additional illustrations would be shown as slides, a list of these should be included.

Lectures should be prepared in accordance with the requirements of the Literature Committee for publication in THE STRUCTURAL ENGINEER. Candidates may obtain a copy of these requirements on application to the Secretary.

4. Six copies of each Lecture should be submitted and should be addressed to the Secretary of the Institution.

5. The closing date for the receipt of entries by the Institution is Monday, March 31st, 1952.

LONDON GRADUATES' AND STUDENTS' SECTION

A meeting of the London Graduates' and Students' Section will be held at 11, Upper Belgrave Street, London, S.W.1, on Tuesday, December 11th, 1951, at 6 p.m., when Dr. P. W. Abeles (Member) will give a lecture on Prestressed Concrete.

Hon. Secretary : D. B. Rogers, 4, Portland Rise, Finsbury Park, N.4.

BRANCH NOTICES

LANCASHIRE AND CHESHIRE BRANCH

Twenty members of the Branch took part in a most interesting visit to the construction of the new double-line tunnel at Woodhead, on September 26th. Mr. J. D. Dempster, the Resident Engineer, took the visitors into the Dunford Bridge end of the tunnel, where work included excavating, drilling and rock blasting. The party was then conducted to the Woodhead end, where work was in a much more advanced stage. Several interesting features were the stockpiling of aggregates, batching plant, concrete pump and the moving formwork for the tunnel. A short visit was then paid to the shaft which drops into the centre position of the three-mile long tunnel. The shaft will later be used for ventilating the tunnel. The party returned to the Resident Engineer's office, and Mr. Dempster very ably answered various questions put by the visitors.

The thanks of the party are due to British Railways, Sir William Halcrow and Partners, Mr. J. D. Dempster and his assistants, and all who were concerned in making the visit so interesting.

The Annual Dinner and Dance will be held at Longford Hall, Stretford, on Tuesday, January 22nd, 1952, at 6 p.m.

The following meetings will be held at the Reynolds Hall, College of Technology, Manchester, at 6 p.m. (preceded by tea at 5.45 p.m.).

Wednesday, January 30th, 1952

Three short lectures by Mr. W. H. Rosier, Mr. A. S. Sinclair, and Mr. C. Thirsk (Associate-Members), followed by discussion.

Monday, February 25th, 1952

"A New Approach to the Elastic Analysis of Two Dimensional Rigid Frames," by Mr. Arthur Bolton, B.Sc. (Graduate).

Hon. Secretary : A. S. Sinclair, A.M.I.Struct.E., 28, Kenwood Road, Stretford, Lancs.

MIDLAND COUNTIES BRANCH

The following meetings will be held :—

Friday, January 25th, 1952

"Notes on Soil Mechanics," by Mr. J. Kolbuszewski, Ph.D., D.I.C., F.G.S., at James Watt Memorial Institute, Birmingham, 6 p.m.

Monday, February 25th, 1952

"Some Aspects of Soil Mechanics with reference to Foundations," by Dr. G. G. Meyerhof, A.M.I.C.E. (Associate-Member), at Derby, 7 p.m.

Hon. Secretary : E. R. Deeley, A.M.I.Struct.E., Arranmoor, Adshead Road, Dudley, Worcs.

GRADUATES' AND STUDENTS' SECTION

The following meetings will be held :—

Saturday, December 8th, 1951

Social Function.

Monday, December 31st, 1951

The following films will be shown : "Building of the South Bank Exhibition," "Prelude to Power," "H.M.S.O. Sighthill, Edinburgh," at James Watt Memorial Institute, Birmingham, 7 p.m.

Wednesday, January 30th, 1952

Annual General Meeting, followed by a film and a talk on "Welding as Applied to Structural Engineering," by Mr. F. Brooksbank, B.A. (Graduate), at James Watt Memorial Institute, Birmingham, 7 p.m.

Hon. Secretary : M. H. Evans, B.Sc., 42, Church Hill Road, Handsworth, Birmingham, 20.

NORTHERN COUNTIES BRANCH

The following meetings will be held :—

Tuesday, December 4th, 1951

"Factory Extension at Silvertown," by Mr. Frank G. Etches, B.Sc.(Eng.), A.M.I.C.E. (Member), at the Cleveland Scientific and Technical Institute, Middlesbrough, 6.30 p.m.

Wednesday, December 5th, 1951

The above meeting will be repeated at the Neville Hall, Newcastle, at 6.30 p.m.

Wednesday, January 9th, 1952

Joint Meeting with the Institution of Civil Engineers. "Reconstruction of Houdon-on-Tyne Gas Works," by Mr. W. R. Garrett, A.M.I.C.E. (Associate-Member), at Middlesbrough, 6.30 p.m.

Wednesday, January 16th, 1952

The above meeting will be repeated at Newcastle, at 6.30 p.m.

Hon. Secretary : Ian MacGregor, M.I.Struct.E., 9, Ellison Place, Newcastle-upon-Tyne, 1.

NORTHERN IRELAND BRANCH

The following meetings will be held :—

Tuesday, December 4th, 1951

"Prestressed Concrete Beams and Slabs," by Mr. A. H. K. Roberts, B.A., M.I.C.E.I. (Member), at the College of Technology, Belfast, 7.30 p.m.

Friday, January 11th, 1952

Annual Dinner and Social Function.

Tuesday, February 5th, 1952

"Some Economies in the Fabrication of Steel Sections," by Mr. R. Montgomery (Associate-Member), at the College of Technology, Belfast, 7.30 p.m.

Hon. Secretary : S. G. Duckworth, M.I.Struct.E., "Lisleen," 13, Finaghy Road North, Belfast.

SCOTTISH BRANCH

The following meeting will be held :—

Monday, December 10th, 1951

Joint Meeting with The Institution of Civil Engineers, at Royal Technical College, Glasgow. "Some Impressions of Technical Education in the United States," by Dr. D. S. Anderson.

Hon. Secretary : D. G. Drummond, B.Sc., A.M.I.C.E., M.I.Struct.E., 11, Woodside Terrace, Glasgow, C.3.

SOUTH-WESTERN COUNTIES BRANCH

The next meeting will be held at Newton Abbot on Friday, January 11th, 1952, when Mr. Wallace A. Evans (Member), will give a paper on "The Reconstruction of Abertillery Bridge."

Hon. Secretary : E. W. Howells, M.I.Struct.E., c/o Messrs. T. L. Harding & Sons, Ltd., 10/12, Market Street, Torquay.

WALES AND MONMOUTHSHIRE BRANCH

The following meetings will be held :

Tuesday, December 4th, 1951

Structural Engineering questions answered, at the South Wales Institute of Engineers, Cardiff, 6.30 p.m.

Friday, December 14th, 1951

"Piling, with particular regard to Sea Defence Works," by Mr. J. Owen Lake (Associate-Member), at the County Buildings, Colwyn Bay, 6 p.m.

Wednesday, February 13th, 1952

"Completed Abertillery Bridge," by Mr. Wallace A. Evans (Member), at the Mackworth Hotel, Swansea, 6.30 p.m.

Friday, February 15th, 1952

A meeting will be held at Colwyn Bay, details of which will be announced later.

Tuesday, February 19th, 1952

"Completed Abertillery Bridge," by Mr. Wallace A. Evans (Member), at Cardiff, 6.30 p.m.

Hon. Secretary : E. R. Steward, A.M.I.Struct.E., Edrom, Ashleigh Road, Blackpill, Swansea.

WESTERN COUNTIES BRANCH

The following meetings will be held :—

Friday, December 7th, 1951

"Some Recent Developments in the Bulk Handling of Raw Materials for Steelworks," by Lt.-Col. E. Ward, T.D., A.M.I.Mech.E. (Associate-Member), at Bristol University, 6 p.m.

Friday, January 4th, 1952

"The Fabrication of Steel Structures," by Mr. O. H. Willey, at Bristol University, 6 p.m.

Friday, February 1st, 1952

Combined Meeting with the Institution of Civil Engineers. "Some Effects of Recent Developments on the Design and Construction of Concrete Structures," by Dr. A. R. Collins, M.B.E., A.M.I.C.E. (Associate-Member).

Wednesday, February 20th, 1952

Annual Dinner at the Royal Hotel, Bristol.

Hon. Secretary : C. E. Saunders, M.I.Struct.E., Dunkery, Edward Road, Walton St. Mary, Clevedon, Somerset.

YORKSHIRE BRANCH

The First Ordinary Meeting of the Session was held at the Great Northern Hotel, Leeds, on October 17th, 1951, at 6.30 p.m. Mr. H. C. Husband was in the Chair, and the meeting was attended by the President, Mr. Walter C. Andrews, and the Secretary, Major R. F. Maitland.

The Chairman introduced the President of the Institution, who installed Mr. A. Robb as the new Branch Chairman. Mr. Robb expressed the thanks and appreciation of the Branch to the retiring Chairman, and then gave his address, which covered many aspects of concrete design and construction.

A vote of thanks to Mr. Robb was proposed by Mr. D. R. S. Wilson, and seconded by Mr. J. Dossor.

The Secretary of the Institution, Major Maitland, spoke on the work of the past year and gave the meeting some facts and figures demonstrating the continued progress and increased membership of the Institution.

The following meetings have been arranged :—

Wednesday, December 12th, 1951

"The New House of Commons—Foundations and Structure," by Mr. S. Vaughan, B.Sc., M.I.C.E. (Vice-President). This will be a combined meeting with the West Yorkshire Society of Architects and will be held at the Great Northern Hotel, Leeds, at 6.30 p.m.

Wednesday, January 16th, 1952

"Structural Engineering at Abbey Works," by Mr. A. V. Hooker, A.M.I.C.E. (Associate-Member).

Friday, February 8th, 1952

Annual Dinner and Dance, Parkway Hotel, Otley Road, Leeds, 7 p.m.

Hon. Secretary : E. Wrigley, A.M.I.Struct.E., 17, The Drive, Alwoodley, Leeds.

UNION OF SOUTH AFRICA BRANCH

Branch Hon. Secretary : A. E. Tait, B.Sc., A.M.I.C.E., A.M.I.Struct.E., P.O. Box 3306, Johannesburg. During week-days, Mr. Tait can be contacted in the City Engineer's Department, City Hall, Johannesburg. 'Phone 34-1111, ext. 257.

Natal Section Hon. Secretary : E. G. Bennett, A.M.I.Struct.E., c/o Reinforcing Steel Co., Ltd., P.O. Box 49, Merebank, Durban.

Cape Section Hon. Secretary : R. Stubbs, M.I.Struct.E., P.O. Box 1692, Cape Town.